Chapter 1

MEASUREMENTS

Learning Objectives

At the end of this chapter the students will be able to:

1. Understand what is Physics.
2. Understand that all physical quantities consist of a numerical magnitude and a unit.
3. Recall the following base quantities and their units; mass (kg), length (m), time (s), current (A), temperature (K), luminous intensity (cd) and amount of substance (mol).
4. Describe and use base units, supplementary units, and derived units.
5. Understand and use the scientific notation.
6. Use the standard prefixes and their symbols to indicate decimal sub-multiples or multiples to both base and derived units.
7. Understand and use the conventions for indicating units.
8. Understand the distinction between systematic errors and random errors.
9. Understand and use the significant figures.
10. Understand the distinction between precision and accuracy.
11. Assess the uncertainty in a derived quantity by simple addition of actual, fractional or percentage uncertainties.
12. Quote answers with correct scientific notation, number of significant figures and units in all numerical and practical work.
13. Use dimensionality to check the homogeneity of physical equations.
14. Derive formulae in simple cases using dimensions.

Eversince man has started to observe, think and reason he has been wondering about the world around him. He tried to find ways to organize the disorder prevailing in the observed facts about the natural phenomena and material things in an orderly manner. His attempts resulted in the birth of a single discipline of science, called natural philosophy. There was a huge increase in the volume of scientific knowledge up till the beginning of nineteenth century and it was found necessary to classify the study of nature into two branches, the biological sciences which deal with living things and physical sciences which concern with non-living things. Physics is an important and basic part of physical sciences besides its other disciplines such as chemistry, astronomy, geology etc. Physics is an experimental science and the scientific method emphasizes the need of accurate measurement of various measurable features of different phenomena or of man made objects. This chapter emphasizes the need of thorough understanding and practice of measuring techniques and recording skills.

Areas of Physics

Mechanics
Heat & thermodynamics
Electromagnetism
Optics
Sound
Hydrodynamics
Special relativity
General relativity
Quantum mechanics
Atomic physics
Molecular physics
Nuclear physics
Solid-state physics
Particle physics
Superconductivity
Super fluidity
Plasma physics
Magneto hydrodynamics
Space physics

1.1 INTRODUCTION TO PHYSICS

At the present time, there are three main frontiers of fundamental science. First, the world of the extremely large, the universe itself, Radio telescopes now gather information from the far side of the universe and have recently detected, as radio waves, the "firelight" of the big bang which probably started off the expanding universe nearly 20 billion years ago. Second, the world of the extremely small, that of the particles such as, electrons, protons, neutrons, mesons and others. The third frontier is the world of complex matter. It is also the World of "middle-sized" things, from molecules at one extreme to the Earth at the other. This is all fundamental physics, which is the heart of science.

Interdisciplinary areas of Physics

Astrophysics
Biophysics
Chemical physics
Engineering physics
Geophysics
Medical physics
Physical oceanography
Physics of music

But what is physics? According to one definition, physics deals with the study of matter and energy and the relationship between them. The study of physics involves investigating such things as the laws of motion, the structure of space and time, the nature and type of forces that hold different materials together, the interaction between different particles, the interaction of electromagnetic radiation with matter and so on.

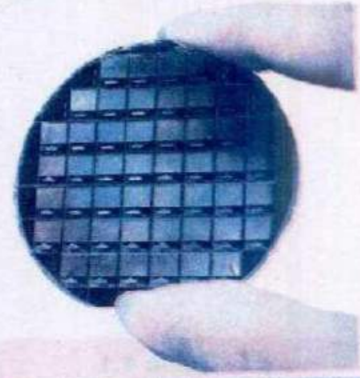
By the end of 19th century many physicists started believing that every thing about physics has been discovered. However, about the beginning of the twentieth century many new experimental facts revealed that the laws formulated by the previous investigators need modifications. Further researches gave birth to many new disciplines in physics such as nuclear physics which deals with atomic nuclei, particle physics which is concerned with the ultimate particles of which the matter is composed, relativistic mechanics which deals with velocities approaching that of light and solid state physics which is concerned with the structure and properties of solids, but this list is by no means exhaustive.

Physics is most fundamental of all sciences and provides other branches of science, basic principles and fundamental laws. This overlapping of physics and other fields gave birth to new branches such as physical chemistry, biophysics, astrophysics, health physics etc. Physics also plays an important role in the development of technology and engineering.

Science and technology are a potent force for change in the outlook of mankind. The information media and fast means of communications have brought all parts of the world in close contact with one another. Events in one part of the world immediately reverberate round the globe.

We are living in the age of information technology. The computer networks are products of chips developed from the basic ideas of physics. The chips are made of silicon. Silicon can be obtained from sand. It is upto us whether we make a sandcastle or a computer out of it.

Do You Know?



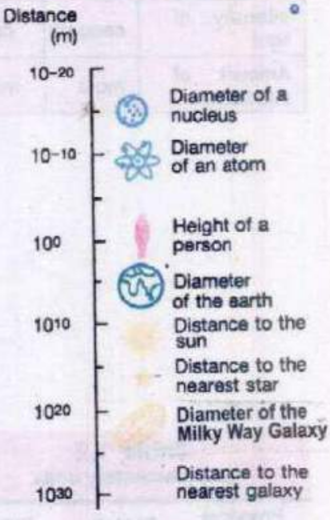
Computer chips are made from wafers of the metalloid silicon, a semiconductor.

1.2 PHYSICAL QUANTITIES

The foundation of physics rests upon physical quantities in terms of which the laws of physics are expressed. Therefore, these quantities have to be measured accurately. Among these are mass, length, time, velocity, force, density, temperature, electric current, and numerous others.

Physical quantities are often divided into two categories: base quantities and derived quantities. Derived quantities are those whose definitions are based on other physical quantities. Velocity, acceleration and force etc. are usually viewed as derived quantities. Base quantities are not defined in terms of other physical quantities. The base quantities are the minimum number of those physical quantities in terms of which other physical quantities can be defined. Typical examples of base quantities are length, mass and time.

For Your Information



Order of magnitude of some distances

The measurement of a base quantity involves two steps: first, the choice of a standard, and second, the establishment of a procedure for comparing the quantity to be measured with the standard so that a number and a unit are determined as the measure of that quantity.

An ideal standard has two principal characteristics: it is accessible and it is invariable. These two requirements are often incompatible and a compromise has to be made between them.

1.3 INTERNATIONAL SYSTEM OF UNITS

In 1960, an international committee agreed on a set of definitions and standard to describe the physical quantities. The system that was established is called the System International (SI).

Due to the simplicity and convenience with which the units in this system are amenable to arithmetical manipulation, it is in universal use by the world's scientific community and by most nations. The system international (SI) is built up from three kinds of units: base units, supplementary units and derived units.

Base Units

There are seven base units for various physical quantities namely: length, mass, time, temperature, electric current, luminous intensity and amount of a substance (with special reference to the number of particles).

The names of base units for these physical quantities together with symbols are listed in Table 1.1. Their standard definitions are given in the Appendix 1.

Table 1.1

Physical Quantity	SI Unit	Symbol
Length	metre	m
Mass	kilogram	kg
Time	second	s
Electric current	ampere	A
Thermodynamic temperature	kelvin	K
Intensity of light	candela	cd
Amount of substance	mole	mol

Supplementary Units

The General Conference on Weights and Measures has not yet classified certain units of the SI under either base units or derived units. These SI units are called supplementary units. For the time being this class contains only two units of purely geometrical quantities, which are plane angle and the solid angle (Table1.2).

Table 1.2
Supplementary units

Physical Quantity	SI Unit	Symbol
Plane angle	radian	rad
Solid angle	steradian	sr

Radian

The radian is the plane angle between two radii of a circle which cut off on the circumference an arc, equal in length to the radius, as shown in Fig. 1.1 (a).

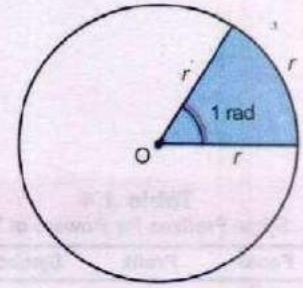


Fig. 1.1(a)

Steradian

The steradian is the solid angle (three-dimensional angle) subtended at the centre of a sphere by an area of its surface equal to the square of radius of the sphere. (Fig. 1.1 b).

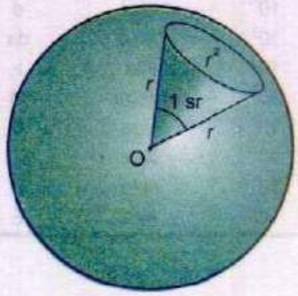


Fig. 1.1(b)

Derived Units

SI units for measuring all other physical quantities are derived from the base and supplementary units. Some of the derived units are given in Table. 1.3.

Table 1.3

Physical quantity	Unit	Symbol	In terms of base units
Force	newton	N	$kg\ m\ s^{-2}$
Work	joule	J	$N\ m = kg\ m^2\ s^{-2}$
Power	watt	W	$J\ s^{-1} = kg\ m^2\ s^{-3}$
Pressure	pascal	Pa	$N\ m^{-2} = kg\ m^{-1}\ s^{-2}$
Electric charge	coulomb	C	A s

Scientific Notation

Numbers are expressed in standard form called scientific notation, which employs powers of ten. The internationally accepted practice is that there should be only one non-zero digit left of decimal. Thus, the number 134.7 should be written as 1.347×10^2 and 0.0023 should be expressed as 2.3×10^{-3}.

Conventions for Indicating Units

Use of SI units requires special care, more particularly in writing prefixes.

Following points should be kept in mind while using units.

(i) Full name of the unit does not begin with a capital letter even if named after a scientist e.g.,newton.

Table 1.4 Some Prefixes for Powers of Ten		
Factor	**Prefix**	**Symbol**
10^{-18}	atto	a
10^{-15}	femto	f
10^{-12}	pico	p
10^{-9}	nano	n
10^{-6}	micro	μ
10^{-3}	milli	m
10^{-2}	centi	c
10^{-1}	deci	d
10^{1}	deca	da
10^{3}	kilo	k
10^{6}	mega	M
10^{9}	giga	G
10^{12}	tera	T
10^{15}	peta	P
10^{18}	exa	E

(ii) The symbol of unit named after a scientist has initial capital letter such as N for newton.

(iii) The prefix should be written before the unit without any space, such as 1×10^{-3} m is written as 1 mm. Standard prefixes are given in table 1.4.

(iv) A combination of base units is written each with one space apart. For example, newton metre is written as N m.

(v) Compound prefixes are not allowed. For example, 1μμF may be written as 1pF.

(vi) A number such as 5.0×10^4 cm may be expressed in scientific notation as 5.0×10^2 m.

(vii) When a multiple of a base unit is raised to a power, the power applies to the whole multiple and not the base unit alone. Thus, $1\ km^2 = 1\ (km)^2 = 1 \times 10^6\ m^2$.

(viii) Measurement in practical work should be recorded immediately in the most convenient unit, e.g., micrometer screw gauge measurement in mm, and the mass of calorimeter in grams (g). But before calculation for the result, all measurements must be converted to the appropriate SI base units.

1.4 ERRORS AND UNCERTAINTIES

All physical measurements are uncertain or imprecise to some extent. It is very difficult to eliminate all possible errors or uncertainties in a measurement. The error may occur due to (1) negligence or inexperience of a person (2) the faulty apparatus (3) inappropriate method or technique. The uncertainty may occur due to inadequacy or limitation of an instrument, natural variations of the object being measured or natural imperfections of a person's senses. However, the uncertainty is also usually described as an error in a measurement. There are two major types of errors.

(i) Random error **(ii) Systematic error**

Random error is said to occur when repeated measurements of the quantity, give different values under the same conditions. It is due to some unknown causes. Repeating the measurement several times and taking an average can reduce the effect of random errors.

Systematic error refers to an effect that influences all measurements of a particular quantity equally. It produces a consistent difference in readings. It occurs to some definite rule. It may occur due to zero error of instruments, poor calibration of instruments or incorrect markings etc. Systematic error can be reduced by comparing the instruments with another which is known to be more accurate. Thus for systematic error, a correction factor can be applied.

1.5 SIGNIFICANT FIGURES

As stated earlier physics is based on measurements. But unfortunately whenever a physical quantity is measured, there is inevitably some uncertainty about its determined value. This uncertainty may be due to a number of reasons. One reason is the type of instrument, being used. We know that every measuring instrument is calibrated to a certain smallest division and this fact put a limit to the degree of accuracy which may be achieved while measuring with it. Suppose that we want to measure the length of a straight line with the help of a metre rod calibrated in millimetres. Let the end point of the line lies between 10.3 and 10.4 cm marks. By convention, if the end of the line does not touch or cross the midpoint of the smallest division, the reading is confined to the previous division. In case the end of the line seems to be touching or have crossed the midpoint, the reading is extended to the next division.

By applying the above rule the position of the edge of a line recorded as 12.7 cm with the help of a metre rod calibrated in millimetres may lie between 12.65 cm and 12.75 cm. Thus in this example the maximum uncertainty is ± 0.05 cm. It is, in fact, equivalent to an uncertainty of 0.1 cm equal to the least count of the instrument divided into two parts, half above and half below the recorded reading.

The uncertainty or accuracy in the value of a measured quantity can be indicated conveniently by using significant figures. The recorded value of the length of the straight line

For Your Information

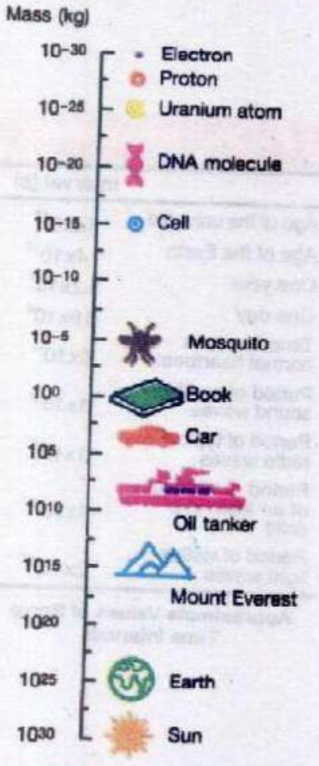
	Interval (s)
Age of the universe	5×10^{17}
Age of the Earth	1.4×10^{17}
One year	3.2×10^{7}
One day	8.6×10^{4}
Time between normal heartbeats	8×10^{-1}
Period of audible sound waves	1×10^{-3}
Period of typical radio waves	1×10^{-6}
Period of vibration of an atom in a solid	1×10^{-13}
Period of visible light waves	2×10^{-15}

Approximate Values of Some Time Intervals

i.e. 12.7 cm contains three digits (1, 2, 7) out of which two digits (1 and 2) are accurately known while the third digit i.e. 7 is a doubtful one. As a rule:

In any measurement, the accurately known digits and the first doubtful digit are called significant figures.

Interesting Information



Order of magnitude of some masses.

In other words, a significant figure is the one which is known to be reasonably reliable. If the above mentioned measurement is taken by a better measuring instrument which is exact upto a hundredth of a centimetre, it would have been recorded as 12.70 cm. In this case, the number of significant figures is four. Thus, we can say that as we improve the quality of our measuring instrument and techniques, we extend the measured result to more and more significant figures and correspondingly improve the experimental accuracy of the result. While calculating a result from the measurements, it is important to give due attention to significant figures and we must know the following rules in deciding how many significant figures are to be retained in the final result.

(i) All digits 1,2,3,4,5,6,7,8,9 are significant. However, zeros may or may not be significant. In case of zeros, the following rules may be adopted.

a) A zero between two significant figures is itself significant.

b) Zeros to the left of significant figures are not significant. For example, none of the zeros in 0.00467 or 02.59 is significant.

c) Zeros to the right of a significant figure may or may not be significant. In decimal fraction, zeros to the right of a significant figure are significant. For example, all the zeros in 3.570 or 7.4000 are significant. However, in integers such as 8,000 kg, the number of significant zeros is determined by the accuracy of the measuring instrument. If the measuring scale has a least count of 1 kg then there are four significant figures written in scientific notation as 8.000×10^3 kg. If the least count of the scale is 10 kg, then the number of significant figures will be 3 written in scientific notation as 8.00×10^3 kg and so on.

d) When a measurement is recorded in scientific notation or standard form, the figures other than the powers of ten are significant figures. For example, a measurement recorded as 8.70×10^4 kg has three significant figures.

(ii) In multiplying or dividing numbers, keep a number of significant figures in the product or quotient not more than that contained in the least accurate factor i.e., the factor containing the least number of significant figures. For example, the computation of the following using a calculator, gives

$$\frac{5.348 \times 10^{-2} \times 3.64 \times 10^{4}}{1.336} = 1.45768982 \times 10^{3}$$

As the factor 3.64×10^4, the least accurate in the above calculation has three significant figures, the answer should be written to three significant figures only. The other figures are insignificant and should be deleted. While deleting the figures, the last significant figure to be retained is rounded off for which the following rules are followed.

a) If the first digit dropped is less than 5, the last digit retained should remain unchanged.

b) If the first digit dropped is more than 5, the digit to be retained is increased by one.

c) If the digit to be dropped is 5, the previous digit which is to be retained is increased by one if it is odd and retained as such if it is even. For example, the following numbers are rounded off to three significant figures as follows. The digits are deleted one by one.

43.75	is rounded off as	43.8
56.8546	is rounded off as	56.8
73.650	is rounded off as	73.6
64.350	is rounded off as	64.4

Do You Know?

Mass can be thought of as a form of energy. In fact the mass is highly concentrated form of energy. Einstein's famous equation, $E=mc^2$ means

Energy = mass x speed of Light^2

According to this equation 1 kg mass is actually 9×10^{16} J energy.

Following this rule, the correct answer of the computation given in section (ii) is 1.46×10^3.

(iii) In adding or subtracting numbers, the number of decimal places retained in the answer should equal the smallest number of decimal places in any of the quantities being added or subtracted. In this case, the number of significant figures is not important. It is the position of decimal that matters. For example, suppose we wish to add the following quantities expressed in metres.

	i)	ii)
	72.1	2.7543
	3.42	4.10
	0.003	1.273
	75.523	8.1273
Correct answer:	75.5 m	8.13 m

In case (i) the number 72.1 has the smallest number of decimal places, thus the answer is rounded off to the same position which is then 75.5 m. In case (ii), the number 4.10 has the smallest number of decimal places and hence, the answer is rounded off to the same decimal positions which is then 8.13 m.

For your information

We use many devices to measure physical quantities, such as length, time, and temperature. They all have some limit of precision.

1.6 PRECISION AND ACCURACY

In measurements made in physics, the terms precision and accuracy are frequently used. They should be distinguished clearly. The precision of a measurement is determined by the instrument or device being used and the accuracy of a measurement depends on the fractional or percentage uncertainty in that measurement.

For example, when the length of an object is recorded as 25.5 cm by using a metre rod having smallest division in millimetre, it is the difference of two readings of the initial and final positions. The uncertainty in the single reading as discussed before is taken as ± 0.05 cm which is now doubled and is called absolute uncertainty equal to ±0.1cm. Absolute uncertainty, in fact, is equal to the least count of the measuring instrument.

Precision or absolute uncertainty (least count) = ± 0.1 cm

$$\text{Fractional uncertainty} = \frac{0.1\,\text{cm}}{25.5\,\text{cm}} = 0.004$$

$$\text{Percentage uncertainty} = \frac{0.1\,\text{cm}}{25.5\,\text{cm}} \times 100 = 0.4\%$$

Another measurement taken by vernier callipers with least count as 0.01 cm is recorded as 0.45 cm. It has

Precision or absolute uncertainty (least count) = ± 0.01 cm

$$\text{Fractional uncertainty} = \frac{0.01\,\text{cm}}{0.45\,\text{cm}} = 0.02$$

$$\text{Percentage uncertainty} = \frac{0.1\,\text{cm}}{0.45\,\text{cm}} \times 100 = 2.0\%$$

Thus the reading 25.5 cm taken by metre rule is although less precise but is more accurate having less percentage uncertainty or error.

Whereas the reading 0.45 cm taken by vernier callipers is more precise but is less accurate. In fact, it is the relative measurement which is important. The smaller a physical quantity, the more precise instrument should be used. Here the measurement 0.45 cm demands that a more precise instrument, such as micrometre screw gauge, with least count 0.001 cm, should have been used. Hence, we can conclude that :

A precise measurement is the one which has less absolute uncertainty and an accurate measurement is the one which has less fractional or percentage uncertainty or error.

For your information

Colour printing uses just four colours- cyan, magenta, yellow and black to produce the entire range of colours. All the colours in this book have been made from just these four colours.

1.7 ASSESSMENT OF TOTAL UNCERTAINTY IN THE FINAL RESULT

To assess the total uncertainty or error, it is necessary to evaluate the likely uncertainties in all the factors involved in that calculation. The maximum possible uncertainty or error in the final result can be found as follows. The proofs of these rules are given in Appendix 2.

1. For addition and subtraction

Absolute uncertainties are added: For example, the distance x determined by the difference between two separate position measurements

$x_1 = 10.5 \pm 0.1$ cm and $x_2 = 26.8 \pm 0.1$ cm is recorded as

$$x = x_2 - x_1 = 16.3 \pm 0.2 \text{ cm}$$

2. For multiplication and division

Percentage uncertainties are added. For example the maximum possible uncertainty in the value of resistance R of a conductor determined from the measurements of potential difference V and resulting current flow I by using $R = V/I$ is found as follows:

$$V = 5.2 \pm 0.1 \text{ V}$$

$$I = 0.84 \pm 0.05\text{A}$$

The %age uncertainty for V is $= \frac{0.1 \text{ V}}{5.2 \text{ V}} \times 100 =$ about 2%.

The %age uncertainty for I is $= \frac{0.05 \text{ A}}{0.84 \text{ A}} \times 100 =$ about 6%.

Hence total uncertainty in the value of resistance R when V is divided by I is 8%. The result is thus quoted as

$$R = \frac{5.2 \text{ V}}{0.84 \text{ A}} = 6.19 \text{ VA}^{-1} = 6.19 \text{ ohms with a \% age uncertainty of 8\%}$$

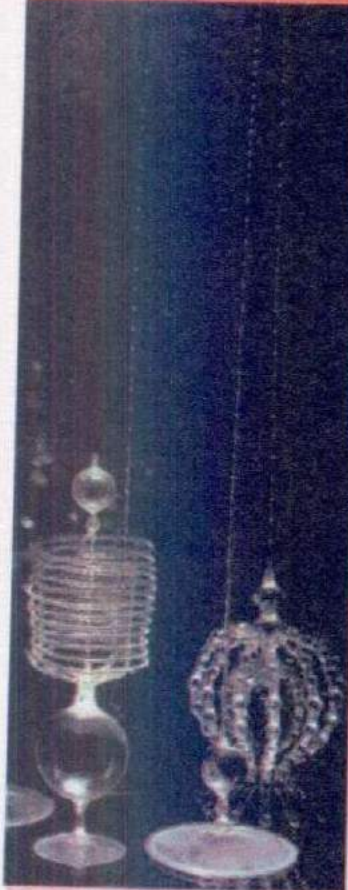
that is $R = 6.2 \pm 0.5$ ohms

The result is rounded off to two significant digits because both V and R have two significant figures and uncertainty, being an estimate only, is recorded by one significant figure.

3. For power factor

Multiply the percentage uncertainty by that power. For example, in the calculation of the volume of a sphere using

For your information



These are not decoration pieces of glass but are the earliest known exquisite and sensitive thermometers, built by the Accademia del Cimento (1657-1667), in Florence. They contained alcohol, some times coloured red for easier reading.

$$V = \frac{4}{3}\pi r^3$$

%age uncertainty in V = 3 x % age uncertainty in radius r.

As uncertainty is multiplied by power factor, it increases the precision demand of measurement. If the radius of a small sphere is measured as 2.25 cm by a vernier callipers with least count 0.01 cm, then

the radius r is recorded as

$$r = 2.25 \pm 0.01 \text{ cm}$$

Absolute uncertainty = Least count = $\pm$ 0.01 cm

$$\text{\%age uncertainty in } r = \frac{0.01\text{ cm}}{2.25\text{ cm}} \times 100 = 0.4\%$$

Total percentage uncertainty in V = 3 x 0.4 = 1.2%

Thus volume $\quad V = \frac{4}{3}\pi r^3$

$$= \frac{4}{3} \times 3.14 \times (2.25\text{ cm})^3$$

$$= 47.689\text{ cm}^3 \text{ with } 1.2\% \text{ uncertainty}$$

Thus the result should be recorded as

$$V = 47.7 \pm 0.6\text{ cm}^3$$

4. For uncertainty in the average value of many measurements.

(i) Find the average value of measured values.

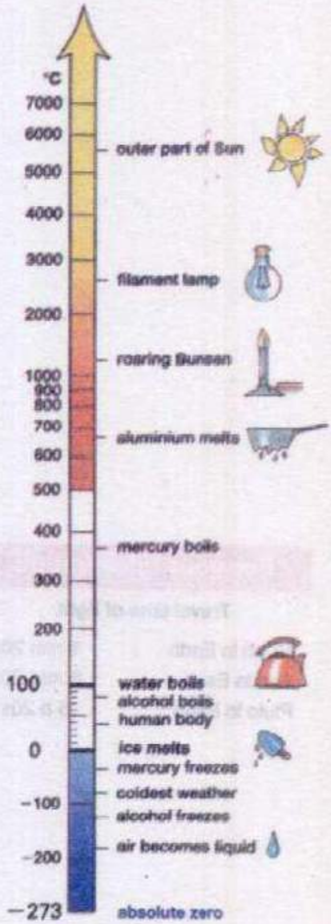
(ii) Find deviation of each measured value from the average value.

(iii) The mean deviation is the uncertainty in the average value.

For example, the six readings of the micrometer screw gauge to measure the diameter of a wire in mm are

1.20,1.22,1.23,1.19,1.22,1.21.

Interesting Information



Some Specific Temperatures

Then $\text{Average} = \dfrac{1.20 + 1.22 + 1.23 + 1.19 + 1.22 + 1.21}{6}$

$= 1.21$ mm

The deviation of the readings, which are the difference without regards to the sign, between each reading and average value are 0.01, 0.01, 0.02, 0.02, 0.01, 0,

$$\text{Mean of deviations} = \frac{0.01 + 0.01 + 0.02 + 0.02 + 0.01 + 0}{6}$$

$= 0.01$ mm

Thus, likely uncertainty in the mean diametre 1.21 mm is 0.01 mm recorded as 1.21 ± 0.01 mm.

5. For the uncertainty in a timing experiment

The uncertainty in the time period of a vibrating body is found by dividing the least count of timing device by the number of vibrations. For example, the time of 30 vibrations of a simple pendulum recorded by a stopwatch accurate upto one tenth of a second is 54.6 s, the period

$$T = \frac{54.6\text{ s}}{30} = 1.82\text{ s with uncertainty } \frac{0.1\text{s}}{30} = 0.003\text{ s}$$

Thus, period T is quoted as $T = 1.82 \pm 0.003$ s

Hence, it is advisable to count large number of swings to reduce timing uncertainty.

For your information

Travel time of light

Moon to Earth	1 min 20s
Sun to Earth	8 min 20s
Pluto to Earth	5 h 20s

Example 1.1: The length, breadth and thickness of a sheet are 3.233m, 2.105 m and 1.05 cm respectively. Calculate the volume of the sheet correct upto the appropriate significant digits.

Solution: Given length $l = 3.233$ m

Breadth $b = 2.105$m

Thickness $h = 1.05\text{cm} = 1.05 \times 10^{-2}$ m

Volume $V = l \times b \times h$

$= 3.233\text{m} \times 2.105\text{m} \times 1.05 \times 10^{-2}\text{m}$

$$= 7.14573825 \times 10^{-2}\ m^3$$

As the factor 1.05 cm has minimum number of significant figures equal to three, therefore, volume is recorded upto 3 significant figures, hence, $V = 7.15 \times 10^{-2} m^3$

Example 1.2: The mass of a metal box measured by a lever balance is 2.2 kg. Two silver coins of masses 10.01 g and 10.02 g measured by a beam balance are added to it. What is now the total mass of the box correct upto the appropriate precision.

Solution: Total mass when silver coins are added to box

$$= 2.2\ \text{kg} + 0.01001\ \text{kg} + 0.01002\ \text{kg}$$

$$= 2.22003\ \text{kg}$$

Since least precise is 2.2 kg, having one decimal place, hence total mass should be to one decimal place which is the appropriate precision. Thus the total mass = 2.2 kg.

Example 1.3: The diameter and length of a metal cylinder measured with the help of vernier callipers of least count 0.01 cm are 1.22 cm and 5.35 cm. Calculate the volume V of the cylinder and uncertainty in it.

Solution: Given data is

Diameter $d = 1.22$ cm with least count 0.01 cm

Length $l = 5.35$ cm with least count 0.01 cm

Absolute uncertainty in length = 0.01 cm

$$\text{\%age uncertainty in length} = \frac{0.01\ \text{cm}}{5.35\ \text{cm}} \times 100 = 0.2\%$$

Absolute uncertainty in diameter = 0.01 cm

$$\text{\%age uncertainty in diameter} = \frac{0.01\ \text{cm}}{1.22\ \text{cm}} \times 100 = 0.8\%$$

As volume is $V = \dfrac{\pi d^2 l}{4}$

For Your Information

Atomic Clock

The cesium atomic frequency standard at the National Institute of Standards and Technology in Colorado (USA). It is the primary standard for the unit of time.

$\therefore$ total uncertainty in $V = 2$ (%age uncertainty in diameter)

+ (%age uncertainty in length)

$= 2 \times 0.8 + 0.2 = 1.8\%$

Then $V = \dfrac{3.14 \times (1.22\,\text{cm})^2 \times 5.35\,\text{cm}}{4} = 6.2509079\ \text{cm}^3$ with 1.8% uncertainty

Thus $\quad V = (6.2 \pm 0.1)\ \text{cm}^3$

Where $6.2\ \text{cm}^3$ is calculated volume and 0.1cm^3 is the uncertainty in it.

1.8 DIMENSIONS OF PHYSICAL QUANTITIES

Each base quantity is considered a dimension denoted by a specific symbol written within square brackets. It stands for the qualitative nature of the physical quantity. For example, different quantities such as length, breadth, diameter, light year which are measured in metre denote the same dimension and has the dimension of length [L]. Similarly the mass and time dimensions are denoted by [M] and [T], respectively. Other quantities that we measure have dimension which are combinations of these dimensions. For example, speed is measured in metres per second. This will obviously have the dimensions of length divided by time. Hence we can write.

$$\text{Dimensions of speed} = \frac{\text{Dimension of length}}{\text{Dimension of time}}$$

$$[\,v\,] = \frac{[L]}{[T]} = [\,L\,][\,T^{-1}] = [LT^{-1}]$$

Similarly the dimensions of acceleration are

$$[\,a\,] = [\,L\,][\,T^{-2}] = [LT^{-2}]$$

and that of force are

$$[\,F\,] = [\,m\,][\,a\,] = [\,M\,][\,L\,T^{-2}] = [MLT^{-2}]$$

Using the method of dimensions called the dimensional analysis, we can check the correctness of a given formula or an equation and can also derive it. Dimensional analysis makes use of the fact that expression of the dimensions can be manipulated as algebraic quantities.

(i) Checking the homogeneity of physical equation

In order to check the correctness of an equation, we are to show that the dimensions of the quantities on both sides of the equation are the same, irrespective of the form of the formula. This is called the principle of homogeneity of dimensions.

Example 1.4: Check the correctness of the relation $v = \sqrt{\frac{F \times l}{m}}$ where v is the speed of transverse wave on a stretched string of tension F, length l and mass m.

Solution:

Dimensions of L.H.S. of the equation = $[v] = [LT^{-1}]$

Dimensions of R.H.S. of the equation = $([F] \times [l] \times [m^{-1}])^{1/2}$

$$= ([MLT^{-2}] \times [L] \times [M^{-1}])^{1/2} = [L^2 T^{-2}]^{1/2} = [LT^{-1}]$$

Since the dimensions of both sides of the equation are the same, equation is dimensionally correct.

(ii) Deriving a possible formula

The success of this method for deriving a relation for a physical quantity depends on the correct guessing of various factors on which the physical quantity depends.

Example 1.5: Derive a relation for the time period of a simple pendulum (Fig. 1.2) using dimensional analysis. The various possible factors on which the time period T may depend are :

i) Length of the pendulum (l)

ii) Mass of the bob (m)

iii) Angle θ which the thread makes with the vertical

iv) Acceleration due to gravity (g)

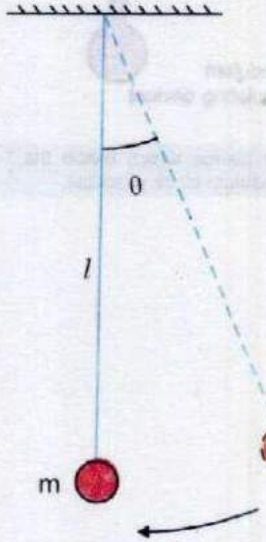
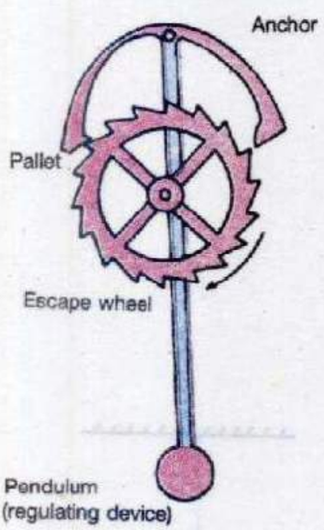


Fig. 1.2

Do You Know?



The device which made the pendulum clock practical.

Solution:

The relation for the time period T will be of the form

$$T \propto m^a \times l^b \times \theta^c \times g^d$$

or $$T = \text{constant } m^a\, l^b\, \theta^c\, g^d \quad \ldots\ldots\ldots \quad (1.1)$$

where we have to find the values of powers a, b, c and d.

Writing the dimensions of both sides we get

$$[T] = \text{constant} \times [M]^a [L]^b [LL^{-1}]^c [LT^{-2}]^d$$

Comparing the dimensions on both sides we have

$$[T] = [T]^{-2d}$$

$$[M]^0 = [M]^a$$

$$[L]^0 = [L]^{b+d+c-c}$$

Equating powers on both the sides we get

$$-2d = 1 \qquad \text{or} \qquad d = -\frac{1}{2}$$

$$a = 0 \qquad \text{and} \qquad b + d = 0$$

or $$b = -d = \frac{1}{2} \qquad \text{and} \qquad \theta = [L\,L^{-1}]^c = [L^0]^c = 1$$

Substituting the values of a, b, θ and d in Eq. 1.1

$$T = \text{constant} \times m^0 \times l^{1/2} \times 1 \times g^{-1/2}$$

Or $$T = \text{constant} \sqrt{\frac{l}{g}}$$

The numerical value of the constant cannot be determined by dimensional analysis, however, it can be found by experiments.

Example 1.6: Find the dimensions and hence, the SI units of coefficient of viscosity η in the relation of Stokes' law for the drag force F for a spherical object of radius r moving with velocity v given as $F = 6\pi\eta r v$

Solution: 6π is a number having no dimensions. It is not accounted in dimensional analysis. Then

$$[F] = [\eta r v]$$

or $$[\eta] = \frac{[F]}{[r][v]}$$

Substituting the dimensions of F, r, and v in R.H.S.

$$[\eta] = \frac{[MLT^{-2}]}{[L][LT^{-1}]}$$

or $$[\eta] = [ML^{-1}T^{-1}]$$

Thus, the SI unit of coefficient of viscosity is kg m^{-1} s^{-1}

SUMMARY

- Physics is the study of entire Physical World.
- The most basic quantities that can be used to describe the Physical World are mass, length and time. All other quantities, called derived quantities, can be described in terms of some combinations of the base quantities.
- The internationally adopted system of units used by all the scientists and almost all the countries of the World is International System (SI) of Units. It consists of seven base units, two supplementary units and a number of derived units.
- Errors due to incorrect design or calibrations of the measuring device are called systematic errors. Random errors are due to unknown causes and fluctuations in the quantity being measured.
- The accuracy of a measurement is the extent to which systematic error make a measured value differ from its true value.
- The accuracy of a measurement can be indicated by the number of significant figures, or by a stated uncertainty.
- The significant figures or digits in a measured or calculated quantity are those digits that are known to be reasonably reliable.
- The result of multiplication or division has no more significant figures than any factor in the input data. Round off your calculator result to correct number of digits.
- In case of addition or subtraction the precision of the result can be only as great as the least precise term added or subtracted.
- Each basic measurable physical property represented by a specific symbol written within square brackets is called a dimension. All other physical quantities can be derived as combinations of the basic dimensions.
- Equations must be dimensionally consistent. Two terms can be added only when they have the same dimensions.

QUESTIONS

1.1 Name several repetitive phenomenon occurring in nature which could serve as reasonable time standards.

1.2 Give the drawbacks to use the period of a pendulum as a time standard.

1.3 Why do we find it useful to have two units for the amount of substance, the kilogram and the mole?

1.4 Three students measured the length of a needle with a scale on which minimum division is 1mm and recorded as (i) 0.2145 m (ii) 0.21 m (iii) 0.214m. Which record is correct and why?

1.5 An old saying is that "A chain is only as strong as its weakest link". What analogous statement can you make regarding experimental data used in a computation?

1.6 The period of simple pendulum is measured by a stop watch. What type of errors are possible in the time period?

1.7 Does a dimensional analysis give any information on constant of proportionality that may appear in an algebraic expression? Explain.

1.8 Write the dimensions of (i) Pressure (ii) Density

1.9 The wavelength λ of a wave depends on the speed v of the wave and its frequency f. Knowing that

$$[\lambda]=[L], \qquad [v]=[LT^{-1}] \qquad \text{and} \qquad [f]=[T^{-1}]$$

Decide which of the following is correct, $f = v\lambda$ or $f = \frac{v}{\lambda}$.

NUMERICAL PROBLEMS

1.1 A light year is the distance light travels in one year. How many metres are there in one light year: (speed of light = $3.0 \times 10^8\ ms^{-1}$).

(Ans: 9.5×10^{15}m)

1.2 a) How many seconds are there in 1 year?

b) How many nanoseconds in 1 year?

c) How many years in 1 second?

[Ans. (a) 3.1536×10^7s, (b) 3.1536×10^{16}ns (c) 3.1×10^{-8} yr]

1.3 The length and width of a rectangular plate are measured to be 15.3 cm and 12.80 cm, respectively. Find the area of the plate.

(Ans: 196 cm^2)

1.4 Add the following masses given in kg upto appropriate precision. 2.189, 0.089, 11.8 and 5.32.

(Ans: 19.4 kg)

1.5 Find the value of '*g*' and its uncertainty using $T = 2\pi\sqrt{\frac{l}{g}}$ from the following measurements made during an experiment

Length of simple pendulum l = 100 cm.

Time for 20 vibrations = 40.2 s

Length was measured by a metre scale of accuracy upto 1 mm and time by stop watch of accuracy upto 0.1 s.

(Ans: 9.76 ± 0.06 ms^{-2})

1.6 What are the dimensions and units of gravitational constant G in the formula $F = G\,\frac{m_1 m_2}{r^2}$

(Ans: $[M^{-1}L^3T^{-2}]$, Nm^2kg^{-2})

1.7 Show that the expression $v_f = v_i + at$ is dimensionally correct, where v_i is the velocity at $t = 0$, a is acceleration and v_f is the velocity at time t.

1.8 The speed v of sound waves through a medium may be assumed to depend on (a) the density ρ of the medium and (b) its modulus of elasticity E which is the ratio of stress to strain. Deduce by the method of dimensions, the formula for the speed of sound.

(Ans: v = Constant $\sqrt{\frac{E}{\rho}}$)

1.9 Show that the famous "Einstein equation" $E = mc^2$ is dimensionally consistent.

1.10 Suppose, we are told that the acceleration of a particle moving in a circle of radius r with uniform speed v is proportional to some power of r, say r^n, and some power of v, say v^m, determine the powers of r and v?

(Ans: n = -1, m = 2)