UNIT

9 Areas and Volumes

- Pythagoras Theorem
- Area
- Volume

After completion of this unit, the students will be able to:

- state Pythagoras theorem.
- solve right angled triangle using Pythagoras theorem.
- find the area of
  - A triangle when three sides are given (apply Hero's formula),
  - A triangle whose base and altitude are given.
  - An equilateral triangle when its side is given.
  - A rectangle when its two sides are given.
  - A parallelogram when base and altitude are given.
  - A square when its side is given.
  - Four walls of a room when its length, width and height are given.
- find the cost of turfing a square/rectangular field.
- find the number of tiles, of given dimensions, required to pave the footpath of given width carried around the outside of a rectangular plot.
- find the area of circle and a semi circle when radius is given.
- find the area enclosed by two concentric circles whose radii are given.
- solve real life problems related with areas of triangle, rectangle, square, parallelogram and circle.
- find the volume of:
  - A cube when its edge is given.
  - A cuboid when its breadth and height are given.
  - A right circular cylinder whose base radius and height are given.
  - A right circular cone whose radius and height are known.
  - A sphere and a hemisphere when radius is given.
- solve real life problems related to volume of cube, cuboid, cylinder, cone and sphere.

SUMMARY

1- An angle bisector of a triangle is a line segment that bisects an angle of the triangle and has its other end on the side opposite to that angle.

2- Every triangle has three angle bisectors, one for each angle.

3- An altitude of a triangle is the line segment from one vertex, perpendicular to the opposite side.

4- Every triangle has three altitudes, one from each vertex.

5- A line-segment which bisects any side of a triangle and makes a right angle with the sides at its mid point is called the perpendicular bisector of the side of a triangle.

6- Every triangle has three perpendicular sides bisectors, one for each side.

7- The point at which the three angle bisectors of a triangle meet is called the incenter of the triangle.

8- The point at which the three altitudes of a triangle meet is called the orthocenter of the triangle.

9- The point of intersection of the three perpendicular bisectors of the sides of a triangle is called the circum-center of the triangle.

10- The point at which the three medians of a triangle meet is called the centroid of the triangle.

11- A line coplanar with a circle intersecting the circle at one point only is called the tangent line to the circle.

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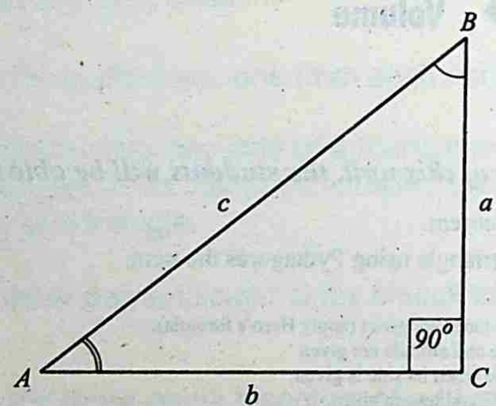
9.1 PYTHAGORAS THEOREM

Pythagoras Theorem :-

The square of the hypotenuse of a right triangle is equal to the sum of the squares of the two sides.

$$\boxed{c^2 = a^2 + b^2}$$



EXAMPLE-1

The sides of a right triangle are 5cm and 12cm.
Find the hypotenuse.

SOLUTION: Given: $a = 5cm$, $b = 12cm$,

Let the length of hypotenuse be c.

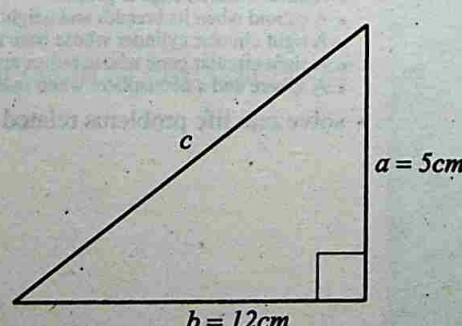
Then by pythagoras theorem.

$$c^2 = a^2 + b^2$$

$$= (5)^2 + (12)^2$$

$$= 25 + 144 = 169$$

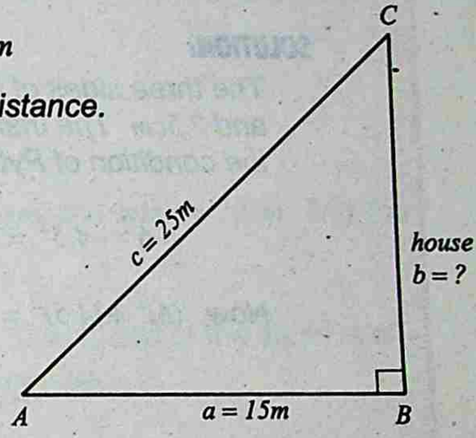
$$c = 13cm$$



EXAMPLE-2

A 25m ladder leans against a house with its foot 15m from the house. How far is the top of the ladder from the ground?

SOLUTION: Given: $a = 15m$, $c = 25m$



Let b represents the desired distance.

Then $a^2 + b^2 = c^2$

$$b^2 = c^2 - a^2$$

$$= (25)^2 - (15)^2$$

$$= 625 - 225$$

$$= 400$$

$$b = 20m$$

EXAMPLE-3

If 30,72,78 represent the lengths of the sides of a triangle. Is triangle a right triangle?

SOLUTION: Given: $a = 30$, $b = 72$, $c = 78$

We have pythagoras theorem, it states: $c^2 = a^2 + b^2$

$$R.H.S = a^2 + b^2 = (30)^2 + (72)^2$$

$$= 900 + 5184$$

$$= 6084$$

$$L.H.S = c^2 = (78)^2$$

$$= 6084$$

$$R.H.S = L.H.S$$

Thus triangle is a right triangle.

EXAMPLE-4

The sides of a triangle are of lengths $6cm$, $4.5cm$ and $7.5cm$. Is this triangle a right triangle? If so, which side is the hypotenuse?

SOLUTION:

The three sides of the triangle are given to be $6cm$, $4.5cm$ and $7.5cm$. The triangle will be a right triangle if it satisfies the condition of Pythagoras theorem

$$6^2 + 4.5^2 = 7.5^2$$

Now $(6)^2 + (4.5)^2 = 36 + 20.25$

$$= 56.25 \quad = (7.5)^2$$

Since the relation $6^2 + 4.5^2 = 7.5^2$ is satisfied, therefore, the triangle whose sides are $6cm$, $4.5cm$, $7.5cm$ is a right triangle.

Also $7.5^2 = 6^2 + 4.5^2$

$\therefore$ The side of length $7.5cm$ is the hypotenuse of the triangle.

EXERCISE – 9.1

1- Find the third side of each right triangle with legs a and b and hypotenuse c.
   (i) $a = 3, \quad b = 4, \quad c = ?$
   (ii) $a = 5, \quad c = 13, \quad b = ?$
   (iii) $b = 5, \quad c = 61, \quad a = ?$

2- If the legs of a right triangle are $2ab$ and $a^2 - b^2$, prove that the hypotenuse is $a^2 + b^2$.

3- Find the hypotenuse of the right isosceles triangle each of whose legs is l.

4- Find the hypotenuse of a right isosceles triangle whose legs are $8cm$.

5- If the numbers represent the lengths of the sides of a triangle, which triangles are right triangles?
(i) 3, 4, 5
(ii) 9, 17, 25
(iii) 11, 61, 60

6- ΔABC is right angled at C. If $m\overline{AC} = 9cm$ and $m\overline{BC} = 12cm$, find the length $\overline{AB}$, using Pythagoras theorem.

7- The hypotenuse of a right triangle is $25cm$. If one of the sides is of length $24cm$, find the length of the other side.

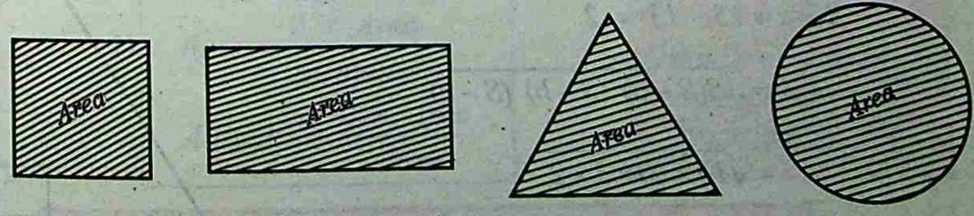
8- A ladder $17m$ long when set against the wall of a house just reaches a window at a height of $15m$ from the ground. How far is the lower end of the ladder from the base of the wall?

9- The two legs of a right triangle are equal and the square of the hypotenuse is 50. Find the length of each leg.

10- The sides of a triangle are $15cm$, $36cm$ and $39cm$. Show that it is a right angled triangle.

9.2 AREAS

The surface inside the boundary of a shape is called area.



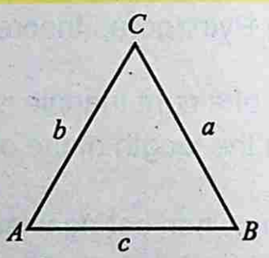
9.2.1 The Area of a Triangle

Area of a Triangle when all the three sides are given

A triangle ABC with sides a,b,c and

$$2S = a+b+c \quad \Rightarrow \quad S = \frac{a+b+c}{2},$$

where 'S' is half the perimeter of a triangle.



Then area of any triangle is $A=\sqrt{S(S-a)\ (S-b)\ (S-c)}$

This is called **Hero's Formula** for finding the area of a triangle.

EXAMPLE *Find the area of a triangle whose sides are 5,12 and 13.*

SOLUTION: Given : $a=5, b=12, c=13$ then

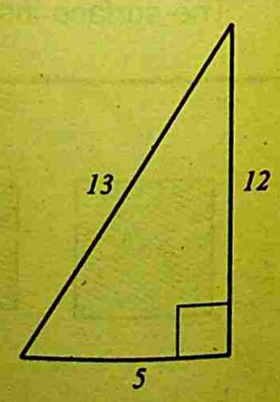
$2S=a+b+c$

$2S=5+12+13=30 \quad \Rightarrow \quad S=15$

$S-a=15-5=10,$

$S-b=15-12=3,$

$S-c=15-13 = 2$



$A=\sqrt{S(S-a)\ (S-b)\ (S-c)}$

$=\sqrt{15\times10\times3\times2}$

$=\sqrt{900} \ = \ 30$ sq.units

Note:- In this case, the triangle is a right triangle with base 5, altitude 12 and hypotenuse 13.

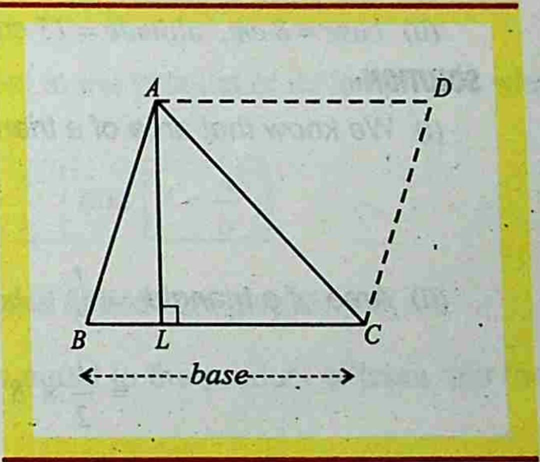
Hence area $(A) = \frac{1}{2} \times (base) \times (altitude)$

$$= \frac{1}{2}(5)(12) = \frac{60}{2} \quad A = 30 \text{ square unit.}$$

Note: Area of a triangle is denoted by A.

Area of a Triangle when base and Altitude are given

Draw any triangle ABC as shown in the figure. Let $\overline{BC}$ be its base and let $\overline{AL} \perp \overline{BC}$. Then $\overline{AL}$ is the corresponding altitude. Through A and C draw line parallel to $\overline{BC}$ and $\overline{BA}$ respectively, intersecting each other at a point D. Then, clearly $ABCD$ is a parallelogram with base $\overline{BC}$ and corresponding altitude $\overline{AL}$.



Area of $\Delta ABC = \frac{1}{2}$ (Area of Parallelogram $ABCD$) $= \frac{1}{2}(\overline{BC} \times \overline{AL})$

$$= \frac{1}{2}(b \times h)$$ (where b is the base and h is the altitude.)

Thus, we have $Area\ of\ \Delta = \frac{1}{2} \times Base \times Altitude$

$$Base = \frac{2 \times Area}{Altitude}$$

$$Altitude = \frac{2 \times Area}{Base}$$

IMPORTANT
Altitude of a triangle is its height and denoted by 'h'.

Notation for base is 'b' and 'h' for the altitude.

EXAMPLE-1 Find the altitude of a triangle whose base is $16\ cm$ and area is $34\ cm^2$

SOLUTION: Altitude of the triangle $= \dfrac{2 \times Area}{base}$

Here $area = 34\ cm^2$ and $base = 16\ cm$

$$Altitude = \frac{2 \times Area}{Base} = \left(\frac{2 \times 34}{16}\right) = 4.25\ cm$$

IMPORTANT

The side opposite to a right angle in a right angled triangle is its hypotenuse.

EXAMPLE-2 Find the area of triangles whose

(i) $base = 18\ cm,\ altitude = 3.5\ cm$

(ii) $base = 8\ cm,\ altitude = 15\ cm$

SOLUTION:

(i) We know that area of a triangle $= \dfrac{1}{2} \times base \times altitude$

$$= \frac{1}{2} \times 18 \times 3.5 = 31.5\ cm^2$$

(ii) Area of a triangle $= \dfrac{1}{2} \times base \times altitude$

$$= \frac{1}{2} \times 8 \times 15 = 60\ cm^2$$

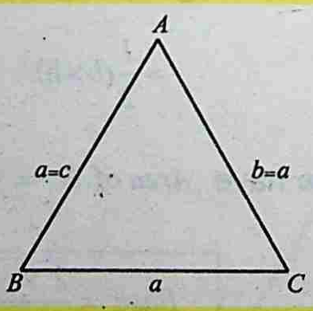
Area of an Equilateral Triangle when its side is given :-

In an equilateral $\triangle ABC, a = b = c$.

Therefore, $S = \dfrac{a+a+a}{2} = \dfrac{3a}{2}$

$S-a = \dfrac{a}{2}$, $S-b = \dfrac{a}{2}$,

$S-c = \dfrac{a}{2}$

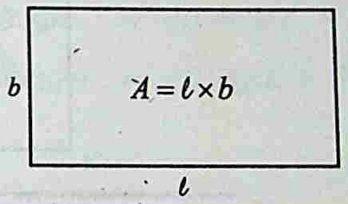


$$\triangle = \sqrt{S(S-a)(S-b)(S-c)} = \sqrt{\frac{3a}{2}.\frac{a}{2}.\frac{a}{2}.\frac{a}{2}} = \frac{\sqrt{3}\ a^2}{4}$$

Thus area of an equilateral $\triangle ABC$ is $\dfrac{\sqrt{3}\ a^2}{4}$

Area of a Rectangle when its two sides are given

Consider a rectangle as shown in the figure.



Length of the rectangle = ℓ
Width of the rectangle = b.

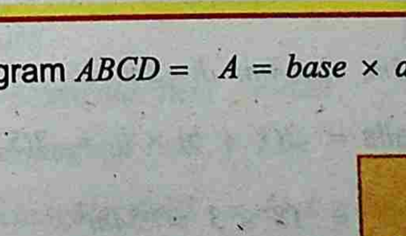
The Area of a rectangle is equal to the product of its length and width.

$A = \ell \times b$

Thus $b = \dfrac{A}{\ell}$ and $\ell = \dfrac{A}{b}$

Area of a Parallelogram when base and Altitude are given:-

The area of a parallelogram is equal to the product of base and the altitude drawn to the base.



Area of a parallelogram $ABCD$ = A = *base* × *altitude*

$A = b \times h$

Base = $b = \dfrac{A}{h}$

Altitude = $h = \dfrac{A}{b}$

IMPORTANT

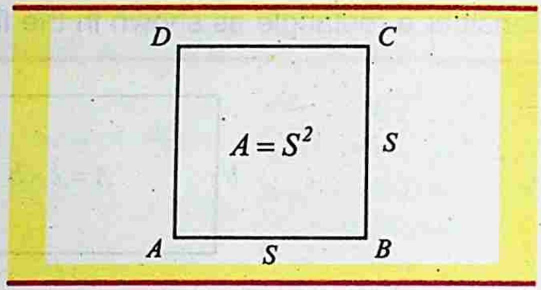
Area of a triangle:

$A = \dfrac{1}{2} \times base \times altitude$

Area of a Square when its side is given:-

The area of a square $ABCD$ is equal to the square of one of its sides.

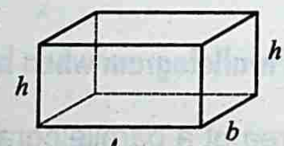
$$A = Side \times Side$$
$$= S \times S = S^2$$
$$\boxed{A = S^2}$$
$$Side = S = \sqrt{A}$$



Unit of Area is square unit of length like: cm^2, m^2, km^2.

Area of four Walls of a Room:

We can find the area of four walls of a room when its length, breadth and height are given.



Let length of the room $= \ell$

Width of the room $= b$

Height of the room $= h$

Area of four walls $= h\times\ell + b\times h + h\times\ell + b\times h$

$$= 2(h\times\ell) + 2(b\times h) = 2(h\times\ell + b\times h)$$
$$= 2h(\ell + b)$$

EXAMPLE *The length, width, and height of a room are 5m, 4m, 3m respectively. Find the cost of white-washing on all the walls of the room at the rate Rs 7.50 per m^2.*

SOLUTION: Given: $\ell = 5m$, $b = 4m$, $h = 3m$

Area of the four walls $= 2(\ell + b) \times h = 2(5 + 4) \times 3$

$$= 18 \times 3 = 54m^2$$

Therefore, cost of white-washing at the rate of Rs 7.50/m^2 $= 7.5 \times 54 = Rs\ 405$

Things to Remember:

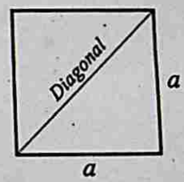
1- Area of rectangle = *(Length × Width)*

2- Diagonal of a rectangle = $\sqrt{(Length)^2 + (Width)^2}$

3- Perimeter of a rectangle = $2(Length + Width)$

4- Area of a square = a^2
where a = *side of the square*

5- Diagonal of a square = $\sqrt{2}a$



6- Area of a square = $\frac{1}{2}(Diagonal)^2$

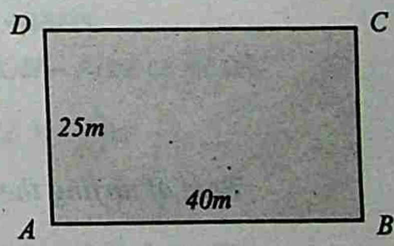
7- Perimeter of a square = $4 \times Side$

9.2.2 Areas of Rectangular and Square Fields

Rectangular paths are generally around (outside or inside) a rectangular field or in the form of central paths. We shall explain the method to calculate their areas through some examples.

EXAMPLE-1

A rectangular field is of length 40m and width 25m.
Find the total cost of turfing the field, if the cost of turfing the field is Rs.16per m^2.



SOLUTION: *Let us represent the field by rectangle ABCD as shown in the figure.*

Length of the rectangular field $= 40m$

Width of the rectangular field $= 25m$

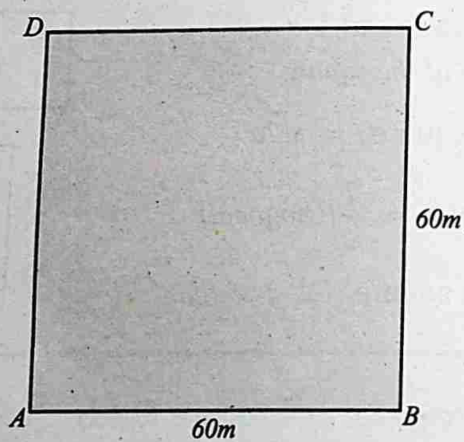
Area of the rectangular field $= A = \ell \times b = 40 \times 25 = 1000\ m^2$

Rate of turfing $= Rs.\ 16 per\ m^2$

Total cost $= 16 \times 1000 = Rs.\ 16000$

EXAMPLE-2

The boundary of a square field with side of 60m.
Find the area of the field.
Also find the cost of turfing the square field at the rate of Rs 5.00 per m^2



SOLUTION:

Let us represent the square field by ABCD as shown in figure.

Length of the side of the square field = 60m

Area of the square = $A = side \times side$

$= 60 \times 60$

$= 3600m^2$

Rate of turfing the square field = Rs. 5per m^2

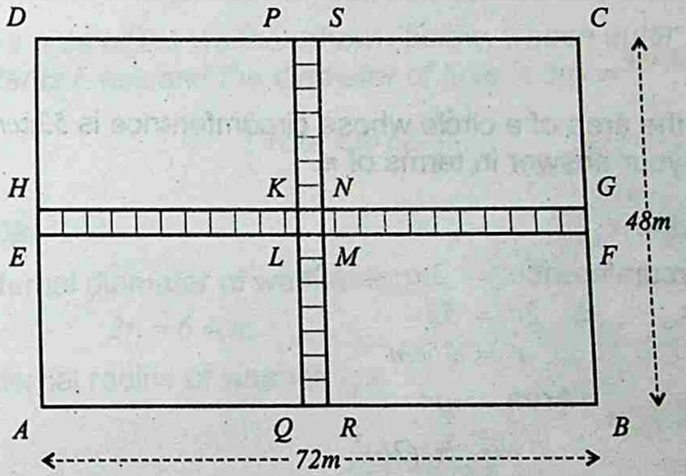
$\therefore$ *Cost of turfing the square field* = 3600×5

= 18000 rupees

9.2.3

EXAMPLE-3

Two cross roads, each $2m$ wide, run at right angles through the center of a rectangular park of length $72m$ and width $48m$ such that each is parallel to one of the sides of the rectangular field. Find the area of the roads. Also find the number of tiles required to beautify this road where each tile having area of $4\ m^2$.



SOLUTION:

In figure, rectangular field $ABCD$ represents the park and rectangle $PQRS$ and $EFGH$ represent the roads.

Area of the roads = Area of $PQRS$ + Area $EFGH$ – Area of $KLMN$

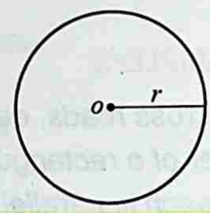
$$= [(48 \times 2) + (72 \times 2) - (2 \times 2)]m^2$$

$$= (96 + 144 - 4)m^2$$

$$= 236\ m^2$$

$\therefore$ Number of tiles required $= \frac{236}{4} = 59$ tiles

9.2.4 Area of a Circle



The circumference of circle = $2\pi r$ where the radius of the circle is 'r'.
Area of a circle $= \pi r^2$

Note: In examples and exercises, where π is not specified, use the value stored in the calculator.

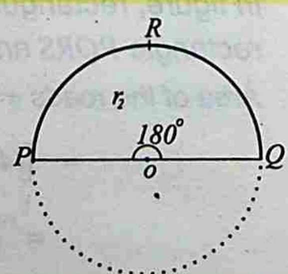
EXAMPLE

Find the area of a circle whose circumference is $52\pi cm$. Give your answer in terms of π.

SOLUTION:

$$\text{Circumference} = 2\pi r = 52\pi$$
$$\Rightarrow 2r = 52$$
$$\Rightarrow r = 26cm$$
$$\text{Area} = \pi r^2$$
$$= \pi (26)^2$$
$$= 676\pi \, cm^2$$

Area of a Semicircle:-



A semicircle is half of a circle, bounded by a diameter and half of the circumference.
Also a sector with an angle of $180°$ at the center of the circle is a semicircle.

In the figure,

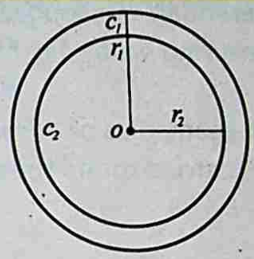
Length of arc $PQ = \frac{1}{2}$ of the circumference of the circle.

Area of sector $PRQ = \frac{1}{2}$ of the area of the circle.

$$\text{Area of semicircle} = \frac{1}{2}(\pi r^2).$$

9.2.5 Area of Concentric Circles

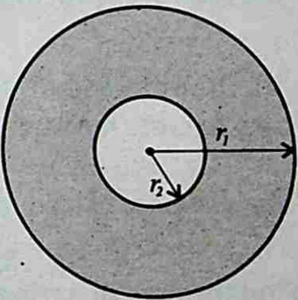
Circles with same center but different radii are called concentric circles. In the figure, c_1, c_2 are two concentric circles with same center 'O' but different radii r_1 and r_2.



EXAMPLE

Find the area of the washer shown below, whose outer diameter is 6.4cm and the diameter of hole is 3.6cm.

$$\left(Take\ \pi\ to\ be\ \frac{22}{7}\right)$$



SOLUTION:

External diameter of washer is

$$2r_1 = 6.4cm$$

External radius of washer, $r_1 = \dfrac{6.4}{2}$

$$r_1 = 3.2cm$$

Internal radius of washer $= r_2 = \dfrac{3.6}{2} = 1.8cm$

$\therefore$ The area of the washer $= \pi r_1^2 - \pi r_2^2$

$$= \pi(3.2)^2\,cm^2 - \pi(1.8)^2\,cm^2$$

$$= \pi\left[(3.2)^2 - (1.8)^2\right]cm^2$$

$$= \pi(10.24 - 3.24)cm^2$$

$$= (\pi \times 7)cm^2$$

$$= \frac{22}{7} \times 7 = 22cm^2$$

EXERCISE – 9.2

1- A verandah $40m$ long, $15m$ wide is to be paved with stones each measuring $6m$ by $5m$. Find the number of stones.

2- How many tiles of $40cm^2$ will be required to pave the footpath $1m$ wide carried round the outside of a grassy plot $28m$ by $18m$?

3- Find the area of a room $5.49m$ long and $3.87m$ wide. What is the cost of carpeting the room if the rate of carpet is $Rs\ 10.50$ per m^2?

4- The area of a rectangular rice field is 2.5 hectares and its sides are in the ratio $3:2$. Find the perimeter of the field.

5- The area of a square playground is $4500\ m^2$. How long will a man take to cross it diagonally at the speed of $3km$ per hour ?

6- The diagonal of a square is $14cm$. Find its area.

7- Find the area of a triangle whose sides are.

(i) 120cm, 150cm and 200cm

(ii) 50dm, 78dm and 112dm

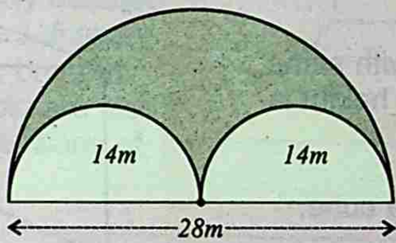
8- The perimeter of a triangular field is $540m$ and its sides are in the ratio $25:17:12$. Find the area of the triangle.
Hint: Let the sides be $25x$, $17x$, $12x$ meters.
Then $25x + 17x + 12x = 540 \Rightarrow 54x = 540 \Rightarrow x = 10$
The sides are $250m$, $170m$, $120m$

9- Find the area of a parallelogram if its two adjacent sides are $12cm$ and $14cm$ and diagonal is $18cm$.
Hints:
Let ABCD is a $||^{m}$ in which $m\overline{AB} = 12\ cm$, $m\overline{BC} = 14cm$, $m\overline{AC} = 18cm$
Find area of $\Delta\ ABC$.
Area of $||^{m} = 2(\text{Area of } \Delta\ ABC)$

10- Find the area of the following washers whose external and Internal diameters are:

(i) $15cm$ and $13cm$ *(ii)* $1.2m$ and $0.9m$
(iii) $40mm$ and $33mm$.

11- Find the area of the shaded region.



12- Find the area of an equilateral triangle whose side is $8m$.

13- The side of an equilateral triangle is $6cm$. Find its area.

14- Find the area of the right triangle with legs $12cm$ and $35cm$.

15- The base of a rectangle is three times its altitude. The area is $147cm^2$. Find the dimensions of the rectangle.

16- Find the base of the parallelogram whose attitude is $18cm$ and whose area is $3m^2$.

17- The area of a parallelogram is $144cm^2$. Find the altitude if the base is $2m$ long.

18- Find the area of the rectangle $2m$ long and $18cm$ wide.

19- The area of an equilateral triangle is $4\sqrt{3}\ cm^2$. Find the length of a side.

9.3 VOLUMES

In this topic we study some figures which are not plane. The simplest of these figures are cubes and cuboids. These figures do not lie completely in a plane, such figures are called solids, *(three dimensional figures)*.

Cube and Cuboid

Cube :-

A six faces figure, with same length, breadth and height is called a cube.

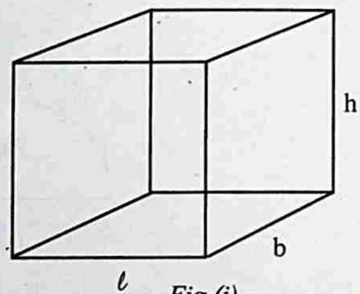


Fig (i)

The given figure is a cube,

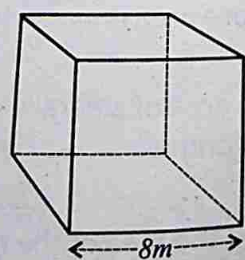
Length of the cube $= \ell$
Breadth of the cube $= b$
Height of the cube $= h$
where $\ell = b = h$,
therefore,

Volume of a cube $= V = \ell \times \ell \times \ell$ or $V = \ell^3$ *cubic unit*

EXAMPLE

Find the volume of the cube whose edge is 8m.

SOLUTION:



Given edge of the cube $= 8m$

Volume $= \ell^3$

$V = 8 \times 8 \times 8 = 8^3$

$V = 512\, m^3$

Dimensions:

Length has <u>one dimension</u>.
Area has <u>two dimensions</u>.
Volume has <u>three dimensions</u>.

Cuboid :-

A six faces figure which has length, breadth and height is called cuboid, *(or rectangular parallelopiped)*.

Figure *(ii)* represents a cuboid.

The length, breadth and height of a cuboid are usually denoted by the letter symbols ℓ, b and h respectively. Length, breadth and height of a cuboid are also called the three dimensions of the cuboid.

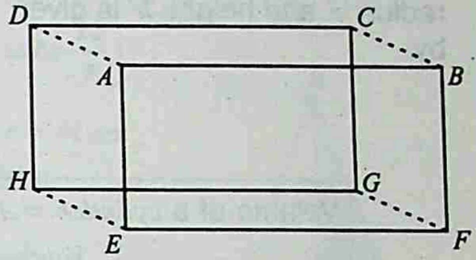


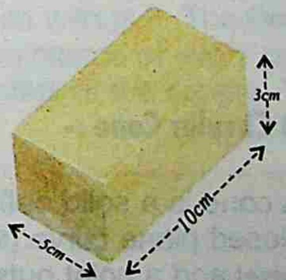
Fig (ii)

Volume of a cuboid of length ℓ, breadth b and height h is

$$V = \ell \times b \times h$$

EXAMPLE

Find the volume of a block of wood whose length, breadth and height are respectively 10cm, 5cm and 3cm.



SOLUTION:

Given:

Length of the block of wood = 10cm

Breadth of the block of wood = 5cm

Height of the block of wood = 3cm

$$V = \ell \times b \times h$$
$$= 10 \times 5 \times 3$$
$$= 150\ cm^3$$

Volume of a Cuboid and a Cube :-

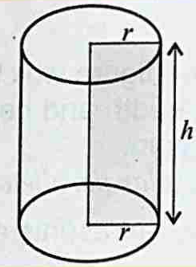
1- Length, breadth and height must be expressed in the same units.

2- From above formula, we also observe that:

Length $\ell = \dfrac{v}{b \times h}$	Breadth $b = \dfrac{v}{\ell \times h}$	Height $h = \dfrac{v}{\ell \times b}$

Volume of Right Circular Cylinder :-

The volume of a cylinder of base radius 'r' and height 'h' is given by.



$$\text{Volume of a } cylinder = Area\ of\ base \times height = \pi r^2 \times h$$
$$Volume = \pi r^2 h$$

EXAMPLE-1

Find the radius of the cylinder with volume $12320\ cm^3$ and height $20\ cm$.

SOLUTION: Given $v = 12320\ cm^3,\ h = 20cm,\ r = ?$

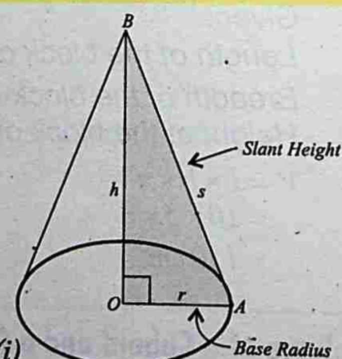
$$v = \pi \times r^2 \times h \Rightarrow r^2 = \frac{v}{\pi h}$$

$$r^2 = \frac{12320}{\frac{22}{7} \times 20} = \frac{12320 \times 7}{22 \times 20} = \frac{616 \times 7}{22} = 196$$

$$r = 14\ cm$$

Right Circular Cone :-

A cone is a solid defined by a closed plane curve *(forming the base)* and a point outside the plane *(the vertex)*. A right circular cone can be generated by rotating the right-angled triangle BOA as shown in *fig(i)* about $\overline{OB}$, which represents the height of the cone. The base of the cone is a circle with radius $\overline{OA}$.



fig(i)

B is the vertex of the cone and $\overline{BA}$ is the slant height.

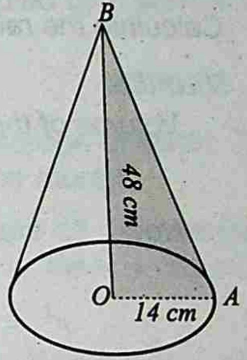
Volume of a $cone = \frac{1}{3} \times area\ of\ base \times height$

$$Volume\ of\ a\ cone = v = \frac{1}{3} \pi \times r^2 \times h$$

EXAMPLE

A cone has a circular base of radius 14cm, a height of 48cm, calculate the volume of the cone.

SOLUTION: $\left(\text{Take } \pi \text{ to be } \frac{22}{7}\right)$



Given radius of the base $= r = 14$ cm

Height of the cone $= h = 48$cm

Volume of the cone $= \frac{1}{3}\pi r^2 h$

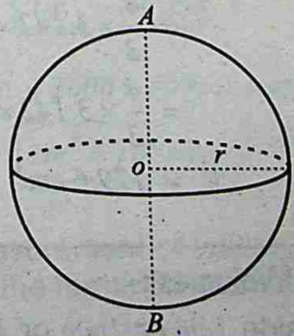
$= \frac{1}{3}\times\frac{22}{7}\times(14)^2\times 48$

$= \frac{1}{3}\times\frac{22}{7}\times 196\times 48$

$= 9856cm^3$

Sphere :-

A sphere is a body or space bounded by surface where every point on the surface is equidistant from a fixed point with in it. The fixed point is called the center of the sphere. The distance of every point on the surface to the fixed point is called the radius of the sphere. This radius is usually denoted by '*r*'.



Volume of the sphere $= \frac{2}{3}\times 2\pi r^3$

$V = \frac{4}{3}\pi r^3$, where r is the radius of the sphere.

Hemispheres :-

If a sphere is cut into half, the two portions are called hemispheres.

EXAMPLE-1

Calculate the radius of a sphere of volume $850\ m^3$ take π to be $\frac{22}{7}$

SOLUTION:

Volume of the a sphere $= 850\ m^3$

Radius $= ?$

Now $V = \frac{4}{3}\pi r^3$

$$r^3 = \frac{3V}{4\pi} \Rightarrow r^3 = \frac{3 \times 850 \times 7}{4 \times 22}$$

$$\Rightarrow r^3 = 202.8409$$

$$\Rightarrow r = (202.8409)^{\frac{1}{3}} = 5.88m$$

EXAMPLE-2

Find the volume of a sphere with radius 3.5cm.

SOLUTION:

Radius of a sphere $= r = 3.5cm$

Volume of a sphere $= V = \frac{4}{3}\pi r^3$

$$= \frac{4}{3} \times 3.142 \times (3.5)^3$$

$$= \frac{4}{3} \times 3.142 \times 3.5 \times 3.5 \times 3.5$$

$$V = 179.6\ cm^3$$

Some Standard Units of Volumes :-

If we take a cube of side $1cm$ or $1mm$ or $1m$ as a standard unit of measuring volumes, then we express the volume as:

cubic centimeters: $(cm)^3$

cubic millimeters: $(mm)^3$

cubic meters: $(m)^3$

Real Life Problems Related to Volume

A solid region has a magnitude or size or measure. The measure or magnitude of a solid region is called its volume.

In other words, the measure of the space occupied by a solid is called its volume.

For example, consider the real life problems.

1. A rectangular overhead tank is built for storage of water. The greater the volume the more water can be stored.
2. A rectangular tin box is to be made to store oil. The greater the volume of the cuboidal region, the more is the quantity of oil it can store.

Remember that:

1- As $1cm = 10mm$,

Therefore, $1cm^3 = 10 \times 10 \times 10\ mm^3$

$1cm^3 = 1000\ mm^3$

2- $1m^3 = 100 \times 100 \times 100\ cm^3$

$= 1000000\ cm^3$

$\boxed{1m^3 = 10^6\ cm^3}$

Also $1m^3 = 1000 \times 1000 \times 1000\ mm^3$

$\boxed{1m^3 = 10^9\ mm^3}$

3- For measurement of volumes of liquids, we use the terms liters (ℓ) and milliliters (ml).

$1cm^3 = 1m\ell$

$1000\ cm^3 = 1\ell$

and $1\ m^3 = 1000000\ cm^3 = 1000\ \ell$

$\boxed{1\ m^3 = 1k\ell \quad (1\ \text{kiloliter})}$

EXAMPLE-1

Find in liters, the volume (capacity) of a storage tank whose length, breadth and depth are respectively 6.3m, 4.5m and 3.6m.

SOLUTION:

Length of the tank $= 6.3\ m$

Breadth of the tank $= 4.5\ m$

Height of the tank $= 3.6\ m$

Volume of the tank $= \ell \times b \times h = 6.3 \times 4.5 \times 3.6 m^3$

$= 102.06\ m^3$

Volume of the tank $(m^3) = 102.06 \times 100 \times 100 \times 100$

$= 102060000\ cm^3$

$= 102060$ *litres* $(\therefore 1000\ cm^3 = 1\ litre)$

EXAMPLE-2

Capacity of a tank is 60kl. If the length, breadth of the tank are respectively 5m, and 4m, find its depth.

SOLUTION:

Volume of the tank $= 60\ k\ell = 60000\ liter = 60m^3$

Length of the tank $= 5\ m$

Breadth of the tank $= 4\ m$

Let depth of the tank $= d$

Now, volume of the tank $= length \times breadth \times depth$

Therefore, depth of the tank $= \dfrac{volume}{length \times breadth}$

$= \dfrac{60}{20} (\because 60000\ \ell t = 60\ m^3)$

$= 3m$

EXERCISE – 9.3

Find the Volume of the Solids

1- A cube of a side $4cm$.

2- A cube whose total area is $96cm^2$.

3- A rectangular box with length $4m$ breadth $3m$ and height $2m$.

4- Right cylinder, with radius of base $4cm$, altitude $10cm$, use $\pi = \frac{22}{7}$.

5- Circular cone, with radius of base $3cm$, altitude $10cm$.

6- Sphere, with radius $3cm$.

7- Right circular cylinder, with circumferences of base $4cm$, altitude $1m$.

8- Cone with altitude $9cm$, radius of base $6cm$.

Review Exercise-9

I- **Encircle the Correct Answer.**

1. If the square of the hypotenuse of a right triangle is equal to the sum of the squares of the other sides, it is called:
   (a) *Pythagoras theorem* (b) *Scalene triangle*
   (c) *Equilateral triangle* (d) *Isosceles triangle*

2. Area of a triangle when all the three sides are given is:
   (a) $\frac{1}{2}bh$ (b) bh
   (c) $\sqrt{s(s-a)(s-b)(s-c)}$ (d) $\frac{a+b+c}{2}$

3. Area of an equilateral triangle with side 'a' is:
   (a) $\frac{1}{2}bh$ (b) bh (c) $\frac{\sqrt{3}a^2}{4}$ (d) $\frac{\sqrt{3}a^2}{2}$

4. Area of a rectangle is:
   (a) $l \times b$
   (b) $\frac{1}{2} \times l + b$
   (c) $\frac{1}{3} \times l + b$
   (d) l^2

5. Area of a square with side 'S' is:
   (a) S
   (b) $4S$
   (c) $2S$
   (d) S^2

6. Area of a circle with radius 'r' is:
   (a) r^2
   (b) $2\pi r$
   (c) πr^2
   (d) $\pi^2 r$

7. Area of a semi-circle is:
   (a) $\frac{\pi r^2}{2}$
   (b) πr^2
   (c) $\pi^2 r$
   (d) $2\pi r$

8. Volume of a cube with edge 'l' is:
   (a) l^2
   (b) $3l$
   (c) l^3
   (d) l^4

9. Volume of a right circular cylinder is:
   (a) $\frac{\pi r^2 h}{3}$
   (b) $\frac{\pi r^2 h}{2}$
   (c) $\pi r^2 h$
   (d) $\frac{4}{3}\pi r^2$

II- Fill in the blanks.

1. If the square of the hypotenuse of a right triangle is equal to sum of the squares of the sides, then it is called______ theorem.
2. The surface inside the boundary of a shape is called__________.
3. Area of a triangle = __________.
4. Hero's formula for a triangle is $A =$__________.
5. An equilateral triangle with side 'a' has area = __________.
6. Area of a rectangle = __________.

7. Area of a circle = ______________.

8. Volume of a cube with edge 'l' is ______________.

9. Volume of a cuboid = ______________.

10. Volume of a right circular cone = ______________.

SUMMARY

Pythagoras Theorem: The square of the hypotenuse of a right triangle is equal to the sum of the squares of the legs.

Area: The space inside the boundary of a shape.

Area of a Triangle: $A = \frac{1}{2} \times base \times altitude$

Area of a Triangle: $A = \sqrt{s(s-a)(s-b)(s-c)}$

$s = \frac{a+b+c}{2}$, a, b, c are the sides of a triangle.

Area of an equilateral triangle: $A = \frac{\sqrt{3}a^2}{4}$, where '$a$' is the side of the triangle.

Area of a rectangle: $A = length \times breadth$.

Area of a square: $A = side \times side$.

Area of a parallelogram: $A = base \times altitude$.

Area of a circle: $A = \pi r^2$

Circumference of a circle: $C = 2\pi r$

Area of a semi-circle: $A = \frac{1}{2}(\pi r^2)$

Area of a washer: $A = \pi\left[r_1^2 - r_2^2\right]$

r_1 is the radius of outer circle.

r_2 is the radius of inner circle.

Volume: The space inside the boundary of a three dimensional shape.

Volume of a cube: $V = l^3, l$ is the length of edge.

Volume of a cuboid: $V = l \times b \times h$

$l = length$, $b = breadth$, $h = height$

Volume of a right circular cylinder: $V = \pi r^2 h$

$h = height\ of\ the\ cylinder$

$r = radius\ of\ the\ base$

Volume of a right circular cone: $V = \frac{1}{3}\pi r^2 h$

$h = height\ of\ the\ cone$

$r = radius\ of\ the\ base$

Volume of sphere: $V = \frac{4}{3}\pi r^3$

Volume of a hemi-sphere: $V = \frac{2}{3}\pi r^3$