UNIT

10 INTRODUCTION TO COORDINATE GEOMETRY

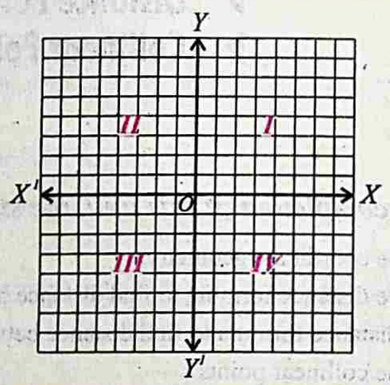
- **Introduction To Coordinate Geometry**
- **Distance Formula**
- **Collinear Points**

After completion of this unit, the students will be able to:

- define coordinate geometry.
- derive distance formula to find distance between two points given in cartesian plane.
- use distance formula to find distance between two given points.
- define collinear points.
- distinguish between collinear and non-collinear points.
- use distance formula to show that given three (or more) points are collinear.
- use distance formula to show that the given three non-collinear points form:
  - An equilateral triangle.
  - An isosceles triangle.
  - A right angled triangle.
  - A scalene triangle.

10.1 DISTANCE FORMULA

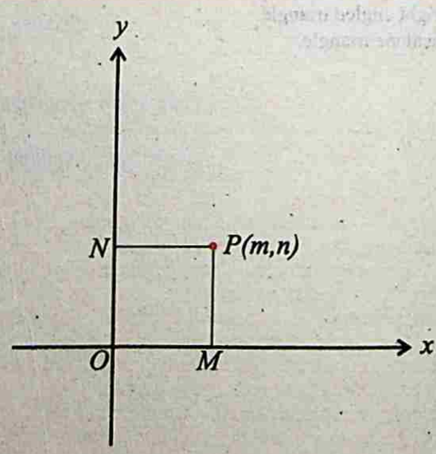
In 17th Century, Descartes, a French Mathematician introduced a plane. A set of infinite number of points, called Cartesian Plane. Every point in a plane can be located in terms of a pair of numbers related to two number axes in the plane, which are perpendicular to each other and intersect at the origin.



The plane is called a cartesian plane, and the axes are designated the horizontal $(\overrightarrow{OX})$ and vertical $(\overrightarrow{OY})$ axes, or the x-axis and the y-axis.

[illegible] axes divide the [illegible] into four quadrants [illegible] shown in the figure.

[illegible] in the plane and [illegible] wn through P [illegible] es, the lines will [illegible] two points, [illegible] coordinate of $M(m)$ [illegible] axis is called the x-coordinate or abscissa of P, and the coordinate of $N(n)$ on the y-axis is called the y-coordinate or ordinate of P.



The two numbers (m,n) are called the coordinates of P with respect to the coordinate axes. The letters m and n stand for numbers, and since the x-coordinate is always written first, such a pair is called an ordered pair of numbers. That is, the pair $(3,2)$ is not the same as the pair $(2,3)$.

Remember that:

(i) A point in a number plane determines a unique ordered pair of numbers.
(ii) With every ordered pair of numbers a unique point is associated in the plane.

Since numbers to the right of the origin on the horizontal axis and numbers above the origin on the vertical axis are taken as positive, therefore:

(i) A point in the *Ist* quadrant is characterized by the fact that both its coordinates are positive.

(ii) A point in the *IInd* quadrant has its abscissa negative and its ordinate positive.

(iii) A point in the *IIIrd* quadrant has both coordinates negative.

(iv) A point in the *IVth* quadrant has its abscissa positive and its ordinate negative.

(v) Points on the axes do not lie in any quadrant.

(vi) Points on the positive x-axis have a positive abscissa, and their ordinate is "0".

(vii) Points on the negative x-axis have a negative abscissa, and their ordinate is "0".

(viii) Points on the positive y-axis have a positive ordinate, and abscissa is "0".

(ix) Points on the negative y-axis have a negative ordinate, and their abscissa is "0".

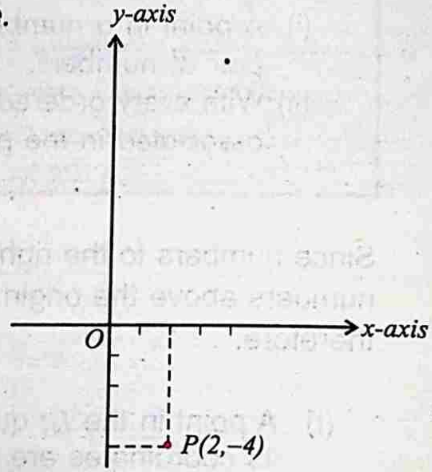
(x) The origin has the coordinates $(0, 0)$.

EXAMPLE-1

Locate (2,–4) in the co-ordinate plane.

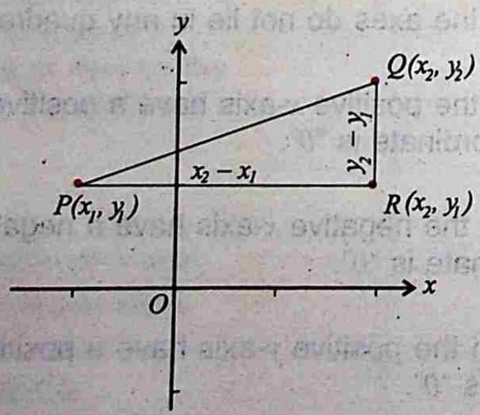
SOLUTION:

In this problem abscissa is positive, therefore it would be towards the right of the origin, and the ordinate is negative, so it would be below the origin, therefore the given point P is as shown in the figure.



10.1.2 Distance Between Two Points

Consider the points $P(x_1, y_1)$ and $Q(x_2, y_2)$ in the cartesian plane as shown in the figure. To find the length of the segment $\overline{PQ}$, we form a right triangle by drawing through P a line parallel to x-axis and through Q a line parallel to y-axis and let these lines meet at $R(x_2, y_1)$.



By Pythagoras theorem, we have.

$$\overline{|PQ|}^2 = \overline{|PR|}^2 + \overline{|RQ|}^2$$

$$\overline{|PQ|}^2 = |(x_2 - x_1)|^2 + |(y_2 - y_1)|^2$$

$$= (x_2 - x_1)^2 + (y_2 - y_1)^2$$

Hence $\overline{|PQ|} = \pm\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

As we are only interested in the length of the segment and not in the direction, therefore we only consider the positive sign.

Hence distance between two points $P(x_1, y_1)$ and $Q(x_2, y_2)$ is given by:

$$d = \overline{|PQ|} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

10.1.3 Use of Distance Formula

EXAMPLE-1

What kind of a triangle has vertices $A(6,-2)$, $B(1,-2)$ and $C(-2,2)$?

SOLUTION:

Given $A(6,-2)$, $B(1,-2)$ and $C(-2,2)$. Using distance formula,

$$\overline{|AB|} = \sqrt{(1-6)^2 + (-2+2)^2} = \sqrt{5^2 + 0} = \sqrt{25} = 5$$

$$\overline{|AC|} = \sqrt{(-2-6)^2 + (2+2)^2} = \sqrt{8^2 + 4^2} = \sqrt{64+16} = \sqrt{80} = 4\sqrt{5}$$

$$\overline{|BC|} = \sqrt{(-2-1)^2 + (2+2)^2} = \sqrt{3^2 + 4^2} = \sqrt{9+16} = \sqrt{25} = 5$$

Since $\overline{|AB|} = \overline{|BC|} = 5$

Thus, triangle is an isosceles.

EXAMPLE-2

Express by an equation the fact that the distance from $P(x,y)$ to $A(2,3)$ is twice the distance from $P(x,y)$ to $B(3,4)$

SOLUTION:

Given $A(2,3)$, $B(3,4)$ and $P(x,y)$, where $P(x,y)$ be any point, According to the condition of the question.

$|\overline{AP}| = 2\,|\overline{BP}|$, using distance formula.

$$\sqrt{(x-2)^2+(y-3)^2} = 2\sqrt{(x-3)^2+(y-4)^2}$$

Taking square on both sides

$$(x-2)^2+(y-3)^2 = 4\left[(x-3)^2+(y-4)^2\right]$$

$$x^2-4x+4+y^2-6y+9 = 4\left[x^2-6x+9+y^2-8y+16\right]$$

$$x^2+y^2-4x-6y+13 = 4x^2+4y^2-24x-32y+100$$

$$3x^2+3y^2-20x-26y+87=0$$

EXAMPLE-3

The vertices of a triangle are $A(1,1)$, $B(5,5)$ and $C(9,1)$. Prove that the triangle is a right triangle.

SOLUTION:

Given $A(1,1)$, $B(5,5)$ and $C(9,1)$.using distance formula,

$$|\overline{AB}| = \sqrt{(5-1)^2+(5-1)^2} = \sqrt{4^2+4^2} = \sqrt{32}$$

$$|\overline{AC}| = \sqrt{(9-1)^2+(1-1)^2} = \sqrt{8^2} = \sqrt{64}$$

$$|\overline{BC}| = \sqrt{(9-5)^2+(1-5)^2} = \sqrt{4^2+4^2} = \sqrt{32}$$

By Pythagoras theorem,

$$|\overline{AB}|^2 + |\overline{BC}|^2 = 32 + 32$$

$$= 64$$

$$= |\overline{AC}|^2$$

Thus $\Delta\, ABC$ is a right triangle.

10.2 COLLINEAR POINTS

10.2.1 Collinear Points

Collinear points are points which are the elements of the set of points forming a straight line. In the given figure (*i*) the points *A,B,C,D,.....* are collinear. If three points are collinear, then one of the points must be lying in between the other two points.

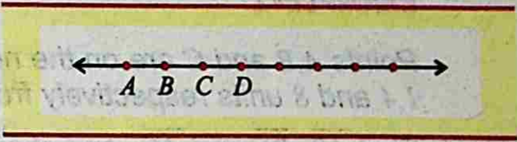


Fig (i)

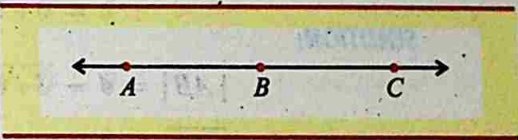
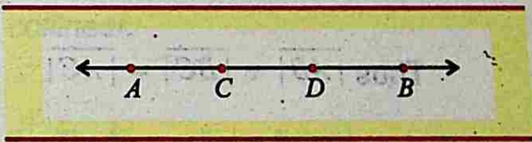


Fig (ii)

In the figure (*ii*) '*B*' is the point in between the point *A* and *C*.
In this case, $\overline{|AB|} + \overline{|BC|} = \overline{|AC|}$.

10.2.2 Collinear and Non-Collinear Points

A line-segment is a subset of a line, consisting of two end points and the set of infinite number of points between them on the line.



In the given figure, $\overline{CD}$ is the line-segment which is a sub-set of a line AB (or $\overleftrightarrow{AB}$). The point C and D are on the line AB and are collinear.

The three or more than three points which are not present on the same straight line are called non-collinear points.

In the given figure *P,Q* and *R* are non-collirear points.

Remember that:

In the figure *P,Q,R* are non-collirear points.
(i) Two points are always collinear.
(ii) Three points may or may not be collinear.

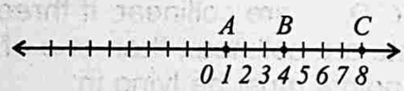
10.2.3 Collinearity of Three Points

EXAMPLE-1

Points A,B and C are on the number line at a distance of 1,4 and 8 units respectively from the origin.

Find $\overline{AB}$, $\overline{BC}$ and $\overline{AC}$, and show that $\overline{AB}+\overline{BC}=\overline{AC}$

SOLUTION:



$$|\overline{AB}| = 4 - 1$$

$$|\overline{AB}| = 3$$

$$|\overline{BC}| = 8 - 4 = 4$$

$$|\overline{AC}| = 8 - 1 = 7$$

Now $|\overline{AB}| + |\overline{BC}| = 3 + 4$

$$= 7 = |\overline{AC}|$$

Thus $|\overline{AB}| + |\overline{BC}| = |\overline{AC}|$

EXAMPLE-2

Show that the points A(1,4) B(5,6) and, C(9,8) are collinear.

SOLUTION:

Given A(1,4) B(5,6) and, C(9,8) .

Using distance formula, we have.

$$|\overline{AB}|=\sqrt{(5-1)^2+(6-4)^2} = \sqrt{4^2+2^2} = \sqrt{20} = 2\sqrt{5}$$

$$|\overline{BC}|=\sqrt{(9-5)^2+(8-6)^2} = \sqrt{4^2+2^2} = \sqrt{20} = 2\sqrt{5}$$

$$|\overline{AC}|=\sqrt{(9-1)^2+(8-4)^2} = \sqrt{8^2+4^2} = \sqrt{80} = 4\sqrt{5}$$

Now $|\overline{AB}| + |\overline{BC}| = 2\sqrt{5} + 2\sqrt{5}$

$$= 4\sqrt{5}$$

$$= |\overline{AC}|$$

Thus, the points A,B, and C are collinear.

EXAMPLE-3

Show that the points $A(4,3)$, $B(-2,3)$ and $B(-6,3)$ are collinear.

SOLUTION:

Given $A(4,3)$, $B(-2,3)$ and $B(-6,3)$.

Using distance formula, we have.

$$|\overline{AB}| = \sqrt{(-2-4)^2 + (3-3)^2} = \sqrt{36+0} = 6$$

$$|\overline{BC}| = \sqrt{(-6-2)^2 + (3-3)^2} = \sqrt{16+0} = 4$$

$$|\overline{AC}| = \sqrt{(-6-4)^2 + (3-3)^2} = \sqrt{100} = 10$$

$$\text{Now} \quad |\overline{AB}| + |\overline{BC}| = 6+4$$

$$= 10$$

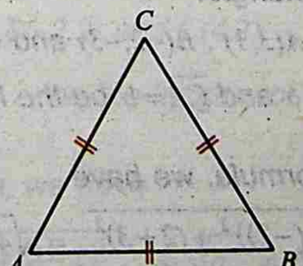
$$= |\overline{AC}|$$

Thus, the points A,B, and C are collinear.

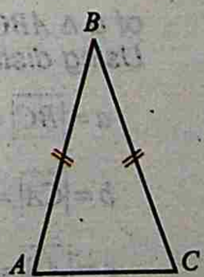
10.2.4 Use of Distance Formula (for The Non-collinear Points)

We use the distance formula to show that the given three non-collinear points form:-

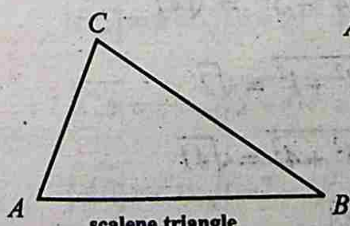
- *a right angle triangle*
- *an isosceles triangle*
- *an equilateral triangle*
- *a scalene triangle*



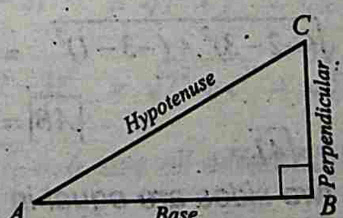
equilateral triangle



isosceles triangle



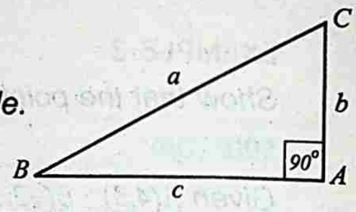
scalene triangle



right triangle

EXAMPLE-1

Show that the points $A(-1,2)$, $B(7,5)$ and $C(2,-6)$ are vertices of a right triangle.



SOLUTION: Given $A(-1,2)$, $B(7,5)$, $C(2,-6)$.

Let a,b,c denote the lengths of the sides $\overline{BC}, \overline{CA}$, and $\overline{AB}$ respectively of $\Delta\, ABC$, using distance formula

$$|\overline{PQ}| = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

we have

$$a = |\overline{BC}| = \sqrt{(2-7)^2+(-6-5)^2} = \sqrt{5^2+11^2} = \sqrt{146}$$

$$b = |\overline{CA}| = \sqrt{(2-(-1))^2+(-6-2)^2} = \sqrt{3^2+8^2} = \sqrt{73}$$

$$c = |\overline{AB}| = \sqrt{(7-(-1))^2+(+5-2)^2} = \sqrt{(8)^2+(3)^2} = \sqrt{64+9} = \sqrt{73}$$

$$\text{clearly } |\overline{AB}|^2 + |\overline{CA}|^2 = c^2+b^2$$

$$= 73 + 73 = 146 = a^2$$

$$= |\overline{BC}|^2$$

Thus, ΔCAB, is a right triangle with right angle at A.

EXAMPLE-2

Show that the points $A(3,1)$ $B(-2,-3)$ and $C(2,2)$ are vertices of an Isosceles triangle.

SOLUTION: Given $A(3,1)$, $B(-2,-3)$ and $C(2,2)$.

Let $\overline{AB} = c$, $\overline{BC} = a$ and $\overline{CA} = b$ be the lengths of the sides of a $\Delta\, ABC$.

Using distance formula, we have

$$a = |\overline{BC}| = \sqrt{(2-(-2))^2+(2+3)^2} = \sqrt{4^2+5^2} = \sqrt{41}$$

$$b = |\overline{CA}| = \sqrt{(3-2)^2+(1-2)^2} = \sqrt{1^2+1^2} = \sqrt{2}$$

$$c = |\overline{AB}| = \sqrt{(-2-3)^2+(-3-1)^2} = \sqrt{5^2+4^2} = \sqrt{41}$$

$$|\overline{AB}| = |\overline{BC}|$$

Here $c = a = \sqrt{41}$

That is, the two sides are equal in length.

Thus, $\Delta\, ABC$ is an Isosceles triangle.

EXAMPLE-3

Show that the points $A(-3,0)$, $B(3,0)$ and $C(0,3\sqrt{3})$ are the vertices of an equilateral triangle.

SOLUTION: Given $A(-3,0)$, $B(3,0)$ and $C(0,3\sqrt{3})$.

Using distance formula, we have,

$$|\overline{AB}| = \sqrt{(-3-3)^2+(0-0)^2} = \sqrt{(-6)^2} = \sqrt{36} = 6$$

$$|\overline{BC}| = \sqrt{(3-0)^2+(0-3\sqrt{3})^2} = \sqrt{9+27} = \sqrt{36} = 6$$

$$|\overline{AC}| = \sqrt{(-3-0)^2+(0-3\sqrt{3})^2} = \sqrt{9+27} = \sqrt{36} = 6$$

Here $|\overline{AB}| = |\overline{BC}| = |\overline{AC}| = 6$

That is, three sides of ΔABC are equal in length.

Thus, ΔABC is an equilateral triangle.

EXAMPLE-4

Show that the points $A(5,3)$ $B(-2,2)$ and $C(4,2)$ are vertices of a scalene triangle.

SOLUTION: Given $A(5,3)$ $B(-2,2)$ and $C(4,2)$.

Let $\overline{BC} = a$, $\overline{CA} = b$, $\overline{AB} = c$ be the lengths of the sides of a ΔABC.

Using distance formula, we have

$$|\overline{BC}| = a = \sqrt{(4+2)^2+(2-2)^2} = \sqrt{6^2} = 6$$

$$|\overline{CA}| = b = \sqrt{(5-4)^2+(3-2)^2} = \sqrt{1^2+1^2} = \sqrt{2}$$

$$|\overline{AB}| = c = \sqrt{(-2-5)^2+(2-3)^2} = \sqrt{7^2+1^2} = \sqrt{50} = 5\sqrt{2}$$

As, $|\overline{AB}| = c$, $|\overline{BC}| = a$, $|\overline{CA}| = b$ are all different in length.

Thus ΔABC is a scalene triangle.

EXERCISE – 10.1

1- Describe the location of these points on the number plane.

(i) $(1,0)$ (ii) $(0,4)$ (iii) $(-2,4)$ (iv) $(3,6)$
(v) $(-4,0)$ (vi) $(-8,-8)$ (vii) $(7,-5)$ (viii) $(-8,10)$
(ix) $(0,-7)$ (x) $(8,-3)$

2- Find the distance between the following pairs of points.

(i) $(2,1)$, $(-4,3)$ (ii) $(-1,3)$, $(-2,-1)$
(iii) $(7,-2)$, $(-2,3)$ (iv) $(a,-b)$, $(b,-a)$

3- Express by an equation, the fact, that the point $P(x,y)$ is equidistant from $A(2,4)$ and $B(6,8)$.

4- Show that the points $A(5,4)$, $B(4,-3)$, $C(-2,5)$ are equidistant from $D(1,1)$.

5- Find the point on the x-axis which is equidistant from $(2,4)$ and $(6,8)$.
(Hint: call the point $(x,0)$. Find x.)

6- Show that the points $A(0,2)$, $B(3,-2)$ and $C(0,-2)$ are vertices of a right triangle.

7- Show that the points $A(-1,1)$, $B(3,2)$, $C(7,3)$ are collinear.

8- Show that the points $A(6,1)$, $B(2,7)$ and $C(-6,-7)$ are vertices of a right triangle.

9- Show that the points $A(2,4)$, $B(6,2)$, $C(4,3)$ are collinear.

10- Show that the points $A(4,-2)$, $B(-2,4)$ and $C(5,5)$ are vertices of an isosceles triangle.

11- Show that the points $A(-2,11)$, $B(-6,-3)$ and $C(4,-9)$ are of a scalene triangle.

12- Show that the points $A(6,1)$, $B(2,7)$ and $C(-6,7)$ are of a scalene triangle.

13- Show that the points $A(2,-5)$, $B(-4,-3)$ and $C(-1,5)$ are of an equilateral triangle.

Review Exercise-10

I- Encircle the Correct Answer.

1. $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ is called

(a) *distance formula* (b) *collinear points*
(c) *non-collinear points* (d) *equal points*

2. A point in a cartesian plane determines a unique ordered pair of:

(a) *set* (b) *abscissa* (c) *numbers* (d) *ordinate*

3. In the plane with every ordered pair is associated:

(a) *a unique point* (b) *zero* (c) *two points* (d) *four points*

4. Points lying on the same line are called:

(a) *non-collinear* (b) *collinear* (c) *equal* (d) *overlapping*

5. Points which do not lie on the same straight line are called:

(a) *non-collinear* (b) *collinear* (c) *equal* (d) *zero*

6. Point on the axis do not lie in any:

(a) *a plane* (b) *line* (c) *quadrant* (d) *circle*

7. The co-ordinates of the origin are:

(a) *0* (b) *(1,0)* (c) *(0,0)* (d) *(0,1)*

8. Points on the negative x-axis have negative:

(a) *abscissa* (b) *ordinate* (c) *value* (d) *fraction*

9. A point in 4th quadrant has its ordinate:

(a) *positive* (b) *negative* (c) *zero* (d) *one*

10. A point in the first quadrant is characterized by the fact that both its co-ordinates are:

(a) *zero* (b) *positive*
(c) *negative* (d) *positive and negative oth*

II- Fill in the blanks.

1. $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ is called ____________
2. A point in a cartesian plane determines a ____________ ordered pair of numbers.
3. With every ordered pair is associated a ____________ point in the plane .
4. Points lying on the same line are called ____________ points.
5. Points which do not lie on the same straight line are called ____________ points.
6. Points on the axes do not lie in any____________.
7. The origin has the co-ordinates____________.
8. Points on the negative x-axis have negative abscissa and their ordinate is____________ .
9. A point in the 4th quadrant has its abscissa positive and its ordinate____________.
10. A point in the first quadrant is characterized by the fact, that both its co-ordinates are ____________.

SUMMARY

Distance Formula: $d = |\overline{PQ}| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

1- A point in a number plane determines a unique ordered pair of numbers.

2- With every ordered pair of numbers a unique point is associated in the plane.

Collinear points: Points lying on the same straight line are called collinear points.

Non-Collinear points: Points which do not lie on a same straight line are called non-collinear points.

Exercise 1.1

1- 9,8 **2-** -11 **3-** 9 **4-** 11,71 **5-** 1,0 **6-** 18.86 **7-** 804.57 **8-** 25,1

9- $\frac{2y}{3x^2}$ **10-** $\frac{25a}{14b^2}$ **11-** $\frac{4a^3b^3}{5a^2+3b}$ **12-** $\frac{2m}{3x^5-4m^2x^3}$ **13-** $\frac{5}{c+d}$

14- $\frac{x+y}{-3}$ **15-** $\frac{2x^3-x^2y+xy^2}{x^3-x^2y+xy^2-y^3}$ **16-** $\frac{4x^2-2x}{x^2-1}$ **17-** $\frac{3x-1}{x^3-7x-6}$

18- $\frac{2x-3y}{2x+3y}$ **19-** $\frac{x-2y}{x^2-y^2}$ **20-** $\frac{x-2y}{xy-y^2}$ **21-** $\frac{2}{x-1}$

22- $\frac{37x+1}{x^2-12x+27}$ **23-** $\frac{x^2-4x+4}{x^2+2x}$ **24-** $\frac{-(x+6)}{x+1}$ **25-** $\frac{x^3+x^2+20x}{x^2+4x-5}$

26- $\frac{3x+4}{2x+1}$ **27-** $\frac{4x^3-x}{2x^2-1}$ **28-** $\frac{x}{x^3-2x^2+2x-1}$ **29-** $\frac{x}{3x-9}$

30- x **31-** 1 **32-** $x-1$

Exercise 1.2

1- $2x^2+8y^2$ **2-** $50x^2+18y^2$ **3-** $24\ell m$ **4-** ℓ^8-m^8 **5-** $a^3b^3-\frac{1}{a^3b^3}-3ab+\frac{3}{ab}$

6- $4x^2+9y^2+4+12xy+12y+8x$ **7-** $8p^3+12p^2q+6pq^2+q^3$

8- $9p^2+q^2+r^2+6pq+2qr+6pr$ **9-** $8x^3+36x^2y+54xy^2+27y^3$

10- $(x+y-1)(x^2+y^2+2xy+x+y+1)$ **11-** $(x-y+4)(x^2+y^2-2xy-4x+4y+16)$

12- $(2x+3y)(4x^2-6xy+9y^2)$ **13-** $(x+3y)(x^2-3xy+9y^2)(x-3y)(x^2+3xy+9y^2)$

14- $(2a+b)(2a-b)(4a^2-2ab+b^2)(4a^2+2ab+b^2)$ **15-** 4 **17-** 17, 4 **18-** 14

19- 133 **20-** 118 **21-** 20 **22-** 46

Exercise 1.3

1- (i) $\frac{\sqrt{5}}{5}$, (ii) $\frac{7\sqrt{6}}{3}$, (iii) $\frac{\sqrt{42}}{7}$ 2- (i) $3\sqrt{2}$, (ii) $35\sqrt{2}$, (iii) $4\sqrt{15}-6\sqrt{6}-2\sqrt{10}+6$

(iv) $30-6\sqrt{5}+5\sqrt{2}-\sqrt{10}$ (v) $5\sqrt{3}-\sqrt{15}-10+2\sqrt{5}$ (vi) $35+7\sqrt{2}+5\sqrt{3}+\sqrt{6}$

3- (i) $2-\sqrt{3}$ (ii) $\frac{4+\sqrt{5}}{11}$ (iii) $2\sqrt{3}(\sqrt{7}-\sqrt{5})$ (iv) $\frac{x+y-2\sqrt{xy}}{x-y}$

(v) $\frac{105-10\sqrt{7}}{59}$ (vi) $5+2\sqrt{6}$ (vii) $\frac{29(11-3\sqrt{5})}{76}$ (viii) $\frac{3\sqrt{7}-2\sqrt{3}}{3}$

4- (i) $2\sqrt{5}$ (ii) 18 5- (i) $2\sqrt{3}$ (ii) 14 6- (i) $-2\sqrt{2}$ (ii) 10

7- (i) $\frac{24-6\sqrt{2}}{7}$ (ii) $\left(\frac{-18+8\sqrt{2}}{7}\right)$ 8- (i) 40 (ii) 36

9- (i) $\frac{2b^2-a^2+2b\sqrt{b^2-a^2}}{a^2}$ (ii) $\frac{a-\sqrt{a^2-9}}{3}$

Review Exercise 1

I- Encircle the Correct Answer.

1- *b* 2- *b* 3- *d* 4- *c* 5- *d* 6- *a* 7- *d* 8- *b* 9- *c* 10- *a*

II- Fill in the blanks.

1- *rational number* 2- *rational expression* 3- $4ab$ 4- $2(a^2+b^2)$

5- $(a+b)^3$ 6- $(a-b)^3$ 7- a^3-b^3 8- a^3+b^3 9- *surd* 10- 2

Exercise 2.1

1- $(x+y)(3a-7b)$ 2- $(a-x)(x+y)$ 3- $(a-3)(a^2+1)$

4- $(x-1)(x^2+x-y)$ 5- $(x+2y)(3a-4b)$ 6- $(a-b)(2a+c)$

7- $(a-b)(a+c)$ 8- $(4-a^3)(2-a)$ 9- $(4x-3a)^2$

10- $(1-7x)^2$ 11- $5(2x-1)^2$ 12- $2ab(a-b)^2$

13- $(x+\frac{1}{2})^2$ 14- $(x-\frac{1}{x})^2$ 15- $5x(x-3)^2$ 16- $(a+b)(a+b+2c)$

Exercise 2.2

1- $(x+y+a)\ (x+y-a)$

2- $(2a+b+3c)\ (2a+b-3c)$

3- $(x+3a+4b)\ (x+3a-4b)$

4- $(y+x-c)\ (y-x+c)$

5- $(x+y+2xy)\ (x+y-2xy)$

6- $(a-2b-3ac)\ (a-2b+3ac)$

7- $(x-y-a+b)\ (x-y+a-b)$

8- $(y^2+2y+2)\ (y^2-2y+2)$

9- $(z^2+8y^2-4yz)\ (z^2+8y^2+4yz)$

10- $(x^3-6x+18)\ (x^3+6x+18)$

11- $(z^2-3z+4)\ (z^2+3z+4)$

12- $(2x-y)(x-y)(2x+y)(x+y)$

Exercise 2.3

1- $(x+4)\ (x+5)$

2- $(x-2)\ (x+7)$

3- $(x-1)\ (x+6)$

4- $(x-3)\ (x-4)$

5- $(x-13)\ (x+12)$

6- $(x-2)\ (x+1)$

7- $(x-15)\ (x+6)$

8- $(a-17)\ (a+5)$

9- $(7-x)\ (x+14)$

10- $(y-19)\ (y+8)$

11- $(x+1)\ (2x+1)$

12- $(x+1)\ (3x+2)$

13- $(x-1)\ (2x+1)$

14- $(2x+3)\ (3x-1)$

15- $(x+2)\ (1-2x)$

16- $(2-x)\ (4+5x)$

17- $(u-2)\ (3u-4)$

18- $(2x-3)\ (5x+4)$

19- $(x-6)\ (5x-2)$

20- $(4x-\sqrt{3})\ (\sqrt{3}x+2)$

Exercise 2.4

1- $(2x-y)\ (4x^2+2xy+y^2)$

2- $(3x+1)\ (9x^2-3x+1)$

3- $(1-7x)\ (1+7x+49x^2)$

4- $(ab+8)\ (a^2b^2-8ab+64)$

5- $(3-10y)\ (9+30y+100y^2)$

6- $(3x-4y)\ (9x^2+12xy+16y^2)$

7- $(xy+z)\ (x^2y^2-xyz+z^2)$

8- $(6p-7)\ (36p^2+42p+49)$

9- $(2x-\frac{1}{3})\ (4x^2+\frac{2}{3}x+\frac{1}{9})$

10- $(a+b)\left[a^2-ab+b^2+1\right]$

11- $(a-b)\left[1-(a^2+ab+b^2)\right]$

12- $x(1-2y)\ (1+2y+4y^2)$

13- $(x-y)(x+y)(x^2+y^2)(x^2+xy+y^2)(x^2-xy+y^2)(x^4-x^2y^2+y^4)$

14- $(1-\frac{4p}{q})\,(1+\frac{4p}{q}+\frac{16p^2}{q^2})$

15- $(1+4u)\,(1-4u+16u^2)$

16- $(2x+3y)\,(4x^2+9y^2-6xy-3)$

17- $(z+5)\,(z^2-5z+25)$

18- $(x+y)\,(x^2-xy+y^2)\,(x^6-x^3y^3+y^6)$

19- $(m+n)(m-n)\,(m^2+mn+n^2)(m^2-mn+n^2)$

20- $x(2x-a)(2x+a)\,(4x^2+2ax+a^2)(4x^2-2ax+a^2)$

21- $(x-3a)\,(x^2+3ax+9a^2)$

22- $(x+3a)\,(x^2-3ax+9a^2)$

Exercise 2.5

1- 3 2- –6 3- –47 4- 0 5- –84 6- *yes* 7- *yes* 8- *no* 9- *no*

10- *yes* 11- *no* 12- *yes* 13- *yes* 14- *no* 15- *no* 16- *yes* 17- *yes*

18- *no* 19- $k = 1$ 20- $k = 1$

Review Exercise 2

I- **Encircle the Correct Answer.**

1- *b* 2- *c* 3- *d* 4- *a* 5- *c* 6- *b* 7- *a* 8- *a* 9- *a* 10- *a*

II- **Fill in the blanks.**

1- *one* 2- *two* 3- *three* 4- $(x-3)\,(x+3)$ 5- $(x+1)\,(x+3)$

6- $(x+2)(x^2-2x+4)$ 7- $(x-2)(x^2+2x+4)$ 8- 3 9- 11 10- 0

Exercise 3.1

1- ab 2- $3qr$ 3- $4xy^2z^2$ 4- $7ab$ 5- $3x^2y^2$

6- $2abc$ 7- $x+4$ 8- x^2-y^2 9- $t+3$ 10- $x-2$

11- $1+x$ 12- $x-2$ 13- $x+1$ 14- $x\,(x+3)$ 15- $5abc$

ANSWERS

Exercise 3.2

1- x^2-x+1 2- $2x^2+3x-2$ 3- $2(x-1)$ 4- $9x(x+3)$ 5- $(x-1)^2(x+1)$ 6- $(x-2)$

7- $(x-1)$ 8- $(3x-5)$ 9- $2x+1$ 10- $(x+3)$

Exercise 3.3

1- $420a^4x^4y^4$ 2- $15a^4b^3c^5$ 3- $12abc$ 4- $x^2y^2z^2$

5- $p^2q^2(p-q)(p+q)(p^2+pq+q^2)$ 6- $(x+4)(x-4)(x^2-4x+16)$

7- $(x-2)(x+3)(x+1)(x-1)$ 8- $(y+3)(y-2)(y+3)(y-3)$

9- $(1+y)(1-y)(1-2y)(y^2-y+1)$ 10- $(x-y)(x+y)(x^2+y^2)(x^4+x^2y^2+y^4)$

11- $(x+1)(x^2-x+1)(x^2+x+1)^2$

12- $(x+y)(x^2+y^2)(x-y)(x^2-xy+y^2)(x^4-x^2y^2+y^4)$ 13- $(2x+3)(x+1)^2(x+3)$

14- $x^2(x+3)(x-2)(x-3)$ 15- $(x+y)^2(x+2y)^2$

Exercise 3.4

1- x^2+1 ; x^4-1 2- $(x^2-4),(x-3)(x^3-x^2-4x+4)$ 3- $2x^2+1$; $2x^4-x^2-1$

4- $2x^2+3x-2$; $(3x-1)(8x^4+6x^3-15x^2+9x-2)$

5- $(3x^2+8x-3);(2x^2-3x+1)(3x^4+17x^3+27x^2+7x-6)$

6- $(x^2+2x-3);(2x^2-x-5)(2x^4+x^3-20x^2-7x+24)$

7- (x^3-1) ; $(x-1)(x^4+x^3-x-1)$

9- $x^2-12x+35$ 10- $(6x^2+x-2)$ 11- $x+4$ 12- $(x+1)$; $(x^3+1)(x^4+x^3-x-1)$

14- $x^3-7x^2+16x-12$ 15- $x^4+8x^3+11x^2-32x-60$ 16- x^5-x^4-4x+4

Exercise 3.5

1- $\dfrac{2(2a+1)}{a(a+1)(a+2)}$ 2- $\dfrac{2ax+x-3a-6a^2}{(x-2a)(x-3a)}$ 3- $2+a^4$ 4- $\dfrac{1}{x^4+x^2+1}$

5- $\frac{2b^2(a-c)}{(a+b)(b+c)}$ 6- $\frac{6x^3}{x^6-1}$ 7- $\frac{2a^3}{a^2-b^2}$ 8- 1 9- 1 10- 1

11- $\frac{a}{a-b}$ 12- $\frac{a+1}{a+2}$

Exercise 3.6

1- $\pm(4x+3y)$ 2- $\pm(x-3)(x-4)(x-5)$ 3- $\pm(x+1)(x+7)(2x-3)$ 4- $\pm(x^2+6x+4)$

5- $\pm(4x^2+16x+11)$ 6- $\pm(x+\frac{1}{x}-5)$ 7- $\pm(t+\frac{1}{t}-2)$ 8- $\pm(x^2+\frac{1}{x^2}-2)$

9- $\pm(2x^2+3x+4)$ 10- $\pm\left(\frac{3x}{2y}-\frac{1}{2}-\frac{2y}{3x}\right)$ 11- $x = 8$ 12- $\ell = 4$, $m = 10$

Review Exercise 3

I- **Encircle the Correct Answer.**

1- *a* 2- *c* 3- *c* 4- *a* 5- *b* 6- *a* 7- *c* 8- *c* 9- *a* 10- *a*

II- **Fill in the blanks.**

1- *two* 2- *two* 3- *H.C.F* 4- *L.C.M* 5- *H.C.F* 6- *second expression*

7- $2x+1$ 8- $x+2$ 9- $2x^2y^3$ 10- $6x^2y^2z$

Exercise 4.1

1- (*i*) 8, (*ii*) 80, (*iii*) 11, (*iv*) 2 2- $\frac{5}{2}$ 3- 2 4- -7 5- -2

6- 3 7- 4 8- 4 9- 3 10- {4} 11- {9} 12- {18} 13- {8}

14- { } 15- { } 16- {13, 5} 17- {8} 18- {13} 19- {101} 20- {15}

Exercise 4.2

1- ± 9 2- $-1, 7$ 3- $-6, 4$ 4- $-1, 4$ 5- $\frac{5}{3}, \frac{-13}{3}$ 6- $x<7$

7- $x > -3$ 8- $x < -1$ 9- $x < -10$ 10- $x > -\frac{17}{9}$ 11- $x < -21$

12- $x > -12\frac{5}{7}$ 13- $x \geq 6$ 14- $x \leq 1\frac{7}{18}$ 15- $x \geq 1\frac{1}{2}$ 16- $x \geq 0$

Review Exercise 4

I- **Encircle the Correct Answer.**

1- a 2- c 3- c 4- c 5- c 6- a 7- c 8- a

II- **Fill in the Blanks.**

1- $>$ 2- $>$ 3- $<$ 4- $<$ 5- $>$ 6- $>$ 7- $>$ 8- $<$

9- $<$ 10- $<$ 11- $<$ 12- $>$

Exercise 5.1

1- $-2, 6$ 2- $1, 5$ 3- $-8, 1$ 4- $2, 3$ 5- $2, \frac{4}{3}$ 6- $-8, \frac{1}{2}$ 7- $3, -4$

8- $3, -\frac{1}{3}$ 9- $2, -\frac{1}{2}$ 10- $2, \frac{-4}{5}$ 11- $2, \frac{-3}{2}$ 12- $\frac{-1}{2}, \frac{4}{5}$ 13- $1, \frac{-1}{2}$

14- $5 \pm 2\sqrt{7}$ 15- $3 \pm 2\sqrt{3}$ 16- $\frac{-1 \pm \sqrt{5}}{2}$ 17- $-3 \pm 2\sqrt{3}$ 18- $\frac{2 \pm \sqrt{2}}{2}$

19- $\frac{3 \pm \sqrt{3}}{2}$ 20- $\frac{-5 \pm \sqrt{73}}{6}$ 21- $\frac{-m \pm \sqrt{m^2 - 4n}}{2}$ 22- $\frac{3 \pm 4\sqrt{15}}{11}$ 23- $-2 \pm \sqrt{17}$

24- $\frac{10 \pm 4\sqrt{15}}{5}$ 25- $\{13, -2\}$

Exercise 5.2

1- $2, 3$ 2- $\frac{3}{4}, \frac{1}{2}$ 3- $-1, \frac{2}{3}$ 4- $-1, \frac{3}{2}$ 5- $-5, 3$ 6- $3, \frac{-7}{2}$ 7- $\pm\sqrt{10}$

8- $\pm 2\sqrt{6}$ 9- ± 8 10- $\frac{5}{3}$ 11- $0, -5$ 12- $5, \frac{1}{3}$

Exercise 5.3

1- 5, 7 **2-** 8,10 **3-** 9,18 **4-** 5 **5-** 5, 6 **6-** 12, 13 **7-** 7, 9
8- 4, 8 *or* 8, 4

Review Exercise 5

I- **Encircle the Correct Answer.**

1- *a* **2-** *b* **3-** *b* **4-** *a* **5-** *c* **6-** *c* **7-** *c* **8-** *c* **9-** *b* **10-** *b*

II- **Fill in the blanks.**

1- *quadratic* **2-** *quadratic formula* **3-** $x(2x-3)$ **4-** $\{-1,3\}$ **5-** *three*

6- *quadratic formula* **7-** $\{2, 3\}$ **8-** $\{\pm 3\}$ **9-** $(x-2)(x+2)(x^2+4)$ **10-** $\{\pm 1\}$

Exercise 6.1

1- $2-by-2, 3-by-1, 3-by-2$ **2-** $2-by-2, 3-by-3, 1-by-3$

3- 5 **4-** $B=F$, $G=J$, $H=K$, $C=E$, $A=D$

Exercise 6.2

1- Row matrix = *A*, Column matrix = *C*, Square matrices = *B,D,E,F* Rectangular matrices = *A*, *C*, *G*
2- Diagonal matrix are *A,B,C,D,E,F,G* Scalar matrix are *B*, *D*, *E*, *G*, Identity is *D*

3- $\begin{bmatrix} 3 & -1 \\ 4 & 4 \end{bmatrix}, \begin{bmatrix} -3 & -1 \\ -2 & 4 \end{bmatrix}, \begin{bmatrix} a & c \\ -b & d \end{bmatrix}, \begin{bmatrix} l & p & a \\ m & q & b \\ n & r & c \end{bmatrix}$ **4-** A, C **5-** A, C, E **6-** C **7-** A

Exercise 6.3

1- *(i)* $\begin{bmatrix} 2 & 4 & 9 \\ 3 & 8 & 11 \\ 5 & 13 & 1 \end{bmatrix}$ *(ii)* $\begin{bmatrix} 2 & 2 & -1 \\ -1 & 2 & -1 \\ 3 & 5 & 5 \end{bmatrix}$ *(iii)* $\begin{bmatrix} -2 & -2 & 1 \\ 1 & -2 & 1 \\ -3 & -5 & -5 \end{bmatrix}$ *(iv)* $\begin{bmatrix} 4 & 9 & 23 \\ 8 & 19 & 28 \\ 11 & 30 & 0 \end{bmatrix}$

(v) $\begin{bmatrix} 6 & 5 & -8 \\ -5 & 3 & -9 \\ 8 & 11 & 17 \end{bmatrix}$ *(vi)* $\begin{bmatrix} 2 & 1 & -6 \\ -3 & -1 & -7 \\ 2 & 1 & 7 \end{bmatrix}$ **2-** $-A = \begin{bmatrix} -4 & -3 \\ -2 & -6 \end{bmatrix}, -B = \begin{bmatrix} -\sqrt{2} & -3 \\ -4 & -\sqrt{3} \end{bmatrix},$

$-C=\begin{bmatrix}-1\\7\\-4\end{bmatrix}, -D=\begin{bmatrix}-1&0&2\\0&-3&-4\\-2&1&3\end{bmatrix}, -E=[-2\ -5\ 3]$ **4-** $-1,2$ **6-** $X=\begin{bmatrix}2&1\\-\frac{4}{3}&2\end{bmatrix}$

7- $a=2, b=-4, c=4, d=3, e=4, f=2$ **8-** $w=-1, x=1, y=7, z=-8$

9- $\begin{bmatrix}-a&-b\\-c&-d\end{bmatrix}$

Exercise 6.4

9- $[12\ 13]$ **10-** $\begin{bmatrix}5\\-3\end{bmatrix}$ **11-** $\begin{bmatrix}2&4\\1&-5\end{bmatrix}$ **12-** $\begin{bmatrix}1&-9\\-3&17\end{bmatrix}$

13- $\begin{bmatrix}10&1\\-2&10\end{bmatrix}$ **14-** $\begin{bmatrix}10&-14\\15&3\end{bmatrix}$ **15-** $a=\frac{10}{7}, b=0$

Exercise 6.5

1- $(i)\ uy-vx$ $(ii)\ -13$ $(iii)\ 0$ $(iv)\ \frac{13}{64}$

2- *(i) singular* *(ii) non-singular* *(iii) non - singular*

3- (i) $\begin{bmatrix}3&-2\\-1&1\end{bmatrix}$ (ii) $\begin{bmatrix}3&-1\\-5&2\end{bmatrix}$ (iii) $\begin{bmatrix}\frac{1}{2}&0\\\frac{1}{6}&\frac{1}{3}\end{bmatrix}$ *(iv) Inverse does not exists*

(v) $\begin{bmatrix}4&\frac{-3}{2}\\-1&\frac{1}{2}\end{bmatrix}$ (vi) $\begin{bmatrix}-1&0\\0&-1\end{bmatrix}$ (vii) $\begin{bmatrix}\frac{3}{5}&\frac{4}{5}\\\frac{-4}{5}&\frac{3}{5}\end{bmatrix}$ **4-** $M^{-1}=\begin{bmatrix}-2&1\\\frac{3}{2}&\frac{-1}{2}\end{bmatrix}$

Exercise 6.6

1- $\frac{-9}{2}$ **2-** $(i)\ (3,1)$ $(ii)\ \left(\frac{7}{3},\frac{3}{2}\right)$ $(iii)\ \left(\frac{-1}{2},\frac{2}{5}\right)$ $(iv)\ (-1,5)$ $(v)\ (0,2)$ $(vi)\ (-2,6)$

3- $(3,-1)$ **4-** $(i)(-1,2)$ $(ii)(1,-1)$ $(iii)(4,-1)$ *(iv) No Solution* $(v)(2,-1)$

$(vi)\left(\frac{31}{21},\frac{59}{21}\right)$

5- $(i)\ 2x-y=2, 5x+2y=4$ $(ii)\ -5x+2y=2, 2x-3y=-1$

$(iii)\ -4x+y=1, 5x+4y=-1$ $(iv)\ 0.8x-0.6y=1, 0.6x+0.8y=2$

ANSWERS

Review Exercise 6

I- **Encircle the Correct Answer.**

1- a 2- a 3- a 4- c 5- a 6- c 7- b 8- c 9- c 10- c

II- **Fill in the blanks.**

1- *order* 2- *row matrix* 3- *same order* 4- *same* 5- *equal* 6- 1

7- *associative* 8- *skew symmatric* 9- $B^t A^t$ 10- $B^{-1}A^{-1}$

Exercise 7.1

1- (i) 130° (ii) 115° (iii) 42° (iv) 30° (v) 108° (vi) 20° 2- $105^\circ, 75^\circ$ 3- 70°

4- $-0^\circ, 100^\circ$ 5- $70^\circ, 30^\circ$ 6- $x + 90^\circ + 30^\circ = 180^\circ \Rightarrow x = 60^\circ$ 7- (i) $a = 40^\circ$

(ii) $c = 35^\circ, d = 145^\circ$ (iii) $e = 29^\circ, f = 151^\circ$ (iv) $b = 135^\circ$

(v) $q = 77^\circ, P = 103^\circ, r = 103^\circ$ (vi) $j = 30^\circ, k = 150^\circ, l = 30^\circ$

(vii) $g = 140^\circ, h = 40^\circ, i = 140^\circ$ (viii) $k = 145^\circ$

(ix) $P = 58^\circ, M = 122^\circ, N = 122^\circ$ (x) $a = 158^\circ, b = 112^\circ$

Exercise 7.2

1- (a) $(\angle 1, \angle 2), (\angle 3, \angle 4)$ (b) $(\angle 1, \angle 6), (\angle 3, \angle 8), (\angle 2, \angle 7), (\angle 5, \angle 4)$ (c) none

(d) $(\angle 1, \angle 8), (\angle 1, \angle 4), (\angle 4, \angle 7), (\angle 7, \angle 8), (\angle 5, \angle 6), (\angle 5, \angle 2), (\angle 2, \angle 3), (\angle 3, \angle 6)$.

(e) $(\angle 1, \angle 7), (\angle 4, \angle 8), (\angle 5, \angle 3), (\angle 2, \angle 6)$

2- (a) $(\angle l, \angle n), (\angle m, \angle r)$ (b) $(\angle p, \angle n), (\angle m, \angle s), (\angle q, \angle r), (\angle l, \angle t)$ (c) none

(d) $(\angle p, \angle m), (\angle n, \angle s), (\angle q, \angle l), (\angle r, \angle t), (\angle q, \angle p), (\angle l, \angle m), (\angle r, \angle n), (\angle t, \angle s)$

(e) $(\angle p, \angle l), (\angle m, \angle q), (\angle n ; \angle t), (\angle s, \angle r)$

Exercise 7.3

1- yes, no, yes 2- yes 3- yes 4- 10*cm*, 12*cm*, 14*cm*, 16*cm*, 18*cm*

5- 6*cm*, 12*cm*, 18*cm*, 21*cm* 6- 15*cm*, 21*cm*, 9*cm*, 12*cm*, 1:3

7- $\overline{AB} \cong \overline{DE}, \overline{AC} \cong \overline{DF}, \overline{BC} \cong \overline{EF}$ $\angle A \cong \angle D, \angle B \cong \angle E, \angle C \cong \angle F$

8- No: size may be different 9- yes: size and shape are same

Exercise 7.4

1- (a) (i) $\overline{AB} \cong \overline{FD}$ (ii) $\overline{BC} \cong \overline{DE}$ (iii) $\overline{AC} \cong \overline{FE}$ (iv) $\angle A \cong \angle F$ (v) $\angle B \cong \angle D$ (vi) $\angle C \cong \angle E$

(b) $\angle R$ (c) $\overline{EF}$ (d) S.A.S≅SAS (e) ASA≅ ASA

2- (i) $\Delta ABC \cong \Delta DFE$ by S.S.S ≅ S.S.S (ii) $\Delta XYZ \cong \Delta DFE$ by S.S.A ≅ S.S.A

(iii) $\Delta ABC \cong \Delta CDA$, by A.S.A ≅ A.S.A (iv) $\Delta PQT \cong \Delta SRT$ by S.A.S ≅ S.A.S

3- $\overline{AD}\cong\overline{DA}$, $\overline{DB}\cong\overline{AC}$, $\overline{AB}\cong\overline{DC}$, $\angle BAD\cong\angle CDA$, $\angle ADB\cong\angle DAC$, $\angle ABD\cong\angle DCA$,
Condition used $S.S.S\cong S.S.S$, $m\angle ADB = 40^\circ$

4- (i) Similar Triangles (ii) Similar Parrallogram (iii) Similar Triangles

5- $\overline{MN}\Leftrightarrow\overline{PQ}$, $\overline{NO}\Leftrightarrow\overline{QR}$, $\overline{PR}\Leftrightarrow\overline{MO}$, $\angle 1\Leftrightarrow\angle 4$

Exercise 7.5

(i) rectangle (ii) square (iii) quadrilateral (iv) bisect (v) congruent

Exercise 7.6

(i) circle (ii) radius (iii) chord (iv) diameter (v) samicircle
(vi) major arc (vii) radius (viii) sector (ix) secant line (x) right angle

Review Exercise 7

I- **Encircle the Correct Answer.**

1- b 2- c 3- b 4- b 5- a 6- c 7- a 8- b 9- c 10- c

II- **Fill in the blanks.**

1- adjacent 2- supplementary 3- obtuse 4- vertical 5- 180° 6- each other
7- congruent 8- scalene 9- diameter 10- right

Review Exercise 8

I- **Encircle the Correct Answer.**

1- c 2- c 3- c 4- c 5- a 6- a 7- a 8- c 9- c 10- a

II- **Fill in the blanks.**

1- concurrent 2- concurrent 3- concurrent 4- concurrent 5- altitude
6- mediam 7- angle bisector 8- three 9- three 10- three

Exercise 9.1

1- (i) 5 (ii) 12 (iii) $4\sqrt{231}$ 3- $\sqrt{2}l$ 4- $8\sqrt{2}$

5- (i) right triangle (ii) not right Δ (iii) right Δ 6- 15cm 7- 7cm 8- 8m 9- 5

ANSWERS

Exercise 9.2

1- *20 stones* 2- *24000 stones* 3- *Rs. 223* 4- *645.50m* 5- *1mm 54sec*

6- $98\ cm^2$ 7- (i) $8967cm^2$ (ii) $16.8m^2$ 8- $9000m^2$ 9- $16\sqrt{110}\ m^2$

10- (i) $44cm^2$ (ii) 0.5 (iii) $401.14\ mm^2$ 11- $154m^2$ 12- $16\sqrt{3}m^2$ 13- $9\sqrt{3}cm^2$

14- $210cm^2$ 15- *7cm , 21cm* 16- *1666.67cm* 17- *72cm* 18- $3600cm^2$ 19- *4cm*

Exercise 9.3

1- $64cm^3$ 2- $64cm^3$ 3- $24m^3$ 4- $502.86cm^3$ 5- $94.3cm^3$ 6- $113.1cm^3$

7- $127.3cm^3$ 8- $339.4cm^3$

Review Exercise 9

I- **Encircle the Correct Answer.**

1- *a* 2- *c* 3- *c* 4- *a* 5- *d* 6- *c* 7- *a* 8- *c* 9- *c*

II- **Fill in the blanks.**

1- *Pythagoras* 2- *Area* 3- $\frac{1}{2}\times base\times altitude$ 4- $\sqrt{S(s-a)(s-b)(s-c)}$

5- $\frac{\sqrt{3}a^2}{4}$ 6- $\ell\times b$ 7- πr^2 8- l^3 9- $l\times b\times h$ 10- $\frac{1}{3}\pi r^2 h$

Exercise 10.1

1- (i) lies on $\overrightarrow{OX}$ (ii) lies on $\overrightarrow{OY}$ (iii) lies in QII (iv) In Q I

(v) on $\overrightarrow{OX}$ (vi) in QIII (vii) in QIV (viii) In QII

(ix) on $\overrightarrow{OY}$ (x) in QIV

2- (i) $2\sqrt{10}$ (ii) $\sqrt{17}$ (iii) $\sqrt{106}$ (iv) $(a-b)\sqrt{2}$ 3- $x+y-10=0$

5- (10,0)

Review Exercise 10

I- **Encircle the Correct Answer.**

1- *a* 2- *c* 3- *a* 4- *b* 5- *a* 6- *c* 7- *c* 8- *a* 9- *b* 10- *b*

II- **Fill in the blanks.**

1- *Distance formula* 2- *unique* 3- *unique* 4- *collinear* 5- *non-collinear*

6- *quadrant* 7- *(0,0)* 8- *zero* 9- *negative* 10- *positive*

GLOSSARY

Unit-1 ALGEBRAIC FORMULAS AND APPLICATIONS

Formula: Where we have a rule to calculate some quality, we write the rule as a formula.

$$(a \pm b)^2 = a^2 \pm 2ab + b^2$$

$$(a+b)^2 + (a-b)^2 = 2(a^2 + b^2)$$

$$(a+b)^2 - (a-b)^2 = 4ab$$

$$(a+b+c)^2 = (a^2 + b^2 + c^2 + 2ab + 2bc + 2ac$$

$$(a \pm b)^3 = a^3 \pm 3ab(a \pm b) \pm b^3$$

$$(x+y)(x-y)(x^2 - xy + y^2)(x^2 - xy + y^2)$$

Surd: A surd is an irrational number that contains an irrational square root.

Pure Surd: A surd which has unity only as rational factor, the other factor being irrational is called a pure surd.

Mixed surd: A surd which has rational factor other than unity, the other factor being irrational is called mixed surd.

Similar surd: Surds having the same irrational factor are called similar or like surd.

Unlike surd: Surd having no common irrational factor are know as unlike surd.

Rationalizing Factor: When the product of two surd is rational, then each one of them is called the rationalizing factor of the other.

Unit-2 FACTORIZATION

Linear Polynomial: A polynomial of degree *"1"* is called a linear polynomial.

Quadratic Polynomial: A polynomial of degree *"2"* is called a quadratic polynomial.

Cubic Polynomial: A polynomial of degree *"3"* is called a cubic polynomial.

Types of Factorization: $kx + ky + kz,\ ax + ay + bx + by,\ a^2 \pm 2ab + b^2$

$a^2 - b_1^2\ (a^2 \pm 2ab + b^2) - c^2,\ a^4 + a^2b^2 + b^4$ or $a^4 + b^4$,

$x^2 + px + q,\ x^2 + bx + c,$

$a^3 + 3a^2bx + 3ab^2 + b^3,\ a^3 - 3a^2b + 3ab^2 - b^3,$

$a^3 \pm b^3.$

Remainder Theorem: If a polynomial P(x) of degree $n \geq 1$ is divided by a polynomial ' x-a' where 'a' is any constant, them remainder is P(x).

Remainder Theorem: If a polynomial P(x) is divided by ' x-a' such that P(a) = 0, then ' x-a' is a factor of P(x).

Unit-3 ALGEBRAIC MANIPULATION

H.C.F: The *H.C.F* of two or more their two algebraic expressions is the expression of highest degree which divides each of them without remainder.

L.C.M: The least common multiple of two or more algebraic expressions is the expression of lowest degree which is divisible by each of them without reminder.

Unit-4 LINEAR EQUATIONS AND INEQUALITIES

Linear Equation: An equation that can be written in the form $ax + b = 0,\ a \neq 0$ where a and b are constants and x is a variable is called a linear equation in one variable.

Solution of a Linear equation : Any value of the variable, which makes the equation a true statement is called the solution of a linear equation.

Absolute Valve: For each real number *'x'* the absolute value of *x*, denoted by |x|, is defined by:

$$|x| = \begin{cases} x, & if \ x > 0 \\ 0, & if \ x = 0 \\ -x, & if \ x < 0 \end{cases}$$

Linear Inequalities: Two algebraic expressions joined by an inequality symbol such as $>, <, \leq, \geq$ is called an inequality.

Tricheotomy Property: If $x, y \in R$, then either $x > y$ or $x = y$ or $x < y$.

Transitive Property: If $x, y, z \in R$, then $x > y$ and $y > z \Rightarrow x > z$.

Additive Property: If $\forall a, b, c, d \in R$, then $a > b$ and $c > d \Rightarrow a + b > b + d$ and $c < d$ and $c < d \Rightarrow a + c < b + d$.

Multiplicative Property: $\forall a, b, c, d \in R$, $a > b$ and $c > d \Rightarrow ac > bd$ and $a < d$ and $c > d \Rightarrow ac > bd$.

Unit-5 QUADRATIC EQUATIONS

Quadratic Equation: A quadratic equation in one variable is an equation that can be written in the form $ax^2 + bx + c = 0$, where $a \neq 0$. Here *'x'* is a variable, where as *a,b* and c are real numbers.

Solution of quadratic Equation: We can solve a quadratic equation by *(i)* factorization *(ii)* completing the square method.

Quadratic Formula: $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Unit-6 MATRICES AND DETERMINANTS

Matrix: A rectangular array of number, enclosed by a pair of brackets and subject to certain rule is called a matrix.

Order of a matrix: The number of rows and columns in a matrix determine its order.

Row matrix: A matrix consisting of one row only is called a row matrix.

Column matrix: A matrix consisting of one column only is called a column matrix.

Square matrix: In a square matrix, the number of rows and columns are equal.

Rectangular matrix: In a rectangular matrix, number of rows and columns are not same.

Zero or null matrix: If all elements in a matrix are zero, the matrix is called a zero or null matrix.

Unit or Identity matrix: In an identity matrix, the diagonal elements are unity and off diagonal elements are all zero.

Transpose of a matrix: A matrix obtained by interchanging rows into columns is called transpose of a matrix.

Symmetric matrix: A matrix A is said to be symmetric, if $A' = A$.

Skew-Symmetric matrix : A matrix A is said to be skew-symmetric, if $A' = A$.

Determinant: A real number associated with a square matrix is called determinant of a square matrix.

Singular matrix: If the determinant of a square matrix is zero, it is called a singular matrix, other wise non-singular matrix.Adjoint of a square matrix of order 2×2 In the adjoint of a square matrix of order 2×2, the diagonal elements are interchanged, where as the sign of foo diagonal elements are changed. Multiplicative inverse of a square matrix, A matrix B is said to be multiplications inverse of 'A', is $AB=I$.

Unit-7 FUNDAMENTALS OF GEOMETRY

Angle: An angle is the union of two rays with common end point.

Right Angle: A right angle contains 90^o.

Straight Angle: A straight angle contains 180^o.

Acute Angle: An acute angle contains more than 0^o and less than 90^o.

Obtuse Angle: An obtuse angle contains more than 90^o and less than 180^o.

Reflex Angle: An reflex angle contains more than 180^o and less than 360^o.

Equal Angle: Equal angle are angle with equal measures.

Adjacent Angle: Two angles with the common vertex and a common side between them.

Complementary Angle: Two angles whose sum is a right angle.

Supplementary Angle: Two angles whose sum is a straight angle.

Vertical Angle: Two non adjacent angles, each less than a straight angle, formed by two intersecting lines.

Result 1: The sum of the angles of a triangle is a straight angle.

2: If two angles are complements of equal angles, they are equal.

3: If two angles are supplements of the same angle, they are equal.

4: Two lines parallel to a third line are parallel to each other.

5: If three parallel lines in percept congruent segments of one transversal, they intersect congruent segment of every transversal.

6: If a line bisect one side of a triangle and parallel to a second side. It bisect the third side.

Transversal: A transversal is a line that intersects two lines in different points.

Congruent Figures: Two geometrical figures which have the same size and shape are congruent.

Polygon: A polygon is a closed broken line in a plane.

Equilateral Triangle: A triangle with three equal sides.

Isosceles Triangle: A triangle with two equal sides.

Scalene Triangle: A triangle with no equal side.

Right Triangle: A triangle containing one right angle.

Obtuse Triangle: A triangle containing one obtuse angle.

Acute Triangle: A triangle containing three acute angle.

Equiangular Triangle: A triangle containing three equal angle.

Properties for congruency between two Triangle: *(i)* $SSS \cong SSS$ *(ii)* $SAS \cong SAS$ *(iii)* $ASA \cong ASA$ *(iv)* $AAS \cong AAS$ *(v)* $RHS \cong RHS$

Quadrilateral: A polygon with four sides.

Parallelogram: A quadrilateral with two pairs of parallel sides.

Rectangle: A parallelogram containing a right angle.

Square: A equilateral rectangle.

Circle: A set of points in a plane which are at a constant distance from a fired point.

Radius: A segment joining the center to any point on the circle.

Diameter: A chord that passer through the center.

Arc: A portion of a circle consisting of two end points and the set of points on the circle between them.

Semi Circle: An arc which is half of a circle.

Minor Arc: An arc less than a semi-circle.

Major Arc: An arc greater than a semi-circle.

Equal Circles: Circles having equal radii and equal diameters.

Secant Line: A line which intersects a circle in two points.

Tangent: A line perpendicular to the radius of a circle at its outer extremity.

Sector: A circular region bounded by an arc of a circle and its two corresponding radical segments.

Concyclic Points: Points lying on the circumference of the same circle.

Concentric Circles: Circles in the same plane with same center and different radii.

Central Angle: An angle formed at the center of the circle by two radii.

Result: *(1)* Angle in a semi-circle is a right angle.
(2) Angle in the segment are equal.
(3) All angles inscribed in the same arc are equal in measure.

Unit-8 PRACTICAL GEOMETRY

1- An angle bisects of a triangle is a line-segment that bisects and angle of the triangle and has its other and on the sides opposite to that angle.
2- Every triangle has tree angle bisectors, one for each angle.
3- An altitude of a triangle is the line-segment from one vertex, perpendicular to line containing the opposite side.
4- Every triangle has three altitudes, one from each vertex.`
5- A line-segment which bisect any side of a triangle and make a right angle with the sides as its mid point is called the perpendicular bisectors of the side of a triangle.
6- Every triangle has three perpendicular sides bisectors, one for each side.
7- The point at which the three angle bisectors of a triangle meet is called the incenter of the triangle.
8- The point at which the three altitudes of a triangle meet is called the other center of the triangle.
9- The point of intersection of the three perpendicular bisects of the sides of a triangle is called the circum-center of the triangle.
10- The point at which the three medians of a triangle meet is called the centroid circle in of a triangle.
11- A line coplanar with a circle intersecting the circle at one point only is called the tangent line to the circle.

Unit-9 AREAS AND VOLUMES

Pythagoras Theorem: The squares of the hypotenuse of a right triangle is equal to the sum of the squares of the legs.

Area: The space inside the boundary of a shape.

Area of a Triangle: $A = \frac{1}{2} \times base \times altitude.$

Area of a Triangle: $A = \sqrt{S(s-a)(s-b)(s-c)}$ $S = \frac{a+b+c}{2}$, a, b, c are the side of a triangle.

Area of an Equilateral Triangle: $A = \frac{\sqrt{3}a^2}{4}$, where '$a$' is the side of the triangle.

Area of a Rectangle: $A = length \times breadth.$

Area of a Square: $A = side \times side.$

Area of a Parallelogram: $A = base \times altitude.$

Area of a Circle: $A = \pi r^2.$

Circumference of a Circle: $C = 2\pi r.$

Area of a Semi-Circle: $A = \frac{1}{2}(\pi r^2)$

Area of a Concentric Circle: $A = \left[r_1^2 - r_2^2\right]$

r_1 is the radius of onter circle.
r_2 is the radius of inner circle.

Volume: The/space/inside/the/boundary/of/a/three/dimensional/shape.

VolumeofaCube: $V = l^3, l$ *is* the length of edge.

VolumeofaCuboid: $V = l \times b \times h \quad l = length \quad b = breadth \quad h = height$

VolumeofaRightCircular Cylinder: $V = \pi r^2 h$

$h = height\ of\ the\ cylinder$

$r = radius\ of\ the\ base$

VolumeofaRightCircular Cone: $V = \frac{1}{3}\pi r^2 h$

$h = height\ of\ the\ cone$

$r = radius\ of\ the\ base$

VolumeofSphere: $V = \frac{4}{3}\pi r^3$

VolumeofaHemisphere: $V = \frac{2}{3}\pi r^3$

Unit-10 INTRODUCTIONOFCOORDINATEGEOMETRY

DistanceFormula: $d - |\overline{PQ}| \quad \sqrt{(x_2 \quad x_1)^2 \quad (y_2 = y_1)^2}$

1- A point/in/a/number/plane/determines/a/unique/ordered/pair/of/numbers.
2- With/every/ordered/pair/of/numbers/is/associated/a/unique/point/in/the/place.

Collinear Points: Points/lying/on/the/same/straight/line/are/called/collinear/points.

Non-Collinear Points: Points/which/do/not/lie/on/a/same/straight/line/are/called/non-collinear/points.

SYMBOLS

Symbol	Standsfor	Symbol	Standsfor
$<$	is/less/than	$\because$	because///as
$>$	is/greater/than	$\therefore$	therefore///so
$\leq$	is/less/than/or/equal/to	$:$	ratio
$\geq$	is/greater/than/or/equal/to	$::$	is/proportional/to
$=$	is/equal/to	∞	varies
$\neq$	is/not/equal/to	$\vert$	tally/mark
$\nless$	is/not/less/than	Σ	summation
$\ngtr$	is/not/greater/than	$\overline{AB}$	line/segment*AB*
$\in$	belongs/to	$\overrightarrow{AB}$	ray*AB*
$\forall$	for/all	$\overleftrightarrow{AB}$	line
$\sqrt{}$	square/root	$\angle$	angle
$\vert x\vert$	absolute/value/ofx	$\triangle$	triangle
$\Rightarrow$	implies/that	$\sim$	is/similar/to
$\Leftrightarrow$	if/and/only/if	$\cong$	is/congruent/to
$\wedge$	and	$\approx$	is/approximately/equal/to
$\cup$	union	$\vert\vert$	is/parallel/to
$\vee$	or	$\widehat{AB}$	arc*AB*
$\cap$	infer/section	$\leftrightarrow$	correspondence

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The Author of this book Prof. Dr. Muhammad Ozair Ahmed, Chairman, Department of Mathematics, University of Engineering and Technology (UET) Lahore, has completed his Ph.D from University of Western Ontorio, Canada. Beside his 4 years teaching experience in colleges, has 30 years experience of teaching in University. He also taught as teaching Assistant in University of Western Ontorio Canada. He has been the President of Punjab Mathematical Society and attended many National and International Seminars on Mathematics.

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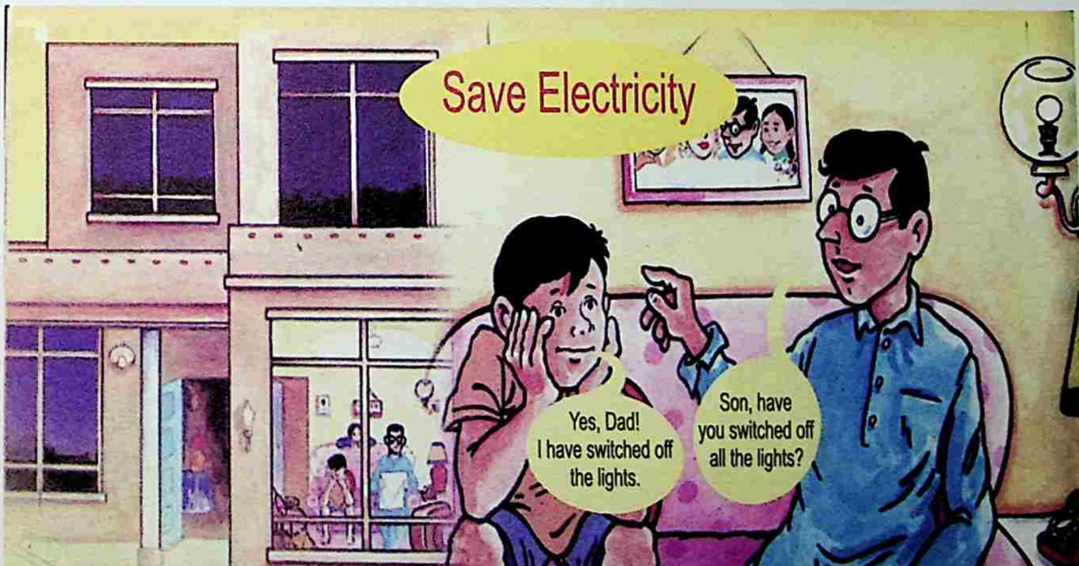
Mr. Maqsood Raza Ahmad, Associate Professor of Mathematics obtained the Degree of M.Sc in Applied Mathematics from University of the Punjab, Lahore, securing first Position in the Government College, Lahore and was awarded Roll of Honour. In 1979 he joined as lecturer in the Department of Mathematics, University of Balochistan Quetta. There he taught M.Sc classes for a period of almost one year. After having selection by P.P.S.C he appointed in College Cadre in the late 1980. In 1982, he was awarded Postgraduate Scholarship for Bulgaria. In 1990 he was nominated for Commonwealth Scholarship for Postgraduate Studies. In 1994 he was awarded the degree of M.Phil (Fluid Dynamics) by the University of Engineering and Technology, Lahore. The Publications Mr. Maqsood Raza Ahmad has to his credit includes six research papers of postgraduate level in the field of fluid dynamics and more than 20 books of Primary, Middle, Secondary and Intermediate Level. He is presently teaching at Government Shalimar Degree College, Baghbanpura Lahore.

Everybody is sitting in one room whereas the whole house is fully lit.



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