UNIT

4 LINEAR EQUATIONS AND INEQUALITIES

- Linear Equations
- Equation Involving Absolute Value
- Linear Inequalities
- Solving Linear Inequalities

After completion of this unit, the students will be able to:

- recall linear equation in one variable.
- solve linear equation with rational coefficients.
- reduce equations, involving radicals, to simple linear form and find their solutions.
- define absolute value.
- solve the equation, involving absolute value in one variable.
- define inequalities $(>,<)$ and $(\geq, \leq)$.
- recognize properties of inequalities (i.e. trichotomy, transitive, additive and multiplicative).
- solve linear inequalities with rational coefficients.

4.1 LINEAR EQUATIONS

A statement in which sign of equality " = " is used to link two algebraic expressions is called an equation. An equation involving only a linear polynomial is called a linear equation. Equation $ax + b = 0$, $a \neq 0$ is a linear equation in one variable in standard form.

For example:

(i) $7x+3 = 5$ (ii) $\frac{3}{2}x+4 = \frac{1}{3}$

(iii) $\frac{1}{2}(t+3)-2t = 5$ (iv) $\frac{5}{3}y+4 = \frac{y-2}{4}$

4.1.1 Linear Equation in One Variable

Any equation that can be written in the form:

$$ax+b = 0, \quad a \neq 0 \ldots\ldots\ldots\ldots\ldots (1)$$

where a and b are constants and x is a variable, is called a linear equation *(or first degree equation)* in one variable.

Equation *(1)* always has a solution:

$$ax+b = 0, \qquad a \neq 0$$
$$ax = -b$$
$$x = \frac{-b}{a}$$ *is the solution of the equation (1)*

EXAMPLE

Verify that $x = 2$ is a root of the equation $5x-12 = -2$

SOLUTION:

Substituting $x = 2$ in the given equation, we get

$$L.H.S = 5x-12 = 5\times(2)-12$$
$$= 10-12 = -2 = R.H.S$$

RULES FOR SOLVING AN EQUATION:

(i) *Same quantity can be added or subtracted to both sides of an equation without changing the equality.*

(ii) *Both sides of an equation may be multiplied by a same non-zero number without changing the equality.*

(iii) *Both sides of an equation may be divided by a same non-zero number without changing the equality.*

(iv) *TRANSPOSITION:*

Any term of an equation may be taken to the other side with its sign changed , without effecting the equality, is called transposition.

EXAMPLE

Solve: $5x-6 = 4x-2$

SOLUTION: We have $5x-6 = 4x-2$

or $5x-4x = -2+6$ [Transposing $4x$ to L.H.S and -6 to R.H.S]

Thus $x = 4$ is a solution of the given equation.

CHECK: Substituting $x = 4$ in the given equation, we get

$$L.H.S = 5\times(4)-6 = 20-6 = 14$$

$$R.H.S = 4\times(4)-2 = 16-2 = 14$$

$$\therefore L.H.S = R.H.S$$

Hence $x = 4$ is solution of the given equation.

4.1.2 Solution of a Linear Equation

Any value of the variable which makes the equation a true statement, is called the solution *(or root of the equation)*.

Solving an equation means to find a value of the variable which satisfies the equation.

EXAMPLE-1

Solve: $3x+\frac{1}{5} = 2-x$

SOLUTION: We have $3x+\frac{1}{5} = 2-x$

or $3x+x = 2 - \frac{1}{5}$ (Transposing $-x$ to L.H.S and $\frac{1}{5}$ to R.H.S)

or $4x = \frac{9}{5}$

or $\frac{1}{4}\times 4x = \frac{1}{4} \times \frac{9}{5}$ (dividing both sides by 4)

$x = \frac{9}{20}$

Thus $x = \frac{9}{20}$ is solution of the given equation.

CHECK: Substituting $x = \frac{9}{20}$ in the given equation, we get

$$L.H.S = 3\times\frac{9}{20}+\frac{1}{5} = \frac{27}{20}+\frac{1}{5} = \frac{31}{20}$$

$$R.H.S = 2 - \frac{9}{20} = \frac{31}{20}$$

$$\therefore L.H.S = R.H.S$$

Hence $x = \frac{9}{20}$ is solution of the given equation.

EXAMPLE-2

Solve: $2y+\frac{11}{4} = \frac{1}{3}y+2$

SOLUTION: We have $2y+\frac{11}{4} = \frac{1}{3}y+2$

$2y - \frac{1}{3}y = 2 - \frac{11}{4}$ (Transposing $\frac{1}{3}y$ to L.H.S and $\frac{11}{4}$ to R.H.S)

$\frac{5}{3}y = \frac{-3}{4}$

$\frac{3}{5} \times \frac{5y}{3} = \frac{3}{5} \times \left(\frac{-3}{4}\right)$ (Multiplying both sides by $\frac{3}{5}$)

$y = \frac{-9}{20}$

Thus, $y = \frac{-9}{20}$ is solution of the given equation.

CHECK: Substituting $y = \frac{-9}{20}$ in the given equation, we get

$$L.H.S = 2 \times \left(\frac{-9}{20}\right) + \frac{11}{4} = \frac{-9}{10} + \frac{11}{4} = \frac{37}{20}$$

$$R.H.S = \frac{1}{3} \times \left(\frac{-9}{20}\right) + 2 = \frac{-3}{20} + 2 = \frac{37}{20}$$

$$\therefore\ L.H.S = R.H.S$$

Hence $y = \frac{-9}{20}$ is solution of the given equation.

EXAMPLE-3

Solve: $\frac{1}{4}x + \frac{1}{6}x = \frac{1}{2}x + \frac{3}{4}$

SOLUTION: $L.C.M$ of the denominators $4,6,2,4$ is 12.
Multiplying both sides by 12, we get

$$3x + 2x = 6x + 9$$

or $5x = 6x + 9$

or $6x - 5x = -9$ [Transposing $5x$ and 9]

or $x = -9$

Thus $x = -9$ is solution of the given equation.

CHECK: Substituting $x = -9$ in the given equation, we get

$$L.H.S = \frac{1}{4} \times (-9) + \frac{1}{6} \times (-9) = \frac{-9}{4} - \frac{3}{2} = \frac{-15}{4}$$

$$R.H.S = \frac{1}{2} \times (-9) + \frac{3}{4} = \frac{-9}{2} + \frac{3}{4} = \frac{-15}{4}$$

$$\therefore\ L.H.S = R.H.S$$

Hence $x = -9$ is solution of the given equation.

EXAMPLE-4

Solve: $\frac{5x-4}{8} - \frac{x-3}{5} = \frac{x+6}{4}$

SOLUTION: We have $\frac{5x-4}{8} - \frac{x-3}{5} = \frac{x+6}{4}$

Multiplying both sides by 40, the L.C.M of 8,5,4, we get

$$5(5x-4) - 8(x-3) = 10(x+6)$$

or $25x - 20 - 8x + 24 = 10x + 60$

or $17x + 4 = 10x + 60$

or $17x - 10x = 60 - 4$ [Transposing $10x$ to L.H.S and 4 to R.H.S]

or $7x = 56$

or $x = \frac{56}{7} = 8$ [Multiplying both sides by $\frac{1}{7}$]

Thus $x = 8$ is solution of the given equation.

CHECK: Substituting $x = 8$ in the given equation, we get

$$L.H.S = \frac{5\times 8-4}{8} - \frac{8-3}{5} = \frac{36}{8} - 1 = \frac{28}{8} = \frac{7}{2}$$

$$R.H.S = \frac{8+6}{4} = \frac{14}{4} = \frac{7}{2}$$

$\therefore$ L.H.S = R.H.S

Hence $x = 8$ is solution of the given equation.

What we need to know ?

- How to solve linear equations with unknown values on both sides.
- How to interpret the terms in expressions and formulae.
- How to solve linear equations with negative signs in the equation.

EXAMPLE-5

Solve: $x-\left[2x-\dfrac{3x-4}{7}\right]=\dfrac{4x-27}{3}-3$

SOLUTION: We have, $x-\left[2x-\dfrac{3x-4}{7}\right]=\dfrac{4x-27}{3}-3$

By removing the brackets, we get,

$$x-2x+\frac{3x-4}{7}=\frac{4x-27}{3}-3$$

or $$-x+\frac{3x-4}{7}=\frac{4x-27}{3}-3$$

Multiplying both sides by 21, the L.C.M of 7,3, we get

$$-21x+3(3x-4)=7(4x-27)-63$$

or $$-21x+9x-12=28x-189-63$$

or $$-12x-12=28x-252$$

or $$-12x-28x=-252+12 \quad \text{[by Transposition]}$$

or $$-40x=-240$$

or $$x=6 \quad \text{[dividing both sides by } -40 \text{]}$$

Thus $x=6$ is solution of the given equation.

CHECK: Substituting $x=6$ in the given equation, we get

$$L.H.S=6-\left[2\times6-\frac{3\times6-4}{7}\right]=6-\left(12-\frac{14}{7}\right)=6-(12-2)$$

$$=6-10=-4$$

$$R.H.S=\frac{4\times6-27}{3}-3=\frac{-3}{3}-3=-1-3=-4$$

$$\therefore L.H.S=R.H.S$$

Hence $x=6$ is solution of the given equation.

What we need to know ?

- How to operate with negative sign present outside a bracket.

EXAMPLE-6

Solve: $0.3x+0.4 = 0.28x+1.16$

SOLUTION: We have $0.3x+0.4 = 0.28x+1.16$

or $0.3x-0.28x = 1.16-0.4$ *[by Transposition]*

or $0.02x = 0.76$

or $x = \frac{0.76}{0.02} = \frac{76}{2} = 38$

Thus $x = 38$ is solution of the given equation.

CHECK: By substituting $x = 38$ in the given equation, we get

$L.H.S = 0.3\times 38+0.4 = 11.4+0.4 = 11.8$

$R.H.S = 0.28\times 38+1.16 = 10.64+1.16 = 11.8$

$\therefore\ L.H.S = R.H.S$

Hence $x = 38$ is solution of the given equation.

EXAMPLE-7

Solve: $3x-2(2x-5) = 2(x+3)-8$

SOLUTION: We have $3x-2(2x-5) = 2(x+3)-8$

$$3x-4x+10 = 2x+6-8$$
$$-x+10 = 2x-2$$
$$3x-2 = 10$$
$$3x = 12$$
$$x = 4$$

Thus $x = 4$ is solution of the given equation.

CHECK: By substituting $x = 4$ in the given equation, we get

$$3(4)-2(2\times 4-5) = 2(4+3)-8$$
$$12-2(8-5) = 2(7)-8$$
$$12-6 = 14-8$$
$$6 = 6$$

Hence $x = 4$ is solution of the given equation.

4.1.3 Equations Involving Radicals

In solving an equation such as

$$\sqrt{x-1} = 5 \qquad ....(1)$$

Squaring both sides $x-1 = 25$

$$x = 26$$

which is solution of (1)

For any natural numbers x and y

$$\sqrt{x} \times \sqrt{y} = \sqrt{xy}$$

or the other way round

$$\sqrt{xy} = \sqrt{x}\,\sqrt{y}$$

However, if we do the same thing to

$$\sqrt{x-1} = -5 \qquad ....(2)$$

$x-1 = 25$ *(Squaring both sides)*

$x = 26$

which is not a solution of (2)

Since $5 \neq -5$

Equations involving radicals may have extraneous roots, which are not the solutions of the original equation.

Similarly , we note that

$$\{x \mid x=5\} = \{5\}$$

$$\{x \mid x^2 = 25\} = \{-5, 5\}$$

Where as we see that the solution set of $x = 5$ is a subset of the solution set of the equation.
We get by squaring each member of $x = 5$.

It is important to remember that any new equation obtained by raising both members of an equation to the same power may have solutions (called **extraneous solutions**). That are not solutions of the original equation. On the other hand, any solution of the original equation must be among those of the new equation.

Thus, every solution of the new equation must be checked in the original equation to eliminate the extraneous solutions.

EXAMPLE-1

Solve: $x+\sqrt{x-4} = 4$

SOLUTION: $x+\sqrt{x-4} = 4$

$\sqrt{x-4} = 4-x$ *(Isolating the radical on one side)*

$x-4 = 16-8x+x^2$ *(Squaring both sides)*

$x^2-9x+20 = 0$ *(Solving quadratic equation)*

$(x-5)(x-4) = 0$

$x = 5,4$

Check to eliminate extraneous roots (if any)

$x = 5 \quad , \quad x = 4$

$5+\sqrt{5-4} = 4 \quad , \quad 4+\sqrt{4-4} = 4$

$5+1 \neq 4 \quad , \quad 4 = 4$

Therefore, $x = 5$ is not a solution $\quad$ *$x = 4$ is a solution*

$\therefore$ *Solution set* $= \{4\}$

What we need to know ?

- Some formulae have squares and square roots in them.
- Squares and square roots are the inverse of each other.
- To remove a square, take the square root of each side.
- To remove a square root, square each side.

EXAMPLE-2

Solve: $\sqrt{3x-2} - \sqrt{x} = 2$

SOLUTION: $\sqrt{3x-2} = 2+\sqrt{x}$

$3x-2 = 4+4\sqrt{x}+x$ *(Squaring both sides)*

$3x-x-2-4 = 4\sqrt{x}$

$2x-6 = 4\sqrt{x}$

$x-3 = 2\sqrt{x}$ *(Dividing both sides by 2)*

$x^2-6x+9 = 4x$ *(Again squaring both sides)*

$x^2-10x+9 = 0$

$(x-9)(x-1) = 0$

$x = 1,9$

Check to eliminate extraneous solutions (if any)

$x = 1$, $x = 9$

$\sqrt{3\times 1-2} - \sqrt{1} = 2$, $\sqrt{3\times 9-2} - \sqrt{9} = 2$

$\sqrt{1} - \sqrt{1} = 2$, $\sqrt{25} - 3 = 2$

$0 \neq 2$, $5-3 = 2$

Therefore, $x = 1$ is not a solution *$x = 9$ is the solution*

$\therefore$ *Solution set* $= \{9\}$

Remember that:

$s = \sqrt{t+r}$: Remove the square root, square each side.

$s^2 = t+r$: Now subtract the 'r' from both sides.

$s^2-r = t$ *This can be written* $t = s^2-r$

EXERCISE – 4.1

Solve:

1. (i) $3x + 20 = 44$ (ii) $\frac{4x}{5} - \frac{3x}{4} = 4$

   (iii) $3x + 3(x+1) = 69$ (iv) $(90 - 9x) + 27 = 90 + 9$

2. $3(x+3) = 14 + x$

3. $3(2x+5) = 25 + x$

4. $9x - 3 = 3(2x - 8)$

5. $3(2x - 1) = 5(x - 1)$

6. $2(7x - 6) = 3(1 + 3x)$

7. $\frac{10x - 1}{2x + 5} = 3$

8. $\frac{2x + 1}{x + 5} = 1$

9. $\frac{5x + 3}{x + 6} = 2$

10. $y - 6 + \sqrt{y} = 0$

11. $x = 15 - 2\sqrt{x}$

12. $m - 13 = \sqrt{m + 7}$

13. $\sqrt{5n + 9} = n - 1$

14. $3 + \sqrt{2x - 1} = 0$

15. $\sqrt{x + 5} + 7 = 0$

16. $\sqrt{2x - 1} - \sqrt{x - 4} = 2$

17. $\sqrt{x + 1} = 3$

18. $\sqrt{2x - 1} = 5$

19. $\sqrt{x - 1} = 10$

20. $\sqrt{3x + 4} = 7$

4.2 EQUATIONS INVOLVING ABSOLUTE VALUE

In this section, we learn to solve the linear equations involving absolute value.

4.2.1 Absolute Value:

For each real number x, the absolute value of x, denoted by $|x|$, is defined by the formula:

$$|x| = \begin{cases} x & \text{if } x > 0 \\ 0 & \text{if } x = 0 \\ -x & \text{if } x < 0 \end{cases}$$

For example:

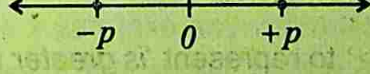
$$|8| = 8$$

$$|-8| = -(-8) = 8$$

4.2.2 Equations Involving Absolute Value:

Using the above definition , we will not find it difficult to show that for $p > 0$.

$$|x| = p \Leftrightarrow x = \pm p$$

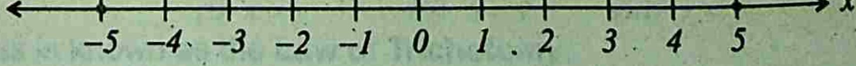


EXAMPLE

Solve (i) $|x| = 5$ (ii) $|x-3| = 5$ (iii) $|x+2| = 3$

SOLUTION:

(i) $|x| = 5 \Rightarrow x = \pm 5$



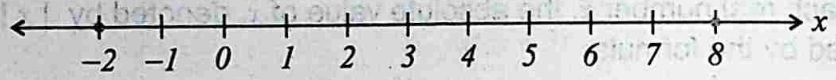
(ii) $|x-3| = 5 \Rightarrow x-3 = \pm 5$

$x-3 = 5$ or $x-3 = -5$

$x = 8$ or $x = -5+3$

$x = -2$

$x = -2$ or 8



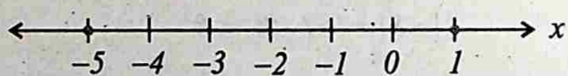
(iii) $|x+2| = 3 \Rightarrow x+2 = \pm 3$

$x+2 = 3$ or $x+2 = -3$

$x = 3-2$ or $x = -3-2$

$x = 1$ or $x = -5$

$x = 1$ or -5



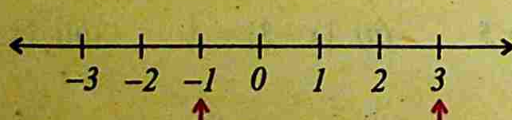
4.3 LINEAR INEQUALITIES

We know about ordering of numbers on the number line. A number on the number line is greater than any number on its left and less than any number on its right.

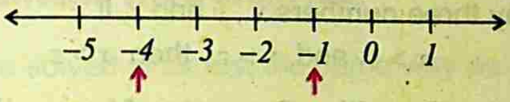
4.3.1 Inequalities (>, <) and (≥, ≤):

We use the symbol '>' to represent *'is greater than'* and the symbol '<' to represent *'is less than'*.

For example:



3 lies on the right of −1, hence 3 is greater than −1 and we write 3 > −1.



−4 lies on the left of −1, hence −4 is less than −1 and we write −4 < −1.

We write $a < b$, read "*a is less than b*" if and only if there exists a positive real number p such that

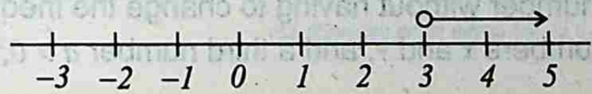
$$a + p = b;$$

We write $a > b$, and read "*a is greater than b*". We write $a \le b$ if and only if $a < b$ or $a = b$ and we write $a \ge b$ if and only if $a > b$ or $a = b$

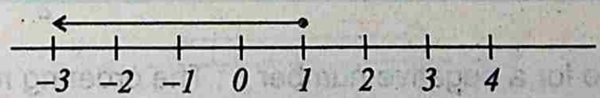
The symbols "<", ">", "≤" and "≥" are called **order relations** or **inequality symbols.**

Two algebraic expressions joined by an inequality symbol, such as $7(3x-2)+\frac{x}{5} < 2x-\frac{2}{3}$ is called an inequality statement or simply an inequality.

$x > 3$ means x can take any value greater than 3. It cannot be 3. It is shown on the number line.



$x \le 1$ means x can take any value less than or equal to 1. This includes 1. It is shown on the number line.



4.3.2 Properties of Inequalities

TRICHOTOMY: Consider any two numbers, x and y, on the number line. One and only one of the following statements must be true.

(i) $x > y$ (ii) $x = y$ (iii) $x < y$

This is known as the **Law of Trichotomy**.

TRANSITIVE: For any three numbers x , y and z, if

$$x > y \text{ and } y > z, \text{ then } x > z$$

This is known as the **Transitive Property of Inequality**.

For example: If $x = 10,\ y = 5$ and $z = 2$,
then $10 > 5,\ 5 > 2$ and $10 > 2$

ADDITIVE: We can add or subtract a positive number from both sides of an inequality without any change in the inequality sign. For any two numbers x and y and a positive number 'a',

If $x > y$ (e.g. $5 > 3$ and $2 > 0$),
then $x + a > y + a$ (e.g. $5 + 2 > 3 + 2$)
$x - a > y - a$ (e.g. $5 - 2 > 3 - 2$)

This is also true for a negative number b;

If $x > y$ (e.g. $5 > 3$ and $-2 < 0$),
then $x + b > y + b$ (e.g. $5 + (-2) > 3 + (-2)$)
$x - b > y - b$ (e.g. $5 - (-2) > 3 - (-2)$)

MULTIPLICATIVE: We can multiply and divide both sides of an inequality by a positive number without having to change the inequality sign. For any two numbers x and y, and a third number $a > 0$,

If $x > y$ (e.g. $5 > 3$ and $2 > 0$),
then $ax > ay$ (e.g. $2 \times 5 > 2 \times 3$) and $\frac{x}{a} > \frac{y}{a}$

This is not true for a negative number b ; The ordering relation is reversed when multiplied or divided by a negative number.

If $x > y$ and $b < 0$ (e.g. $5 > 3$ and $-2 < 0$),
then $bx < by$ (e.g. $(-2) \times 5 < (-2) \times 3$)
and $\frac{x}{b} < \frac{y}{b}$ (e.g. $\frac{5}{-2} < \frac{3}{-2}$)

4.4 SOLVING LINEAR INEQUALITIES:

Inequalities are solved in almost the same way as equations.

EXAMPLE-1

Solve the following inequalities

(i) $x+3<7$ (ii) $2x-1>5$ (iii) $6-x>4$

SOLUTION:

(i) $x+3<7$

$x+3-3<7-3$ (Subtracting 3 from both sides)

$x<4$

(ii) $2x-1>5$

$2x-1+1>5+1$ (Adding 1 to both sides)

$2x>6$

$x>3$ (Dividing both sides by 2)

(iii) $6-x>4$

$6-x-6>4-6$ (Subtracting 6 from both sides)

$-x>-2$

$x<2$ (Multiplying both sides by -1 ; also change $>$ into $<$.)

How to include the points in the solution of an inequality?

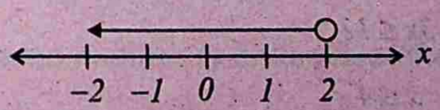


Fig (i)

It is convenient to represent the solution of an inequality, for example, $x<2$ by using the number line as shown in fig (i). The small empty circle shows that the value '2' is not included as a possible answer where as any value to the left of 2 is included.

EXAMPLE-2

Solve the inequality: $\frac{1}{3}x > \frac{1}{4}(x-1)$

SOLUTION: $\frac{1}{3}x > \frac{1}{4}(x-1)$

$$12 \times \frac{1}{3}x > 12 \times \frac{1}{4}(x-1)$$

$$4x > 3(x-1)$$

$$4x > 3x - 3$$

$$4x - 3x > -3$$

$$x > -3$$

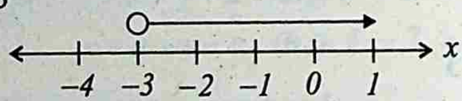


Fig (ii)

The solution is shown by the number line in fig (ii).

EXAMPLE-3

Solve the inequality: $x - 7 \leq 5 - 2x$

SOLUTION: $x - 7 \leq 5 - 2x$

$$x + 2x - 7 \leq 5$$

$$3x \leq 5 + 7$$

$$3x \leq 12$$

$$x \leq 4$$

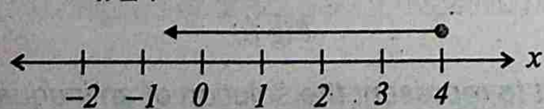


Fig (iii)

The filled circle shows that the point '4' is also included in the solution.

EXAMPLE-4

Solve and graph $\frac{4x-3}{3}+8>6+\frac{3x}{2}$

SOLUTION: $\frac{4x-3}{3}+8>6+\frac{3x}{2}$

$$6\times\frac{4x-3}{3}+6\times 8>6\times 6+6\times\frac{3x}{2}$$

$$8x-6+48>36+9x$$

$$8x+42>36+9x$$

$$8x-9x+42>36$$

$$-x>36-42$$

$$-x>-6$$

$$x<6$$

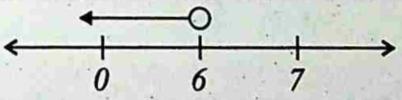


Fig (iv)

The solution is shown by the number line in fig (iv)

Remember that:

Most inequalities are written in algebra. Inequalities are solved in a very similar way to equations. This means we can:

- Add the same number to both sides of an inequality.
- Subtract the same number from both sides of an inequality.
- Multiply or divide both sides of an inequality by any positive number.

EXERCISE – 4.2

Solve:

1. $|x| = 9$
2. $|x-3| = 4$
3. $|x+1| = 5$
4. $|2x-3| = 5$
5. $|3x+4| = 9$
6. $3(x-2) < 2x+1$
7. $3(x+5) > 2(x+2)+8$
8. $\frac{1}{2}(2-x) > \frac{1}{4}(3-x)+\frac{1}{2}$
9. $\frac{x-2}{4}+\frac{2}{3}<\frac{x-4}{6}$
10. $\frac{3x+4}{5} - \frac{x+1}{3} > 1 - \frac{x+5}{3}$
11. $\frac{x+1}{2} - \frac{x+3}{3} > \frac{x+1}{4}+1$
12. $\frac{x+3}{4} - \frac{x+2}{5} < 1+\frac{x+5}{6}$
13. $\frac{1}{2}x \geq 1+\frac{1}{3}x$
14. $\frac{1}{4}(2x+3) \leq (7-4x)$
15. $\frac{4}{3}(2x+3) \geq 10 - \frac{4x}{3}$
16. $\frac{x-2}{4} - \frac{x-5}{6} \geq \frac{1}{3}$

Review Exercise-4

I- **Encircle the Correct Answer.**

1. An equation that can be written in the form $ax + b = 0, a \neq 0$, where a and b are constants and x is variable is called:
   (a) *linear equation* (b) *inequality*
   (c) *solution* (d) *constant*

2. Any value of the variable which makes the equation a true statement is called the:
   (a) *equation* (b) *inequality*
   (c) *solution* (d) *variable*

3. For each number 'x' the absolute valve of x is denoted by:

(a) x (b) $-x$

(c) $|x|$ (d) 0

4. The symbol $\geq$ stands for:

(a) *greater than* (b) *greater than and equal to*

(c) *less than or equal to* (d) *equal to*

5. The symbol $\leq$ stands for:

(a) *less than* (b) *greater than and equal to*

(c) *less than or equal to* (d) *equal to*

6. Solution of $|x-3| = 5$ is:

(a) $\{8, -2\}$ (b) $\{-8, -2\}$

(c) $\{8, 2\}$ (d) $\{-8, 2\}$

7. Solution of $|x| = 3$ is:

(a) 3 (b) -3

(c) ± 3 (d) 0

8. Solution of $|x-1| = 4$ is:

(a) $\{5, -3\}$ (b) $\{-5, -3\}$

(c) $\{-5, 3\}$ (d) $\{5, 3\}$

I- Fill in the blanks with '>' '=' or '<' to make each of the statement correct.

1. If $15 > 10$ and $10 > p$, then 15 ______ p.

2. If $-3 > x$ and $x > y$, then -3 ______ y.

3. If $a < 60$ and $60 < b$, then a ______ b.

4. If $x + 1 = y$, then x ______ y.

5. If $m - 2 = n$, then m ______ n.

6. If $x > y$, then $4x$ ______ $4y$.

7. If $x > y$, then $\frac{x}{10}$ ______ $\frac{y}{10}$.

8. If $x > y$, then $(-2)x$ ______ $(-2)y$.

$\frac{x}{-3}$ ______ $\frac{y}{-3}$.

< 0, then p ______ 0.

$-3)u$ ______ 0.

______ 0.

SUMMARY

Linear Equation: An equation that can be written in the form $ax + b = 0$, $a \neq 0$ where a and b are constants and x is a variable, is called a linear equation in one variable.

Solution of a Linear equation : Any value of the variable, which makes the equation a true statement, is called the solution of a linear equation.

Absolute Valve: For each real number 'x' the absolute value of x, denoted by $|x|$, is defined by:

$$|x| = \begin{cases} x, & \text{if } x > 0 \\ 0, & \text{if } x = 0 \\ -x, & \text{if } x < 0 \end{cases}$$

Linear Inequalities: Two algebraic expressions joined by an inequality symbol such as $>, <, \leq, \geq$ is called an inequality.

Trichotomy Property: If $x, y \in R$ then either $x > y$ or $x = y$ or $x < y$.

Transitive Property: If $x, y, z \in R$, then $x > y$ and $y > z \Rightarrow x > z$.

Additive Property: $\forall a, b, c \in R$. If $a > b$, then $a + c > b + c$ and $a - c > b - c$.

Multiplicative Property: $\forall a, b, c \in R$. Let $a > b$. Then $ac > bc$ if $c > 0$ and $ac < bc$ if $c < 0$.