UNIT

10

BASIC STATISTICS

- **Frequency Distribution**
- **Cumulative Frequency**
- **Measure of Central Tendency**
- **Measure of Dispersions**

After completion of this unit, the students will be able to:

- Construct grouped frequency table.
- Construct histograms with equal and unequal class intervals.
- Construct a frequency polygon.
- Construct a cumulative frequency table.
- Draw a cumulative frequency polygon.
- Calculate (for ungrouped and grouped data):
  - Arithmetic mean by definition and using deviations from assumed mean,
  - Median, mode, geometric mean, harmonic mean.
- Recognize properties of arithmetic mean.
- Calculate weighted mean and moving averages.
- Estimate median, quartiles and mode graphically.
- Measure range, variance and standard deviation.

10.1 FREQUENCY DISTRIBUTION

Frequency

The number of times each value appears is called the frequency of that value. For example if *3* students get marks from *10%* to *20%*, the frequency is *3* if *5* students get marks from *20%* to *30%*, frequency is *5*.

Frequency Table

The table which gives the frequency of each score is called a frequency table.

10.1.1 Grouped Frequency Table

The local health authority wanted to collect some information about the heights of five years old children in their area. At their first visit to school for medical examination, the height of each child of five years was recorded, e.g., the following figures are the recorded heights *(in cm to the nearest cm)* of ninety children.

99	107	102	98	115	95	106	110	108	105
118	102	114	108	94	104	113	102	105	95
105	110	109	101	106	108	107	107	101	109
108	105	116	109	114	110	97	110	113	116
112	101	92	105	104	115	111	103	110	99
93	104	103	113	107	94	102	117	116	104
99	114	106	114	98	109	107	104	106	107
109	113	112	100	109	113	118	104	94	114
107	96	108	103	112	106	115	111	115	101

In the table the heights are listed in a random order *(the same order the children came for the medical examination)*. We are to arrange and organize this data in some manner for clear understanding to get any result. We group the heights as:

$90 - 94cm$; $95 - 99cm$; $100 - 104cm$;

$105 - 109cm$; $110 - 114cm$; $115 - 119cm$

The heights are arranged in order of magnitude. Counting the number of heights is each group gives the following frequency table.

Heights *(cm)*	Tally	Frequency
90 – 94	𝍸	*5*
95 – 99	𝍸 IIII	*9*
100 – 104	𝍸 𝍸 𝍸 II	*17*
105 – 109	𝍸 𝍸 𝍸 𝍸 𝍸 III	*28*
110 – 114	𝍸 𝍸 𝍸 𝍸 I	*21*
115 – 119	𝍸 𝍸	*10*
		Total: 90

10.1.2 Histogram

When a bar chart is constructed so that the area of each bar is proportional to the number of items in each group, it is called a histogram.

Histogram with equal class intervals

Consider the following examples:

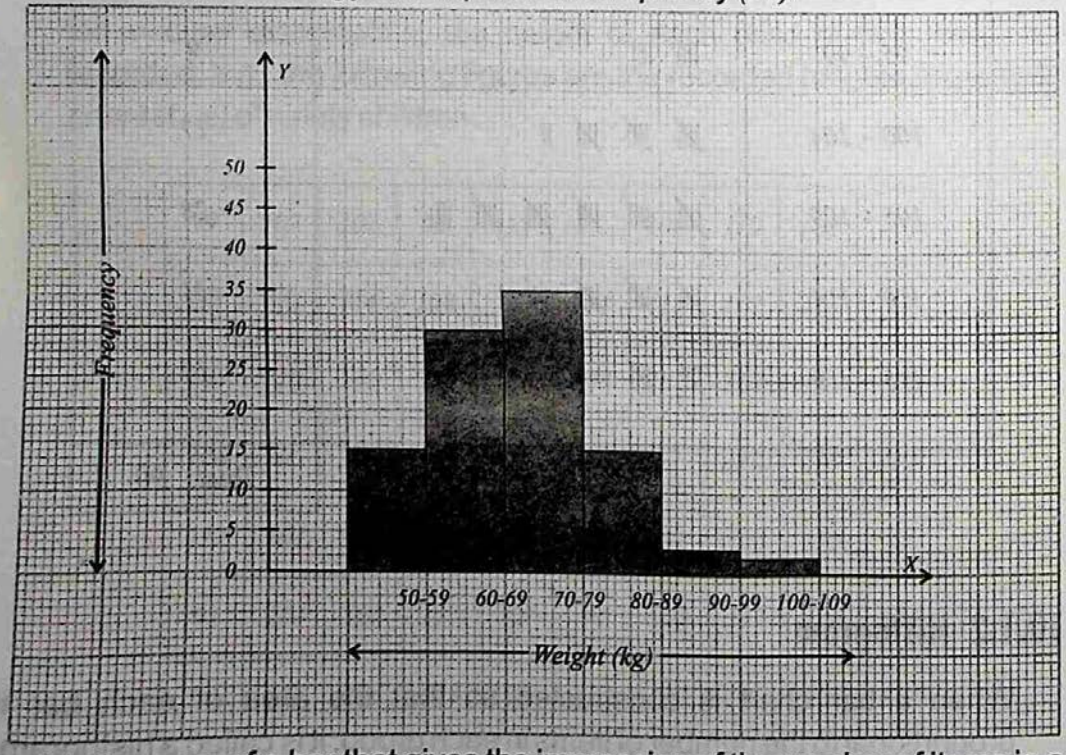
EXAMPLE

Construct a histogram with equal intervals with the help of following frequency.

Weights (kg)	50 – 59	60 – 69	70 – 79	80 – 89	90 – 99	100 – 109
Frequency	15	30	35	15	3	2
					Total:	100

SOLUTION:

The following bar chart illustrates the frequency table.
Weight (kg) has been taken along x-axis towards right of 'O'.
Frequency has been taken above x-axis.
One big square along x-axis represents 10kg weight.
One big square along y-axis represents frequency (10).



It is the area of a bar that gives the impression of the number of items in a group.

The diagram is a histogram with equal intervals. Each group covers the same span, so each bar has the same width. Hence the area of each bar is proportional to the number of items in the group.

Histogram with unequal class intervals

EXAMPLE

Consider the following frequency table which shows the result of a survey on the per month pay of 100 men.

Pay (rupees)	0 – 1999	2000 – 3999	4000 – 5999	6000 – 9999	10000 – 19999
Frequency	20	36	25	14	5

SOLUTION:

The intervals of the group are not equal.
The first three groups each cover a span of Rs.2000.
The fourth group intervals Rs.40000 twice the interval of the first three.
The fifth group intervals Rs.10,000 five times the interval of the first three.

To illustrate this distribution on histogram we must make:

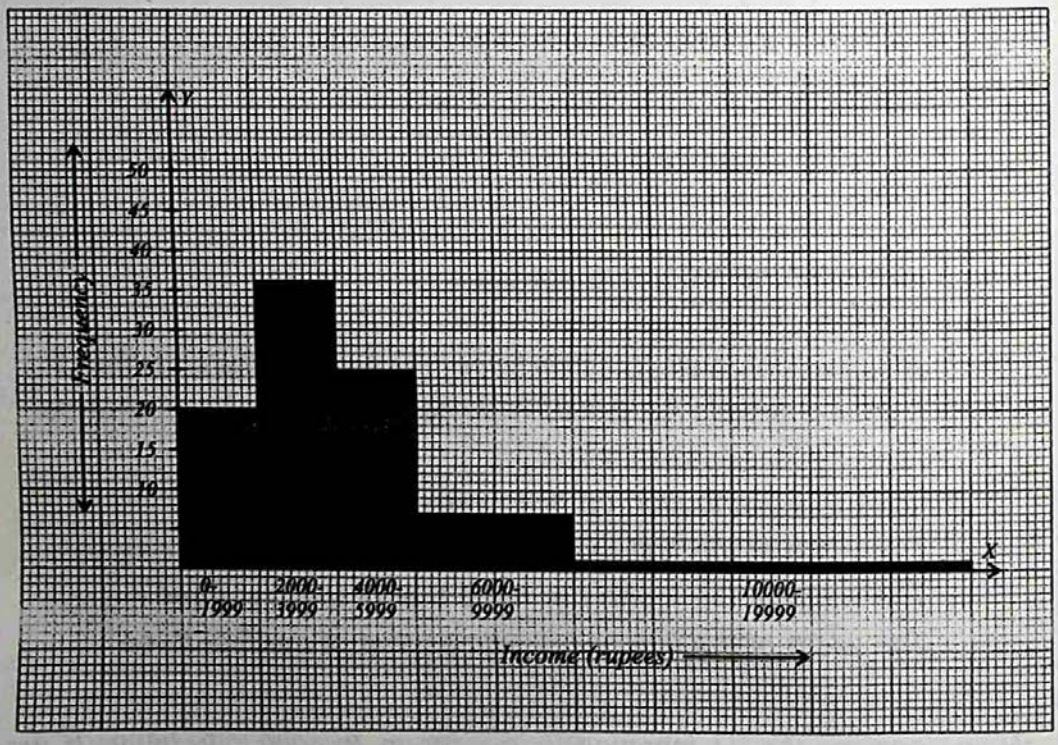
The width of the first three bars equal.
The width of the fourth bar twice that of the first three.
The width of the fifth bar five times that of the first three.

Now consider the 14 incomes in the group Rs.6000 - Rs.9999. If we suppose that these incomes are evenly spread throughout this group, then these are:

Seven men with incomes in the group Rs.6000 - Rs.7999
and seven men with incomes in the group Rs.8000 - Rs.9999.

The fourth bar is therefore 7 units high for the whole group, because the width of this bar is twice that of the others used. Its area is proportional to the frequency of the group.

If we also assume that the income in the fifth group are evenly spread throughout the group, then there is one man with an income in each of the subgroups with interval Rs.2000. The fifth bar is therefore one unit high for the whole group. The area of the fifth bar is proportional to the frequency of the group.



Note: We do not use frequency to label vertical axis, because it is the area of the bar which represents the frequency and not the height.

10.1.3 Frequency Polygon

A frequency polygon is a many sided closed figure, It is constructed by plotting frequencies against their class marks (midpoints) and then joining the resulting points by means of straight lines. A frequency polygon can also be obtained by joining the mid points of the tops of the rectangles in the histogram.

EXAMPLE

The data in the frequency table shows the mass (in kg) of 40 people upon joining a weight loss program.

Represent the given data using a:

(i) Histogram (ii) Frequency Polygon

Class intervals	Frequency
60 —— 70	2
70 —— 80	5
80 —— 90	9
90 —— 100	12
100 —— 110	7
110 —— 120	3
120 —— 130	2
	Total: 40

SOLUTION:

(i) Draw the axes on graph paper.

(ii) Title the graph.

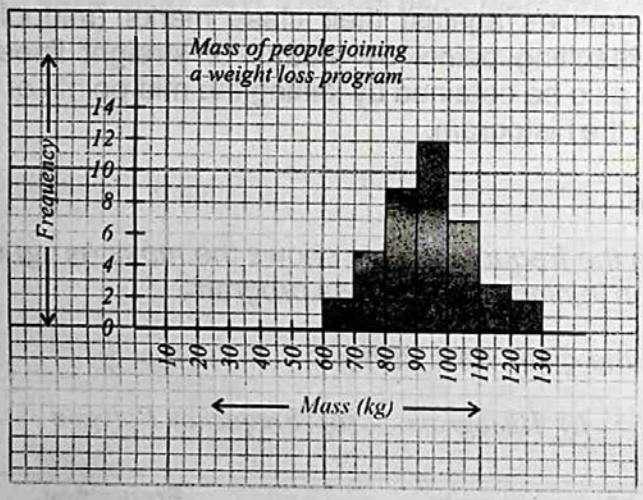
(iii) Label the horizontal axis mass (kg) and the vertical axis frequency.

(iv) Scale the horizontal and vertical axes.

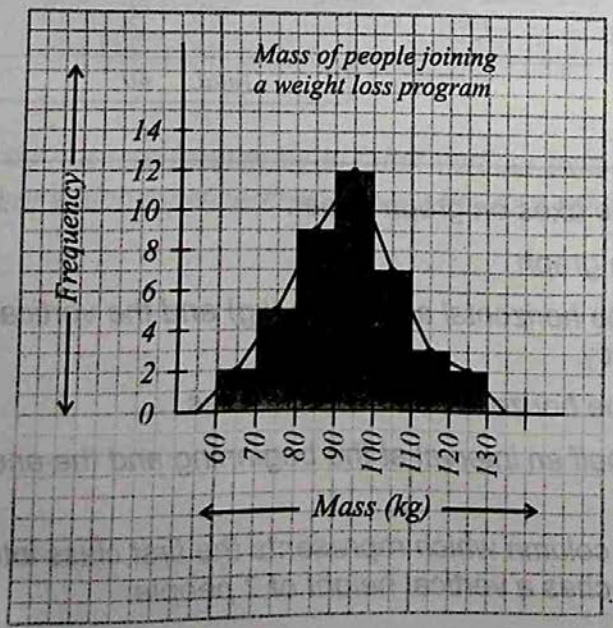
(v) Leave half an interval at the beginning and the end of the graph.

(vi) Draw a column which represents the first class interval and reaches a vertical height of 2 people.

Repeat step *3* for each of the other class intervals.



(ii) Mark the mid points of the tops of the rectangles obtained in the histogram from part A. Join the mid points by straight line intervals. Close the polygon by drawing lines which meet the horizontal axis a half column width before the first column and a half column width after the last column.



EXERCISE – 10.1

1- Fifty Junior school children joined the school's computer club. Their ages were recorded.

10	8	9	10	7	8	8	11	10	9
7	8	9	9	10	11	11	10	9	8
8	7	9	7	10	7	10	8	9	11
10	11	8	10	9	8	9	7	11	10
9	10	10	11	10	11	7	11	10	9

Make a frequency table showing the number of each age and illustrate this information with a bar chart.

2- The local fish and chip shop had *56* customers on Saturday evening. They spent the following amount in rupees.

270	110	45	96	250	490	325	45
382	136	125	450	420	380	150	250
85	250	320	525	218	210	216	120
155	430	250	40	510	150	510	245
320	120	316	150	260	45	180	310
273	280	85	280	318	45	210	282
462	316	218	316	325	45	560	315

use groups *Rs.0-99, Rs.100-199, Rs.200-299, Rs.300-399, Rs.400-499, Rs.500-599* to make a frequency tables illustrate the data with a bar chart.

3- The weights to the nearest gram of *30* bags of popcorn sold at a festival are given as:

69	83	75	65	68	68	73	70	80	79
70	76	63	86	69	65	66	74	86	68
70	60	67	74	65	65	67	88	81	63

Make a frequency table, Illustrate the data with a bar chart.

10.2 CUMULATIVE FREQUENCY

The cumulative frequency shows the total number of observations (scores), which are below a certain value. We explain this with the help of following example:

EXAMPLE-1

All the students of class 9 took a maths test. Here are their marks as percentage. They have been grouped in 10. Find cumulative frequency.

Marks in %	Frequency
1 – 10	3
11 – 20	6
21 – 30	11
31 – 40	13
41 – 50	18
51 – 60	24
61 – 70	14
71 – 80	6
81 – 90	3
91 – 100	2
	Total Frequency: 100

(i) How many students scored *10 %* or less ?

(ii) How many students scored *20 %* or less ?

(iii) How many students scored *30 %* or less ?

(iv) How many students scored *40 %* or less ?

(v) How many students scored *50 %* or less?

(vi) How many students scored *60 %* or less?

(vii) How many students scored *70 %* or less?

(viii) How many students scored *80 %* or less?

(ix) How many students scored *90 %* or less?

(x) How many students scored *100 %* or less?

SOLUTION:

The following table shows cumulative frequencies.

Marks %	Cumulative Frequency	
10 % or less	*3*	*3*
20 % or less	*9*	*3 + 6 = 9*
30 % or less	*20*	*9 + 11 = 20*
40 % or less	*33*	*20 + 13 = 33*
50 % or less	*51*	*33 + 18 = 51*
60 % or less	*75*	*51 + 24 = 75*
70 % or less	*89*	*75 + 14 = 89*
80 % or less	*95*	*89 + 6 = 95*
90 % or less	*98*	*95 + 3 = 98*
100 % or less	*100*	*98 + 2 = 100*

EXAMPLE-2

The marks in a science test are given in the table. Complete the cumulative frequency.

Marks	Frequency	Marks	Cumulative frequency
1 – 10	*5*	*10* or less	
11 – 20	*6*	*20* or less	
21 – 30	*8*	*30* or less	
31 – 40	*16*	*40* or less	
41 – 50	*23*	*50* or less	
51 – 60	*18*	*60* or less	
61 – 70	*12*	*70* or less	
71 – 80	*10*	*80* or less	
81 – 90	*2*	*90* or less	
91 – 100	*0*	*100* or less	

SOLUTION:

Marks	Frequency	Marks	Cumulative Frequency
1 – 10	5	10 or less	5
11 – 20	6	20 or less	11
21 – 30	8	30 or less	19
31 – 40	16	40 or less	35
41 – 50	23	50 or less	58
51 – 60	18	60 or less	76
61 – 70	12	70 or less	88
71 – 80	10	80 or less	98
81 – 90	2	90 or less	100
91 – 100	0	100 or less	100

10.2.2 Cumulative Frequency Polygon

When the cumulative frequencies are plotted against the end points of their respective class intervals and joined together, the resultant graph is called a cumulative frequency polygon, or an **ogive**.

Therefore, an ogive can be considered as a line graph of the cumulative frequency results.

EXAMPLE

The data in the frequency table shows the number of fish caught by 28 competitors in fishing competition.

Construct a cumulative frequency polygon (that is, an ogive) for the given data.

Number of Fish Caught	Frequency
0 —— 5	3
5 —— 10	5
10 —— 15	6
15 —— 20	8
20 —— 25	4
25 —— 30	2
	Total: 28

SOLUTION:

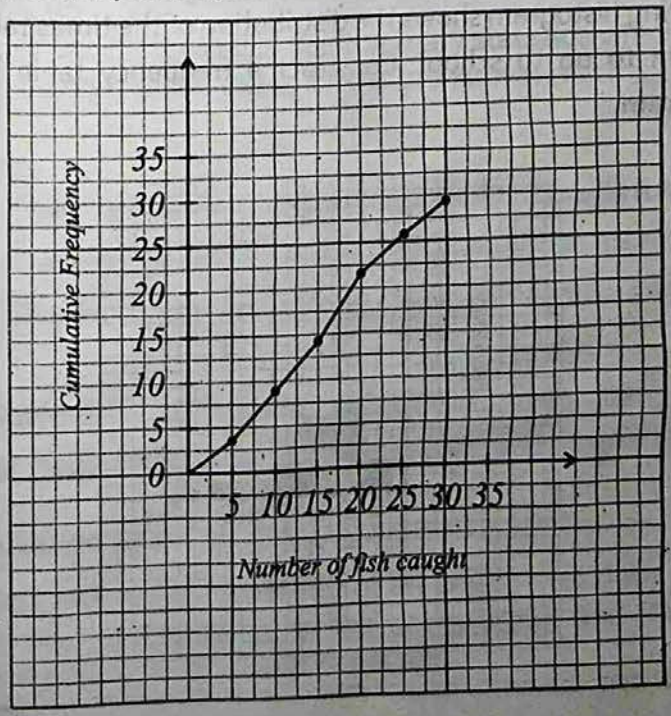
Find cumulative frequencies, draw the axes on graph paper, title the graph, label the horizontal axis "Number of fish caught" and the vertical axis "Cumulative frequency".

Number of fish caught	Frequency *(f)*	Cumulative Frequency *(cf)*
0 —— 5	3	3
5 —— 10	5	3 + 5 = 8
10 —— 15	6	8 + 6 = 14
15 —— 20	8	14 + 8 = 22
20 —— 25	4	22 + 4 = 26
25 —— 30	2	26 + 2 = 28
Total:	$\Sigma f = 28$	

Plot each of the cumulative frequencies against the end point of its respective class intervals.

The first point of an ogive curve has a cumulative frequency value of zero corresponding to the lowest possible value of the initial class interval. Therefore the first point of the ogive will be (0,0).

The next point will be (5,3) followed by (10,8) and so on. Connect each of the plotted points with straight line segments.



EXERCISE – 10.2

1- Draw a histogram to represent the frequency table in each of the following tables.

(i) The table shows the distribution of ages of *100* people attending a school function.

Age (years)	*0-19*	*20-39*	*40-59*	*60-79*	*80-89*
Frequency	*43*	*24*	*17*	*10*	*6*

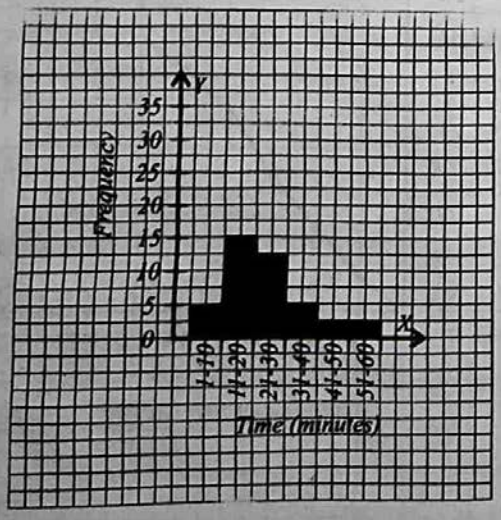
(ii) The table shows the results of a survey on the weekly earnings of *100* sixteen-year old boys.

Weekly earnings	*0-9*	*10-19*	*20-29*	*30-39*	*40-49*	*50-59*
Frequency	*45*	*10*	*11*	*21*	*10*	*3*

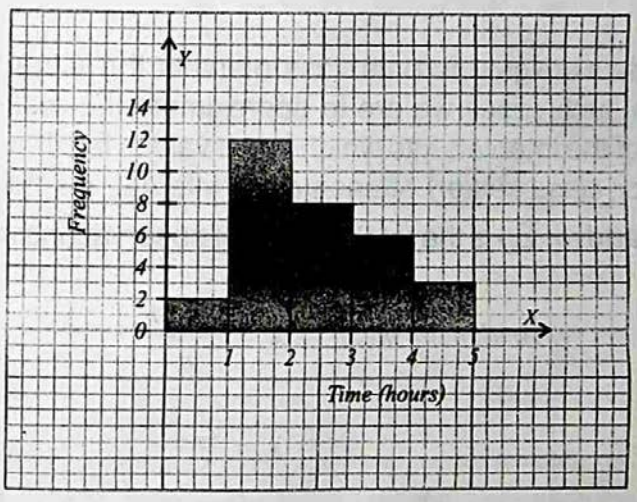
(iii) The table shows the distribution of the average marks of *40* children in the end-of-year examinations.

Average mark	*1-20*	*21-40*	*41-60*	*61-80*	*81-100*
Frequency	*2*	*4*	*19*	*12*	*3*

2- Following histogram shows the distribution of the times taken by *50* children to go to school. Construct a frequency table from the histogram.



3- Following histogram is based on the number of hours that *30* children spent watching television on a particular Saturday. Construct a frequency table from the histogram.



10.3 MEASURES OF CENTRAL TENDENCY

Measures of central tendency are summary statistics which measure the middle (or center) of the data. These are known as mean, median and mode.

(i) The mean is the average of all observations in a set of data.

(ii) The median is the middle observation in an ordered set of data.

(iii) The mode is the most frequent observation in a data set.

10.3.1 Calculate Arithmetic Mean or Mean, Median, Mode, Geometric Mean, Harmonic Mean from Ungrouped/Grouped Data

Arithmetic Mean by definition

By definition the Arithmetic mean of an ungrouped data is obtained by adding all numbers (scores) in the set together and then the total is divided by the number of scores in that set.

$$Arithmetic\ Mean = \frac{Sum\ of\ all\ scores}{Number\ of\ scores}$$

Symbolically this is written as: $\bar{x} = \frac{\sum x}{n}$, Where the symbol "$\Sigma$" indicates the "sum".

When the data are grouped into class intervals, the actual values *(or data)* are lost. In such cases we are to approximate the real values with the midpoints of the intervals into which these values fall. If *'x'* represents the midpoint *(or class center)* of each class interval, f is the corresponding frequency and $(n=\Sigma f)$ is the total number of observations in a set. Then

$$Arithmetic\ mean = \bar{x} = \frac{\Sigma(fx)}{n}$$

$$\bar{x} = \frac{\Sigma fx}{\Sigma f}$$

EXAMPLE-1

The total monthly income of 8 persons are Rs.3000, Rs.4000, Rs.3500, Rs.4500, Rs.3800, Rs.4200, Rs.3600 and Rs.5400. Find their arithmetic mean.

SOLUTION: $x_1 = 3000,\ x_2 = 4000,\ x_3 = 3500,\ x_4 = 4500,$
$x_5 = 3800,\ x_6 = 4200,\ x_7 = 3600,\ x_8 = 5400,$

$$Arithmetic\ mean = \bar{x} = \frac{\Sigma x}{n}$$

$$= \frac{x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8}{8}$$

$$= \frac{3000 + 4000 + 3500 + 4500 + 3800 + 4200 + 3600 + 5400}{8}$$

$$= \frac{32000}{8}$$

$$\bar{x} = Rs.4000$$

EXAMPLE-2

Find the arithmetic mean for the following distribution showing marks obtained by 50 students in mathematics in the annual exam.

Marks	20 – 24	25 – 29	30 – 34	35 – 39	40 – 44	45 – 49	50 – 54
Frequency	1	4	8	11	15	9	2

SOLUTION:

Marks	Frequency (f)	Class Mark (x)	fx
20 – 24	1	22	22
25 – 29	4	27	108
30 – 34	8	32	256
35 – 39	11	37	407
40 – 44	15	42	630
45 – 49	9	47	423
50 – 54	2	52	104

$$\bar{x} = \frac{\Sigma(fx)}{n} = \frac{22+108+256+407+630+423+104}{50}$$

$$= \frac{1950}{50}$$

$$= 39 \text{ marks}$$

Arithmetic Mean using Deviation from Assumed Means

The computation of mean using the formula $\bar{x} = \frac{\Sigma fx}{\Sigma f}$ is easy, provided the values of x_i and f_i are not large. If x_i and f_i are large we can save a considerable time taking deviations from assumed means. If 'A' is an assumed mean (which may be any number) and D_i denotes the deviations of X_i from 'A' i.e.

$$D_i = X_i - A, \text{ then } X_i = D_i + A, \text{ we have}$$

$$\overline{X} = A + \frac{\sum_{i=1}^{k} D_i}{n}$$

$$= A + \frac{\sum_{i=1}^{k} D}{n} \qquad \text{(ungrouped data)}$$

and $$\overline{X} = A + \frac{\sum_{i=1}^{k} f_i D_i}{\Sigma f_i}$$

$$= A + \frac{\Sigma f D}{\Sigma f} \qquad \text{(grouped data)}$$

EXAMPLE-1

Find the arithmetic mean from the following values using formula:

$$\overline{X} = A + \frac{\Sigma D}{n}$$

184, 191, 172, 168, 187, 189, 196, 186, 193, 195.

SOLUTION:

Taking deviation from the assumed mean $A = 180$, $D = X - A$, we get

$D = 184 - 180,\ 191 - 180,\ 172 - 180,\ 168 - 180,\ 187 - 180,\ 189 - 180,$
$196 - 180,\ 189 - 180,\ 193 - 180,\ 195 - 180$

$= 4,\ 11,\ -8,\ -12,\ 7,\ 9,\ 16,\ 9,\ 13,\ 15$

$$\overline{X} = A + \frac{\Sigma D}{n} = 180 + \frac{4+11-8-12+7+9+16+9+13+15}{10}$$

$$= 180 + \frac{64}{10}$$

$$= 180 + 6.4$$

$$= 186.4$$

EXAMPLE-2

Find the mean weight of 120 students for the distribution of weights in the following table. Using formula:

$$\overline{X} = A + \frac{\Sigma f D}{\Sigma f}$$

Weight (Pounds)	Class Mark (X)	Frequency (f)	$D = X - 144.5$	fD
110 – 119	114.5	1	– 30	– 30
120 – 129	124.5	4	– 20	– 80
130 – 139	134.5	17	– 10	– 170
140 – 149	144.5 ←A	28	0	0
150 – 159	154.5	25	10	250
160 – 169	164.5	18	20	360
170 – 179	174.5	13	30	390
180 – 189	184.5	6	40	240
190 – 199	194.5	5	50	250
200 – 209	204.5	2	60	120
210 – 219	214.5	1	70	70
		$n = \Sigma f = 120$	$\Sigma f D = 1680 - 280 = 1400$	

SOLUTION: $\overline{X} = A + \frac{\Sigma f D}{\Sigma f}$

$= 144.5 + \frac{1400}{120}$

$= 144.5 + 1167$

$= 156.17$ pounds

Median (ungrouped data)

The median is middle value of any set of data arranged in numerical order. In the set of n numbers, the median is located at the $\frac{n+1}{2}$ th score.

The median is:

- the middle score for an odd number of scores arranged in numerical order.
- the average of the two middle scores for an even number of scores arranged in numerical order.

EXAMPLE

For the data set 6, 2, 4, 3, 4, 5, 4, 5. Find the median.

SOLUTION:

2 3 4 4 4 5 5 6	*(Check the scores are arranged in numerical order)*
2 3 4 4 4 5 5 6	*{Locate the position of the median using the rule $\frac{n+1}{2}$ where $n = 8$. This places the median as the 4.5th score = 4th score – 0.5 (5th score – 4th score)}*
$Median = \frac{4+4}{2}$	
$= \frac{8}{2}$	
$= 4$	*(obtain the average of two middle scores)*

The median is 4.

Median (grouped data)

The median is obtained by drawing an ogive of the data and estimating the median from the *50th* percentage.

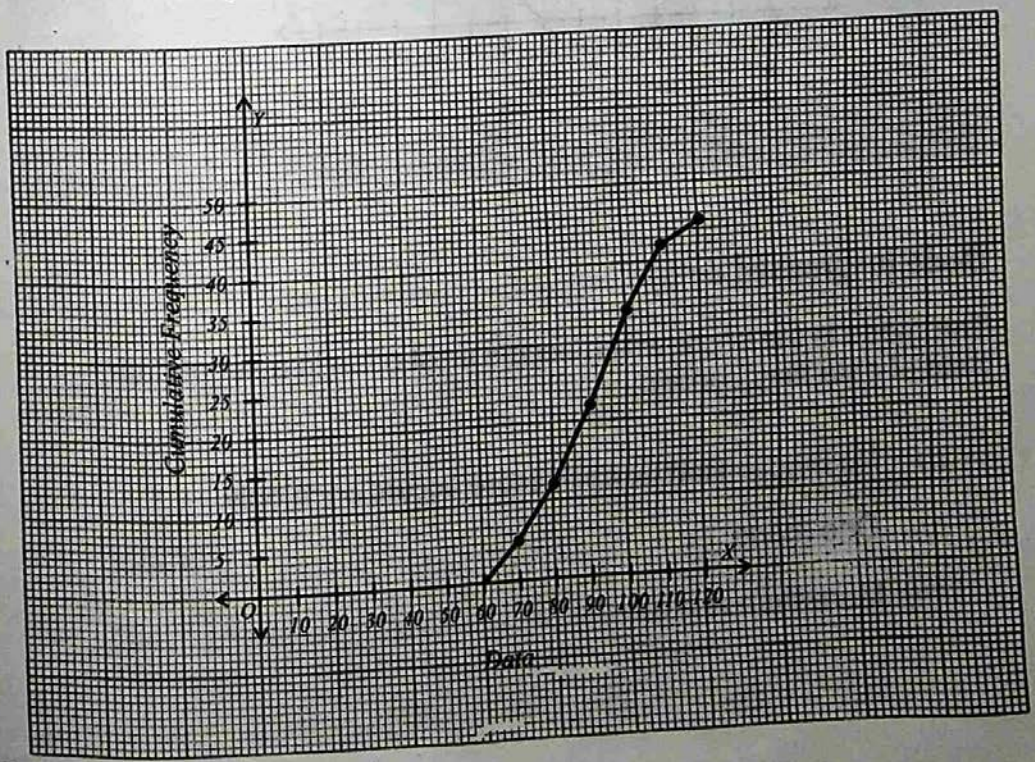
EXAMPLE

For the given data estimate the median.

Class interval	Frequency (f)	Cumulative Frequency (Cf)
60 – 70	5	5
70 – 80	7	12
80 – 90	10	22
90 – 100	12	34
100 – 110	8	42
110 – 120	3	45
Total	45	

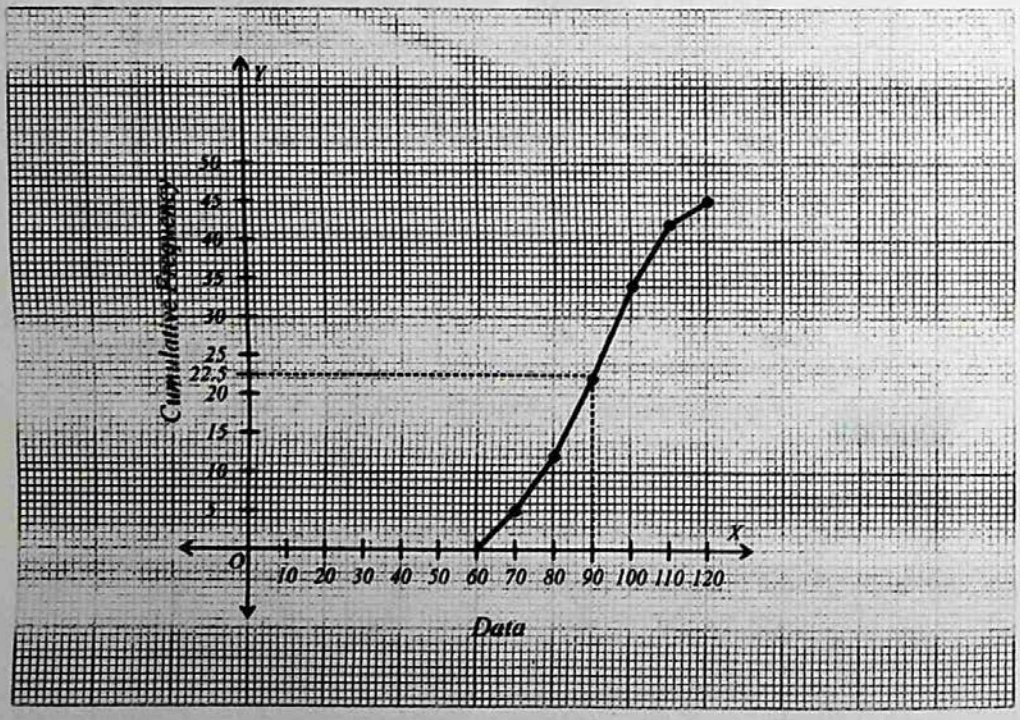
SOLUTION:

Draw the ogive.



Locate the middle of the cumulative frequency axis, that is 22.5 and label it.

Draw a horizontal line from the point to the ogive and then vertically to the horizontal axis. Read the value of the median from the x-axis. The median for the given data is approximately 90.



Mode (ungrouped data)

The mode is the score which occurs most often in a set of data. Sets of data may contain:

(i) No mode: That is, each score occurs once only.

(ii) One mode

(iii) More than one mode

EXAMPLE

For the data set 6, 2, 4, 3, 4, 5, 4, 5, find the mode.

SOLUTION:

We work through the set and see that value in the data which occurs the greatest number of times. The mod is 4.

Mode (grouped data)

We do not find a mode, because exact scores are lost. We can however, find a model class. This is the class interval that has the highest frequency.

EXAMPLE

Find the model class from the following table.

Class interval	Frequency
60 – 70	5
70 – 80	7
80 – 90	10
90 – 100	12
100 – 110	8
110 – 120	3
Total	45

SOLUTION:

The model class is the class interval with the highest frequency.
The model class is the 90 – 100 class interval.

Geometric Mean (ungrouped data)

The geometric mean "G" of a set of n positive values $x_1, x_2, ..., x_n$ is the nth root of the product of the values. Thus

$$G = \sqrt[n]{x_1.x_2.x_3 \cdots, x_n}$$

$$= (x_1.x_2.x_3 \cdots, x_n)^{\frac{1}{n}}$$

EXAMPLE

Geometric mean of 2,4 and 8 is:

SOLUTION: $G = \sqrt[3]{2 \times 4 \times 8} = \sqrt[3]{64}$

$$= [(4)^3]^{\frac{1}{3}}$$

$$= 4$$

Geometric Mean (grouped data)

Let $x_1, x_2, ..., x_k$ represent the class marks in a frequency distribution with $f_1, f_2, ..., f_k$ as the corresponding class frequencies (where $f_1 + f_2 + ... + f_k = \Sigma f = n$). Since x_1 occurs f_1 times and so on. x_k occurs f_k times, then the product of original values may be written as :

$$\underbrace{x_1.x_1....x_1}_{f_1 \text{ times}} \quad \underbrace{x_2.x_2....x_2}_{f_2 \text{ times}} \quad ... \quad \underbrace{x_k.x_k....x_k}_{f_k \text{ times}}$$

or $x_1^{f_1}.\ x_2^{f_2}...\ x_k^{f_k}$ and the geometric mean is:

$$G = \sqrt[n]{x_1^{f_1}.\ x_2^{f_2}...\ x_k^{f_k}}$$

$$= (x_1^{f_1}.\ x_2^{f_2}...\ x_k^{f_k})^{1/n}$$

This is sometimes called the weighted geometric mean with weights $f_1, f_2, \ldots, f_k$.

EXAMPLE

Find the geometric mean for the following frequency distribution.

x	1	2	3	4
f	2	3	4	1

SOLUTION:

Here $\Sigma f = 2+3+4+1 = 10$

$$G = (x_1^{f_1} . x_2^{f_2} \ldots x_k^{f_k})^{1/n}$$

$$= ((1)^2 . (2)^3 . (3)^4 . (4)^1)^{1/10}$$

$$= (1 \times 8 \times 81 \times 4)^{1/10}$$

$$= (2592)^{1/10} = 2.1946$$

Harmonic Mean (ungrouped data)

The harmonic mean H of a set of n values $x_1, x_2, \ldots, x_n$ is the reciprocal of the arithmetic mean of the reciprocal of the values.

$$H = \frac{n}{\frac{1}{x_1} + \frac{1}{x_2} + \ldots + \frac{1}{x_n}} = \frac{n}{\sum_{i=1}^{n}\left(\frac{1}{x_i}\right)} = \frac{n}{\sum\left(\frac{1}{x}\right)}$$

EXAMPLE

Find the harmonic mean of the values 2,4 and 8.

SOLUTION: $H = \dfrac{3}{\frac{1}{2}+\frac{1}{4}+\frac{1}{8}} = \dfrac{3}{\frac{4+2+1}{8}} = \dfrac{3}{\frac{7}{8}} = \dfrac{3\times 8}{7} = \dfrac{24}{7} = 3.43$

The Harmonic Mean (grouped data)

Let $x_1, x_2, \ldots . x_k$ represent the class marks and $f_1, f_2, \cdots f_k$ as the corresponding class frequencies (where $f_1 + f_2 + \cdots + f_k = \sum f_i = n$).

Then the reciprocal of the class marks will be $\frac{1}{x_1}, \frac{1}{x_2}, \cdots, \frac{1}{x_k}$. Since the reciprocals occur with frequencies $f_1, f_2, \cdots f_k$. The total value of the reciprocals in the first class is $\frac{f_1}{x_1}$ in the second class is $\frac{f_2}{x_2}, \cdots$ in the k^{th} class is $\frac{f_k}{x_k}$. Then $\frac{f_1}{x_1} + \frac{f_2}{x_2} + \cdots + \frac{f_k}{x_k} = \sum_{i=1}^{k} \frac{f_i}{x_i} = \sum f(\frac{1}{x})$.

The harmonic mean is defined as:

$$H = \frac{\sum_{i=1}^{k} f_i}{\sum_{i=1}^{k} f_i(\frac{1}{x_i})} = \frac{\sum f}{\sum f(\frac{1}{x})} = \frac{n}{\sum f(\frac{1}{x})}$$

Sometimes this is also called weighted harmonic mean with weights $f_1, f_2, \cdots f_k$.

10.3.2 Properties of Arithmetic Mean

Following are the properties of Arithmetic mean.

(i) The sum of deviations of values from their mean is zero, symbolically:

$$\Sigma (x_i - \overline{X}) = 0 \text{ or } \Sigma f_i (x_i - \overline{X}) = 0$$

(ii) If n_1 values have mean $\overline{x_1}$, n_2 values have mean $\overline{x_2}, \cdots, n_k$ values have mean $\overline{x_k}$, then mean of all the values is:

$$\overline{X} = \frac{n_1\overline{X_1} + n_2\overline{X_2} + \cdots + n_k\overline{X_k}}{n_1 + n_2 + \cdots + n_k}$$

$$= \frac{\sum_{i=1}^{k} n_i \overline{X_i}}{\sum_{i=1}^{k} n_i}$$

(iii) The sum of squares of the deviations of the values x_i from any value *"a"* is minimum if and only if $a = \overline{X}$, symbolically $\Sigma(x_i - a)^2$ is a minimum if and only if $a = \overline{X}$.

10.3.3 Weighted Mean and Moving Average

If $x_1, x_2, \cdots, x_k$ have weights $w_1, w_2, \cdots, w_k$, then the weighted arithmetic mean or the weighted mean (denoted by $\overline{x_w}$) is defined as:

$$\overline{X_w} = \frac{w_1x_1 + w_2x_2 + \cdots + w_kx_k}{w_1 + w_2 + \cdots + w_k}$$

$$= \frac{\sum_{i=1}^{k} w_i x_i}{\sum_{i=1}^{k} w_i}$$

Moving Average

The average calculated by using *"n"* consecutive values of the observed series, for example we have to calculate *3* year moving average, then we take first three values from the series, add them and place against the middle of its time period. Then repeat the operation by dropping 1st value from the beginning and including first value after the preceding total. Mathematical form:

$$a_1 = \frac{1}{3}(y_1 + y_2 + y_3)$$

$$a_2 = \frac{1}{3}(y_2 + y_3 + y_4) \text{ and soon.}$$

Quartiles

We know that the median of an array is the middle value (or the mean of the two middle values). It divides a set of data into two equal parts. There are certain other values which divided a set of data into four equal parts called as first, second and third quartiles. These are denoted by Q_1, Q_2 and Q_3 respectively.

The first and third quartiles are also called as lower and upper quartiles respectively. The second quartile is the median.

$$Q_1 = value\ of \left(\frac{n+1}{4}\right) th\ item.$$

$$Q_2 = value\ of\ 2\left(\frac{n+1}{4}\right) th\ item\ or \left(\frac{n+1}{2}\right) th\ item.$$

$$Q_3 = value\ of \frac{3(n+1)}{4} th\ item.$$

EXAMPLE

Find all the quartiles from the following marks obtained by 20 students in statistic test.

53 74 82 42 39 20 81 68 58 28
67 54 93 70 30 55 36 37 29 61

SOLUTION:

The marks of $n=20$ students are arranged in ascending order as follows:

20 28 29 30 36 37 39 42 53 54
55 58 61 67 68 70 74 81 82 93

Q_1 = value of $\left(\frac{n+1}{4}\right)th$ or $\left(\frac{20+1}{4}\right)th$ or 5.25th item from below.

The value of the 5th item is 36 and that of the 6th item is 37. Thus the first quartile is a value of 0.25th of the way between 36 and 37, which is 36.25.

$$\text{Thus } Q_1 = 36.25$$

Q_2 (median) = value of $2\left(\frac{n+1}{4}\right)th$ or $\frac{2(20+1)}{4}th$ or $\frac{21}{2}th$ or 10.5th item from below.

The value of the 10th item is 54 and that of the 11th item is 55. Thus the second quartile is a value of 0.5th of the way between 54 and 55, which is 54.5.

$$\text{Thus } Q_2 = 54.5$$

Q_3 = value of $\frac{3(n+1)}{4}th$ or $\frac{3(20+1)}{4}th$ or $\frac{3\times 21}{4}th$ or $\frac{63}{4}th$ or 15.75th item from below.

The value of the 15th item is 68 and that of the 16th item is 70. Thus the first quartile is a value of 0.75th of the way between 68 and 70, which is 68+2(0.75)=69.5.

$$\text{Thus } Q_3 = 69.5$$

EXAMPLE

The marks of 100 students in an examination are given in the following table.

Marks	No.of Students	Cumulative Frequency
1 —— 10	2	2
11 —— 20	12	14
21 —— 30	25	39
31 —— 40	29	68
41 —— 50	15	83
51 —— 60	10	93
61 —— 70	4	97
71 —— 80	3	100

Use ogive to estimate the quartile, that is

(i) The upper quartile.

(ii) The lower quartile.

$\frac{3}{4}$ of the total frequency

$= \frac{3}{4} \times 100 = 3 \times 25 = 75$

From the curve the upper quartile = 44

$\frac{1}{4}$ of the total frequency

$= \frac{1}{4} \times 100 = 25$

From the curve, the lower quartile = 25

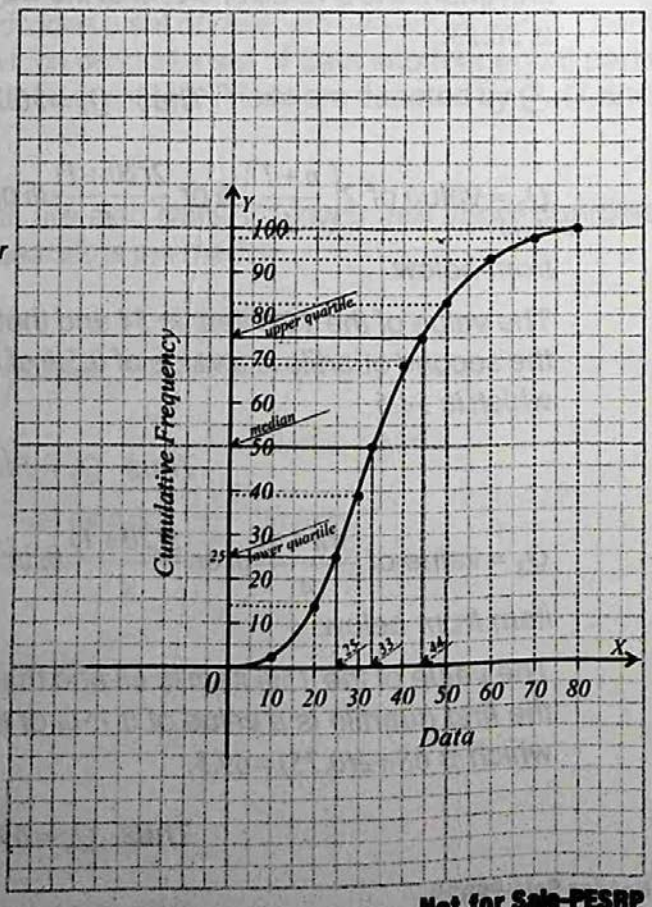
$\frac{1}{2}$ of the total frequency

$= \frac{1}{2} \times 100 = 50$

From the curve the median mark = 33

Two small squares along x-axis represents 10 units.

Two small squares along y-axis represents 10 units.



10.3.4 Estimate Median, Quartile and Mode Graphically, Graphic Location of Median

The approximate value of the median can be located from an ogive (a cumulative frequency polygon). In an ogive, the median is the value of x corresponding to $\frac{n}{2}$. Thus to locate median, we mark $\frac{n}{2}$ along the y-axis and draw a perpendicular from this point of y-axis and extend it so as to intersect the ogive. Then we drop a perpendicular on th x-axis from the point of intersection. The point at which the perpendicular intersects the x-axis is the value of the median.

For the given data estimate the median, Mode

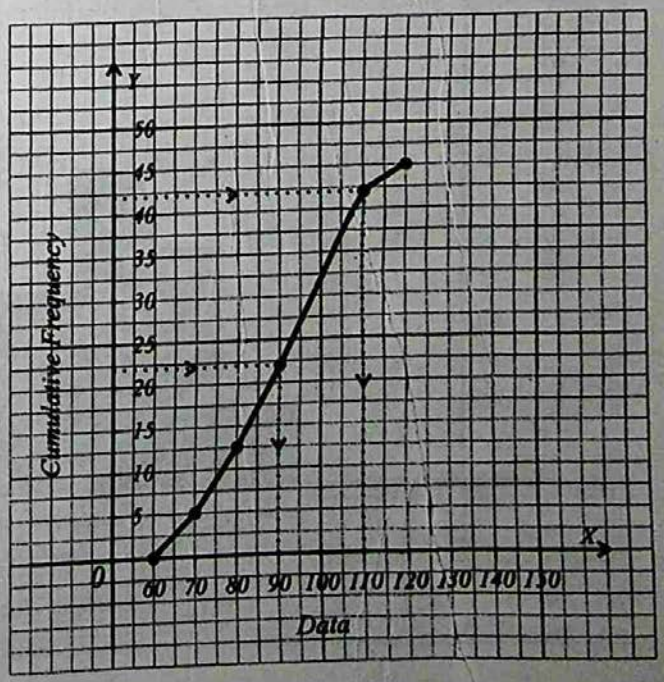
Class-intervals	Frequency	Cumulative Frequency
60 —— 70	5	5
70 —— 80	7	12
80 —— 90	10	22
90 —— 100	12	34
100 —— 110	8	42
110 —— 120	3	45
	45	——

(i) Draw the ogive

(ii) Locate the middle of the cumulative frequency axis, that is 22.5 and label it.

(iii) Draw a horizontal line from this point to the ogive and then vertically to the horizontal axis.

(iv) Read the value of the median from the x-axis. The median for the given data is approximately 90.



EXAMPLE

Calculate mode by graphic method from the given data.

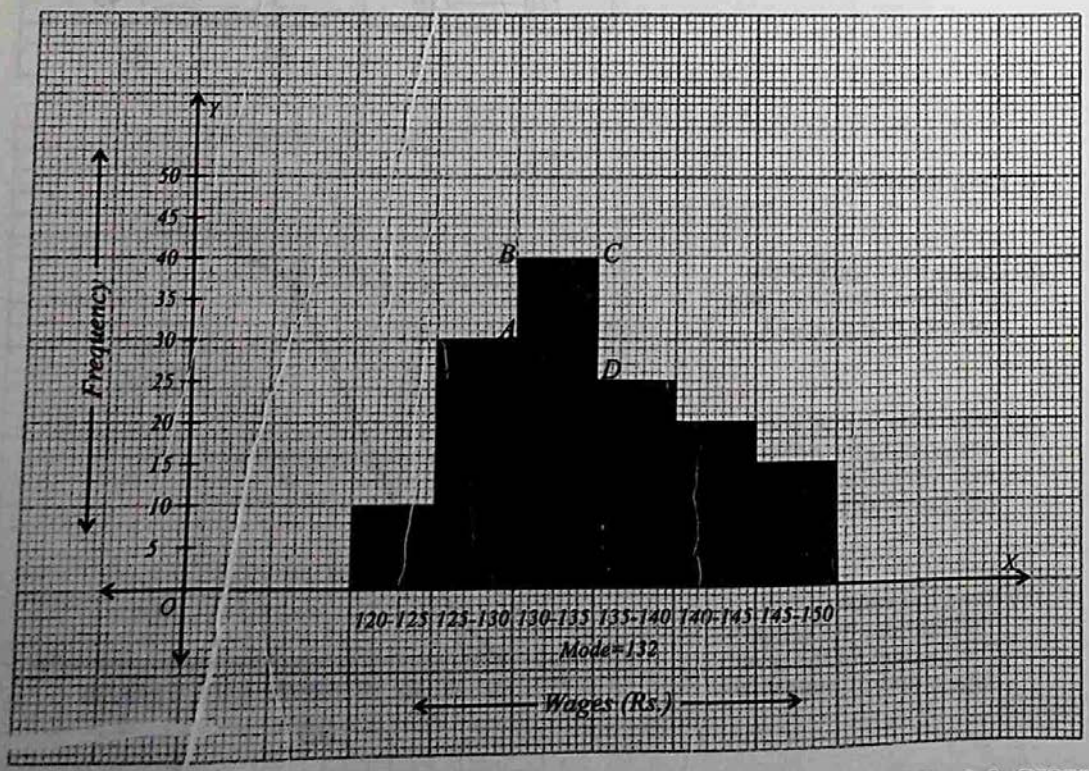
Wages Rs.	120 – 125	125 – 130	130 – 135	135 – 140	140 – 145	145 – 150
f	10	30	40	25	20	15

SOLUTION:

Wages in rupees are taken along x-axis while the frequency along the y-asix. One big square along x-axis represents 5units while one big square along y-axis represents 10units. We join the extreme ends of the bar with maximum frequency 30 and 25 as shown in the figure.

The two line-segments AC and BD cut each other at point P. We drop a perpendicular from P which meets x-axis at M. The value of M is 132.

Thus Mode = 132.



Graphic Location of Quartiles

EXAMPLE

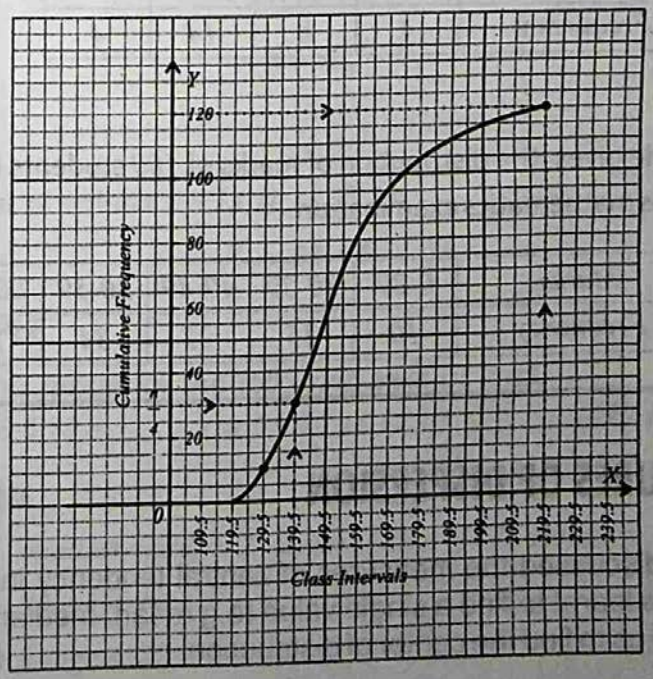
The following table shows a frequency distribution of grades on a final examination in mathematics. Locate quartiles graphically.

Grade	90-99	80-89	70-79	60-69	50-59	40-49	30-39
No of Students	9	32	43	21	11	3	1

SOLUTION:

For the graphic location of quartile, students are shown in the following table. We write the given data as follow:

Grade	Number of Students (f)	Class Intervals	Cumulative Frequency
30-39	1	29.5 – 39.5	1
40-49	3	39.5 – 49.5	4
50-59	11	49.5 – 59.5	15
60-69	21	59.5 – 69.5	36
70-79	43	69.5 – 79.5	79
80-89	32	79.5 – 89.5	111
90-99	9	89.5 – 99.5	120



EXERCISE – 10.3

1- Represent the given data using Frequency polygon.

(i) The table shows the distribution of marks of *30* children in a test.

Marks	*0-39*	*40-49*	*50-79*	*80-99*
Frequency	*8*	*8*	*10*	*4*

(ii) The table shows the distribution of time (in seconds) taken for *40* children to complete the obstacle race.

Time (second)	*1-40*	*41-50*	*51-60*	*61-70*
Frequency	*8*	*15*	*7*	*10*

(iii) The table shows the distribution of weights of *30* bags of chips from a fish and chip shop.

Weights (grams)	*1-50*	*51-60*	*61-70*	*71-80*
Frequency	*4*	*8*	*14*	*4*

(iv) The table gives the distribution of marks of *100* students in an end-of-terms mathematics examination.

Marks	*0-29*	*30-39*	*40-49*	*50-59*	*60-99*
Frequency	*10*	*15*	*25*	*34*	*16*

10.4 MEASURES OF DISPERSION

We have discussed the measures of central tendency in the preceding section, we learnt to find a single value (e.g mean, median, mode) which would help us to find the center of distribution.

For this let us consider the following distributions.

1.	63, 63, 63, 63, 63, 63, 63, 63	$\overline{X} = 63$
2.	62, 62, 62, 63, 63, 64, 64, 64	$\overline{X} = 63$
3.	48, 49, 57, 63, 69, 68, 74, 78	$\overline{X} = 63$
4.	40, 41, 47, 52, 62, 87, 88, 94	$\overline{X} = 63$

The above mentioned distributions have the same mean i:e *63*, but these distributions differ greatly in their dispersion, i.e. the extent to which the values are spread out from the average.

Though there is a great difference in the dispersion of the values of the distribution, yet each of these distributions is described by the same mean, i.e. *63*. We therefore need a measure to see how dispersed the data is.

The measures used for this purpose are called "Measures of Dispersion".

Range

The simplest measure of dispersion is range. Range is the difference between the largest value and the smallest value in the data. If the smallest value is denoted by X_o and the largest value is denoted by X_m then the range denoted by R is given by:

$$R = X_m - X_o$$

For Example

For the set of values *6,8,13,11,18,27,23*

$$R = 27 - 6 = 21$$

EXAMPLE

Find the range for the following sets of data.

(i) 2,9,3,6,12,8 *(ii) 6,8,10,14,13,12* *(iii) 10,18,9,6,22*

SOLUTION:

(i) smallest value = 2, largest value = 12, Range = 12 − 2 = 10

(ii) Range = 14 − 6 = 8

(iii) Range = 22 − 6 = 16

Variance

Variance is defined as the square of the standard deviation, i.e. the mean of the squared deviation from the mean. It is given by:

$$Var = \frac{\Sigma(x-\overline{X})^2}{n} \text{ (for ungrouped data)}$$

$$Var = \frac{\Sigma f(x-\overline{X})^2}{\Sigma f} \text{ (for grouped data)}$$

Standard Deviation

It is defined as the positive square root of the mean of the squared deviations of the values from their mean, The standard deviation of a set of n values, $x_1, x_2, \ldots\ldots x_n$ is denoted by S, where:

$$S = \sqrt{\frac{\Sigma(x-\overline{X})^2}{n}} \text{ (for ungrouped data)}$$

$$S = \sqrt{\frac{\Sigma f(x-\overline{X})^2}{\Sigma f}} \text{ (for grouped data)}$$

where $n = f_1 + f_2 + \cdots + f_k = \Sigma f$

EXAMPLE-1

Find the standard deviation for the values, 1,2,3,4,6,8,11. Also find the variance in this case.

SOLUTION: Here $\overline{X} = \frac{1+2+3+4+6+8+11}{7} = \frac{35}{7} = 5$

$$S = \sqrt{\frac{\Sigma(x-\overline{X})^2}{n}}$$

$$= \sqrt{\frac{(1-5)^2+(2-5)^2+(3-5)^2+(4-5)^2+(6-5)^2+(8-5)^2+(11-5)^2}{7}}$$

$$= \sqrt{\frac{16+9+4+1+1+9+36}{7}} = \sqrt{\frac{76}{7}}$$

$$= \sqrt{10.86} = 3.295$$

Thus standard deviation = $S = 3.63318$

$$\text{Variance} = \frac{\Sigma(x-\overline{X})^2}{n} = \frac{76}{7}$$
$$= 10.86$$

EXAMPLE-2

Find the standard deviation for the frequency distribution of marks obtained by 50 students in English at a certain examination. Also find the variance in this case.

Marks	20–24	25–29	30–34	35–39	40–44	45–49	50–54
Frequency	1	4	8	11	15	9	2

SOLUTION: Here

Marks	Frequency (f)	Class Mark (X)	fx	$x-\overline{X}$	$(x-\overline{X})^2$	$f(x-\overline{X})^2$
20–24	1	22	22	− 17	289	289
25–29	4	27	108	− 12	144	576
30–34	8	32	256	− 7	49	392
35–39	11	37	407	− 2	4	44
40–44	15	42	630	3	9	135
45–49	9	47	423	8	64	576
50–54	2	52	104	13	169	338
	$n = \Sigma f = 50$	$\Sigma f = 259$	$\Sigma fx = 1950$			2350

$$\overline{X} = \frac{\Sigma f x}{\Sigma f} = \frac{1950}{50} = 39 \text{ marks}$$

$$S = \sqrt{\frac{\Sigma f(x-\overline{X})^2}{n}} = \sqrt{\frac{2350}{50}} = \sqrt{47}$$

Thus standard deviation = $S.D = 6.85$

$$\text{Variance} = \frac{\Sigma f(x-\overline{X})^2}{n} = \frac{2350}{50}$$
$$= 47$$

EXERCISE – 10.4

1- Construct a cumulative frequency polygon (that is, an ogive) for the given data.

(i) The table shows the distribution of weights (in kilograms) of *60* boys of ten years of age.

Weight (kg)	*31-36*	*37-39*	*40-42*	*43-45*	*46-54*
Frequency	*8*	*10*	*18*	*12*	*12*

(ii) The table shows the distribution of times taken (in minutes) for *50* children of five years age to eat their school dinners.

Time (minutes)	*4-5*	*6-7*	*8-9*	*10-11*	*12-15*	*16-19*	*20-29*
Frequency	*5*	*4*	*10*	*9*	*6*	*6*	*10*

(iii) The table shows the distribution of the ages of people boarding buses at the bus station between *08.30* to *09.00* in the morning.

Age (years)	*0-9*	*10-19*	*20-29*	*30-39*	*40-69*
Frequency	*10*	*20*	*30*	*20*	*15*

(iv)

Classes	*5-10*	*10-15*	*15-20*	*20-25*	*25-30*
Frequency	*10*	*15*	*20*	*30*	*15*

(v) The table gives the distribution of weights (kilograms) of *100* people.

Weight (kilograms)	*50-59*	*60-69*	*70-79*	*80-89*	*90-99*	*100-109*
Frequency	*15*	*30*	*35*	*15*	*3*	*2*

Review Exercise – 10

1- Encircle the correct answer.

(i) When a bar graph is constructed, so that the area of each bar is proportional to the number of items in each group is called.

(a) *curve* (b) *ogive* (c) *histogram* (d) *bar diagram*

(ii) The summary statistics which measure the middle (or center) of the data is called:

(a) *mean* (b) *mode*

(c) *median* (d) *all of these*

(iii) If all numbers in a set are added together and then the total is divided by the number of scores in the set is called

(a) *mean* (b) *mode* (c) *median* (d) *weighted mean*

(iv) The middle values of data arranged in numerical order is called

(a) *mode* (b) *median* (c) *mean* (d) *geometric mean*

(v) The score which occurs most often in a set of data is called

(a) *mode* (b) *mean* (c) *median* (d) *geometric mean*

(vi) $\overline{X} = \dfrac{x_1 + x_2 + x_3 + \cdots + x_n}{n}$ is called

(a) *mean value of* $x_1, x_2, \cdots, x_n$ (b) *arithmetic mean*

(c) *geometric mean* (d) *weighted mean*

(vii) $H = \dfrac{n}{\Sigma\left(\dfrac{1}{x}\right)}$ is called

(a) *harmonic mean* (b) *mode* (c) *mean* (d) *arithmetic mean*

(viii) $\overline{X}_w = \dfrac{\Sigma wx}{\Sigma w}$ is called

(a) *arithmetic mean* (b) *weighted mean*

(c) *geometric mean* (d) *mean*

(ix) $\Sigma(x_i - \overline{X}) = 0$ is one of the properties of

(a) *arithmetic mean* (b) *geometric mean*

(c) *harmonic mean* (d) *mode*

2- Fill in the blanks.

(i) When a bar graph is constructed, so that the area of each bar is proportion to the number of items in each group is called a _____.

(ii) The summary statistic which measure the middle (or center) for the data is called _____.

(iii) If all numbers in a set are added together and then the total is divided by the number of scores in the set is called _____.

(iv) The middle value of data arranged in numerical order is called _____.

(v) The score which occurs most often in a set of data is called _____.

(vi) $\overline{X} = \frac{x_1 + x_2 + x_3 + \cdots + x_n}{n}$ is called the _____.

(vii) The *nth* root of the product of the values of a set of n positive values is called _____.

(viii) $H = \frac{n}{\Sigma\left(\frac{1}{x}\right)}$ is called the _____.

(ix) $\overline{X}_w = \frac{\Sigma wx}{\Sigma w}$ is called the _____.

(x) $\Sigma(x_i - \overline{X}) = 0$ is one of the properties of _____.

3- Find the standard deviation of the values *2, 3, 6, 8, 11.*

4- Find the standard deviation and variance for a set of ungrouped values, when $n = 15$, $\Sigma(x - \overline{x})^2 = 1444$

5- For the data *3, 5, 6, 8, 8, 9, 10* , find
(i) Mean (ii) Median (iii) Mode

6- Find the mean, median and mode for the set of the value *4, 6, 7, 4, 8, 9, , 7, 10.*

SUMMARY

- When a bar chart is construed, so that the area of each bar is proportional to the number of items in each group is called a histogram.
- Cumulative frequency is a running total of class frequencies.
- When the cumulative frequencies are plotted against the end point of their respective class intervals and joined together, the resultant graph is called a cumulative frequency polygon or ogive.
- These are summary statistics which measure the middle (or center) of the data.
- To obtain the mean of a data, all numbers in the set are added together and then the total is divided by the number of scores in that set.
- The middle value of data arranged in numerical order is called median.
- The mode is the score which occurs most often in a set of data.
- Arithmetic mean of an ungrouped data is obtained by adding all numbers (scores) in the set together and then the sum is divided by the number of scores in that set.
- The geometric mean "G" of a set of n positive values $x_1, x_2, ..., x_n$ is the nth root of the product of the values.
- The harmonic mean H of a set of n values $x_1, x_2, \cdots, x_n$ is the reciprocal of the arithmetic mean of the reciprocal of the values.

SUMMARY

- Variance is defined as the square of the standard deviation, i.e. the mean of the squared deviation from the mean.

- The simplest measure of dispersion is range. Range is the difference between the largest value and the smallest value in the data.

- The average calculated by using "n" consecutive values of the observed series, for example we have to calculate *3* years moving average, then we take first three values from the series, add them and place against the middle of its time period. Then repeat the operation by dropping 1st value from the beginning and including first value after the preceding total.

- Standard deviation is defined as the positive square root of the mean of the squared deviations of the values from their mean, The standard deviation of a set of n values, $x_1, x_2, \ldots\ldots x_n$ is denoted by S.

- If $x_1, x_2, \cdots, x_k$ have weights $w_1, w_2, \cdots, w_k$, then the weighted arithmetic mean or the weighted mean (denoted by $\overline{x_w}$) is defined as:

$$\overline{x_w} = \frac{\sum_{i=1}^{k} w_i x_i}{\sum_{i=1}^{k} w_i}$$

Answers

Exercise 1.1

1- (i) $\frac{19}{20}$ (ii) $\frac{13}{20}$ (iii) $\frac{3}{4}$ (iv) $\frac{1}{4}$ (v) $\frac{14}{25}$ (vi) $\frac{12}{25}$ (vii) $\frac{2}{25}$ (viii) $\frac{67}{200}$ (ix) $\frac{3}{8}$ (x) $\frac{7}{8}$ (xi) $\frac{21}{400}$ (xii) $\frac{17}{40}$

2- (i) 75% (ii) 60% (iii) 16% (iv) 65% (v) 124% (vi) 52.5% (vii) 38.3% (viii) 266.66% (ix) 160% (x) 87.5% (xi) 62.5% (xii) 37.5%

3- (i) 0.47 (ii) 0.58 (iii) 0.92 (iv) 0.08 (v) 0.12 (vi) 1.20 (vii) 1.80 (viii) 1.45 (ix) 0.055 (x) 0.0533 (xi) 0.486 (xii) 0.583

4- (i) 50% (ii) 90% (iii) 125% (iv) 139% (v) 172% (vi) 22% (vii) 264% (viii) 341% (ix) 84.5% (x) 178% (xi) 158% (xii) 6.5%

5- (ii) 80% (iii) $\frac{2}{5}$, 0.4 (iv) $\frac{31}{50}$, 62% (v) $\frac{11}{25}$, 0.44

Exercise 1.2

1- 55% **2-** 18% **3-** 12% **4-** (i) $P = 208, C = 68, I = 48, B = 76$ (ii) 19%

5- Math **6-** 27.78% **7-** 500 pages **8-** Rs.7500 **9-** 20%

Exercise 1.3

1- (i) 4 : 1 (ii) 4 : 1 (iii) 1 : 4 (iv) 1 : 1 (v) 5 : 4 (vi) 6 : 11

2- (i) 10 : 9 (ii) 16 : 15 (iii) 25 : 21 (iv) 13 : 6 (v) 4 : 1 (vi) 1 : 50 (vii) 3 : 4 (viii) 3 : 7 (ix) 6 : 5

3- (i) 14 : 9 (ii) 3 : 25 (iii) 9 : 28

4- (i) 15 : 23 (ii) 23 : 17 (iii) 23 : 12 (iv) 12 : 17 (v) 4 : 5 (vi) 17 : 15

Exercise 1.4

1- 1 : 1 **2-** 16 : 15 **3-** 3 : 4 **4-** (i) 1 : 3 (ii) 1 : 3 (iii) 1 : 9 **5-** 2 : 1

6- (i) 3 : 4 (ii) 2 : 3 (iii) 1 : 2 **7-** 3 : 7 **8-** (i) 8 : 7 (ii) 7 : 1 (iii) 8 : 1

9- (i) 3 : 4 (ii) 9 : 16 **10-** (i) 2 : 1 (ii) 1 : 3

Exercise 1.5

1- *20* 2- *9 suits* 3- *5 liters* 4- *75 km / hr*

5- *8 days* 6- *120 bicycles* 7- *864 fans* 8- *720 soaps*

9- *Rs.2240* 10- *Rs.18200* 11- *110 cows* 12- *2700 bottles*

Review Exercise -1

1- Encircle the Correct Answer.

(i) b (ii) b (iii) a (iv) d (v) c (vi) b (vii) c (viii) a (ix) a (x) b

2- Fill in the blanks.

(i) 450 *(ii)* $\frac{3}{20}$ *(iii) 28 %* *(iv) 67%* *(v) 29 %* *(vi) antecedent*

(vii) consquent *(viii) extremes* *(ix) means* *(x) 10 : 9*

3- *30%* 4- *Rs.7000* 5- *(i) 2:1 (ii) 22 : 5 (iii) 11:3*

6- *2 : 3* 7- *16 days* 8- *Rs.9720*

Exercise 2.1

1- *Rs.27750* 2- *Rs.7500* 3- *Rs.15,000* 4- *Rs.36,250* 5- *Rs.60,000*

6- *Rs.17500* 7- *widow Rs.93750; son Rs.262500; daughter Rs.131250*

8- *widow Rs.50,000; daughter Rs.87500* 9- *Rs.3,75,000*

10- *widow Rs.2,50,000; son Rs.8,75,000*

11- *widow Rs.60,000; son Rs.84,000, daughter Rs.42000* 12- *100,000*

Review Exercise 2

1- Encircle the Correct Answer.

(i) a (ii) c (iii) a (iv) a (v) c (vi) b (vii) a (viii) c (ix) a (x) a

2- Fill in the blanks.

(i) 2.5 % *(ii) 10 %* *(iii) 5 %* *(iv) Rs.5000* *(v) Rs.10,000*

(vi) $\frac{1}{8}$ *(vii)* $\frac{1}{4}$ *(viii)* $\frac{1}{2}$ *(ix) 2:1* *(x)* $\frac{2}{3}$

3- *Rs.37500* 4- *Rs.24500* 5- *widow : Rs.5,62,500, Sons : Rs.19,68,750*

6- *widow : Rs.6,00,000, Son : Rs.8,40,000 each, daughter Rs.4,20,000 each.*

Exercise 3.1

1- (i) Rs.1045 (ii) Rs.1463 (iii) Rs.10560 (iv) Rs.119700 (v) Rs.494.40 (vi) Rs.729.60

2- Rs.2,73,000 3- (i) Rs.640 (ii) Rs.925 (iii) Rs.1560 (iv) Rs.3000 (v) Rs.75

4- $12\frac{1}{2}\%$ 5- 25 % 6- Rs.1260 7- Rs.213 8- Rs.1215

Exercise 3.2

1- (i) Rs.684.32 (ii) Rs.2622 (iii) Rs.364.08 2- (i) Rs.560 (ii) Rs.975 (iii) Rs.2500

3- 5 % 4- 7.5 % 5- Rs.400 6- Rs.920 7- Rs.11730.60 8- 20 %

Exercise 3.3

1- 60,000; 40,000; 1,00,000

2- Ali's share Rs.22,500; Daniyal's share Rs.33,750; Abdullah's share Rs.56,250

3- 180; 240; 300

4- Profit 1st partner Rs.1,00,000; 2nd partner Rs.1,50,000; 3rd partner Rs.3,50,000
Amount 1st partner Rs.600,000. 2nd partner Rs.900,000. 3rd partner Rs.2100,000

5- 39200 6- (a) Rs.70 Total amount Rs.154

Review Exercise 3

1- Encircle the Correct Answer.

(i) c (ii) b (iii) b (iv) b (v) a

2- Fill in the blanks.

(i) cost price (ii) sale price (iii) profit (iv) $\frac{loss}{CP} \times 100$ (v) 100 − discount%

3- 5.5 % 4- 16.6 % 5- 9000; 15000; 9000 6- (i) 60000 (ii) 36000

Exercise 4.1

1- 161.406 2- Rs.111724.5 3- Loss Rs.15

4- (i) Rs.12615 (ii) Rs.12517.42 (iii) Rs.97.58 5- (i) Rs.4.63 (ii) 1.89375 Riyals

6- 15772 7- Profit = Rs.1099.58 8- Profit = Rs.4759.5

9- T.T buying is applicable for Rs.10055.20 (ii) Rs.1857.37

Exercise 4.2

1- Rs.1100 2- Rs.132 3- 10 years 4- 4.25% per year 5- Rs.1019.50 6- $7\frac{1}{2}$%P.A

7- (a) Rs.6720, Rs.18720 (b) 4years, Rs.720 (c) Rs.300, Rs.408 (d) 4%, Rs.4200 (e) Rs.3600, 5%
(f) 7%, Rs.1989 (g) 6%, Rs.540 (h) Rs.1200, $1\frac{1}{2}$ years 8- Rs.16 9- Rs.6180 10- Rs.20,000

11- (i) Rs.94.50 (ii) Rs.257.34 (iii) Rs.1244.03 (iv) Rs.149.4 (v) Rs.2422.94

12- Rs.5829.57 13- Rs.903.13 14- 9550.87

Exercise 4.3

1- Rs.1520.50 2- Rs.88.50 3- Rs.65000 4- Rs.7084 5- Rs.3600
6- Rs.35000 7- Rs.157.49 8- Rs.3040 9- Rs.6480

Exercise 4.4

1-

Amount of policy	Yearly premium	Half yearly premium	Quarterly premium	Monthly premium
(i) 50,000	2250 + 125	1235	641.50	213.75
(ii) 100,000	4500 + 200	2444	1269.00	423.00
(iii) 150,000	6750 + 200	3614	1876.50	625.30
(iv) 200,000	9000 + 200	4784	2484.00	828.00

2- Rs.(50,000 + 1,20,000 + 4500) = Rs. 1,74,500 3- Rs.(150000 + 135000 + 121500 + 0) = Rs. 406500
4- Rs.700,000, Rs.159050 5- Rs.4,32,250 6- Rs.(36,125 + 32512.50) = Rs. 68,637.50
7- Rs.(26,250 + 23,625 + 21,262.50 + 19136.25) = 90273.75 benefit Rs. 9726.25
8- Rs. 800,000 (approx.), Rs.23400, Zero.

Exercise 4.5

1- (a) i. Rs.90 ii. 25% (b) i. Rs.150 ii. $16\frac{2}{3}\%$ (c) i. Rs.3000 ii. 12% 2- (i) Rs.21 (ii) Rs.288
3- (a) i. Rs.236 ii. 18% (b) i. Rs.517.50 ii. 15% (c) i. Rs.1960 ii. 22.5%
4- (a) i. Rs.63 ii. 7% (b) i. Rs.200 ii. 15% (c) i. Rs.75 ii. $16\frac{2}{3}\%$
5- (i) Rs.3716.80 (ii) Rs.516.80 (iii) 16.15%

Review Exercise 4

1- Encircle the Correct Answer.

(i) a (ii) b (iii) a (iv) c (v) a (vi) a (vii) a (viii) c (ix) a

2- Fill in the blanks.

(i) cheque (ii) pay order (iii) ATM (iv) profit (v) rate
(vi) time (vii) mark-up (viii) premium (ix) insurer (x) insured

3- Rs.515,850 4- Rs.700,000, Rs.159,050 5- Markup Rs.2,770 Prinicipal Rs.27,230

Exercise 5.1

1- Rs.28000 2- Rs.6400, Rs.46400 3- Rs.18,00,000, Rs.28,00,000
4- Rs.2,40,000, Rs.3,20,000 5- Rs.11250, Rs.13950 6- Rs.16350 7- Rs.21375
8- Rs.6875

Exercise 5.2

1- (i) Rs.5124.29 (ii) Rs.9168.056 (iii) Rs.3622.84 (iv) Rs.950.12 (v) Rs.14771.5 (vi) Rs.9848.75

2- (i) Rs.1155.39 (ii) Rs.718.09 (iii) Rs.1540 (iv) Rs.1021.41

3- (i) Rs.3153.50 (ii) Rs.3808.00 (iii) Rs.4462.50 (iv) Rs.1606.50 (v) Rs.2856 (vi) Rs.1874.25

Exercise 5.3

1- 48 hrs, Rs.1920 2- Rs.4800 3- Rs.17640 4- Rs.26500 5- Rs.41,500

6- (i) Rs.1975 (ii) Rs.2725 (iii) Rs.2875 7- Rs.10,800

Review Exercise 5

1- Encircle the Correct Answer.

(i) a (ii) b (iii) a (iv) b (v) a (vi) a (vii) c

2- Fill in the blanks.

(i) tax (ii) direct tax (iii) indirect tax (iv) sales tax (v) exercise duty (vi) property tax (vii) income tax (viii) Rs.90,000 (ix) Rs.1200 (x) 4,50,000

3- Rs.19200 4- Rs.24750 5- Rs.15867.37

6- (i) Rs.1017.48 (ii) Rs.736.02 (iii) Rs.989.23 (iv) Rs.1055 7- Rs.70500

Exercise 6.1

1- (i) Redical $\sqrt{3}$; Redicand 3 (ii) Redical $\sqrt{a}$; Redicanda (iii) Redical $\sqrt{11}$; Redicand11 (iv) Redical $\sqrt{6}$; Redicand6 (v) Redical $\sqrt{5}$; Redicand5 (vi) Redical $\sqrt{13}$; Redicand13

2- (i) $a^{3/2}$ (ii) $a^{3/5}$ (iii) $a^{-k/p}$ (iv) $a^{-k/h}$

3- (i) $\sqrt{25}, 5$ (ii) $\sqrt[3]{64}, 4$ (iii) $\sqrt[4]{81}, 3$ (iv) $\sqrt[3]{27}, 3$ (v) $\sqrt[3]{(27)^2}, 9$ (vi) $\sqrt[3]{\frac{1}{8}}, \frac{1}{2}$ (vii) $\sqrt[3]{(1000)^2}$; 100 (viii) $\sqrt{64}$; 8

4- (i) a^8 (ii) a^5 (iii) $3a^3$ (iv) $2a^3$ (v) x^8 (vi) $3x^5$ (vii) $5x^3y^5$ (viii) $(8+y)^{7/2}$ (ix) $2x^{1/2}y^{3/2}$ (x) $\frac{x^{5/4}y^{3/2}}{z^{1/2}}$ (xi) $\frac{2x^{1/3}}{(x+y)^{1/3}}$ (xii) $\frac{y^{n/p}}{a^{m/p}}$

5- (i) $\sqrt{21}$ (ii) $\sqrt[3]{512}$ (iii) $\sqrt[7]{2187}$ (iv) $\frac{1}{\sqrt[18]{2}}$ (v) $\sqrt[5]{59}$ (vi) $\sqrt{\frac{1}{3}}$ (vii) $\sqrt[12]{a^{11}}$ (viii) $\sqrt[4]{x^{24/7}y}$ (ix) $y\sqrt{x^9}$ (x) $\sqrt[6]{x}$ (xi) $\sqrt[4]{x^4y^5}$ (xii) $\sqrt[24]{a^{17}b^{24}}$ (xiii) $x^{29/60}y^{11/10}$

Exercise 6.2

1-

Q.No.	(i)	(ii)	(iii)	(iv)	(v)	(vi)
Base	x	x	$4y$	$x-2$	x	x
Exponent	3	9	3	3	5	2

2- a^6b^9 **3-** $\frac{x^{4/3}}{y}$ **4-** $\frac{1}{y^{2a+3b}}$ **5-** $\frac{1}{2x^{1/2}y^{1/2}}$ **6-** $\frac{4}{9a^2x^2}$ **7-** $16ac^4$

8- $\frac{3}{ba^{1/2}}$ **9-** $\frac{a^{11/2}c^{3/2}}{b^3}$ **10-** b^3 **11-** $\frac{x^{11}y^6}{16}$ **12-** ab^{13} **13-** a^4b^3c

14- $\frac{4b}{9ac^9}$ **15-** $2^{16}\times3^8$ **16-** $2^{14}\times3^7$ **17-** $\frac{1}{2a^4b^6}$ **18-** 2187

19- $\frac{3^7}{2^6}$ **20-** $\frac{2^3}{3^6}$ **21-** $\frac{125}{2187}$ **22-** $a^{7/6}b^{11/12}$ **23-** $a^{5/6}b^{3/2}$ **24-** $a^3b^2c^{3/2}$

25- $a^{1/2}b^{23/72}$ **26-** $a^{11/12}$

27- (i) $4^{4/5}$ (ii) $2^{1/2}$ (iii) $10x^{8/15}$ (iv) $x^{23/20}$ (v) $2y^{5/7}$ (vi) $5x^2$

28- (i) $ab^{3/2}$ (ii) $x^{4/5}y^{5/9}$ (iii) $6a^{8/5}b^{17/15}$ (iv) $2x^{19/28}y^{2/5}$ (v) $x^{19/6}y^{5/6}z^{5/6}$

29- (i) $3^{1/6}$ (ii) $x^{4/3}$ (iii) $\frac{1}{2}x^{3/20}$ (iv) $\frac{5}{4}y^{7/20}$ (v) $x^{5/3}y^{7/5}$ (vi) $a^{7/45}b^{4/15}$

(vii) $2x^{2/15}y^{3/4}$ (viii) $\frac{1}{4}a^{11/20}b^{7/20}$

Exercise 6.3

1- 5.1×10^{-2} **2-** 8.899×10^{1} **3-** 4.24×10^{-1} **4-** 2.566324×10^{6}

5- 7.5×10^{-7} **6-** 8600 **7-** 0.00001345 **8-** 0.00000000051

9- 0.0000000525 **10-** 0.0006365 **11-** 4.8×10^{2} **12-** 1.044×10^{6}

13- 3.126×10^{4} **14-** 4.5×10^{3} m **15-** 6.4×10^{6}

Exercise 6.4

1- (i) 3 (ii) 3 (iii) 0 (iv) $\bar{1}$ (v) $\bar{2}$ (vi) $\bar{4}$

2- (i) 1.7170 (ii) 0.7996 (iii) $\bar{1}$.7873 (iv) $\bar{3}$.7559 (v) $\bar{5}$.4771

3- (i) 0.8044 (ii) $\bar{1}$.8044 (iii) $\bar{3}$.8044 **4-** (i) 0.01090 (ii) 1.444 (iii) 26530

Exercise 6.5

1- (i) 2 (ii) 2 (iii) $\frac{5}{2}$ **2-** (i) 1 (ii) 3 (iii) 4 (iv) 2 (v) 3

3- (i) $\log\frac{1.3472\times 22.79}{5}$ (ii) $\log\frac{22.13\times 0.354\times 7}{3}$ (iii) $\log\frac{57.86\times 4.385}{2.391\times 3.072}$

4- (i) 1.923 (ii) 19.19 (iii) 0.9945

8- (i) 0.3291 (ii) 19.19 (iii) 14.139 (iv) 0.3466 (v) 160.4 (vi) 48.01
(vii) 1964 (viii) 2.082 **9-** 27.71 **10-** 67.39 **11-** 0.1224

Review Exercise 6

1- Encircle the Correct Answer.

(i) b (ii) a (iii) b (iv) c (v) a (vi) b (vii) a (viii) b (ix) d (x) a

2- Fill in the blanks.

(i) radical (ii) radicand (iii) exponent (iv) base (v) common logarithm
(vi) characteristic (vii) mantissa

3- (i) $x^{3/2}y^{-5/6}$ (ii) $a^{7/8}b^{7/12}$ **4-** (i) $x^{1/3}y^{19/24}$ (ii) $\frac{1}{40}$ **6-** (i) 0.2983 (ii) 5.158 (iii) 0.2465

Exercise 7.1

1- (i) 4,5,6 (ii) $-1,8,-27$ (iii) 8,11,14 (iv) $\frac{2}{7},\frac{1}{3},\frac{4}{11}$ (v) $1,\frac{1}{9},\frac{1}{25}$ (vi) 4,5,6
(vii) $\frac{1}{3}\cdot\frac{1}{9}\cdot\frac{1}{27}$ (viii) $-2.1.4$ (ix) 1,3,12

2- (i) 51 (ii) 20160 (iii) $\frac{1}{243}$ (iv) 15 (v) $\frac{5}{11}$ (vi) 17

3- (i) 34,42,51,61 (ii) 63,127,255,511 (iii) 112,288,704,1664
(iv) 25,27,30,33 (v) 20,24,28,32 (vi) 12,14,16,18

Exercise 7.2

1- *(i) 243 (ii) −58 (iii) −82 (iv) 74 (v) −74*

2- *(i) 16 (ii) 132 (iii) 4 (iv) 65 (v) 79 (vi) 193* **3-** *12* **4-** *39*

5- *42* **6-** *20* **7-** $12-11x$ **8-** $3n+24$ **9-** $\left(\frac{3}{3n+1}\right)^2$ **10-** $-2,1,4,7,10,13,\ldots\ldots$

Exercise 7.3

1- *(i) 2* *(ii)* $x+3$ *(iii)* $2\sqrt{7}$ *(iv)* x^2+1 **2-** *0,9* **3-** *13,15,17*

4- $\frac{9\sqrt{2}}{4},\frac{7\sqrt{2}}{2},\frac{19\sqrt{2}}{4}$ **5-** $\frac{38}{7},\frac{41}{7},\frac{44}{7},\frac{47}{7},\frac{50}{7},\frac{53}{7}$ **6-** $\frac{17}{2},9,\frac{19}{2},10,\frac{21}{2},11,\frac{23}{2}$

7- *15* **8-** *70* **9-** *1,17*

Exercise 7.4

1- *8192* **2-** *118098* **3-** *48* **4-** *324* **5-** *(i) 3 (ii) 5 (iii) 10*

6- *576* **7-** *18225* **9-** $\left(\pm\frac{2}{3}\right)^n$ **10-** *2,6,18 or 18,6,2*

11- $\frac{1}{x^{28}}$ **12-** x^{2p-1}

Exercise 7.5

1- *(i)* $\pm 3\sqrt{5}$ *(ii)* ± 6 *(iii)* $\pm 2\sqrt{2}$ **2-** *(i) 2,4 (ii) 9,27* **3-** *(i) 2,4,8 (ii) 2,8,16*

4- *6,12,24,48* **5-** *2,8; 8,2* **6-** *2,18* **8-** *80,40,20,10*

9- *162,54,18* **10-** $\frac{1}{2},2,8,32$ **11-** $-28,14,-7,\frac{7}{2},\frac{-7}{4},\frac{7}{8}$ **12-** $\frac{16}{27},\frac{8}{9},\frac{4}{3},2,3$

Review Exercise 7

1- Encircle the Correct Answer.

(i) b (ii)c (iii) a (iv) a (v) c (vi) b (vii) a (viii)c (ix) a (x) b

2- Fill in the blanks.

(i) a_n (ii) 15 (iii) nth term (vi) 10 (v) $\frac{a+b}{2}$

(vi) common ratio (vii) ar^{n-1} (viii) $\pm\sqrt{ab}$ (ix) 4 (x) $3n + 24$

3- $a_n = 2n + 1,\ 37$ **4-** $a_n = \left(\frac{3}{2n+3}\right)^3$ **5-** 32 **6-** $(3)^{11}$ **7-** 1,2,4,8

8- 2,6,18, or 18,6,2

Exercise 8.1

1. (i) {1,4,6,7,8,9} (ii) {3,4,5,6,7,8,9} (iii) {4,7} (iv) {4}
(v) {1,3,4,5,6,7,8,9} (vi) {4}

8. (i) A (ii) A (iii) A (iv) Φ (v) Φ (vi) $A' \cup B'$
(vii) $A' \cap B'$ (viii) A (ix) Φ (x) Φ

Exercise 8.2

1. {(3,1), (3,3), (5,1) (5,3), (6,1), (6,3)} , {(1,3), (1,5) (1,6), (3,3), (3,5), (3,6)}
Dom $A \times B$ = {3, 5, 6} Range $A \times B$ = {1, 3}
Dom $B \times A$ = {1, 3} Range $B \times A$ = {3, 5, 6}

2. {(−2,−2), (−2,1)}, {(−2,4) (1,−2)} Dom = {−2} Dom = {−2,1}
Range = {−2,1} Range = {4,−2}

3. (i) 2^9 (ii) 2^{12} **4.** {(2,2), (3,3), (3,2)}

Review Exercise 8

1- Encircle the Correct Answer.

(i) b (ii) b (iii) a (iv) b (v) a (vi) a (vii) b (viii) b (ix) c (x) d

2- Fill in the blanks.

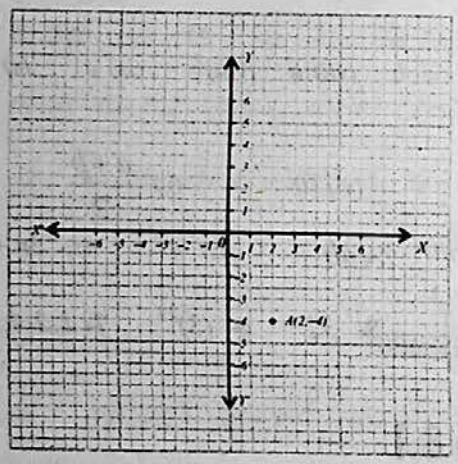
(i) $A' \cap B'$ (ii) $A' \cup B'$ (iii) $(A \cup B) \cup C$ (iv) $(A \cap B) \cap C$
(v) commutative law as under union (vi) commutative law under intersection
(vii) binary relation (viii) {1,3,5} (ix) {2,4,6} (x) set B

6- (i) 2^{16} (ii) 2^6 **7-** Dom (R) = {2,4} Range (R) = {2,5}

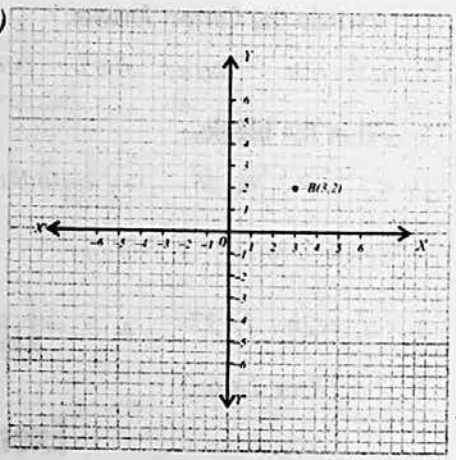
Exercise 9.1

1-

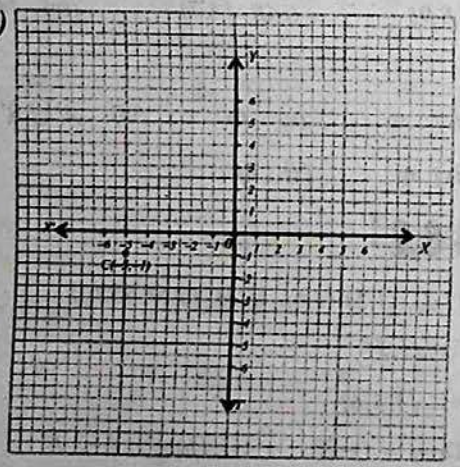
(i)



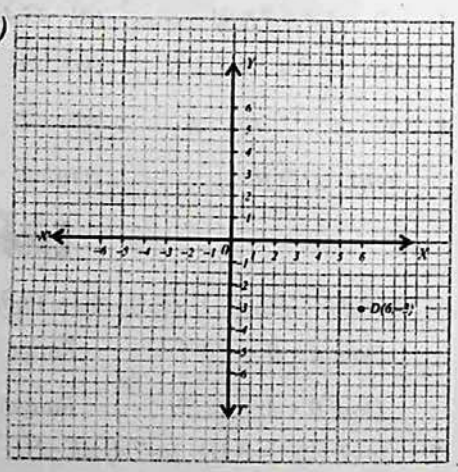
(ii)



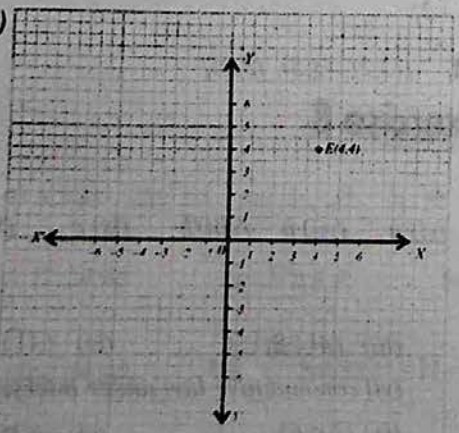
(iii)



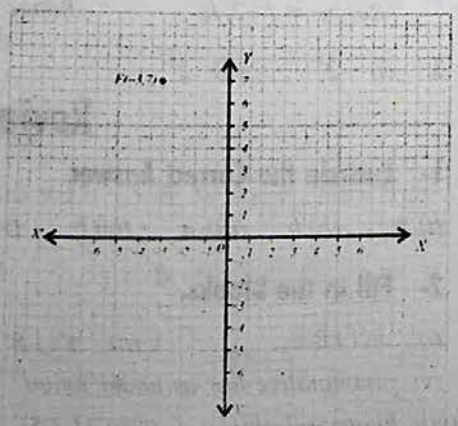
(iv)



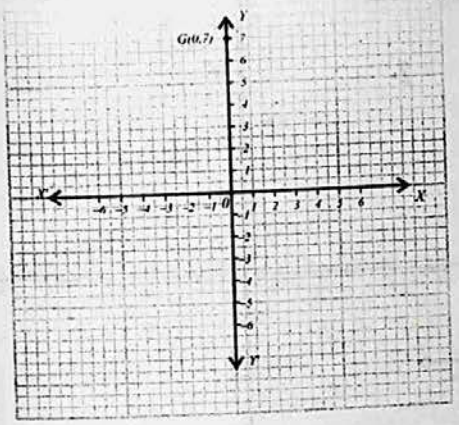
(v)



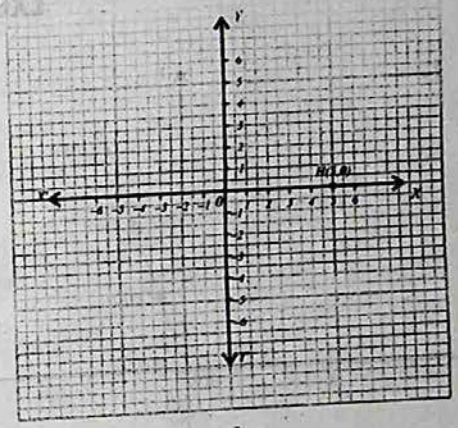
(vi)



(vii)

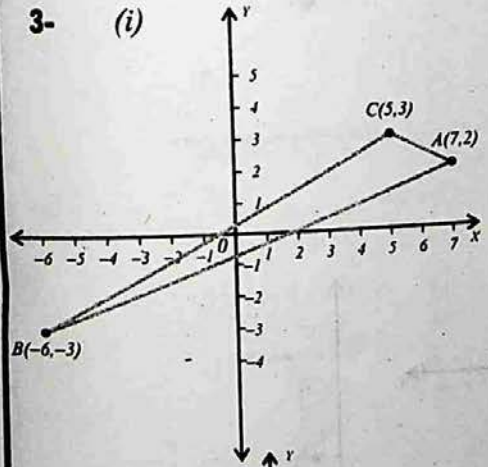


(viii)

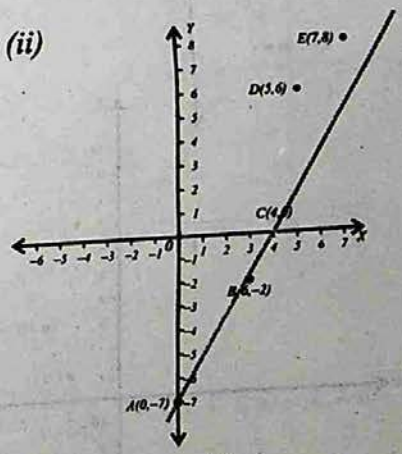


2- (i) (0,0) (ii) (−5,0) (iii) (3,0) (iv) (0,4) (v) (0,−6)

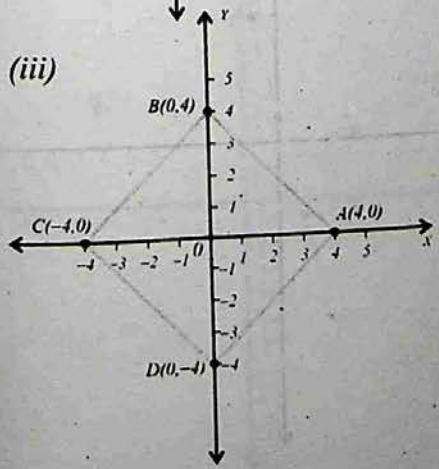
3- (i)



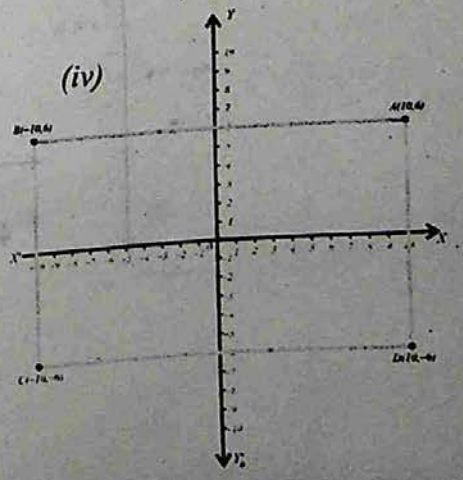
(ii)



(iii)

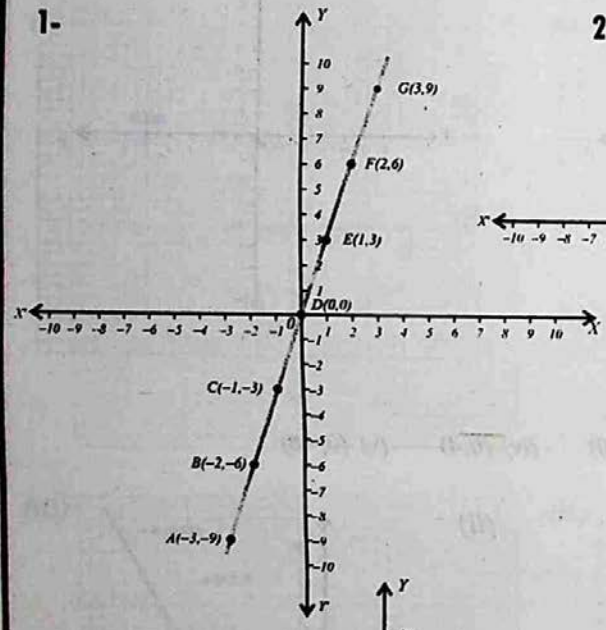


(iv)



Exercise 9.2

1-



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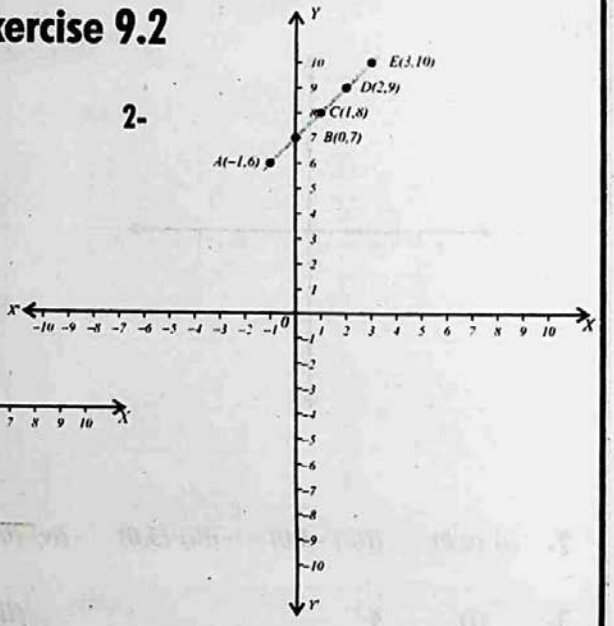
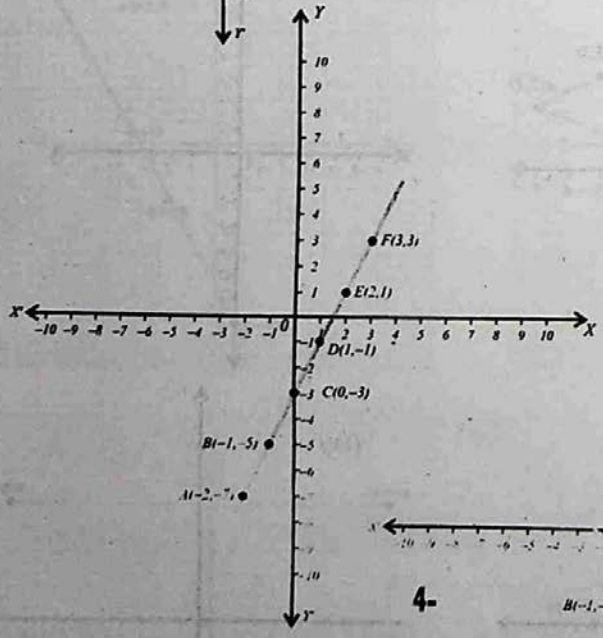


3-



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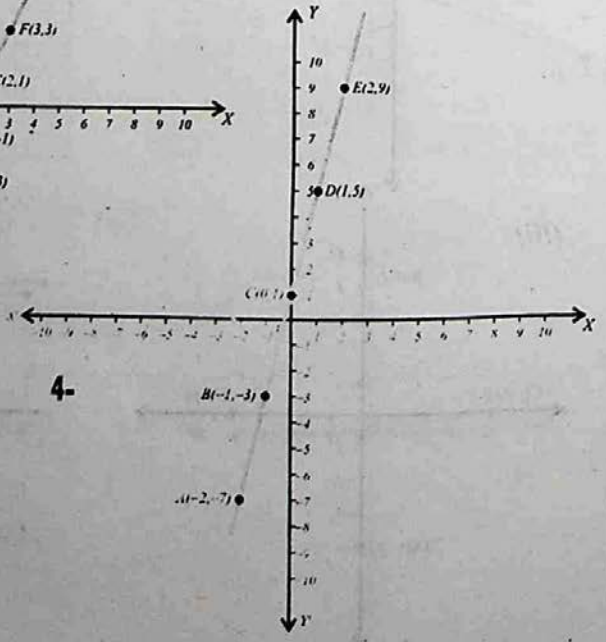
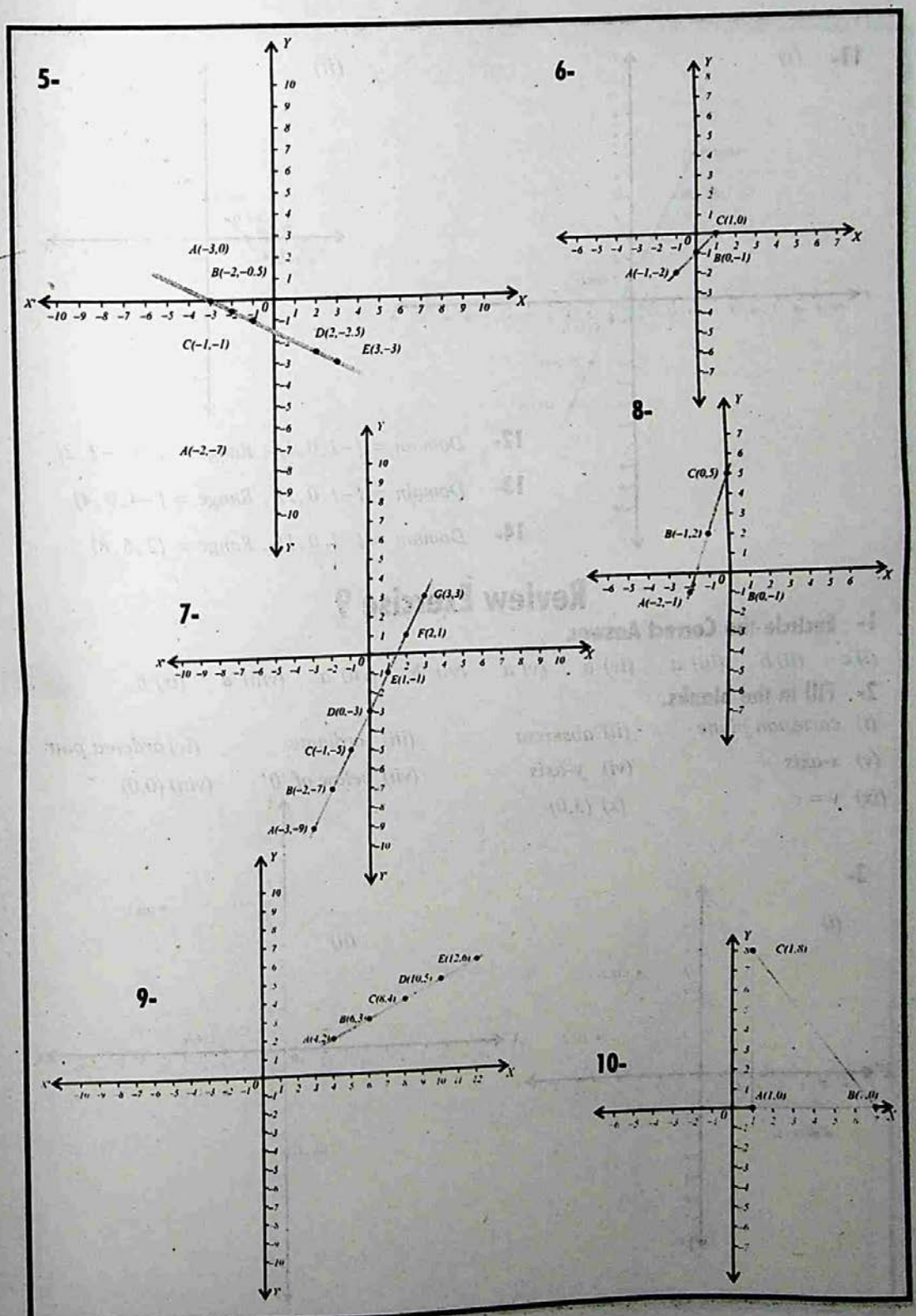
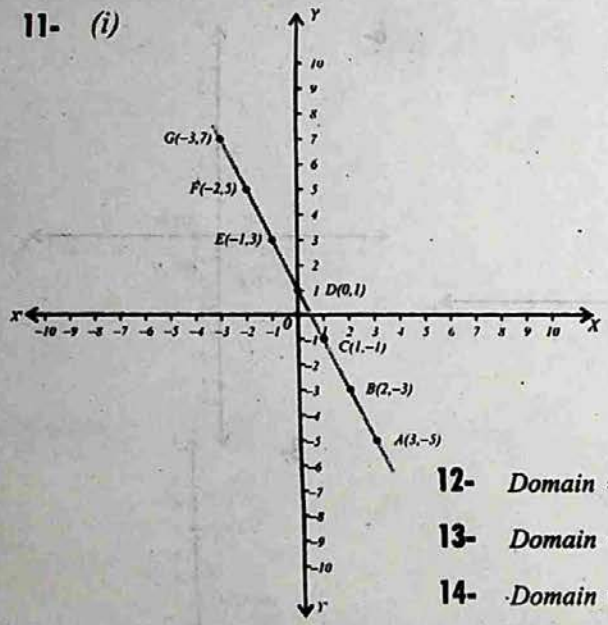
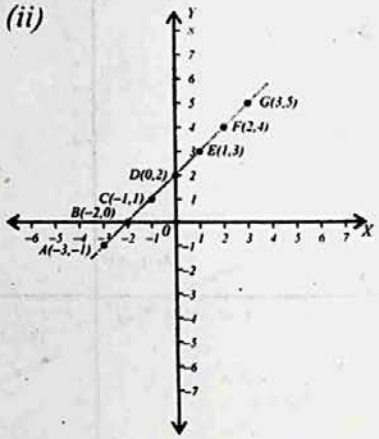




11- *(i)*



(ii)



12- *Domain* $= \{-1, 0, 1\}$, *Range* $= \{-4, -1, 2\}$

13- *Domain* $= \{-1, 0, 1\}$, *Range* $= \{-4, 0, 4\}$

14- *Domain* $= \{-1, 0, 1\}$, *Range* $= \{2, 5, 8\}$

Review Exercise 9

1- Encircle the Correct Answer.

(i) c (ii) b (iii) a (iv) b (v) a (vi) b (vii) a (viii) a (ix) b

2- Fill in the blanks.

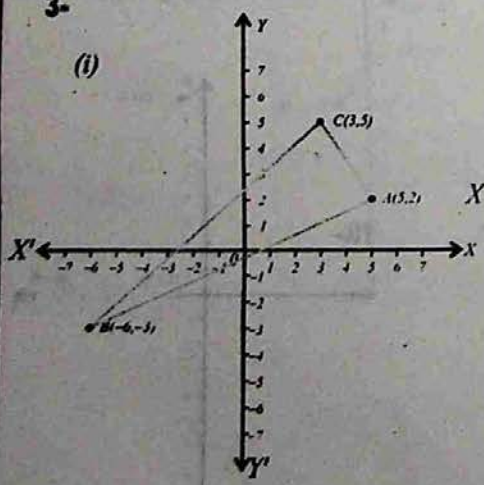
(i) cartesian plane *(ii) abscissa* *(iii) ordinate* *(iv) ordered pair*

(v) x-axis *(vi) y-axis* *(vii) below of '0'* *(viii) (0,0)*

(ix) $y = c$ *(x) (3,0)*

3-

(i)



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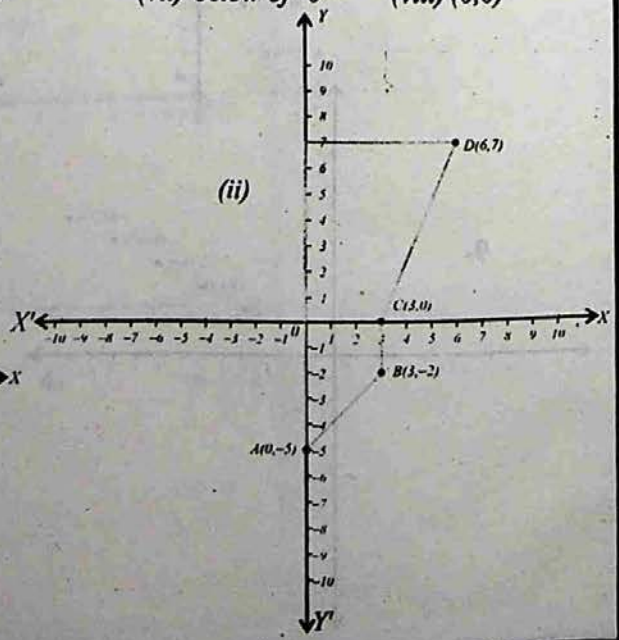

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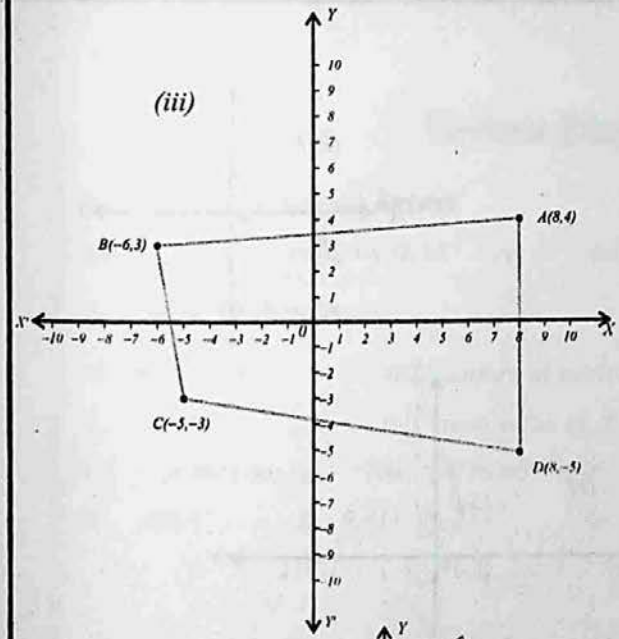
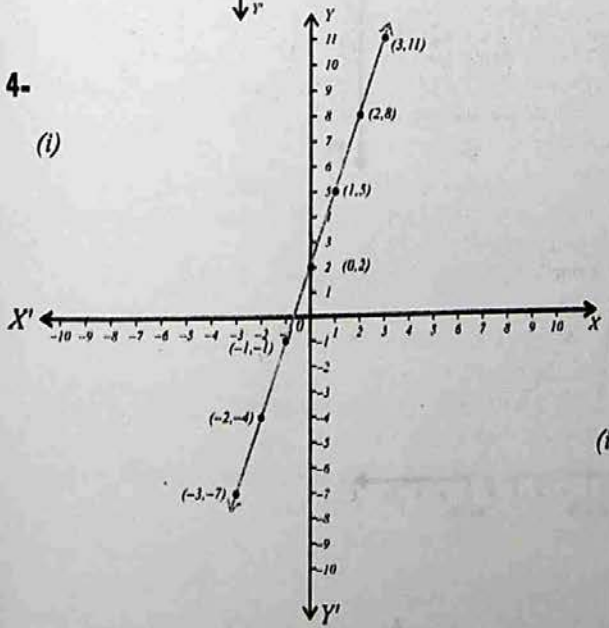


4-

(i)



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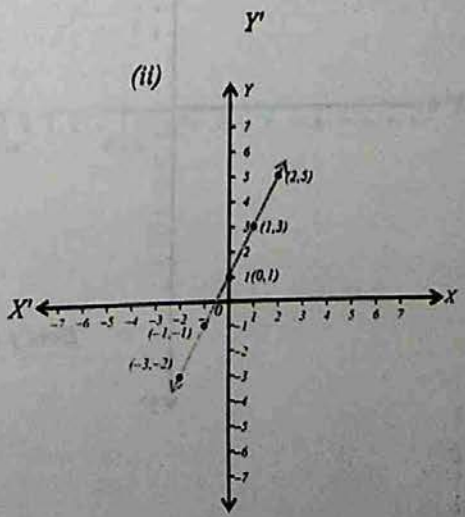
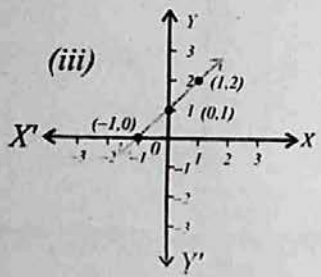
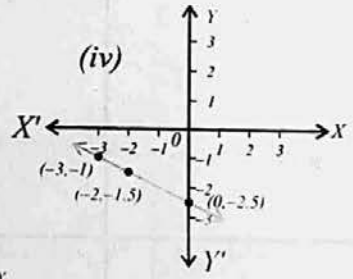


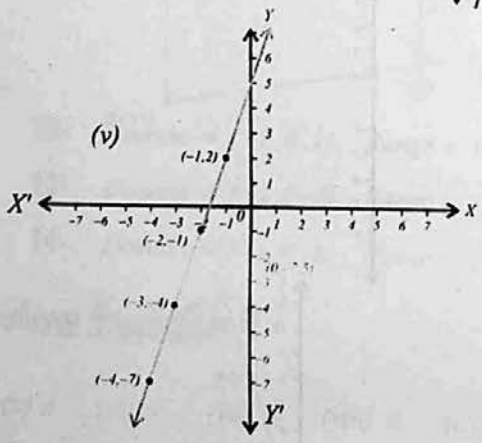
(iii)



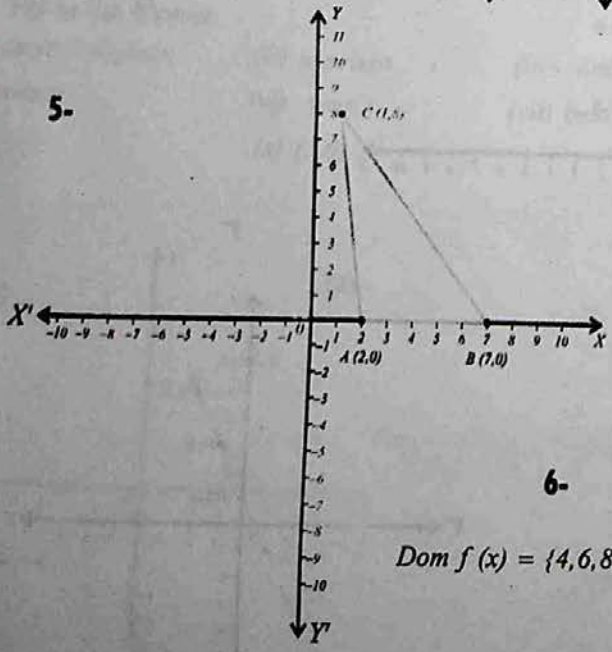
(iv)



(v)



5-



6-

Dom f (x) = {4,6,8,10,12} , Range f (x) = {2,3,4,5,6}

Review Exercise 10

1- Encircle the Correct Answer.

(i) c (ii) d (iii) a (iv) b (v) a (vi) a (vii) a (viii) b (ix) a (x) a

2- Fill in the blanks.

(i) histogram (ii) measure of central tendency (iii) mean (iv) medain

(v) mode (vi) mean value of $x_1, x_2, \ldots\ldots, x_n$

(vii) geometric mean (viii) harmonic mean (ix) weighted mean (x) arithmetic mean

3- *3.286* **4-** *9.811; 96.255* **5-** *7; 8; 8* **6-** *6.875* **7-** *4,7*

GLOSSARY

Unit-1 PERCENTAGE, RATIO AND PROPORTION

Percentage: Percentage means out of hundred.

Ratio: A comparison between the two like quantities is called a ratio.

Antecedent: In a ratio *a:b*, *"a"* is called the antecedent.

Consequent: In a ratio *a:b*, *"b"* is called the consequent.

Proportion: The equality of two ratios is called proportion.

Extremes and Means: If *a:b::c:d*, then *"a"* and *"d"* the first and fourth terms are called extremes and *"b"* and *"c"* the *2nd* and *3rd* terms are called means.

Direct Proportion: The relation between two ratios in which an increase in one quantity causes a proportional increase in the other quantity or decreases in one quantity causes a decrease in another quantity is called direct proportion.

Inverse Proportion: The relationship between two ratios in which increase in one quantity causes a proportional decreases in the other quantity or vice versa is called inverse proportion.

Compound Proportion: The relationship between two or more proportions is called a compound proportion.

Unit-2 ZAKAT, USHR AND INHERITANCE

Zakat: Zakat is a transfer payment which Sahib-e-Nisab Muslims pay at given rates by them according to sharia or through the Islamic rate to the poor and needy in or after month of Rajab.

Sahib-e-Nisab Muslim: A Muslim who owns or keeps in his / her possession at least *7.5* tola gold or *52.5* tola silver or cash money equivalent to value for one year is considered a sahib-e-nisab Muslim.

Exposed wealth: This includes agriculture goods camels, sheep, goats, minerals, business inventories etc.

Unexposed wealth: This includes gold, silver, cash money, liquid assets etc.

Rate of Zakat: Rate of zakat is *2.5 %* of the total value of the goods or money.

Ushr: It is a tax paid at the rate of *10 %* from agriculture produce of land which is irrigated by natural resources and *5 %* by artificial.

Inheritance: Distribution of remaining inheret amongst the heirs according to sharia after the burial of a deceased.

Unit-3 BUSINESS MATHEMATICS

Cost Price: The price at which a particular item is purchased is called cost price. It is denoted by *"CP"*.

Sale Price: The price at which an article is sold out is called the sale price. It is denoted by *"SP"*.

Profit: If the selling price of an article is greater than its cost price, then the difference of these two is the profit earned. It is denoted by *"P"*.

Loss: If the selling price of an article is less than its cost price, then the difference of these two is the loss. It is denoted by *"L"*.

Discount: Some times a rebate is declared on the selling price of an article, this rebate is called the discount.

Marked Price (MP): The price tagged on a card of each and every article in a shop is known as the marked price, It is denoted by "MP".

Unit-4 FINANCIAL MATHEMATICS

Current Account: A running account which continuously remains in operation due to its liquidity.

Saving Account: It is meant to encourage thrift and promote saving among the persons of small means. The bank pays nominal interest half yearly on the basis of monthly balance to the depositors.

PLS Saving Account: Profit and loss sharing account opened with small amount with profit earned on loss sustained at the end of each half year / full year depending upon the mode of payment.

Fixed / Time deposit Account: The deposits kept with the bank in a account for a certain period of time ranging from 3 months to 5 years.

Foreign Currency Accounts: Account maintained with the bank in foreign currency like dollars, pounds and Euro etc.

Negotiable Instruments: It means a promissory note, a bill exchange or cheque payable whether to be order or bearer of the instruments.

Insured: The person or entity whose insurance is being done is called "the insured".

Insurer: The company under taker the act of insurance is called the insurer.

Insured or Insurant: A person to whom an insurance policy issued, the beneficiary in a contract issuance is called insured or insurant.

Insurance Policy: The contract which is executed between two parties is called insurance policy.

Premium: The periodic installment to be paid by the insured is called premium.

Maturity: The time-period agreed upon by both the parties (insured and insurer is called maturity.)

Bonus: The agreed amount to be paid back on maturity or expiry of the agreed period, includes the actual amount paid in installments plus profit is termed as bonus.

Cheque: A bill of exchange drawn on a specific banker and not expressed to be payable other wise on demand.

Pay Order: A cheque like instruerent issued by bank on the request of its customers.

Bank Draft: An order to pay money, drawn by one office of a bank upon another office of the same bank for a sum of money payable to order on demand.

On-Line Banking: This system indicates that a direct connection is made to centralize computer system for authorization or validation before a transaction is executed.

ATM Card: It is a payment card issued to a person for activating automated teller machine computer based terminal which allow consumers to make deposits and with draws.

Credit Card: A card indicating that the holder has been granted a line of credit enabling the holder to make purchases and or with draw cash.

ATM: A machine installed by the bank to dispense cash to its account holders.

Profit: Profit is the amount which is paid by the bank on the deposits maintained by the client with the bank.

Principal: The amount / capital borrowed or lent is called principal.

Rate: The percentage of interest charged is called rate.

Time: The period of the loan or deposit is called the time.

Amount: When the interest is added to the principal, the sum is called the amount.

Mark Up: The interest earned by the bank is named as mark up.

Leasing: Lease is a contract where by the owner of an asset, the lessor, gives the hirer, the lessee, the right to use the asset for a specific period in exchange of rental payment.

Down Payment: The customer is required to deposit the payment with the bank along with the application form.

Unit-5 CONSUMER MATHEMATICS

Tax: Money that must be paid to the state, charged as a proportion of income and profits or added to the cost of same goods and services.

Direct Tax: These are the taxes which are charged on income, property and profits in the from of income tax, property tax.

Indirect Tax: Indirect taxes include duties, motor vehicle taxes, goods and services taxes *(GST)* general sale tax and value added taxes etc.

Sales Tax: When we buy article we have to pay a certain amount of tax as the value added tax in addition to the price of the article. This tax usually given as a certain percentage of the selling price. In Pakistan sales tax of *17%* is imposed on goods bought and services rendered.

Excise Duty: It is the form of a tax which the buyer pay on a manufactured item at the time of purchase.

Property Tax: A property tax is changed on the owner of land, house, flats or building at a standard rate of *16%* on annual value of the property.

Income Tax: It is the tax charged on all taxable incomes during the year from *1st* July to next *30th* June.

Unit-6 EXPONENTS AND LOGARITHMS

Rational Number: A number of the form $\frac{p}{q}, q \neq 0, p, q \in I$ is called a rational number.

Irrational Number: A number of form $\sqrt{2}, \sqrt{3}, \sqrt{5}, \pi$.. are called irrational numbers.

Radicals: If $\sqrt[n]{a}$ is irrational, where "a" is rational number, then $\sqrt[n]{a}$ is called a radical.

Radicands: The symbol $\sqrt[n]{\ }$ is called radicands .

Base and Exponent: For any real number "a" and a positive integer "n" we define $a^n = a \times a \times a \times \ldots\ldots \times a$ *(n times)*. Here "a" is called the base and "n" the exponent.

Pure Radical: A radical which has unity (one) only as rational factor, the other factor being irrational is called a pure radical.

Mixed Radical: A radical which has a rational factor other than unity, the other factor being irrational is called a mixed radical.

Common Logarithm: The logarithm calculated to the base *10* is called a common logarithm.

Characteristic and Mantissa: The logarithm of a number consists of two parts, the integral part is called the characteristic and the decimal part is called the mantissa.

Unit-7 ARITHMETIC AND GEOMETRIC SEQUENCES

Sequence: A sequence is a function whose domain D is a set of positive integers.

Arithmetic Sequence: A sequence in which each term is obtained from the previous term by adding a fixed number is called an arithmetic sequence.

Arithmetic Mean: A number "A" is said to be an arithmetic mean between the two numbers a and b if a, A, b is arithmetic sequence.

Geometric Sequence: A sequence in which each term is obtained from the previous term by multiplying with a common ratio is called a geometric sequence.

Geometric Mean: A number "G" is said to be a geometric mean between the two numbers a and b if a, G, b is a geometric sequence.

Unit-8 SETS AND FUNCTIONS

Set: A collection of well defined distinct objects is called a set.

$N = \{1, 2, 3, ...\}$ is called set of natural numbers.

$M = \{0, 1, 2, 3, ...\}$ is called set of whole numbers.

$I = \{....., -1, 0, 1, 2, 3, ...\}$ is called set of integers.

$Q = \{p/q, q \neq 0, p, q \in I\}$ is called set of rational numbers.

$Q' = A$ set of irrational numbers.

$R = QUQ' =$ A set of real numbers.

Universal Set: If there are some sets under consideration there happens to be a set, which is a super set of each one of the given sets, such a set is called the universal set denoted by U.

Complement of a Set: Let A be a sub-set of a universal set U. Then complement of A is denoted by A' or $U-A$ is the set of all those element of U which are not in A.

Binary Relation: Let A and B any two sets, then any sub-set the cartesian product $A \times B$ is called a binary relation from A to B.

Function: Any binary relation 'f' between two non-empty sets A and B such that:

(i) $Dom\ f = A$

(ii) First element in any two of the ordered pairs of f are not repeated, then f is called a function from A to B.

Unit-9 LINEAR GRAPHS

Plane: The walls of a class room, the black board, the top of the desk and the top of the table are all examples of a plane.

Cartesian Plane: A Cartesian plane consists of two number lines OX and OY intersecting at right angle at "O".

Ordinate: The perpendicular distance of a point from X-axis is called an ordinate.

Unit-10 BASIC STATISTICS

Histogram: When a bar chart is construed, so that the area of each bar is proportional to the number of items in each group is called a histogram.

Cumulative Frequency: Cumulative frequency is running total of class frequency.

Cumulative Frequency Polygon or (ogive): When the cumulative frequencies are plotted against the end point of their respective class intervals are joined together, the resultant graph is called a cumulative frequency polygon or ogive.

Measures of Central Tendency: These are summary statistics which measure the middle (or center) of the data.

Mean: To obtain the mean of a ungrouped data, all numbers in the set are added together and then the total is divided by the number of scores in that set.

Median: The middle value of data arranged in numerical order is called median.

Mode: The mode is the score which occurs most often in a set of data.

Arithmetic Mean: It is defined as the value obtained by dividing the sum of the values by their numbers. Thus the mean value of $x_1, x_2, \ldots\ldots, x_n$ denoted by $\bar{x}$ is :

$$\bar{x} = \frac{x_1, x_2, \ldots\ldots, x_n}{n}$$

Properties of arithmetic mean or mean: *(i)* The sum of deviations of values from their mean is zero. Symbolically: $\sum(x_i - \bar{x}) = 0$ or $\sum f_i(x_i - \bar{x}) = 0$

(ii) $x = \dfrac{\sum n\bar{x}}{\sum n}$

(iii) $\sum(x_i - a)^2$ is a minimum if and only if $a = \bar{x}$.

SYMBOLS

Symbol	Stands for	Symbol	Stands for
$<$	is less than	$\because$	because / as
$>$	is greater than	$\therefore$	therefore / so
$\leq$	is less than or equal to	:	ratio
$\geq$	is greater than or equal to	::	is proportional to
$=$	is equal to	$\propto$	varies
$\neq$	is not equal to	|	tally mark
$\not<$	is not less than	Σ	summation
$\not>$	is not greater than	$\overline{AB}$	line segment AB
$\in$	belongs to / element	$\overrightarrow{AB}$	ray AB
$\forall$	for all	$\overleftrightarrow{AB}$	line
$\sqrt{}$	square root	$\angle$	angle
$\lvert x \rvert$	absolute value of x	$\triangle$	triangle
$\Rightarrow$	implies that	$\sim$	is similar to
$\Leftrightarrow$	if and only if	$\cong$	is congruent to
$\wedge$	and	$\approx$	is approximately equal to
$\cup$	union	$\parallel$	is parallel to
$\vee$	or	$\overset{\frown}{AB}$	arc AB
$\cap$	intersection	$\leftrightarrow$	correspondence
$\notin$	not element / not	U	universal sett

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LOGARITHMS

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
10	0000	0043	0086	0128	0170	0212	0253	0294	0334	0374	4	9	13	17	21	26	30	34	38
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11	0414	0453	0492	0531	0569	0607	0645	0682	0719	0755	4	8	12	15	19	23	27	31	35
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27	4314	4330	4346	4362	4378	4393	4409	4425	4440	4456	2	3	5	6	8	9	11	13	14
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35	5441	5453	5465	5478	5490	5502	5514	5527	5539	5551	1	2	4	5	6	7	9	10	11
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37	5682	5694	5705	5717	5729	5740	5752	5763	5775	5786	1	2	3	5	6	7	8	9	10
38	5798	5809	5821	5832	5843	5855	5866	5877	5888	5899	1	2	3	5	6	7	8	9	10
39	5911	5922	5933	5944	5955	5966	5977	5988	5999	6010	1	2	3	4	5	7	8	9	10
40	6021	6031	6042	6053	6064	6075	6085	6096	6107	6117	1	2	3	4	5	6	8	9	10
41	6128	6138	6149	6160	6170	6180	6191	6201	6212	6222	1	2	3	4	5	6	7	8	9
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43	6335	6345	6355	6365	6375	6385	6395	6405	6415	6425	1	2	3	4	5	6	7	8	9
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45	6532	6542	6551	6561	6571	6580	6590	6599	6609	6618	1	2	3	4	5	6	7	8	9
46	6628	6637	6646	6656	6665	6675	6684	6693	6702	6712	1	2	3	4	5	6	7	7	8
47	6721	6730	6739	6749	6758	6767	6776	6785	6794	6803	1	2	3	4	5	5	6	7	8
48	6812	6821	6830	6839	6848	6857	6866	6875	6884	6893	1	2	3	4	4	5	6	7	8
49	6902	6911	6920	6928	6937	6946	6955	6964	6972	6981	1	2	3	4	4	5	6	7	8

LOGARITHMS

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
50	6990	6998	7007	7016	7024	7033	7042	7050	7059	7067	1	2	3	3	4	5	6	7	8
51	7076	7084	7093	7101	7110	7118	7126	7135	7143	7152	1	2	3	3	4	5	6	7	8
52	7160	7168	7177	7185	7193	7202	7210	7218	7226	7235	1	2	2	3	4	5	6	7	7
53	7243	7251	7259	7267	7275	7284	7292	7300	7308	7316	1	2	2	3	4	5	6	6	7
54	7324	7332	7340	7348	7356	7364	7372	7380	7388	7396	1	2	2	3	4	5	6	6	7
55	7404	7412	7419	7427	7435	7443	7451	7459	7466	7474	1	2	2	3	4	5	5	6	7
56	7482	7490	7497	7505	7513	7520	7528	7536	7543	7551	1	2	2	3	4	5	5	6	7
57	7559	7566	7574	7582	7589	7597	7604	7612	7619	7627	1	2	2	3	4	5	5	6	7
58	7634	7642	7649	7657	7664	7672	7679	7686	7694	7701	1	1	2	3	4	4	5	6	7
59	7709	7716	7723	7731	7738	7745	7752	7760	7767	7774	1	1	2	3	4	4	5	6	7
60	7782	7789	7796	7803	7810	7818	7825	7832	7839	7846	1	1	2	3	4	4	5	6	6
61	7853	7860	7868	7875	7882	7889	7896	7903	7910	7917	1	1	2	3	4	4	5	6	6
62	7924	7931	7938	7945	7952	7959	7966	7973	7980	7987	1	1	2	3	3	4	5	6	6
63	7993	8000	8007	8014	8021	8028	8035	8041	8048	8055	1	1	2	3	3	4	5	5	6
64	8062	8069	8075	8082	8089	8096	8102	8109	8116	8122	1	1	2	3	3	4	5	5	6
65	8129	8136	8142	8149	8156	8162	8169	8176	8182	8189	1	1	2	3	3	4	5	5	6
66	8195	8202	8209	8215	8222	8228	8235	8241	8248	8254	1	1	2	3	3	4	5	5	6
67	8261	8267	8274	8280	8287	8293	8299	8306	8312	8319	1	1	2	3	3	4	5	5	6
68	8325	8331	8338	8344	8351	8357	8363	8370	8376	8382	1	1	2	3	3	4	4	5	6
69	8388	8395	8401	8407	8414	8420	8426	8432	8439	8445	1	1	2	2	3	4	4	5	6
70	8451	8457	8463	8470	8476	8482	8488	8494	8500	8506	1	1	2	2	3	4	4	5	6
71	8513	8519	8525	8531	8537	8543	8549	8555	8561	8567	1	1	2	2	3	4	4	5	5
72	8573	8579	8585	8591	8597	8603	8609	8615	8621	8627	1	1	2	2	3	4	4	5	5
73	8633	8639	8645	8651	8657	8663	8669	8675	8681	8686	1	1	2	2	3	4	4	5	5
74	8692	8698	8704	8710	8716	8722	8727	8733	8738	8745	1	1	2	2	3	4	4	5	5
75	8751	8756	8762	8768	8774	8779	8785	8791	8797	8802	1	1	2	2	3	3	4	5	5
76	8808	8814	8820	8825	8831	8837	8842	8848	8854	8859	1	1	2	2	3	3	4	5	5
77	8865	8871	8876	8882	8887	8893	8899	8904	8910	8915	1	1	2	2	3	3	4	4	5
78	8921	8927	8932	8938	8943	8949	8954	8960	8965	8971	1	1	2	2	3	3	4	4	5
79	8976	8982	8987	8993	8998	9004	9009	9015	9020	9025	1	1	2	2	3	3	4	4	5
80	9031	9036	9042	9047	9053	9058	9063	9069	9074	9079	1	1	2	2	3	3	4	4	5
81	9085	9090	9096	9101	9106	9112	9117	9122	9128	9133	1	1	2	2	3	3	4	4	5
82	9138	9143	9149	9154	9159	9165	9170	9175	9180	9186	1	1	2	2	3	3	4	4	5
83	9191	9196	9201	9206	9212	9217	9222	9227	9232	9238	1	1	2	2	3	3	4	4	5
84	9243	9248	9253	9258	9263	9269	9274	9279	9284	9289	1	1	2	2	3	3	4	4	5
85	9294	9299	9304	9309	9315	9320	9325	9330	9335	9340	1	1	2	2	3	3	4	4	5
86	9345	9350	9355	9360	9365	9370	9375	9380	9385	9390	1	1	2	2	3	3	4	4	5
87	9395	9400	9405	9410	9415	9420	9425	9430	9435	9440	0	1	1	2	2	3	3	4	4
88	9445	9450	9455	9460	9465	9469	9474	9479	9484	9489	0	1	1	2	2	3	3	4	4
89	9494	9499	9504	9509	9513	9518	9523	9528	9533	9538	0	1	1	2	2	3	3	4	4
90	9542	9547	9552	9557	9562	9566	9571	9576	9581	9586	0	1	1	2	2	3	3	4	4
91	9590	9595	9600	9605	9609	9614	9619	9624	9628	9633	0	1	1	2	2	3	3	4	4
92	9638	9643	9647	9652	9657	9661	9666	9671	9675	9680	0	1	1	2	2	3	3	4	4
93	9685	9689	9694	9699	9703	9708	9713	9717	9722	9727	0	1	1	2	2	3	3	4	4
94	9731	9736	9741	9745	9750	9754	9759	9763	9768	9773	0	1	1	2	2	3	3	4	4
95	9777	9782	9786	9791	9795	9800	9805	9809	9814	9818	0	1	1	2	2	3	3	4	4
96	9823	9827	9832	9836	9841	9845	9850	9854	9859	9863	0	1	1	2	2	3	3	4	4
97	9868	9872	9877	9881	9886	9890	9894	9899	9903	9908	0	1	1	2	2	3	3	4	4
98	9912	9917	9921	9926	9930	9934	9939	9943	9948	9952	0	1	1	2	2	3	3	4	4
99	9956	9961	9965	9969	9974	9978	9983	9987	9991	9996	0	1	1	2	2	3	3	3	4

ANTI LOGARITHMS

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.00	1000	1002	1005	1007	1009	1012	1014	1016	1019	1021	0	0	1	1	1	1	2	2	2
.01	1023	1026	1027	1030	1033	1035	1038	1040	1042	1045	0	0	1	1	1	1	2	2	2
.02	1047	1050	1052	1054	1057	1059	1062	1064	1067	1069	0	0	1	1	1	1	2	2	2
.03	1072	1074	1076	1079	1081	1084	1086	1089	1091	1094	0	0	1	1	1	1	2	2	2
.04	1096	1099	1102	1104	1107	1109	1112	1114	1117	1119	0	0	1	1	1	2	2	2	2
.05	1122	1125	1127	1130	1132	1135	1138	1140	1143	1146	0	1	1	1	1	2	2	2	2
.06	1148	1151	1153	1156	1159	1161	1164	1167	1169	1172	0	1	1	1	1	2	2	2	2
.07	1175	1178	1180	1183	1186	1189	1191	1194	1197	1199	0	1	1	1	1	2	2	2	2
.08	1202	1205	1208	1211	1213	1216	1219	1222	1225	1227	0	1	1	1	1	2	2	2	3
.09	1230	1235	1236	1239	1242	1245	1247	1250	1253	1256	0	1	1	1	1	2	2	2	3
.10	1259	1262	1265	1268	1271	1274	1276	1279	1282	1285	0	1	1	1	1	2	2	2	3
.11	1288	1291	1294	1297	1300	1303	1306	1309	1312	1315	0	1	1	1	2	2	2	2	3
.12	1318	1321	1324	1327	1330	1334	1337	1340	1343	1346	0	1	1	1	2	2	2	2	3
.13	1349	1352	1355	1358	1361	1365	1368	1371	1374	1377	0	1	1	1	2	2	2	3	3
.14	1380	1384	1387	1390	1393	1396	1400	1403	1406	1409	0	1	1	1	2	2	2	3	3
.15	1413	1416	1419	1422	1426	1429	1432	1435	1439	1442	0	1	1	1	2	2	2	3	3
.16	1445	1449	1452	1455	1459	1462	1466	1469	1472	1476	0	1	1	1	2	2	2	3	3
.17	1479	1483	1486	1489	1493	1496	1500	1503	1507	1510	0	1	1	1	2	2	2	3	3
.18	1514	1517	1521	1524	1528	1531	1535	1538	1542	1545	0	1	1	1	2	2	2	3	3
.19	1549	1552	1556	1560	1563	1567	1570	1574	1578	1581	0	1	1	1	2	2	3	3	3
.20	1585	1589	1592	1596	1600	1603	1607	1611	1614	1618	0	1	1	1	2	2	3	3	3
.21	1622	1626	1629	1633	1637	1641	1644	1648	1652	1656	0	1	1	2	2	2	3	3	3
.22	1660	1663	1667	1671	1675	1679	1683	1687	1690	1694	0	1	1	2	2	2	3	3	3
.23	1698	1702	1706	1710	1714	1718	1722	1726	1730	1734	0	1	1	2	2	2	3	3	4
.24	1738	1742	1746	1750	1754	1758	1762	1766	1770	1774	0	1	1	2	2	2	3	3	4
.25	1778	1782	1786	1791	1795	1799	1803	1807	1811	1816	0	1	1	2	2	2	3	3	4
.26	1820	1824	1828	1832	1837	1841	1845	1849	1854	1858	0	1	1	2	2	3	3	3	4
.27	1862	1866	1871	1875	1879	1884	1888	1892	1897	1901	0	1	1	2	2	3	3	3	4
.28	1905	1910	1914	1919	1923	1928	1932	1936	1941	1945	0	1	1	2	2	3	3	4	4
.29	1950	1954	1959	1963	1968	1972	1977	1982	1986	1991	0	1	1	2	2	3	3	4	4
.30	1995	2000	2004	2009	2014	2018	2023	2028	2032	2037	0	1	1	2	2	3	3	4	4
.31	2042	2046	2051	2056	2061	2065	2070	2075	2080	2084	0	1	1	2	2	3	3	4	4
.32	2089	2094	2099	2104	2109	2113	2118	2123	2128	2133	0	1	1	2	2	3	3	4	4
.33	2138	2143	2148	2153	2158	2163	2168	2173	2178	2183	0	1	1	2	2	3	3	4	4
.34	2188	2193	2198	2203	2208	2213	2218	2223	2228	2234	1	1	2	2	3	3	4	4	5
.35	2239	2244	2249	2254	2259	2265	2270	2275	2280	2286	1	1	2	2	3	3	4	4	5
.36	2291	2296	2301	2307	2312	2317	2323	2328	2333	2339	1	1	2	2	3	3	4	4	5
.37	2344	2350	2355	2360	2366	2371	2377	2382	2388	2393	1	1	2	2	3	3	4	4	5
.38	2399	2404	2410	2415	2421	2427	2432	2438	2443	2449	1	1	2	2	3	3	4	4	5
.39	2455	2460	2466	2472	2477	2483	2489	2495	2500	2506	1	1	2	2	3	3	4	5	5
.40	2512	2518	2523	2529	2535	2541	2547	2553	2559	2564	1	1	2	2	3	4	4	5	5
.41	2570	2576	2582	2588	2594	2600	2606	2612	2618	2624	1	1	2	2	3	4	4	5	5
.42	2630	2636	2642	2649	2655	2661	2667	2673	2679	2685	1	1	2	2	3	4	4	5	6
.43	2692	2698	2704	2710	2716	2723	2729	2735	2742	2748	1	1	2	3	3	4	4	5	6
.44	2754	2761	2767	2773	2780	2786	2793	2799	2805	2812	1	1	2	3	3	4	4	5	6
.45	2818	2825	2831	2838	2844	2851	2858	2864	2871	2877	1	1	2	3	3	4	5	5	6
.46	2884	2891	2897	2904	2911	2917	2924	2931	2938	2944	1	1	2	3	3	4	5	5	6
.47	2951	2958	2965	2972	2979	2985	2992	2999	3006	3013	1	1	2	3	3	4	5	5	6
.48	3020	3027	3034	3041	3048	3055	3062	3069	3076	3083	1	1	2	3	4	4	5	6	6
.49	3090	3097	3105	3112	3119	3126	3133	3141	3148	3155	1	1	2	3	4	4	5	6	6

ANTI LOGARITHMS

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.50	3162	3170	3177	3184	3192	3199	3206	3214	3221	3228	1	1	2	3	4	4	5	6	7
.51	3236	3243	3251	3258	3266	3273	3281	3289	3296	3304	1	2	2	3	4	5	5	6	7
.52	3311	3319	3327	3334	3342	3350	3357	3365	3373	3381	1	2	2	3	4	5	5	6	7
.53	3388	3396	3404	3412	3420	3428	3436	3443	3451	3459	1	2	2	3	4	5	6	6	7
.54	3467	3475	3483	3491	3499	3508	3516	3524	3532	3540	1	2	2	3	4	5	6	6	7
.55	3548	3556	3565	3573	3581	3589	3597	3606	3614	3622	1	2	2	3	4	5	6	7	7
.56	3631	3639	3648	3656	3664	3673	3681	3690	3698	3707	1	2	3	3	4	5	6	7	8
.57	3715	3724	3733	3741	3750	3758	3767	3776	3784	3793	1	2	3	3	4	5	6	7	8
.58	3802	3811	3819	3828	3837	3846	3855	3864	3873	3882	1	2	3	4	4	5	6	7	8
.59	3890	3899	3908	3917	3926	3936	3945	3954	3963	3972	1	2	3	4	5	6	6	7	8
.60	3981	3990	3999	4009	4018	4027	4036	4046	4055	4064	1	2	3	4	5	6	6	7	8
.61	4074	4083	4093	4102	4111	4121	4130	4140	4150	4159	1	2	3	4	5	6	7	8	9
.62	4169	4178	4188	4198	4207	4217	4227	4236	4246	4256	1	2	3	4	5	6	7	8	9
.63	4266	4276	4285	4295	4305	4315	4325	4335	4345	4355	1	2	3	4	5	6	7	8	9
.64	4365	4375	4385	4395	4406	4416	4426	4436	4446	4457	1	2	3	4	5	6	7	8	9
.65	4467	4477	4487	4498	4508	4519	4529	4539	4550	4560	1	2	3	4	5	6	7	8	9
.66	4571	4581	4592	4603	4613	4624	4634	4645	4656	4667	1	2	3	4	5	6	7	9	10
.67	4677	4688	4699	4710	4721	4732	4742	4753	4764	4775	1	2	3	4	5	7	8	9	10
.68	4786	4797	4808	4819	4831	4842	4853	4864	4875	4887	1	2	3	4	6	7	8	9	10
.69	4898	4909	4920	4932	4943	4955	4966	4977	4989	5000	1	2	3	5	6	7	8	9	10
.70	5012	5023	5035	5047	5058	5070	5082	5093	5105	5117	1	2	4	5	6	7	8	9	11
.71	5129	5140	5152	5164	5176	5188	5200	5212	5224	5236	1	2	4	5	6	7	8	10	11
.72	5248	5260	5272	5284	5297	5309	5321	5333	5346	5358	1	2	4	5	6	7	9	10	11
.73	5370	5383	5395	5408	5420	5433	5445	5458	5470	5483	1	3	4	5	6	8	9	10	11
.74	5495	5508	5521	5534	5546	5559	5572	5585	5598	5610	1	3	4	5	6	8	9	10	12
.75	5623	5636	5649	5662	5675	5689	5702	5715	5728	5741	1	3	4	5	7	8	9	10	12
.76	5754	5768	5781	5794	5808	5821	5834	5848	5861	5875	1	3	4	5	7	8	9	11	12
.77	5888	5902	5916	5929	5943	5957	5970	5984	5998	6012	1	3	4	5	7	8	10	11	12
.78	6026	6039	6053	6067	6081	6095	6109	6124	6138	6152	1	3	4	6	7	8	10	11	13
.79	6166	6180	6194	6209	6223	6237	6252	6266	6281	6295	1	3	4	6	7	9	10	11	13
.80	6310	6324	6339	6353	6368	6383	6397	6412	6427	6442	1	3	4	6	7	9	10	12	13
.81	6457	6471	6486	6501	6516	6531	6546	6561	6577	6592	2	3	5	6	8	9	11	12	14
.82	6607	6622	6637	6653	6668	6683	6699	6714	6730	6745	2	3	5	6	8	9	11	12	14
.83	6761	6776	6792	6808	6823	6839	6855	6871	6887	6902	2	3	5	6	8	9	11	13	14
.84	6918	6934	6950	6966	6982	6998	7015	7031	7047	7063	2	3	5	6	8	10	11	13	15
.85	7079	7096	7112	7129	7145	7161	7178	7194	7211	7228	2	3	5	7	8	10	12	13	15
.86	7244	7261	7278	7295	7311	7328	7345	7362	7379	7396	2	3	5	7	8	10	12	13	15
.87	7413	7430	7447	7464	7482	7499	7516	7534	7551	7568	2	3	5	7	9	10	12	14	16
.88	7586	7603	7621	7638	7656	7674	7691	7709	7727	7745	2	4	5	7	9	11	12	14	16
.89	7762	7780	7798	7816	7834	7852	7870	7889	7907	7925	2	4	5	7	9	11	13	14	16
.90	7943	7962	7980	7998	8017	8035	8054	8072	8091	8110	2	4	6	7	9	11	13	15	17
.91	8128	8147	8166	8185	8204	8222	8241	8260	8279	8299	2	4	6	8	9	11	13	15	17
.92	8318	8337	8356	8375	8395	8414	8433	8453	8472	8492	2	4	6	8	10	12	14	15	17
.93	8511	8531	8551	8570	8590	8610	8630	8650	8670	8690	2	4	6	8	10	12	14	16	18
.94	8710	8730	8750	8770	8790	8810	8831	8851	8872	8892	2	4	6	8	10	12	14	16	18
.95	8913	8933	8954	8974	8995	9016	9036	9057	9078	9099	2	4	6	8	10	12	15	17	19
.96	9120	9141	9162	9183	9204	9226	9247	9268	9290	9311	2	4	6	8	11	13	15	17	19
.97	9333	9354	9376	9397	9419	9441	9462	9484	9506	9528	2	4	7	9	11	13	15	17	20
.98	9550	9572	9594	9616	9638	9661	9683	9705	9727	9750	2	4	7	9	11	13	16	18	20
.99	9772	9795	9817	9840	9863	9886	9908	9931	9954	9977	2	5	7	9	11	14	16	18	20