

UNIT

7

ARITHMETIC AND GEOMETRIC SEQUENCES

- ▶ Sequence
- ▶ Arithmetic Sequence
- ▶ Arithmetic Mean
- ▶ Geometric Sequence
- ▶ Geometric Mean

After completion of this unit, the students will be able to:

- ▶ Define a sequence (progression) and its terms.
- ▶ Know that a sequence can be constructed from a formula or an inductive definition.
- ▶ Identify arithmetic sequence.
- ▶ Find the n th or the general term of an arithmetic sequence.
- ▶ Solve problems involving arithmetic sequence.
- ▶ Know arithmetic mean between two numbers.
- ▶ Insert n arithmetic means between two numbers.
- ▶ Identify a geometric sequence.
- ▶ Find the n th or the general term of a geometric sequence.
- ▶ Solve problems involving geometric sequence.
- ▶ Know geometric mean between two numbers.
- ▶ Insert n geometric means between two numbers.

7.1 SEQUENCE (Progression)

In our daily life, we often observe things which increase or decrease progressively by fixed amounts. For example:

- 1- Number of days pass in a year by *7 days* every week.
- 2- Our age increases by *12 months* every year.
- 3- The price of a thing increases by a fixed amount, as you increase the number of units of that thing one-by-one.

In order to study such situations from daily life, let us consider the concept of a sequence. A sequence is an arrangement of numbers written in definite order according to some specific rule. A sequence is also called progression.

Look at the following number patterns.

$$i- \quad 1, 3, 5, 7, 9, \dots$$

$$ii- \quad 2, 4, 6, 8, 10, \dots$$

$$iii- \quad 1, 4, 9, 16, 25, \dots$$

$$iv- \quad 1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \dots$$

$$v- \quad 1, \frac{1}{3}, \frac{1}{9}, \frac{1}{27}, \frac{1}{81}, \dots$$

From these number patterns, it can be noticed that each successive number, can be found by applying a specific rule that justifies the position of succeeding one. This shows that all the numbers of each pattern are in a definite order.

From (i), the rule is:
Start with 1, then add 2 to each term to get the next term.

From (ii), the rule is:
Start with 2, then add 2 to each term to get the next term.

From (iii), the rule is:
Square each number of 1, 2, 3, 4, 5,...

From (iv), the rule is:
Start with 1, and multiply each term by $\frac{1}{2}$ to get the next term.

From (i), to (iv) we say each pattern form a number sequence. The number in a sequence are the terms of the sequence.

To represent a sequence, a special notation a_n is adopted and the symbol $\{a_n\}$ or $a_1, a_2, a_3, \dots, a_n, \dots$ is used. (Read the final dots "... as "and so forth").

$$\begin{aligned} a_1 &= 1st \text{ term,} \\ a_2 &= 2nd \text{ term,} \\ a_3 &= 3rd \text{ term,} \\ &\dots\dots\dots \\ &\dots\dots\dots \\ a_n &= nth \text{ term or general term.} \end{aligned}$$

7.1.1 Finite and Infinite Sequences

Look at the following number patterns.

$$(i) \quad 1, 2, 3, 4, \dots$$

$$(ii) \quad 1, 3, 5, 7, \dots, 15$$

$$(iii) \quad 1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \dots$$

$$(iv) \quad 2, 4, 6, 8, \dots, 20$$

$$(v) \quad 1, 4, 7, 10, \dots$$

$$(vi) \quad 1, \frac{1}{3}, \frac{1}{9}, \frac{1}{27}, \dots$$

If there is a last term in a sequence, it is called a finite sequence. In the above examples, (ii) and (iv) are finite sequences.

If there is no last term in a sequence, it is called an infinite sequence. In the given examples (i), (iii), (v), (vi) are infinite sequences.

7.1.2 Construction of a Sequence from a Formula

Now we write the sequence with the help of n th term.

If $a_n = 2n + 3$, $n = 1, 2, 3, \dots, 8$ then

$$a_1 = 2 \times 1 + 3 = 2 + 3 = 5$$

$$a_2 = 2 \times 2 + 3 = 4 + 3 = 7$$

$$a_3 = 2 \times 3 + 3 = 6 + 3 = 9$$

$$a_4 = 2 \times 4 + 3 = 8 + 3 = 11$$

$$a_5 = 2 \times 5 + 3 = 10 + 3 = 13$$

$$a_6 = 2 \times 6 + 3 = 12 + 3 = 15$$

$$a_7 = 2 \times 7 + 3 = 14 + 3 = 17$$

$$a_8 = 2 \times 8 + 3 = 16 + 3 = 19$$

The sequence is $5, 7, 9, 11, 13, 15, 17, 19$.

The terms of the sequence $\{a_n\}$ have been written by assigning the values $1, 2, 3, \dots, 8$ to n . For example:

If $a_n = (-1)^{n+1} (n+3)$ and $n = 1, 2, 3, 4$ then

$$a_1 = (-1)^{1+1} (1+3) = (-1)^2 (4) = 1 \times 4 = 4$$

$$a_2 = (-1)^{2+1} (2+3) = (-1)^3 (5) = -1 \times 5 = -5$$

$$a_3 = (-1)^{3+1} (3+3) = (-1)^4 (6) = 1 \times 6 = 6$$

$$a_4 = (-1)^{4+1} (4+3) = (-1)^5 (7) = -1 \times 7 = -7$$

The sequence is : $4, -5, 6, -7$.

With the help of n th term, we can write any desired term by giving a particular value to " n ".

EXERCISE - 7.1

1- Write the first three terms of the following:

(i) $a_n = n + 3$

(ii) $a_n = (-1)^n n^3$

(iii) $a_n = 3n + 5$

(iv) $a_n = \frac{n+1}{2n+5}$

(v) $a_n = \frac{1}{(2n-1)^2}$

(vi) $a_n = n + 3 = 2$

(vii) $a_n = \frac{1}{3^n}$

(viii) $a_n = 3n - 5$

(ix) $a_n = (n+1)a_{n-1}, a_1 = 1$

2- Find the terms indicated in the following sequences.

(i) $2, 6, 11, 17, \dots, a_8$

(ii) $1, 3, 12, 60, \dots, a_7$

(iii) $1, \frac{1}{3}, \frac{1}{9}, \frac{1}{27}, \dots, a_6$

(iv) $-1, 1, 3, 5, \dots, a_9$

(v) $\frac{1}{3}, \frac{2}{5}, \dots, a_5$

(vi) $1, -3, 5, -7, \dots, a_9$

3- Find the next four terms of the following sequences.

(i) $12, 16, 21, 27, \dots$

(ii) $1, 3, 7, 15, 31, \dots$

(iii) $-1, 2, 12, 40, \dots$

(iv) $9, 11, 14, 17, 19, 22, \dots$

(v) $4, 8, 12, 16, \dots$

(vi) $-2, 0, 2, 4, 6, 8, 10, \dots$

7.2 ARITHMETIC SEQUENCE (Progression)

An arithmetic progression (*abbreviated A.P.*) is a sequence of numbers called terms, each of which after the first is obtained from the preceding one by adding to it a fixed number called "common difference" of the progression.

Let ' a ' be the first term and ' d ' be the common difference in an A.P. Then the second term is $a + d$, the 3rd term is $a + 2d$. In each of these terms the co-efficient of d is one less than the number of the term. Similarly the 10th term is $a + 9d$.

The n th term is $(n - 1)$ th after the 1st term and thus is obtained after ' d ' has been added $(n - 1)$ times, then

$$\text{General term} = n\text{th term} = a_n = a + (n - 1)d.$$

If we take $n = 12$ then 12th term = $a_{12} = a + (12 - 1)d = a + 11d$

EXAMPLE-1

Find the general term and the 14th term of an A.P, whose 1st term is 2 and the common difference is 5.

SOLUTION: Given $a_1 = a = 2$, $d = 5$, we know that:

$$\begin{aligned} a_n &= a + (n-1)d \\ &= 2 + (n-1)5 \\ &= 2 + 5n - 5 \\ &= 2 - 5 + 5n \\ &= 5n - 3 \end{aligned}$$

General term = n th term = $a_n = 5n - 3$

Now putting $n = 14$ in equation $a_n = a + (n-1)d$, we have

$$\begin{aligned} a_{14} &= a + (14-1)d \\ &= 2 + 13 \times 5 \\ &= 2 + 65 \\ &= 67 \\ a_{14} &= 67 \end{aligned}$$

EXAMPLE-2

If 5th term of an A.P is 16 and 20th term is 46, what is the 15th term ?

SOLUTION: Given $a_5 = 16$ and $a_{20} = 46$

Since $a_n = a + (n-1)d$ (1)

Putting $n = 5$ in equation (1), we have

$$a_5 = a + (5-1)d$$

$$16 = a + 4d \text{ (2)}$$

Putting $n = 20$ in equation (1), we have

$$a_{20} = a + (20-1)d$$

$$46 = a + 19d \text{ (3)}$$

Subtracting equation (2) from (3), we have

$$46 - 16 = a - a + 19d - 4d$$

$$30 = 15d \Rightarrow \boxed{d = 2}$$

Putting $d = 2$ in equation (2), we have

$$16 = a + 4 \times 2$$

$$16 = a + 8 \Rightarrow 16 - 8 = a$$

$$\Rightarrow \boxed{a = 8}$$

Putting $n = 15$ in equation (1), we have

$$a_{15} = a + (15 - 1)d$$

$$= 8 + 14 \times 2$$

$$= 8 + 28$$

$$= 36$$

Thus $\boxed{a_{15} = 36}$

EXAMPLE-3

Find the number of terms in an A.P.,

if $a_1 = 3$, $d = 4$, $a_n = 59$

SOLUTION: Given $a_1 = a = 3$, $d = 4$, $a_n = 59$

Since $a_n = a + (n - 1)d$

$$59 = 3 + (n - 1)4$$

$$= 3 + 4n - 4$$

$$= 4n - 1$$

$$60 = 4n$$

$$n = \frac{60}{4} = 15$$

$$\boxed{n = 15}$$

Thus the number of terms in the given A.P is 15.

EXAMPLE-4

If $a_{n-3} = 2n - 12$, find a_n .

SOLUTION: Given $a_{n-3} = 2n - 12$

Putting $n = n + 3$, we have,

$$a_{n+3-3} = 2(n+3) - 12$$

$$a_n = 2n + 6 - 12$$

$$= 2n - 6$$

$$\boxed{a_n = 2n - 6}$$

EXERCISE - 7.2

1- Find the specified term of the following A.P

(i) 3, 7, 11, ..., 61st term.

(ii) -4, -7, -10, ..., a_{19}

(iii) 6, 4, 2, ..., 45th term.

(iv) 9, 14, 19, ..., a_{14}

(v) 11, 6, 1, ..., a_{18}

2- Find the missing element using the formula of A.P

$$a_n = a + (n-1)d$$

(i) $a = 2$, $a_n = 402$, $n = 26$,

(ii) $a_n = 81$, $d = -3$, $n = 18$

(iii) $a = 5$, $a_n = 61$, $n = 15$

(iv) $a = 16$, $a_n = 0$, $d = -\frac{1}{4}$

(v) $a = 10$, $a_n = 400$, $d = 5$

(vi) $a_n = 261$, $d = 4$, $n = 18$

3- Find the 15th term of an A.P where the 3rd term is 8 and the common difference is $\frac{1}{3}$.

4- Which term of an A.P 6, 2, -2, ... is -146 ?

5- Which term of an A.P 5, 2, -1, ... is -118 ?

6- How many terms are there in an A.P, in which $a_1 = a = 11$, $a_n = 68$, $d = 3$.

7- Find the 11th term of an A.P $2-x$, $3-2x$, $4-3x$, ...

8- Find the n^{th} term of an A.P, where $a_{n-5} = 3n + 9$.

9- Find the n^{th} term of an A.P: $\left(\frac{3}{4}\right)^2, \left(\frac{3}{7}\right)^2, \left(\frac{3}{10}\right)^2, \dots$

10- If the n^{th} term of an A.P is $3n - 5$. Find the A.P.

7.3 ARITHMETIC MEAN

A number 'A' is said to be an arithmetic mean between the two numbers 'a' and 'b', if a, A, b is an A.P.

$$A - a = b - A \quad (\text{Common Difference})$$

$$A + A = a + b$$

$$2A = a + b$$

$$A = \frac{a+b}{2}$$

EXAMPLE-1

Find A.M between 4 and 8.

SOLUTION: Given $a = 4, b = 8$

$$A = \frac{a+b}{2}$$

$$= \frac{4+8}{2}$$

$$= \frac{12}{2} = 6$$

$$A = 6$$

A.M represents
Arithmetic Mean

EXAMPLE-2

Find an A.M between $2\sqrt{5}$ and $6\sqrt{5}$.

SOLUTION: Given $a = 2\sqrt{5}, b = 6\sqrt{5}$

$$A = \frac{a+b}{2}$$

$$= \frac{2\sqrt{5} + 6\sqrt{5}}{2}$$

$$= \frac{8\sqrt{5}}{2}$$

$$= 4\sqrt{5}$$

7.3.2 Arithmetic Means Between Two Numbers

Let $A_1, A_2, A_3, \dots, A_n$, be " n " A.Ms between the two numbers a and b , such that $a, A_1, A_2, A_3, \dots, A_n, b$ is an A.P.

Here $a_1 = a$, $a_{n+2} = b$, because there are $n + 2$ terms in an A.P.

Using $a_n = a + (n-1)d$, we have

$$a_{n+2} = a + (n+2-1)d$$

$$b = a + (n+1)d$$

$$b - a = (n+1)d$$

$$\frac{(b-a)}{n+1} = d \quad \text{or} \quad \boxed{d = \frac{b-a}{n+1}}$$

$$A_1 = a + d = a + \frac{b-a}{n+1} = \frac{an+a+b-a}{n+1} = \frac{na+b}{n+1}$$

$$A_2 = a + 2d = a + 2\left(\frac{b-a}{n+1}\right) = \frac{na+a+2b-2a}{n+1} = \frac{na+a+2b}{n+1} = \frac{(n-1)a+2b}{n+1}$$

$$A_3 = a + 3d = a + 3\left(\frac{b-a}{n+1}\right) = \frac{na+a+3b-3a}{n+1} = \frac{na-2a+3b}{n+1} = \frac{(n-2)a+3b}{n+1}, \dots,$$

$$A_n = a + nd = a + n\left(\frac{b-a}{n+1}\right) = \frac{na+a+nb-na}{n+1} = \frac{a+nb}{n+1}$$

EXAMPLE-1

If 8 and 12 are two A.Ms between a and b . Find a and b .

SOLUTION: $a, 8, 12, b$ is an A.P

$$\text{Common difference} = d$$

$$= a_3 - a_2$$

$$= 12 - 8 = 4$$

$$\text{and } b = a_4$$

$$= a_3 + d$$

$$b = 12 + 4 = 16$$

$$a = a_2 - d$$

$$= 8 - 4 = 4$$

$$\text{Thus } a = 4, b = 16$$

EXAMPLE-2

Find three A.Ms between $\sqrt{3}$ and $9\sqrt{3}$.

SOLUTION: Let A_1, A_2, A_3 be three A.Ms between $\sqrt{3}$ and $9\sqrt{3}$

such that $\sqrt{3}, A_1, A_2, A_3, 9\sqrt{3}$ is an A.P.

$$\text{Here } a_1 = a = \sqrt{3}, n = 5, a_5 = 9\sqrt{3}$$

$$\text{Using } a_n = a + (n-1)d$$

$$a_5 = a + (5-1)d$$

$$9\sqrt{3} = a + 4d$$

$$9\sqrt{3} = \sqrt{3} + 4d$$

$$9\sqrt{3} - \sqrt{3} = 4d$$

$$4d = 8\sqrt{3}$$

$$d = 2\sqrt{3}$$

$$\text{Thus } A_1 = a + d = \sqrt{3} + 2\sqrt{3} = 3\sqrt{3}$$

$$A_2 = A_1 + d = 3\sqrt{3} + 2\sqrt{3} = 5\sqrt{3}$$

$$A_3 = A_2 + d = 5\sqrt{3} + 2\sqrt{3} = 7\sqrt{3}$$

Thus $3\sqrt{3}, 5\sqrt{3}, 7\sqrt{3}$ are the required three A.Ms between $\sqrt{3}$ and $9\sqrt{3}$.

EXERCISE - 7.3

- 1- Find *A.M* between:
 - (i) $-3, 7$
 - (ii) $x-1, x+7$
 - (iii) $\sqrt{7}, 3\sqrt{7}$
 - (iv) $x^2+x+1; x^2-x+1$
- 2- If 3 and 6 are two *A.Ms* between a and b , find a and b .
- 3- Find three *A.Ms* between 11 and 19.
- 4- Find three *A.Ms* between $\sqrt{2}$ and $6\sqrt{2}$
- 5- Find six *A.Ms* between 5 and 8.
- 6- Find seven *A.Ms* between 8 and 12.
- 7- If the *A.M* between 5 and b is 10, then find the value of b .
- 8- If the *A.M* between a and 10 is 40, then find the value of " a ".
- 9- If the three *A.Ms* between a and b are 5, 9 and 13, find a and b .

7.4 GEOMETRIC SEQUENCE (Progression)

A geometric progression (abbreviated *G.P*) is a sequence of numbers called terms, each of which after the first is obtained by multiplying the preceding one by a fixed number called common ratio. This common ratio is denoted by ' r ' which cannot be zero in any case. We can obtain common ratio as:

$$r = \frac{a_2}{a_1} = \frac{a_3}{a_2} = \frac{a_4}{a_3} = \dots = \frac{a_n}{a_{n-1}} = \dots$$

Let a be the first term and r be the common ratio in a *G.P*, then the second term is ar . Third term is ar^2 . In each term the exponent of r is one less than the number of the term. Similarly the eighth term is ar^7 and n th term is ar^{n-1} . Thus the general term of *G.P* is $a_n = ar^{n-1}$

EXAMPLE-1

Find the 5th term of a *G.P*, in which $a = 2, r = 3$.

SOLUTION: Given $a = 2, r = 3, n = 5, a_5 = ?$

Since $a_n = ar^{n-1}$

$$a_5 = ar^{5-1}$$

$$\begin{aligned} a_5 &= 2(3)^4 \\ &= 2 \times 81 \\ &= 162 \end{aligned}$$

$$\boxed{a_5 = 162}$$

EXAMPLE-2

If $a_4 = \frac{8}{27}$, $a_7 = \frac{-64}{729}$ in G.P. Find a_{10} .

SOLUTION: Given $a_4 = \frac{8}{27}$, $a_7 = \frac{-64}{729}$,

Here we will find a and r first.

Since $a_n = ar^{n-1}$

$$a_4 = ar^{4-1}$$

$$\frac{8}{27} = ar^3$$

$$ar^3 = \frac{8}{27} \dots\dots(i)$$

and $a_7 = ar^{7-1}$

$$\frac{-64}{729} = ar^6$$

$$ar^6 = \frac{-64}{729} \dots\dots(ii)$$

Now dividing (ii) by (i), we have, $\frac{ar^6}{ar^3} = \frac{-64}{\frac{8}{27}}$

$$r^3 = \frac{-64 \times 27}{729 \times 8} = -\frac{8}{27} = \left(-\frac{2}{3}\right)^3$$

$$r = -\frac{2}{3}, \text{ putting in (i)}$$

$$a \left(-\frac{2}{3}\right)^3 = \frac{8}{27} \Rightarrow a \left(-\frac{2}{3}\right)^3 = \left(\frac{2}{3}\right)^3$$

$$a = -1$$

Then $a_{10} = ar^{10-1}$

$$= (-1) \left(\frac{2}{3}\right)^9 = -\left(\frac{2}{3}\right)^9$$

EXERCISE - 7.4

- 1- Find the 7th term of a G.P 2,8,32, ...
- 2- Find the 11th term of a G.P 2,6,18,...
- 3- Find the 6th term of a G.P $-\frac{3}{2}, 3, -6, \dots$
- 4- Find the 5th term of a G.P 4,-12,36,...
- 5- Find the missing elements of the G.P:
 (i) $r = 10, a_n = 100, a = 1$ (ii) $a_n = 400, r = 2, a = 25$
 (iii) $a = 128, r = \frac{1}{2}, a_n = \frac{1}{4}$
- 6- Find the 11th term of a G.P whose 5th term is 9 and common ratio is 2.
- 7- Find the 13th term of a G.P whose 7th term is 25 and common ratio is 3.
- 8- If a, b, c, d are in G.P, show that, $a - b, b - c, c - d$, are in G.P.
- 9- Find the n^{th} term of a G.P, if $\frac{a_5}{a_3} = \frac{4}{9}$ and $a_2 = \frac{4}{9}$.
- 10- Find three consecutive numbers in G.P, whose sum is 26 and their product is 216.
- 11- Find the 30th term of a G.P $x, 1, \frac{1}{x}, \dots$
- 12- Find the p^{th} term of a G.P x, x^3, x^5, \dots

7.5 GEOMETRIC MEAN (G.M)

A number 'G' is said to be a geometric mean between the two numbers a and b , if a, G, b is a geometric progression.

$$\frac{G}{a} = \frac{b}{G} \text{ (common ratio)}$$

$$G^2 = ab$$

$$G = \pm \sqrt{ab}$$

Positive G.M between a and b is \sqrt{ab} .

EXAMPLE-1

Find the G.M between 3 and 27.

SOLUTION: Given $a = 3$, $b = 27$. then

$$\begin{aligned} G &= \pm\sqrt{ab} \\ &= \pm\sqrt{3 \times 27} \\ &= \pm\sqrt{81} \\ &= \pm 9 \end{aligned}$$

EXAMPLE-2

Find the G.M between $2x^2$ and $8y^4$.

SOLUTION: Given $a = 2x^2$, $b = 8y^4$

$$\begin{aligned} G &= \pm\sqrt{ab} \\ &= \pm\sqrt{2x^2 \times 8y^4} \\ &= \pm\sqrt{16x^2y^4} \\ &= \pm\sqrt{(4xy^2)^2} \\ &= \pm 4xy^2 \end{aligned}$$

7.5.1 'n' Geometric Means Between Two Numbers

Let $G_1, G_2, G_3, \dots, G_n$ be the n G.Ms between the two numbers a and b such that $a, G_1, G_2, G_3, \dots, G_n, b$ is a G.P, there are $n + 2$ terms in this G.P. in which $a_1 = a$, $a_{n+2} = b$,

Using $a_n = ar^{n-1}$

$$a_{n+2} = ar^{n+2-1}$$

$$b = ar^{n+1}$$

$$r^{n+1} = \frac{b}{a}$$

$$r = \left(\frac{b}{a}\right)^{\frac{1}{n+1}}$$

$$G_1 = a \times r = a \times \left(\frac{b}{a}\right)^{\frac{1}{n+1}}$$

$$G_2 = G_1 \times r = ar^2 = a \left(\frac{b}{a}\right)^{\frac{2}{n+1}}$$

$$G_3 = G_2 \times r = ar^3 = a \left(\frac{b}{a}\right)^{\frac{3}{n+1}}$$

$$\vdots \quad \vdots \quad \vdots \quad \vdots$$

$$G_n = G_{n-1} \times r = ar^n = a \times \left(\frac{b}{a}\right)^{\frac{n}{n+1}}$$

EXAMPLE-1

Insert two G.Ms between 4 and $\frac{1}{2}$.

SOLUTION: Let G_1, G_2 be the two G.Ms between 4 and $\frac{1}{2}$ such that:

$4, G_1, G_2, \frac{1}{2}$ is a G.P,

Here $a = 4, n = 4, a_4 = \frac{1}{2}$,

Since $a_n = ar^{n-1}$

$$a_4 = ar^{4-1}$$

$$\frac{1}{2} = ar^3$$

$$ar^3 = \frac{1}{2}$$

$$4r^3 = \frac{1}{2}$$

$$r^3 = \frac{1}{8} = \frac{1}{2^3}$$

$$r^3 = \left(\frac{1}{2}\right)^3$$

$$r = \frac{1}{2}$$

Thus $G_1 = a \times r = 4 \times \frac{1}{2} = 2$

$$G_2 = G_1 \times r = 2 \times \frac{1}{2} = 1$$

EXERCISE - 7.5

- 1- Find *G.M* between: (i) 9 and 5 (ii) 4 and 9 (iii) -2 and -8.
- 2- Insert two *G.Ms* between: (i) 1 and 8 (ii) 3 and 81
- 3- Insert three *G.Ms* between: (i) 1 and 16 (ii) 2 and 32
- 4- Insert four real geometric means between 3 and 96.
- 5- The *A.M* between two numbers is 5 and their positive *G.M* is 4.
Find the numbers.
- 6- The positive *G.M* between two numbers is 6 and the *A.M* between them is 10. Find the numbers.
- 7- Show that the *A.M* between the two numbers 4 and 8 is greater than their geometric mean.
- 8- Insert four geometric means between 160 and 5.
- 9- Insert three geometric means between 486 and 6.
- 10- Insert four geometric means between $\frac{1}{8}$ and 128.
- 11- Insert six geometric means between 56 and $-\frac{7}{16}$.
- 12- Insert five geometric means between $\frac{32}{81}$ and $\frac{9}{2}$.

Review Exercise - 7

1- Encircle the correct answer.

(i) Third term of $a_n = n + 3$, when $n = 0$ is

- (a) 3 (b) 6 (c) 9 (d) 0

(ii) Fourth term of $a_n = \frac{1}{(2n-1)^2}$, when $n = 0$ is

- (a)
- $\frac{1}{7}$
- (b)
- $\frac{1}{49}$
- (c)
- $\frac{1}{81}$
- (d) 0

(iii) For 2, 6, 11, 17, ..., a_5 is

- (a) 24 (b) 30 (c) 21 (d) 22

(iv) Next term of 12, 16, 21, 27 is

- (a) 34 (b) 30 (c) 31 (d) 32

(v) a_6 of 3, 7, 11, ... is

- (a) 3 (b) 19 (c) 23 (d) 20

(vi) A.M between $\sqrt{3}$ and $3\sqrt{3}$ is

- (a)
- $2\sqrt{3}$
- (b)
- $5\sqrt{3}$
- (c)
- $9\sqrt{3}$
- (d)
- $4\sqrt{3}$

(vii) A.M between $2\sqrt{5}$ and $6\sqrt{5}$ is

- (a)
- $4\sqrt{5}$
- (b)
- $3\sqrt{5}$
- (c)
- $5\sqrt{5}$
- (d)
- $7\sqrt{5}$

(viii) a_5 of 2, 6, 18, ... is

- (a) 160 (b) 161 (c) 162 (d) 30

(ix) G.M between -3 and -12 is

- (a)
- ± 6
- (b)
- ± 9
- (c)
- ± 36
- (d)
- ± 3

(x) G.M between 1 and 8 is

- (a)
- $2\sqrt{2}$
- (b)
- $\pm 2\sqrt{2}$
- (c)
- $-2\sqrt{2}$
- (d)
- $\sqrt{2}$

2- Fill in the blanks.

- (i) The general or n th term of a sequence is denoted by _____
- (ii) If $a_n = 2n + 3$, then $a =$ _____
- (iii) In an A.P. $a_n = a + (n-1)d$, is called _____
- (iv) A.M between 5 and 15 is _____
- (v) If a, A, b is an A.P then $A =$ _____
- (vi) In a G.P. " r " is called _____
- (vii) In a G.P. $a_n =$ _____
- (viii) If a, G, b is a G.P, then $G =$ _____
- (ix) Positive geometric mean between 2 and 8 is _____
- (x) The n th term of an A.P when $a_{n-5} = 3n + 9$ _____

3- Find the general term and the 18th term of an A.P, whose first term is 3 and the common difference is 2.

4- Find the n th term of an A.P $\left(\frac{3}{5}\right)^3, \left(\frac{3}{7}\right)^3, \left(\frac{3}{9}\right)^3, \dots$

5- If the A.M between a and 16 is 24. Then find the value of ' a '.

6- Find the 15th term of a G.P. whose 7th term is 27 and common ratio is 3.

7- Insert four Geometric Means between $\frac{1}{2}$ and 16.

8- Find the three consecutive number in G.P, whose sum is 26 and their product is 216.

SUMMARY

- ✦ In a number pattern each successive number can be found by applying a specific rule that justifies the position of succeeding one, that is all the members in a pattern are in a definite order, such a number pattern is called a sequence.

- ✦ A sequence in which each term is obtained from the previous term by adding a fixed number is called an arithmetic sequence.

- ✦ A number " A " is said to be an arithmetic mean between the two numbers a and b if a, A, b is arithmetic sequence.

- ✦ A sequence in which each term is obtained from the previous term by multiplying it with a common ratio is called a geometric sequence.

- ✦ A number " G " is said to be a geometric mean between the two numbers a and b if a, G, b is a geometric sequence.