Unit 6

DIGITAL LOGIC AND DESIGN

- Recall that data is represented using binary pulses (0 and 1)
- Explain that binary pulses have a respective low and high voltage
- Explain the three basic logic gates.
- Construct the 2, 3, 4…n variable truth tables for basic logic gates.
- Explain the universal gates with the help of truth tables.
- Differentiate between basic and universal logic gates.

6.1 DATA REPRESENTATION IN A COMPUTER

Data Representation refers to the form in which data is stored, processed, and transmitted. Digital devices such as smartphones, iPads, and computers store data in digital formats that can be handled by electronic circuitry. These circuits work on two logical binary states i.e., Low and High (1 & 0) pulses.

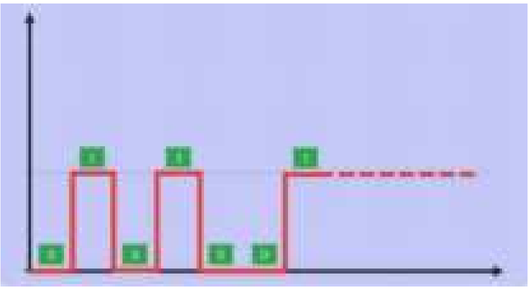


Fig. 6.1 Digital Signal

Logic gates are the electronic circuits in a digital system. Logical gates perform logical operations like AND, OR, NOT, NAND, NOR etc. are shown in Figure No. 6.1

Do You Know?	The Binary logic consists of binary variables and logical operations. The variables are denoted by letters of alphabets such as A, B, C,… ,Z, a,b,c,…z, e.g., with each variable having only two distinct possible logical values 1 and 0, these two values of variables may also be identified by different names (e.g., true and false, Yes or No).

6.2 LOGIC GATES

Logic gates are the electronic circuits in a digital system. Logical gates perform logical operations like AND, OR, NOT, NAND, NOR etc. Logic gates are divided into two categories.

6.2.1 Basic Logic Gates

6.2.2 Universal Logic Gates

6.2.1 Basic Logic Gates:

The logic gate is the basic unit of digital logic circuits, there are mainly three basic gates AND, OR, and NOT and these logical gates perform AND, OR, and NOT operations in the digital system.

Do You Know? A truth table is a tabular representation of all the combinations of values for inputs and their corresponding outputs.

AND GATE:

An AND gate is a digital circuit that has two or more inputs and a single output. AND gate operates on logical multiplication rules. AND operation using variables A and B is represented "A.B", here (.) dot is a logical multiplication sign.

Boolean Expression of AND gate: Y=A.B

Truth table of and operation using two input variables

A	B	Y = A . B
0	0	0
0	1	0
1	0	0
1	1	1

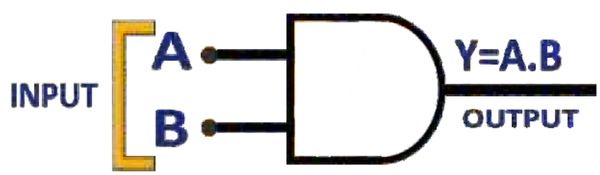


Fig. 6.2
AND Gate using two input variables

Truth Table of AND gate using two input variables A, B and output is Y. If any input is 0, then output Y becomes 0. If all inputs are 1 then output Y becomes 1.

Truth table of and operation using three input variables			
A	B	C	Y = A.B.C
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	0
1	0	0	0
1	0	1	0
1	1	0	0
1	1	1	1

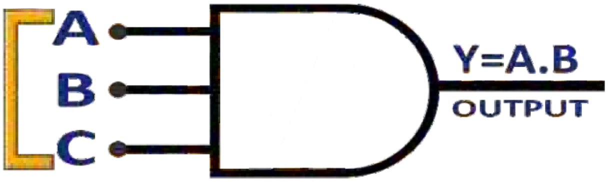


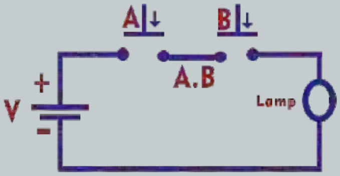
Fig. 6.3
AND Gate using three input variables

AND gate using three input variables A, B, C, and output is Y. If any input is 0, then output Y becomes 0. If all inputs are 1 then output Y becomes 1.

Teachers Note

Teacher gives the concept of implementation of AND operation for the logical input and output using given diagram.

A,B open for logic 0
A,B closed for logic 1
When Lamp is OFF for A.B=0 (Logical)
When Lamp is ON for A.B=1(Logical)



OR GATE:

An OR gate is a digital circuit that has two or more inputs and a single output. OR gate operates on logical Addition rules. Logical OR operation using variables A and B is represented as "**A+B**", here (**+**) is a logical Addition sign. Boolean Expression of OR gate is Y=A+B.

Truth table of or gate operation using two input variables		
A	B	Y = A + B
0	0	0
0	1	1
1	0	1
1	1	1

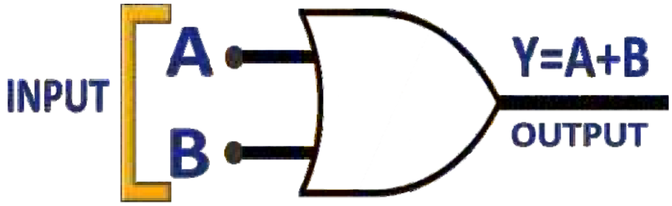


Fig. 6.4 OR Gate using two input variables

Truth Table of OR gate using two input variables A, B and Y is output. If any input is 1 then output Y becomes 1 and if all inputs are 0 then ouput Y becomes 0. Boolean expression of OR gate is Y=A+B.

Truth table of or operation using three input variables			
A	B	C	Y = A+B+C
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	1

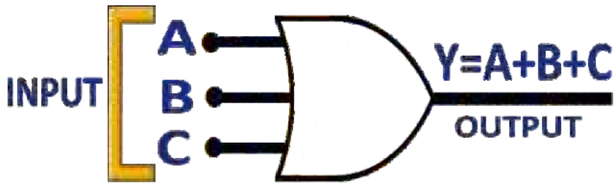
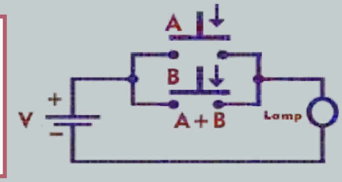


Fig. 6.5
OR Gate using three input variables

Truth Table of OR gate using three input variables A, B, C and Y is output. If any input is 1 then output Y becomes 1 and if all inputs are 0 then ouput Y becomes 0.

Teachers Note	Teacher gives the concept of implementation of AND operation for the logical input and output using given diagram.

A,B open for logic 0
A,B closed for logic 1
When Lamp is OFF for A+B=0 (Logical)
When Lamp is ON for A+B=1 (Logical)



NOT GATE:

A NOT gate is a digital circuit that has a single input and a single output. It is also known as an INVERTER. The output of NOT gate is the logical inversion of input. It is symbolically represented by complement sign (') Right side on top of the input variable or bar(-) sign on top of the variable. Boolean expression of NOT gate is Y = A′ **or** Y = $\overline{A}$

Truth table of not gate operation using two input variables	
A	Y = $\overline{A}$
0	1
1	0

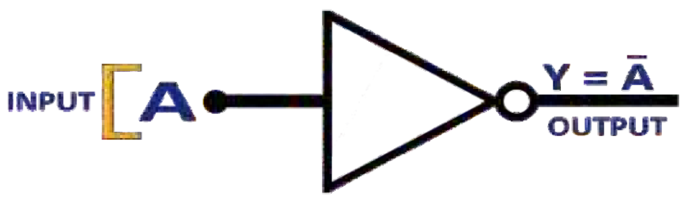


Fig. 6.6 NOT Gate

Truth table of NOT gate is A as input and Y = $\overline{A}$ is output.

6.2.2 Universal Gates:

A **universal gate** is a logic gate which can implement any Boolean function without the need to use any other type of basic gates. The NOR gate and NAND gate are universal gates.

NAND(NOT – AND) GATE:

The NAND gate or "Not AND" gate is the collection of two basic logic gates, the AND gate and the NOT gate connected in series.
Boolean expression of NAND gate is Y= (A.B)′ **or** Y = $\overline{AB}$.

Truth table of nand operation using two input variables		
A	B	Y = $\overline{AB}$
0	0	1
0	1	1
1	0	1
1	1	0

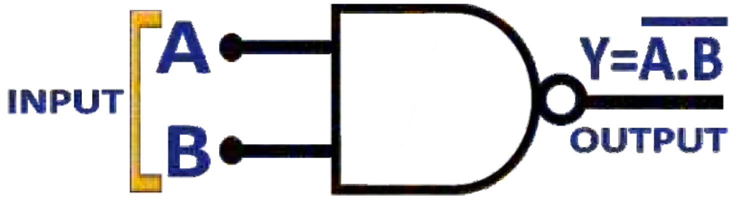


Fig. 6.8 NAND Gate

TheTruth table of the NAND gate using two input variables A, B and Y is the output. When all inputs are "1", the output, Y is "0". If any one of the inputs is "0", then the output Y is "1".

NOR GATE:

A NOR Gate is the collection of OR Gate NOT Gate. The output of NOR gate is inverter OR. The Boolean expression of NOR gate is Y = (A+B)′ **or** Y = $\overline{A+B}$.

Truth table of nor operation using two input variables		
A	B	Y = $\overline{A + B}$
0	0	1
0	1	0
1	0	0
1	1	0

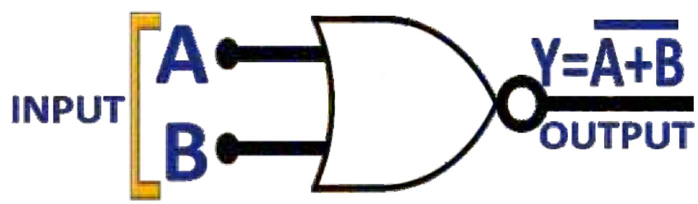


Fig. 6.9 NOR Gate

The Truth table of the NOR gate using two input variables A, B and Y is the output. If both inputs are "0", then the output, Y is "1". If any one of the inputs is "1", then the output Y is "0".

Teachers Note 	Teacher will demonstrate thoroughly the concept and operations of NAND & NOR universal gates. And also compare the symbols of NOT gate and Inverter gate.

Differentiate between basic & universal logic gates

Basic logic gates	Universal logic gates
AND, OR, and NOT gates are the most basic logic gates. By using this set of logic gates, it is possible to implement all the possible Boolean Expressions by using these three gates.	The NAND gate and NOR gate can be called the universal gates, the collection of NAND & NOR gates can be used to achieve any of the basic AND, OR and NOT operations.
Individual logic gates can be connected to form a variety of different combinational logic circuits.	A universal gate is a gate which can implement any Boolean expression without using other type of gate.

- Recall that data is represented using binary pulses (0 and 1)
- Apply 12 rules of Boolean algebra for simplification of any expression
- Design a logic circuit for any Boolean expression
- Derive Boolean expression for any logic circuit

6.3 BOOLEAN ALGEBRA

Boolean algebra was invented by George Boole in 1954. It is a branch of mathematics and it can be used to describe the logical operations and processing binary information. It is based on true or false input values to produce a true or false output value.

6.3.1 Rules of Boolean Algebra:

The Boolean arithmetic rules are pre-defined rules that help to simplify the logical expression. There are 12 basic rules which are invented to simplify the gates. To reduce the number of logic gates needed to perform a particular logic operation we can apply a set of rules. These rules are commonly known as the Laws of Boolean Algebra Expressions.

The following table shows some of the Boolean algebra rules for Boolean Expression Simplification.

Rule No.	Boolean Expression Simplification Rules	Prove these rules with Proof according to their Values
1.	**A + 0 = A**	If A=0 ⟶ 0 +0=0 If A=1 ⟶ 1+0=1
2.	**A + 1 = 1**	If A=0 ⟶ 0+1=1 If A=1 ⟶ 1+1=1
3.	**A . 0 = 0**	If A=0 ⟶ 0.0=0 If A=1 ⟶ 1.0=1
4.	**A . 1 = A**	If A=0 ⟶ 0.1=0 If A=1 ⟶ 1.1=1
5.	**A + A = A**	If A=0 ⟶ 0+0=0 If A=1 ⟶ 1+1=1
6.	**$A + \overline{A} = 1$**	If A=0 ⟶ $0+(\overline{\mathbf{0}})$ ⟶ 0+**1**=1 If A=1 ⟶ $1+(\overline{\mathbf{1}})$ ⟶ 1+**0**=1
7.	**A . A = A**	If A=0 ⟶ 0.0 =0 If A=1 ⟶ 1.1 = 1
8.	**$A . \overline{A} = 0$**	If A=0 ⟶ $0.(\overline{\mathbf{0}})$ ⟶ 0.1=0 If A=1 ⟶ $1.(\overline{\mathbf{1}})$ ⟶ 1.0=0
9.	**$\overline{\overline{A}} = A$**	If A=0 ⟶ $(\overline{\mathbf{0}})$ = 1 ⟶ $(\overline{\mathbf{1}})$ = 0 so, $\overline{\overline{\mathbf{A}}}$ = 0 If A=1 ⟶ $(\overline{\mathbf{1}})$ = 0 ⟶ $(\overline{\mathbf{0}})$ = 1 so, $\overline{\overline{\mathbf{A}}}$ = 1
10.	**A + A . B = A**	If A=0, B=0 ⟶ 0+(0.0) ⟶ 0+0 = 0 If A=1, B=1 ⟶ 1+(1.1) ⟶ 1+1 = 1
11.	**$A + \overline{A} . B = A + B$**	If A & B=0⟶0+$\overline{\mathbf{0}}$.0=0+0⟶0+**1**.0=0+0⟶0+0 = 0+0 Hence **0=0** If A & B=1⟶1+$\overline{\mathbf{1}}$.1=1+1⟶1+**0**.1 =1+1 ⟶1+0=1+1 Hence **1=1**

12.	**(A+B) (A+C)= A + BC**	If A,B& C=1 ⟶ (1+1)(1+1)=1+(1.1) ⟶ **(1)(1) = 1+(1)** ⟶ **1.1=1+1** ⟶ **1=1** If A, B=0 and C=1 ⟶ (0+0) (0+1)=0+0.1 ⟶ (0).(1)=0+0 ⟶ 0.1=0 ⟶ 0=0 Hence **0 = 0**

6.3.2.1 Example to apply Boolean Rules for Simplification expression.

a. $AB+A\overline{B}=A$

Solution:

$AB+A\overline{B}=A$

Take L.H.S

Here take common variable A

$A(B+\overline{B})$

According to 6th Rule of Boolean Algebra is $A+\overline{A}=1$ so, $B+\overline{B}=1$

A. (1) Or A.1 ⟶ According to 4th Rule of Boolean Algebra i.e. A.1= A

A .1=A

A=A

Hence **L.H.S = R.H.S**

A =A **Proved**

b. $(A+B)+(A+\bar{B})=A$

Solution:

$(A+B)+(A+\bar{B})=A$

Take L.H.S

$(A+B)+(A+\bar{B})$

ANDing (Multiplication) of both expressions

$AA+A\bar{B} + BA+B\bar{B}$

Apply Rule 7th of Boolean Algebra i.e. **A.A=A** and Rule 8th of Boolean Algebra i.e. $\mathbf{A.\bar{A}=0}$ **0r** $B\bar{B}$

$A+A\bar{B} + BA + 0$

Take common variable A from $A+A\bar{B} +BA$ expression

$A+A(B+\bar{B})+0$

Apply Rule 6th of Boolean Algebra i.e. $\mathbf{A+\bar{A} = 1}$ **or** $B+\bar{B}=1$

$A+A(1)+0 = A+A+0$

Apply Rule 5thof Boolean Algebra i.e. **A+A = A**

$A+0 = A$

Hence **L.H.S = R.H.S**

A = A **Proved**

6.3.2.2 Draw Logic Circuit of the given Boolean expressions.

a. $Y = \bar{A}BC\,(\overline{A + D})$

Solution:

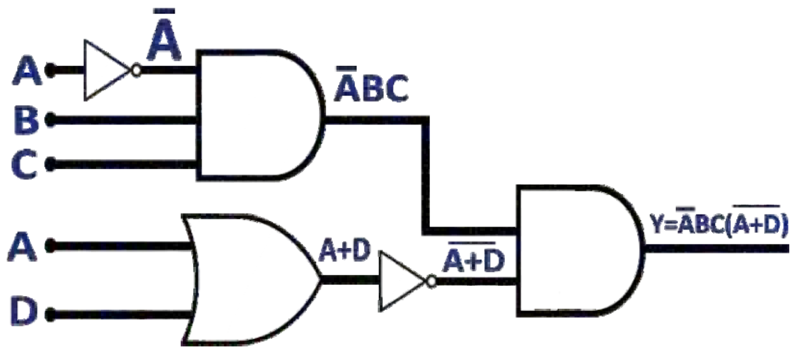


Fig. 6.10 Logic Circuit

Explanation:

Above logic circuit consists of AND, OR and, NOR gates. The expression $\bar{A}$ NOT and B, C gate connected with AND gate and A, D is a connected with OR gate and converted in NOR gate. Finally, $\bar{\mathbf{A}}\mathbf{BC}$ and $\overline{\mathbf{A + D}}$ connected to **AND** gate.

b. $X=\overline{AB(\overline{C + D})}$

Solution:

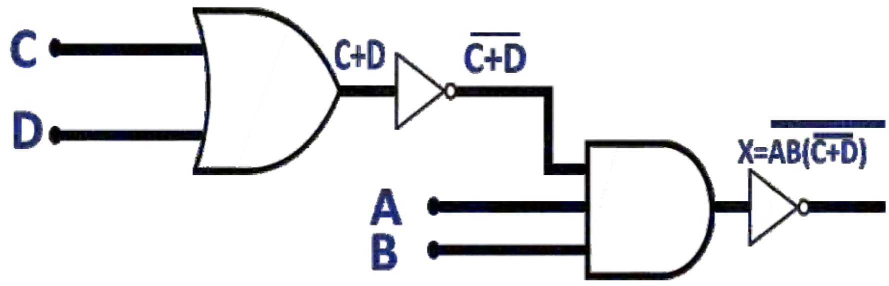


Fig 6.11 Logic Circuit

c. Let:- $Q=(A.B)+(\overline{A+B})$

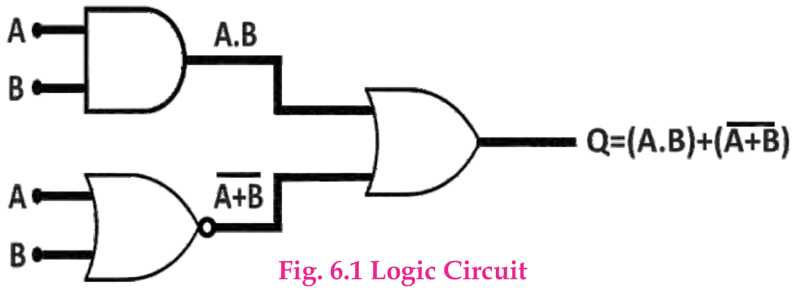
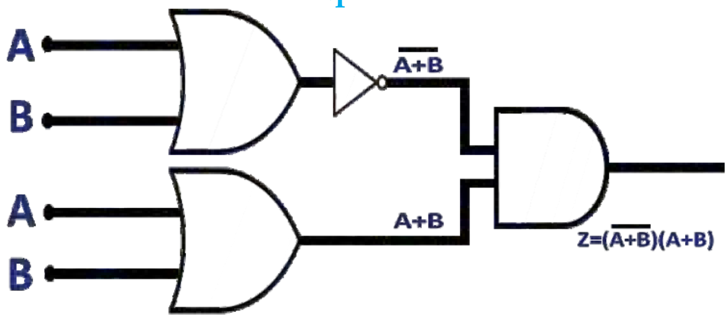


Fig. 6.1 Logic Circuit

6.3.3.3 Derive the Boolean expression from the given circuit and make a truth table of that Boolean expression.



Solution:

Boolean Expression of the above circuit is **Z =$(\overline{A+B})$ (A+B)**

Truth table of the **Z = $(\overline{A+B})$ (A+B)**

INPUTS		TRUTH TABLE		
A	**B**	**A+B**	$\overline{A+B}$	$(\overline{A+B})$.(A+B)
0	0	0	1	0
1	0	1	0	0
0	1	1	0	0
1	1	1	0	0

Teachers Note 	Teacher usually find it difficult to teach Digital Logic & Boolean Algebra since these are abstract concepts. These concepts may be presented to students with the help of images and videos. If students can visualize these concepts, they can better assimilate them.

SUMMARY

- Data Representation refers to the form in which data is stored, processed, and transmitted. Digital devices such as smart phones, iPods, and computers store data in digital formats that can be handled by electronic circuitry.
- Logic Gates are the electronic circuits in a digital system.
- Logical Gates perform logical operations like AND, OR, NOT, NAND, NOR etc.
- The logic gate is the basic unit of digital logic circuits, there are mainly three basic gates AND, OR, and NOT and these logical gates perform AND, OR, and NOT operations in the digital system.
- An AND gate is a digital circuit that has two or more inputs and a single output AND gate operates on logical multiplication rules. Boolean Expression of AND gate: Y=A.B
- An OR gate is a digital circuit that has two or more inputs and a single output. OR gate operates on logical Addition rules.
- Boolean expression of OR gate is Y=A+B.
- A NOT gate is a digital circuit that has a single input and a single output. It is also known as INVERTER.
- Universal Gates are logic gates. They are capable of implementing any Boolean function without requiring any other type of gate. There are two types of universal gates
- A NAND Gate could be construct by connecting a NOT Gate at the Output terminal of the AND Gate. Boolean expression of NAND gate is Y= (A.B)′or Y = $\overline{AB}$.
- A NOR Gate could construct by connecting a NOT Gate at the output terminal of
- The Boolean expression of NOR gate is Y = (A+B)′ or Y = $\overline{A + B}$.
- The Boolean arithmetic rules are pre-defined rules that help to simplify the logical expression. There are 12 Boolean algebra rules.

EXERCISE

A. ENCIRCLE THE CORRECT ANSWER:

1. The universal gate is ______________________.
   - **a.** NAND Gate
   - **b.** AND Gate
   - **c.** OR Gate
   - **d.** None of these
2. The __________ is Inverter.
   - **a.** AND
   - **b.** OR Gate
   - **c.** NOT
   - **d.** None of these
3. In Boolean Algebra, the bar sign (-) indicates ___________________.
   - **a.** OR Operation
   - **b.** NOR Operation
   - **c.** NOT Operation
   - **d.** Both b and c
4. The Boolean Algebra is used for ______________________.
   - **a.** Creating Circuits
   - **b.** Apply 12 rules of Boolean
   - **c.** Simplify the Boolean expression
   - **d.** Differentiate the gates
5. With the combination of three variables, how many outputs are expected altogether?
   - **a.** Three
   - **b.** Six
   - **c.** Eight
   - **d.** Nine
6. **A+A=A** is a ___________________ rule of Boolean Algebra.
   - **a.** 3rd
   - **b.** 6th
   - **c.** 5th
   - **d.** 7th
7. **$A.\bar{A}$**=0 is a ____________________ rule of Boolean Algebra.
   - **a.** 1st
   - **b.** 8th
   - **c.** 6th
   - **d.** 10th
8. Simply form of Boolean expression of **$ABC + AB\bar{C} + \bar{A}B$** is ________________.
   - **a.** A
   - **b.** B
   - **c.** C
   - **d.** $\bar{B}$

9. Simplify form of Boolean expression of $\mathbf{ABC+\bar{A}BC}$ is ____________.

   a. AC **b.** BC

   c. B **d.** $\bar{A}$

10. Boolean expression of the given circuit is ____________________.

   a. $\overline{\mathbf{A + B}}$ **b.** B.A

   c. A.B **d.** $\overline{\mathbf{A\,.\,B}}$

B. RESPOND THE FOLLOWING:

1. Explain all basic logic gates and their operations.
2. Differentiate between NAND and NOR gates.
3. Why do we use Boolean Algebra?
4. Explain the function of Inverter.
5. Explain the purpose of Truth table
6. Simplify the following Boolean expression.
   $\mathbf{Z = AB + A(B + C) + B(B + C)}$

LAB ACTIVITIES

1. Draw a Logic Circuit of 10^{th} Law of Boolean Algebra.
2. Design a Logic Circuit from Boolean expression $Q = (A.B) + (A.B) + (\overline{A + B})$
3. Derive the Boolean expression from the given circuit and make a truth table and simplify that Boolean expression.