Chapter 3

MOTION AND FORCE

Learning Objectives

At the end of this chapter the students will be able to:

1. Understand displacement from its definition and illustration.
2. Understand velocity, average velocity and instantaneous velocity.
3. Understand acceleration, average acceleration and instantaneous acceleration.
4. Understand the significance of area under velocity-time graph.
5. Recall and use equations, which represent uniformly accelerated motion in a straight line including falling in a uniform gravitational field without air resistance.
6. Recall Newton's Laws of motion.
7. Describe Newton's second law of motion as rate of change of momentum.
8. Define impulse as a product of impulsive force and time.
9. Describe law of conservation of momentum.
10. Use the law of conservation of momentum in simple applications including elastic collisions between two bodies in one dimension.
11. Describe the force produced due to flow of water.
12. Understand the process of rocket propulsion (simple treatment).
13. Understand projectile motion in a non-resistive medium.
14. Derive time of flight, maximum height and horizontal range of projectile motion.
15. Appreciate the motion of ballistic missiles as projectile motion.

We live in a universe of continual motion. In every piece of matter, the atoms are in a state of never ending motion. We move around the Earth's surface, while the Earth moves in its orbit around the Sun. The Sun and the stars, too, are in motion. Everything in the vastness of space is in a state of perpetual motion.

Every physical process involves motion of some sort. Because of its importance in the physical world around us, it is logical that we should give due attention to the study of motion.

We already know that motion and rest are relative. Here, in this chapter, we shall discuss other related topics in some more details.

3.1 DISPLACEMENT

Whenever a body moves from one position to another, the change in its position is called displacement. The displacement can be represented as a vector that describes how far and in what direction the body has been displaced from its original position. The tail of the displacement vector is located at the position where the displacement started, and its tip or arrowhead is located at the final position where the displacement ended. For example, if a body is moving along a curve as shown in Fig. 3.1 with A as its initial position and B as its final position then the displacement **d** of the body is represented by **AB**. Note that although the body is moving along a curve, the displacement is different from the path of motion.

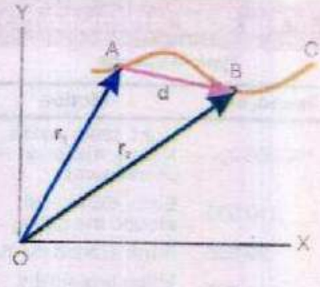


Fig.3.1

If $\mathbf{r}_1$ is the position vector of A and $\mathbf{r}_2$ that of point B then by head and tail rule it can be seen from the figure that

$$\mathbf{d} = \mathbf{r}_2 - \mathbf{r}_1$$

The displacement is thus a change in the position of body from its initial position to its final position.

Its magnitude is the straight line distance between the initial position and the final position of the body.

When a body moves along a straight line, the displacement coincides with the path of motion as shown in Fig. 3.2. (a)

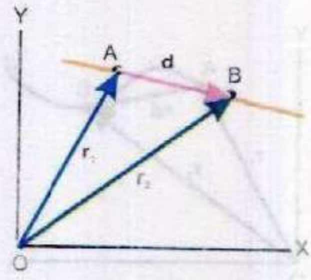


Fig.3.2(a)

3.2 VELOCITY

We have studied in school physics that time rate of change of displacement is known as velocity. Its direction is along the direction of displacement. So if **d** is the total displacement of the body in time t, then its average velocity during the interval t is defined as

$$\mathbf{v}_{av} = \frac{\mathbf{d}}{t} \qquad \text{..........} \qquad (3.1)$$

Average velocity does not tell us about the motion between A and B. The path may be straight or curved and the motion may be steady or variable. For example if a squash ball comes back to its starting point after bouncing off the wall several times, its total displacement is zero and so also is its average velocity.

In such cases the motion is described by the instantaneous velocity.

In order to understand the concept of instantaneous velocity, consider a body moving along a path ABC in xy plane. At any time t, let the body be at point A Fig.3.2(b).Its position is given by position vector $\mathbf{r}_1$. After a short time interval Δt following the instant t, the body reaches the point B which is described by the position vector $\mathbf{r}_2$. The displacement of the body during this short time interval is given by

$$\Delta \mathbf{d} = \mathbf{r}_2 - \mathbf{r}_1$$

The notation Δ (delta) is used to represent a very small change.

The instantaneous velocity at a point A, can be found by making Δt smaller and smaller. In this case $\Delta \mathbf{d}$ will also become smaller and point B will approach A. If we continue this process, letting B approach A, thus, allowing Δt and $\Delta \mathbf{d}$ to decrease but never disappear completely, the ratio $\Delta \mathbf{d}/\Delta t$ approaches a definite limiting value which is the instantaneous velocity. Although Δt and $\Delta \mathbf{d}$ become extremely small in this process, yet their ratio is not necessarily a small quantity. Moreover, while decreasing the displacement vector, $\Delta \mathbf{d}$ approaches a limiting direction along the tangent at A. Therefore,

The instantaneous velocity is defined as the limiting value of $\Delta \mathbf{d}/\Delta t$ as the time interval Δt, following the time t, approaches zero.

For Your Information

Typical Speeds

Speed, ms^{-1}	Motion
300 000 000	Light, radio waves, X-rays, microwaves (in vacuum)
210 000	Earth-Sun travel around the galaxy
29 600	Earth around the Sun
1 000	Moon around the Earth
980	SR-71 reconnaissance jet
333	Sound (in air)
267	Commercial jet airliner
62	Commercial automobile (max.)
37	Falcon in a dive
29	Running cheetah
10	100-metres dash (max.)
9	Porpoise swimming
5	Flying bee
4	Human running
2	Human swimming
0.01	Walking ant

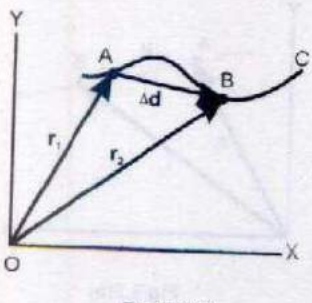


Fig.3.2(b)

Using the mathematical language, the definition of instantaneous velocity $\mathbf{v}_{ins}$ is expressed as

$$\mathbf{v}_{ins} = \lim_{\Delta t \to 0} \frac{\Delta \mathbf{d}}{\Delta t} \quad .... \quad (3.2)$$

read as limiting value of $\Delta\mathbf{d}/\Delta t$ as Δt approaches zero.

If the instantaneous velocity does not change, the body is said to be moving with uniform velocity.

3.3 ACCELERATION

If the velocity of an object changes, it is said to be moving with an acceleration.

The time rate of change of velocity of a body is called acceleration.

As velocity is a vector so any change in velocity may be due to change in its magnitude or a change in its direction or both.

Consider a body whose velocity $\mathbf{v}_1$ at any instant t changes to $\mathbf{v}_2$ in further small time interval Δt. The two velocity vectors $\mathbf{v}_1$ and $\mathbf{v}_2$ and the change in velocity, $\mathbf{v}_2 - \mathbf{v}_1 = \Delta\mathbf{v}$, are represented in Fig. 3.3. The average acceleration $\mathbf{a}_{av}$ during time interval Δt is given by

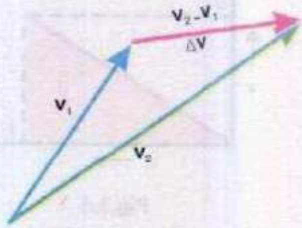


Fig.3.3

$$\mathbf{a}_{av} = \frac{\mathbf{v}_2 - \mathbf{v}_1}{\Delta t} = \frac{\Delta \mathbf{v}}{\Delta t} \quad .... \quad (3.3)$$

As $\mathbf{a}_{av}$ is the difference of two vectors divided by a scalar Δt, $\mathbf{a}_{av}$ must also be a vector. Its direction is the same as that of $\Delta\mathbf{v}$. Acceleration of a body at a particular instant is known as instantaneous acceleration and it is the value obtained from the average acceleration as Δt is made smaller and smaller till it approaches zero. Mathematically, it is expressed as

$$\text{Instantaneous acceleration} = \mathbf{a}_{ins} = \lim_{\Delta t \to 0} \frac{\Delta \mathbf{v}}{\Delta t} \quad .... \quad (3.4)$$

If the velocity of a body is increasing, its acceleration is positive but if the velocity is decreasing the acceleration is negative. If the velocity of the body changes by equal amount in equal intervals of time, the body is said to have uniform acceleration. For a body moving with uniform acceleration, its average acceleration is equal to instantaneous acceleration.

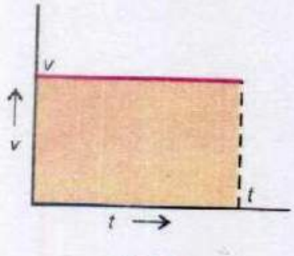


Fig.3.4

3.4 VELOCITY-TIME GRAPH

Graphs may be used to illustrate the variation of velocity of an object with time. Such graphs are called velocity-time graphs. The velocity-time graphs of an object making three different journeys along a straight road are shown in figures 3.4 to 3.6. When the velocity of the car is constant, its velocity-time graph is a horizontal straight line (Fig 3.4). When the car moves with constant acceleration, the velocity-time graph is a straight line which rises the same height for equal intervals of time (Fig 3.5).

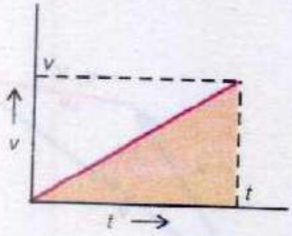


Fig.3.5

The average acceleration of the car during the interval *t* is given by the slope of its velocity-time graph.

When the car moves with increasing acceleration, the velocity-time graph is a curve (Fig 3.6). The point A on the graph corresponds to time *t*. The magnitude of the instantaneous acceleration at this instant is numerically equal to the slope of the tangent at the point A on the velocity-time graph of the object as shown in Fig 3.6.

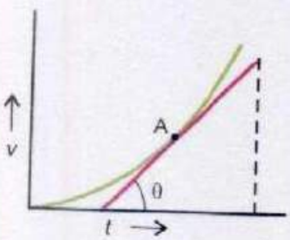


Fig.3.6

The distance moved by an object can also be determined by using its velocity-time graph. For example, Fig 3.4 shows that the object moves at constant velocity **v** for time *t*. The distance covered by the object given by Eq. 3.1 is $v \times t$. This distance can also be found by calculating the area under the velocity-time graph. This area is shown shaded in Fig 3.4 and is equal to $v \times t$. We now give another example shown in Fig 3.5. Here the velocity of the object increases uniformly from 0 to **v** in time *t*. The magnitude of its average velocity is given by

$$v_{av} = \frac{0 + v}{2} = \frac{1}{2}v$$

Distance covered = average velocity x time = $\frac{1}{2} v \times t$

Now we calculate the area under velocity-time graph which is equal to the area of the triangle shaded in Fig 3.5. Its value is equal to 1/2 base x height = 1/2 $v \times t$. Considering the above two examples it is a general conclusion that

The area between the velocity-time graph and the time axis is numerically equal to the distance covered by the object.

Example 3.1: The velocity-time graph of a car moving on a straight road is shown in Fig 3.7. Describe the motion of the car and find the distance covered.

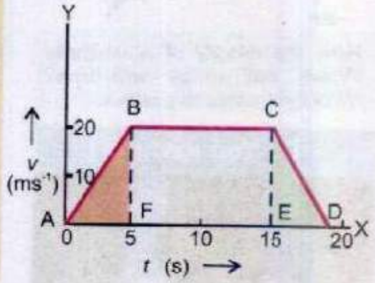


Fig.3.7

Solution: The graph tells us that the car starts from rest, and its velocity increases uniformly to 20 ms^{-1} in 5 seconds. Its average acceleration is given by

$$a = \frac{\Delta v}{\Delta t} = \frac{20\ \text{ms}^{-1}}{5\ \text{s}} = 4\ \text{ms}^{-2}$$

The graph further tells us that the velocity of the car remains constant from 5th to 15th second and it then decreases uniformly to zero from 15th to 19th seconds. The acceleration of the car during last 4 seconds is

$$a = \frac{\Delta v}{\Delta t} = \frac{-20\ \text{ms}^{-1}}{4\ \text{s}} = -5\ \text{ms}^{-2}$$

The negative sign indicates that the velocity of the car decreases during these 4 seconds.

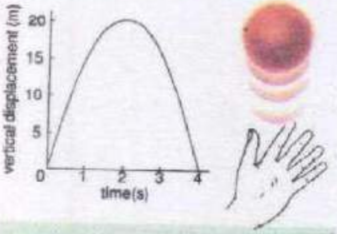
The distance covered by the car is equal to the area between the velocity-time graph and the time-axis. Thus

Distance travelled = Area of ΔABF + Area of rectangle BCEF + Area of ΔCDE

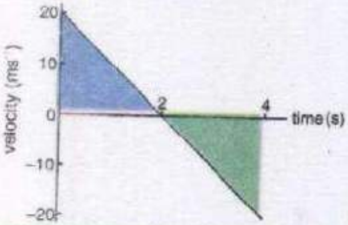
$$= \frac{1}{2} \times 20\ \text{ms}^{-1} \times 5\ \text{s} + 20\ \text{ms}^{-1} \times 10\ \text{s} + \frac{1}{2} \times 20\ \text{ms}^{-1} \times 4\ \text{s}$$

$$= 50\ \text{m} + 200\ \text{m} + 40\ \text{m} = 290\ \text{m}$$

Do You Know?

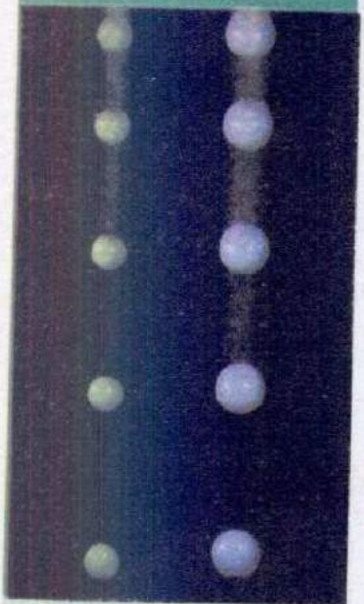


How the displacement of a vertically thrown ball varies with time?



How the velocity of a vertically thrown ball varies with time? Velocity is upwards positive.

Do You Know?



At the surface of the Earth, in situations where air friction is negligible, objects fall with the same acceleration regardless of their weights.

3.5 REVIEW OF EQUATIONS OF UNIFORMLY ACCELERATED MOTION

In school physics we have studied some useful equations for objects moving at constant acceleration.

Suppose an object is moving with uniform acceleration a along a straight line. If its initial velocity is v_i and final velocity after a time interval t is v_f. Let the distance covered during this interval be S then we have

$$v_f = v_i + at \qquad \text{..........} \qquad (3.5)$$

$$S = \frac{(v_f + v_i)}{2} \times t \qquad \text{..........} \qquad (3.6)$$

$$S = v_i t + \frac{1}{2} at^2 \qquad \text{..........} \qquad (3.7)$$

$$v_f^2 = v_i^2 + 2aS \qquad \text{..........} \qquad (3.8)$$

These equations are useful only for linear motion with uniform acceleration. When the object moves along a straight line, the direction of motion does not change. In this case all the vectors can be manipulated like scalars. In such problems, the direction of initial velocity is taken as positive. A negative sign is assigned to quantities where direction is opposite to that of initial velocity.

In the absence of air resistance, all objects in free fall near the surface of the Earth, move towards the Earth with a uniform acceleration. This acceleration, known as acceleration due to gravity, is denoted by the letter **g** and its average value near the Earth surface is taken as 9.8 ms^{-2} in the downward direction.

The equations for uniformly accelerated motion can also be applied to free fall motion of the objects by replacing a by g.

3.6 NEWTON'S LAWS OF MOTION

Newton's laws are empirical laws, deduced from experiments. They were clearly stated for the first time by Sir Isaac Newton, who published them in 1687 in his famous book called "Principia". Newton's laws are adequate for speeds that are low compared with the speed of light.

For very fast moving objects, such as atomic particles in an accelerator, relativistic mechanics developed by Albert Einstein is applicable.

You have already studied these laws in your secondary school Physics. However a summarized review is given below.

An unappreciated anticipation.

No body begins to move or comes to rest of itself.

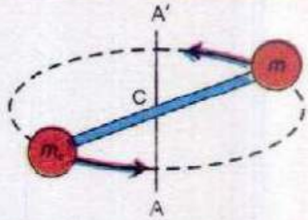
ABU ALI SENA, (980-1037)

Newton's First Law of Motion

A body at rest will remain at rest, and a body moving with uniform velocity will continue to do so, unless acted upon by some unbalanced external force. This is also known as law of inertia. The property of an object tending to maintain the state of rest or state of uniform motion is referred to as the object's inertia. The more inertia, the stronger is this tendency in the presence of a force. Thus,

The mass of the object is a quantitative measure of its inertia.

For Your Information



A measurement of mass independent of gravity. The unknown mass m and a calibrated mass m_c are mounted on a light weight rod. If the masses are equal, the rod will rotate without wobble about its centre.

The frame of reference in which Newton's first law of motion holds, is known as inertial frame of reference. A frame of reference stationed on Earth is approximately an inertial frame of reference.

Newton's Second Law of Motion

A force applied on a body produces acceleration in its own direction. The acceleration produced varies directly with the applied force and inversely with the mass of the body. Mathematically, it is expressed as

$$\mathbf{F} = m\mathbf{a} \quad \ldots\ldots\ldots \quad (3.9)$$

Newton's Third Law of Motion

Action and reaction are equal and opposite. For example, whenever an interaction occurs between two objects, each object exerts the same force on the other, but in the opposite direction and for the same length of time. Each force in action-reaction pair acts only on one of the two bodies, the action and reaction forces never act on the same body.

Point to Ponder

A car accelerates along a road. Which force actually moves the car?

3.7 MOMENTUM

Interesting Information

Throwing a package onto shore from a boat that was previously at rest causes the boat to move out-ward from shore (Newton's third law).

Point to Ponder

Which will be more effective in knocking a bear down.

i. a rubber bullet or
ii. a lead bullet of the same momentum.

We are aware of the fact that moving object possesses a quality by virtue of which it exerts a force on anything that tries to stop it. The faster the object is travelling, the harder is to stop it. Similarly, if two objects move with the same velocity, then it is more difficult to stop the massive of the two.

This quality of the moving body was called the quantity of motion of the body, by Newton. This term is now called linear momentum of the body and is defined by the relation.

$$\text{Linear momentum} = \mathbf{p} = m\,\mathbf{v} \quad \ldots\ldots\ldots \quad (3.10)$$

In this expression **v** is the velocity of the mass *m*. Linear momentum is, therefore, a vector quantity and has the direction of velocity.

The SI unit of momentum is kilogram metre per second (kg m s^{-1}). It can also be expressed as newton second (N s).

Momentum and Newton's Second Law of Motion

Consider a body of mass *m* moving with an initial velocity $\mathbf{v}_i$. Suppose an external force **F** acts upon it for time *t* after which velocity becomes $\mathbf{v}_f$. The acceleration **a** produced by this force is given by

$$\mathbf{a} = \frac{\mathbf{v}_f - \mathbf{v}_i}{t}$$

By Newton's second law, the acceleration is given as

$$\mathbf{a} = \frac{\mathbf{F}}{m}$$

Equating the two expressions of acceleration, we have

$$\frac{\mathbf{F}}{m} = \frac{\mathbf{v}_f - \mathbf{v}_i}{t}$$

or

$$\mathbf{F} \times t = m\,\mathbf{v}_f - m\,\mathbf{v}_i \quad \ldots\ldots\ldots \quad (3.11)$$

where $m\mathbf{v}_i$ is the initial momentum and $m\mathbf{v}_f$ is the final momentum of the body.

The equation 3.11 shows that change in momentum is equal to the product of force and the time for which force is applied. This form of the second law is more general than the form **F** = *m***a**, because it can easily be extended to account for changes as the body accelerates when its mass also changes. For example, as a rocket accelerates, it loses mass because its fuel is burnt and ejected to provide greater thrust.

From Eq. 3.11. $\mathbf{F} = \frac{m\mathbf{v}_f - m\mathbf{v}_i}{t}$

Thus, second law of motion can also be stated in terms of momentum as follows

Time rate of change of momentum of a body equals the applied force.

Impulse

Sometimes we wish to apply the concept of momentum to cases where the applied force is not constant, it acts for very short time. For example, when a bat hits a cricket ball, the force certainly varies from instant to instant during the collision. In such cases, it is more convenient to deal with the product of force and time (**F** × *t*) instead of either quantity alone. The quantity **F** × *t* is called the impulse of the force, where **F** can be regarded as the average force that acts during the time *t*. From Eq. 3.11

$$\text{Impulse} = \mathbf{F} \times t = m\,\mathbf{v}_f - m\,\mathbf{v}_i \quad \ldots\ldots\ldots \quad (3.12)$$

Example 3.2: A 1500 kg car has its velocity reduced from 20 ms^{-1} to 15 ms^{-1} in 3.0 s. How large was the average retarding force?

Solution: Using the Eq 3.11

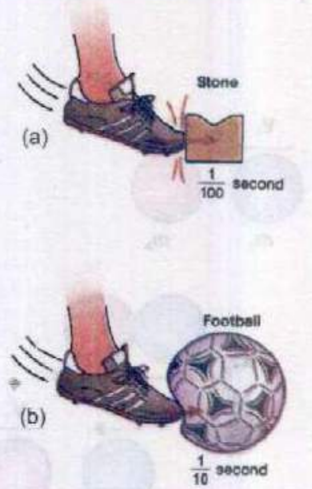
$$F \times t = m\,v_f - m\,v_i$$

$$F \times 3.0\text{ s} = 1500\text{ kg} \times 15\text{ ms}^{-1} - 1500\text{ kg} \times 20\text{ ms}^{-1}$$

$$\text{or } F = -2500\text{ kg ms}^{-2} = -2500\text{ N} = -2.5\text{ kN}$$

The negative sign indicates that the force is retarding one.

Point to Ponder



Which hurt you in the above situations (a) or (b) and think why?

Point to Ponder

Does a moving object have impulse?

Do You Know?

Your hair acts like a crumple zone on your skull. A force of 5N might be enough to fracture your naked skull (cranium), but with a covering of skin and hair, a force of 50 N would be needed.

Law of Conservation of Momentum

Let us consider an isolated system. It is a system on which no external agency exerts any force. For example, the molecules of a gas enclosed in a glass vessel at constant temperature constitute an isolated system. The molecules can collide with one another because of their random motion but, being enclosed by glass vessel, no external agency can exert a force on them.

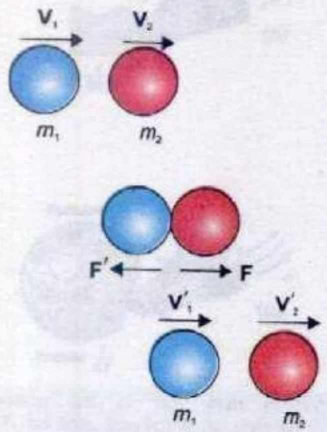


Fig. 3.8

Consider an isolated system of two smooth hard interacting balls of masses m_1 and m_2, moving along the same straight line, in the same direction, with velocities $\mathbf{v}_1$ and $\mathbf{v}_2$ respectively. Both the balls collide and after collision, ball of mass m_1 moves with velocity $\mathbf{v}'_1$ and m_2 moves with velocity $\mathbf{v}'_2$ in the same direction as shown in Fig 3.8.

To find the change in momentum of mass m_1, using Eq 3.11 we have,

$$\mathbf{F}' \times t = m_1 \mathbf{v}'_1 - m_1 \mathbf{v}_1$$

Similarly for the ball of mass m_2, we have

$$\mathbf{F} \times t = m_2 \mathbf{v}'_2 - m_2 \mathbf{v}_2$$

Adding these two expressions, we get

$$(\mathbf{F} + \mathbf{F}')t = (m_1 \mathbf{v}'_1 - m_1 \mathbf{v}_1) + (m_2 \mathbf{v}'_2 - m_2 \mathbf{v}_2)$$

Since the action force $\mathbf{F}$ is equal and opposite to the reaction force $\mathbf{F}'$, we have $\mathbf{F}' = -\mathbf{F}$, so the left hand side of the equation is zero. Hence,

$$0 = (m_1 \mathbf{v}'_1 - m_1 \mathbf{v}_1) + (m_2 \mathbf{v}'_2 - m_2 \mathbf{v}_2)$$

In other words, change in momentum of 1st ball + change in momentum of the 2nd ball = 0

Or $(m_1 \mathbf{v}_1 + m_2 \mathbf{v}_2) = (m_1 \mathbf{v}'_1 + m_2 \mathbf{v}'_2)$ ...... (3.13)

Point to Ponder

What is the effect on the speed of a fighter plane chasing another when it opens fire? What happens to the speed of pursued plane when it returns the fire?

Which means that total initial momentum of the system before collision is equal to the total final momentum of the system after collision. Consequently, the total change in momentum of the isolated two ball system is zero.

For such a group of objects, if one object within the group experiences a force, there must exist an equal but opposite reaction force on some other object in the same group. As a result, the change in momentum of the group of objects as a whole is always zero. This can be expressed in the form of law of conservation of momentum which states that

The total linear momentum of an isolated system remains constant.

In applying the conservation law, we must notice that the momentum of a body is a vector quantity.

Example 3.3: Two spherical balls of 2.0 kg and 3.0 kg masses are moving towards each other with velocities of 6.0 ms^{-1} and 4 ms^{-1} respectively. What must be the velocity of the smaller ball after collision, if the velocity of the bigger ball is 3.0 ms^{-1}?

Solution: As both the balls are moving towards one another, so their velocities are of opposite sign. Let us suppose that the direction of motion of 2 kg ball is positive and that of the 3 kg is negative.

The momentum of the system before collision = $m_1 v_1 + m_2 v_2$

$$= 2\ kg \times 6\ ms^{-1} + 3\ kg \times (-4\ ms^{-1}) = 12\ kgms^{-1} - 12\ kg\ m\ s^{-1} = 0$$

Momentum of the system after collision = $m_1 v'_1 + m_2 v'_2$

$$= 2\ kg \times v'_1 + 3\ kg \times (-3) m s^{-1}$$

From the law of conservation of momentum

$$\begin{bmatrix}\text{Momentum of the system}\\ \text{before collision}\end{bmatrix} = \begin{bmatrix}\text{Momentum of the system}\\ \text{after collision}\end{bmatrix}$$

$$0 = 2\ kg \times v'_1 - 9\ kg\ m\ s^{-1}$$

$$2\ kg \times v'_1 = 9\ kg\ m\ s^{-1}$$

$$v'_1 = 4.5\ m\ s^{-1}$$

Do you wear seat belts?

When a moving car stops quickly, the passengers move forward towards the windshield. Seat belts change the forces of motion and prevent the passengers from moving. Thus the chance of injury is greatly reduced.

Do You Know?

A motorcycle's safety helmet is padded so as to extend the time of any collision to prevent serious injury.

3.8 ELASTIC AND INELASTIC COLLISIONS

When two tennis balls collide then, after collision, they will rebound with velocities less than the velocities before the impact. During this process, a portion of K.E is lost, partly due to friction as the molecules in the ball move past one another when the balls distort and partly due to its change into heat and sound energies.

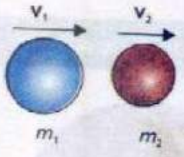
A collision in which the K.E of the system is not conserved, is called the inelastic collision.

Under certain special conditions no kinetic energy is lost in the collision.

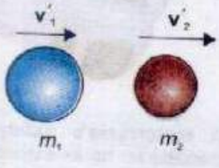
In the ideal case when no K.E is lost, the collision is said to be perfectly elastic.

For example, when a hard ball is dropped onto a marble floor, it rebounds to very nearly the initial height. It looses negligible amount of energy in the collision with the floor.

It is to be noted that momentum and total energy are conserved in all types of collisions. However, the K.E. is conserved only in elastic collisions.



Before collision



After collision

Fig. 3.9

Elastic Collision in One Dimension

Consider two smooth, non-rotating balls of masses m_1 and m_2, moving initially with velocities v_1 and v_2 respectively, in the same direction. They collide and after collision, they move along the same straight line without rotation. Let their velocities after the collision be v_1' and v_2' respectively, as shown in Fig. 3.9.

We take the positive direction of the velocity and momentum to the right. By applying the law of conservation of momentum we have

$$m_1 v_1 + m_2 v_2 = m_1 v_1' + m_2 v_2'$$

$$m_1(v_1 - v_1') = m_2(v_2' - v_2) \quad .... \quad (3.14)$$

As the collision is elastic, so the K.E is also conserved. From the conservation of K.E we have

$$\frac{1}{2}m_1 v_1^2 + \frac{1}{2}m_2 v_2^2 = \frac{1}{2}m_1 v_1'^2 + \frac{1}{2}m_2 v_2'^2$$

or $$m_1(v_1^2 - v_1'^2) = m_2(v_2'^2 - v_2^2)$$

or $$m_1(v_1 + v'_1)(v_1 - v'_1) = m_2(v'_2 + v_2)(v'_2 - v_2) \quad \dots \quad (3.15)$$

Dividing equation 3.15 by 3.14

$$(v_1 + v'_1) = (v'_2 + v_2) \quad \dots\dots\dots \quad (3.16)$$

or $$(v_1 - v_2) = (v'_2 - v'_1) = -(v'_1 - v'_2)$$

We note that, before collision $(v_1 - v_2)$ is the velocity of first ball relative to the second ball. Similarly $(v'_1 - v'_2)$ is the velocity of the first ball relative to the second ball after collision. It means that relative velocities before and after the collision has the same magnitude but are reversed after the collision. In other words, the magnitude of relative velocity of approach is equal to the magnitude of relative velocity of separation.

In equations 3.14 and 3.16, m_1, m_2, v_1 and v_2 are known quantities. We solve these equations to find the values of v'_1 and v'_2, which are unknown. The results are

$$v'_1 = \frac{m_1 - m_2}{m_1 + m_2}v_1 + \frac{2m_2}{m_1 + m_2}v_2 \quad \dots\dots\dots \quad (3.17)$$

$$v'_2 = \frac{2m_1}{m_1 + m_2}v_1 + \frac{m_2 - m_1}{m_1 + m_2}v_2 \quad \dots\dots\dots \quad (3.18)$$

There are some cases of special interest, which are discussed below:

(i) When $m_1 = m_2$

From equations 3.17 and 3.18 we find that

$$v'_1 = v_2$$

and $v'_2 = v_1$ as shown in Fig 3.10

(ii) When $m_1 = m_2$ **and** $v_2 = 0$

In this case the mass m_2 be at rest, then $v_2 = 0$ the equations 3.17 and 3.18 give

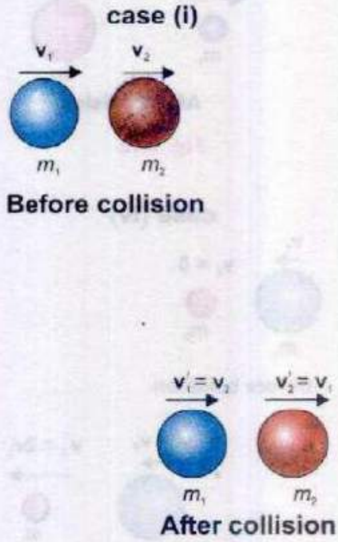


Fig. 3.10

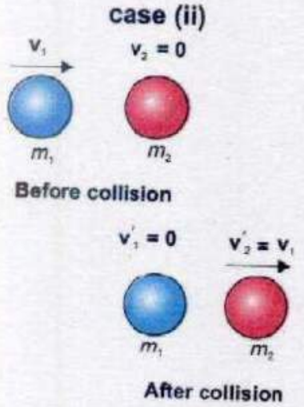


Fig. 3.11

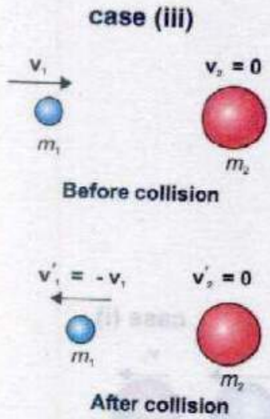


Fig. 3.12

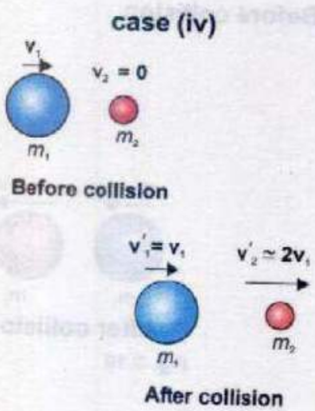


Fig. 3.13

$$v'_1 = \frac{m_1 - m_2}{m_1 + m_2} v_1 \quad ; \quad v'_2 = \frac{2m_1}{m_1 + m_2} v_1$$

When $m_1 = m_2$ then ball of mass m_1 after collision will come to a stop and m_2 will take off with the velocity that m_1 originally has, as shown in Fig 3.11. Thus when a billiard ball m_1, moving on a table collides with exactly similar ball m_2 at rest, the ball m_1 stops while m_2 begins to move with the same velocity, with which m_1 was moving initially.

(iii) When a light body collides with a massive body at rest

In this case initial velocity $v_2 = 0$ and $m_2 >> m_1$. Under these conditions m_1 can be neglected as compared to m_2. From equations 3.17 and 3.18 we have $v'_1 = -v_1$ and $v'_2 = 0$

The result is shown in Fig 3.12. This means that m_1 will bounce back with the same velocity while m_2 will remain stationary. This fact is made used of by the squash player.

(iv) When a massive body collides with light stationary body

In this case $m_1 >> m_2$ and $v_2 = 0$ so m_2 can be neglected in equations 3.17 and 3.18. This gives $v'_1 \simeq v_1$ and $v'_2 \simeq 2v_1$. Thus after the collision, there is practically no change in the velocity of the massive body, but the lighter one bounces off in the forward direction with approximately twice the velocity of the incident body, as shown in Fig. 3.13.

Example 3.4: A 70 g ball collides with another ball of mass 140 g. The initial velocity of the first ball is 9 ms^{-1} to the right while the second ball is at rest. If the collision were perfectly elastic what would be the velocity of the two balls after the collision?

Solution:

$m_1 = 70$ g $\quad v_1 = 9\ ms^{-1} \quad v_2 = 0$

$m_2 = 140$ g $\quad v'_1 = ?\quad v'_2 = ?$

We know that $\quad v'_1 = \dfrac{m_1 - m_2}{m_1 + m_2} v_1$

$$= \frac{70\,g - 140\,g}{70\,g + 140\,g} \times 9\ ms^{-1} = -3\ ms^{-1}$$

$$v'_2 = \frac{2m_1}{m_1 + m_2} v_1$$

$$= \frac{2 \times 70\,g}{70\,g + 140\,g} \times 9\ ms^{-1} = 6\ ms^{-1}$$

Example 3.5: A 100 g golf ball is moving to the right with a velocity of 20 ms^{-1}. It makes a head on collision with an 8 kg steel ball, initially at rest. Compute velocities of the balls after collision.

Solution: We know that

$$v'_1 = \frac{m_1 - m_2}{m_1 + m_2} v_1 \quad \text{and} \quad v'_2 = \frac{2m_1}{m_1 + m_2} v_1$$

Hence

$$v'_1 = \frac{0.1\,kg - 8\,kg}{0.1\,kg + 8\,kg} \times 20\ ms^{-1} = -19.5\ ms^{-1}$$

$$v'_2 = \frac{2 \times 0.1\,kg}{0.1\,kg + 8\,kg} \times 20\ ms^{-1} = 0.5\ ms^{-1}$$

3.9 FORCE DUE TO WATER FLOW

When water from a horizontal pipe strikes a wall normally, a force is exerted on the wall. Suppose the water strikes the wall normally with velocity v and comes to rest on striking the wall, the change in velocity is then $0 - v = -v$. From second law, the force equals the momentum change per second of water. If mass m of the water strikes the wall in time t then force F on the water is

$$F = -\frac{m}{t} v = -\text{mass per second} \times \text{change in velocity} \quad .... \quad (3.19)$$

From third law of motion, the reaction force exerted by the water on the wall is equal but opposite

Hence, $$F = -(-\frac{m}{t} v) = \frac{m}{t} v$$

Do you know?

If another car crashes into back of yours, the head-rest of the car seat can save you from serious neck injury. It helps to accelerate your head forward with the same rate as the rest of your body.

Point to Ponder

In thrill machine rides at amusement parks, there can be an acceleration of **3g** or more. But without head rests, acceleration like this would not be safe. Think why not?

Thus force can be calculated from the product of mass of water striking normally per second and change in velocity. Suppose the water flows out from a pipe at 3 kgs^{-1} and its velocity changes from 5 ms^{-1} to zero on striking the ball, then,

$$\text{Force} = 3\ kgs^{-1} \times (5\ ms^{-1} - 0) = 15 kgms^{-2} = 15\ N$$

Example 3.6: A hose pipe ejects water at a speed of 0.3 ms^{-1} through a hole of area 50 cm^2. If the water strikes a wall normally, calculate the force on the wall, assuming the velocity of the water normal to the wall is zero after striking.

Solution:

$$\begin{bmatrix}\text{The volume of water per} \\ \text{second striking the wall}\end{bmatrix} = 0.005\ m^2 \times 0.3\ m = 0.0015\ m^3$$

Mass per second striking the wall = volume x density

$$= 0.0015\ m^3 \times 1000\ kgm^{-3} = 1.5\ kg$$

Velocity change of water on striking the wall = $0.3 ms^{-1} - 0 = 0.3 ms^{-1}$

Force = Momentum change per second

$$= 1.5\ kgs^{-1} \times 0.3\ ms^{-1} = 0.45\ kgms^{-2} = 0.45\ N$$

3.10 MOMENTUM AND EXPLOSIVE FORCES

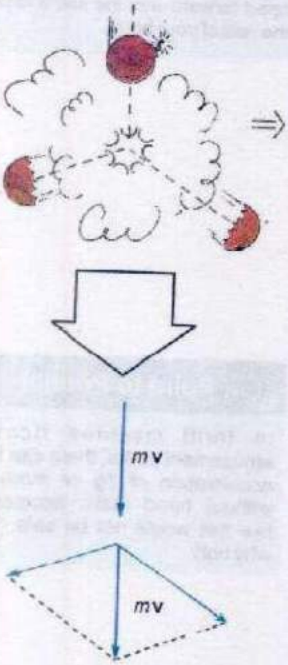


Fig. 3.14

There are many examples where momentum changes are produced by explosive forces within an isolated system. For example, when a shell explodes in mid-air, its fragments fly off in different directions. The total momentum of all its fragments equals the initial momentum of the shell. Suppose a falling bomb explodes into two pieces as shown in Fig. 3.14. The momenta of the bomb fragments combine by vector addition equal to the original momentum of the falling bomb.

Consider another example of bullet of mass m fired from a rifle of mass M with a velocity **v**. Initially, the total momentum of the bullet and rifle is zero. From the principle of conservation of linear momentum, when the bullet is fired, the total momentum of bullet and rifle still remains zero, since no external force has acted on them. Thus if **v**′ is the velocity of the rifle then

$$mv \text{ (bullet)} + Mv' \text{ (rifle)} = 0$$

$$Mv' = -mv \quad \text{or} \quad v' = \frac{-mv}{M} \quad \ldots\ldots\ldots \quad (3.20)$$

The momentum of the rifle is thus equal and opposite to that of the bullet. Since mass of rifle is much greater than the bullet, it follows that the rifle moves back or recoils with only a fraction of the velocity of the bullet.

3.11 ROCKET PROPULSION

Rockets move by expelling burning gas through engines at their rear. The ignited fuel turns to a high pressure gas which is expelled with extremely high velocity from the rocket engines (Fig. 3.15). The rocket gains momentum equal to the momentum of the gas expelled from the engine but in opposite direction. The rocket engines continue to expel gases after the rocket has begun moving and hence rocket continues to gain more and more momentum. So instead of travelling at steady speed the rocket gets faster and faster so long the engines are operating.

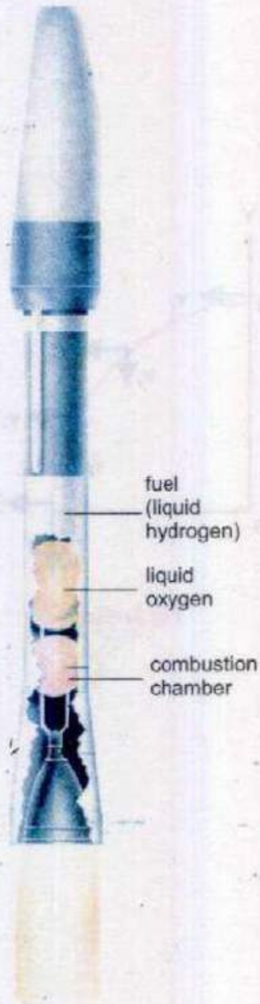


Fig. 3.15

Fuel and oxygen mix in the combustion chamber. Hot gases exhaust the chamber at a very high velocity. The gain in momentum of the gases equals the gain in momentum of the rocket. The gas and rocket push against each other and move in opposite directions.

A rocket carries its own fuel in the form of a liquid or solid hydrogen and oxygen. It can, therefore work at great heights where very little or no air is present. In order to provide enough upward thrust to overcome gravity, a typical rocket consumes about 10000 kgs^{-1} of fuel and ejects the burnt gases at speeds of over 4000 ms^{-1}. In fact, more than 80% of the launch mass of a rocket consists of fuel only. One way to overcome the problem of mass of fuel is to make the rocket from several rockets linked together.

When one rocket has done its job, it is discarded leaving others to carry the space craft further up at ever greater speed.

If m is the mass of the gases ejected per second with velocity **v** relative to the rocket, the change in momentum per second of the ejecting gases is $m\mathbf{v}$. This equals the thrust produced by the engine on the body of the rocket. So, the acceleration '**a**' of the rocket is

$$\mathbf{a} = \frac{m\mathbf{v}}{M} \quad \ldots\ldots\ldots \quad (3.21)$$

where M is the mass of the rocket. When the fuel in the rocket is burned and ejected, the mass M of rocket decreases and hence the acceleration increases.

3.12 PROJECTILE MOTION

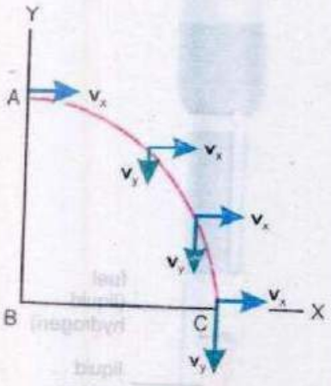


Fig.3.16(a)

Uptill now we have been studying the motion of a particle along a straight line i.e. motion in one dimension. Now we consider the motion of a ball, when it is thrown horizontally from certain height. It is observed that the ball travels forward as well as falls downwards, until it strikes something. Suppose that the ball leaves the hand of the thrower at point A (Fig 3.16 a) and that its velocity at that instant is completely horizontal. Let this velocity be $\mathbf{v}_x$. According to Newton's first law of motion, there will be no acceleration in horizontal direction, unless a horizontally directed force acts on the ball. Ignoring the air friction, only force acting on the ball during flight is the force of gravity. There is no horizontal force acting on it. So its horizontal velocity will remain unchanged and will be $\mathbf{v}_x$, until the ball hits something. The horizontal motion of ball is simple. The ball moves with constant horizontal velocity component. Hence horizontal distance x is given by

$$x = v_x \times t \quad \text{.........} \quad (3.22)$$

The vertical motion of the ball is also not complicated. It will accelerate downward under the force of gravity and hence $\mathbf{a} = \mathbf{g}$. This vertical motion is the same as for a freely falling body. Since initial vertical velocity is zero, hence, vertical distance y, using Eq. 3.7, is given by

$$y = \frac{1}{2} g t^2$$

It is not necessary that an object should be thrown with some initial velocity in the horizontal direction. A football kicked off by a player; a ball thrown by a cricketer and a missile fired from a launching pad, all projected at some angles with the horizontal, are called projectiles.

Projectile motion is two dimensional motion under constant acceleration due to gravity.

In such cases, the motion of a projectile can be studied easily by resolving it into horizontal and vertical components which are independent of each other. Suppose that a projectile is fired in a direction angle θ with the horizontal by velocity v_i as shown in Fig. 3.16 (b). Let components of velocity v_i along the horizontal and vertical directions be $v_i \cos\theta$ and $v_i \sin\theta$ respectively. The horizontal acceleration is $a_x = 0$ because we have neglected air resistance and no other force is acting along this direction whereas vertical acceleration $a_y = g$. Hence, the horizontal component v_{ix} remains constant and at any time t, we have

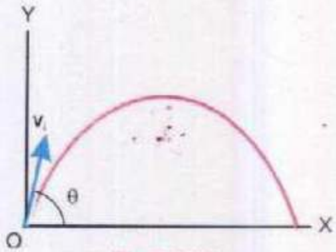


Fig.3.16(b)

$$v_{fx} = v_{ix} = v_i \cos\theta \qquad .... \qquad (3.23)$$

Now we consider the vertical motion. The initial vertical component of the velocity is $v_i \sin\theta$ in the upward direction. Using Eq. 3.5 the vertical component v_{fy} of the velocity at any instant t is given by

$$v_{fy} = v_i \sin\theta - gt \qquad .... \qquad (3.24)$$

The magnitude of velocity at any instant is

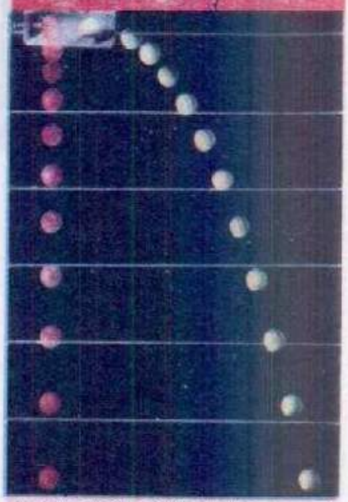
$$v = \sqrt{v_{fx}^2 + v_{fy}^2} \qquad .... \qquad (3.25)$$

The angle ϕ which this resultant velocity makes with the horizontal can be found from

$$\tan\phi = \frac{v_{fy}}{v_{fx}} \qquad .... \qquad (3.26)$$

In projectile motion one may wish to determine the height to which the projectile rises, the time of flight and horizontal range. These are described below.

Interesting Information



A photograph of two balls released simultaneously from a mechanism that allows one ball to drop freely while the other is projected horizontally. At any time the two balls are at the same level, i.e., their vertical displacements are equal.

Height of the Projectile

In order to determine the maximum height the projectile attains, we use the equation of motion

$$2aS = v_f^2 - v_i^2$$

As body moves upward, so $a = -g$, the initial vertical velocity $v_{iy} = v_i \sin\theta$ and $v_{fy} = 0$ because the body comes to rest after reaching the highest point. Since

$$S = \text{height} = h$$

So $$-2\,gh = 0 - v_i^2 \sin^2\theta$$

or $$h = \frac{v_i^2 \sin^2\theta}{2g} \quad \text{.........} \quad (3.27)$$

Time of Flight

The time taken by the body to cover the distance from the place of its projection to the place where it hits the ground at the same level is called the time of flight.

This can be obtained by taking $S = h = 0$, because the body goes up and comes back to same level, thus covering no vertical distance. If the body is projecting with velocity v making angle θ with a horizontal, then its vertical component will be $v_i \sin\theta$. Hence the equation is

$$S = v_i\, t + \tfrac{1}{2}\, gt^2$$

$$0 = v_i \sin\theta\, t - \tfrac{1}{2}\, gt^2$$

$$t = \frac{2\, v_i \sin\theta}{g} \quad \text{.........} \quad (3.28)$$

where t is the time of flight of the projectile when it is projected from the ground as shown in Fig. 3.16 (b).

Point to Ponder

Water is projected from two rubber pipes at the same speed-from one at an angle of 30° and from the other at 60°. Why are the ranges equal?

Range of the Projectile

Maximum distance which a projectile covers in the horizontal direction is called the range of the projectile.

To determine the range R of the projectile, we multiply the horizontal component of the velocity of projection with total time taken by the body after leaving the point of projection. Thus

$$R = v_{ix} \times t \qquad \text{using Eq. 3.28}$$

$$R = \frac{v_i \cos\theta \times 2\, v_i \sin\theta}{g}$$

$$R = \frac{v_i^2}{g}\, 2 \sin\theta \cos\theta$$

But, $2\sin\theta\cos\theta = \sin 2\theta$, thus the range of the projectile depends upon the velocity of projection and the angle of projection.

Therefore, $$R = \frac{v_i^2}{g} \sin 2\theta \quad \text{........ (3.29)}$$

For the range R to be maximum, the factor $\sin 2\theta$ should have maximum value which is 1 when $2\theta = 90^\circ$ or $\theta = 45^\circ$.

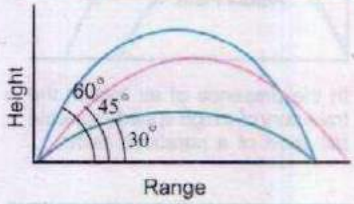
Application to Ballistic Missiles

A ballistic flight is that in which a projectile is given an initial push and is then allowed to move freely due to inertia and under the action of gravity. An un-powered and un-guided missile is called a ballistic missile and the path followed by it is called ballistic trajectory.

As discussed before, a ballistic missile moves in a way that is the result of the superposition of two independent motions: a straight line inertial flight in the direction of the launch and a vertical gravity fall. By law of inertia, an object should sail straight off in the direction thrown, at constant speed equal to its initial speed particularly in empty space. But the downward force of gravity will alter straight path into a curved trajectory. For short ranges and flat Earth approximation, the trajectory is parabolic but the dragless ballistic trajectory for spherical Earth should actually be elliptical. At high speed and for long trajectories the air friction is not negligible and some times the force of air friction is more than gravity. It affects both horizontal as well as vertical motions. Therefore, it is completely unrealistic to neglect the aerodynamic forces.

The shooting of a missile on a selected distant spot is a major element of warfare. It undergoes complicated motions due to air friction and wind etc. Consequently the angle of projection can not be found by the geometry of the situation at the moment of launching. The actual flights of missiles are worked out to high degrees of precision and the result were contained in tabular form. The modified equation of trajectory is too complicated to be discussed here. The ballistic missiles are useful only for short ranges. For long ranges and greater precision, powered and remote control guided missiles are used.

Do You Know ?



For an angle less than 45°, the height reached by the projectile and the range both will be less. When the angle of projectile is larger than 45°, the height attained will be more but the range is again less.

For Your Information

In the presence of air friction the trajectory of a high speed projectile fall short of a parabolic path.

Example 3.7: A ball is thrown with a speed of 30 ms^{-1} in a direction 30° above the horizon. Determine the height to which it rises, the time of flight and the horizontal range.

Solution: Initially

$$v_{ix} = v_i \cos\theta = 30\ ms^{-1} \times \cos 30^\circ = 25.98\ ms^{-1}$$

$$v_{iy} = v_i \sin\theta = 30\ ms^{-1} \times \sin 30^\circ = 15\ ms^{-1}$$

As the time of flight

$$t = \frac{2v_i}{g}\sin\theta$$

So $$t = \frac{2 \times 15\ ms^{-1}}{9.8\ ms^{-2}} = 3.1\ s$$

Height $$h = \frac{v_i^2 \sin^2\theta}{2g}$$

So $$h = \frac{(30\ ms^{-1})^2 \times (0.5)^2}{2 \times 9.8\ ms^{-2}}$$

$$h = 11.5\ m$$

Range $$R = \frac{v_i^2}{g}\sin 2\theta$$

So $$R = \frac{(30\ ms^{-1})^2 \times 0.866}{9.8\ ms^{-2}} = 80m$$

Example 3.8: In example 3.7 calculate the maximum range and the height reached by the ball if the angles of projection are (i) 45° (ii) 60°.

Solution:

(i) Using the equation for height and range we have

height $$h = \frac{v_i^2 \sin^2\theta}{2g}$$

So $$h = \frac{(30\ ms^{-1} \times 0.707)^2}{2 \times 9.8\ ms^{-2}}$$

$$h = 23m$$

Range $$R = \frac{v_i^2}{g} \sin 2\theta$$

or $$R = \frac{v_i^2}{g} \sin 90^\circ$$

or $$R = \frac{(30\ \text{ms}^{-1})^2}{9.8\ \text{ms}^{-2}} \times 1 = 91.8\ \text{m}$$

(ii) Using the equation for height and range we have

height $$h = \frac{v_i^2 \sin^2\theta}{2g}$$

So $$h = \frac{(30\ \text{ms}^{-1} \times .866)^2}{2 \times 9.8\ \text{ms}^{-2}}$$

or $$h = 34.4\ \text{m}$$

Range $$R = \frac{v_i^2}{g} \sin 2\theta$$

or $$R = \frac{v_i^2}{g} \sin 120^\circ$$

or $$R = \frac{(30\ \text{ms}^{-1})^2 \times 0.866}{9.8\ \text{ms}^{-2}} = 80\text{m}$$

SUMMARY

- Displacement is the change in the position of a body from its initial position to its final position.
- Average velocity is the average rate at which displacement vector changes with time.
- Instantaneous velocity is the velocity at a particular instant of time. When the time interval, over which the average velocity is measured, approaches zero, the average velocity becomes equal to the instantaneous velocity at that instant.

$$\mathbf{v}_{ins} = \lim_{\Delta t \to 0} \frac{\Delta \mathbf{d}}{\Delta t}$$

- Average acceleration is the ratio of the change in velocity $\Delta \mathbf{v}$ that occurs within time interval Δt to that time interval.

- Instantaneous acceleration is the acceleration at a particular instant of time. It is the value obtained from the average acceleration as time interval Δt is made smaller and smaller, approaching zero.

$$\mathbf{a} = \lim_{\Delta t \to 0} \frac{\Delta \mathbf{v}}{\Delta t}$$

- The slope of velocity-time graph at any instant represents the instantaneous acceleration at that time.
- The area between velocity-time graph and the time axis is numerically equal to the distance covered by the object.
- Freely falling is a body moving under the influence of gravity alone.
- Acceleration due to gravity near the Earth surface is 9.8 ms^{-2} if air friction is ignored.
- Equations of uniformly accelerated motion are

(i) $v_f = v_i + at$ (ii) $S = \frac{(v_f + v_i)}{2} \cdot t$

(iii) $S = v_i t + \frac{1}{2} at^2$ (iv) $v_f^2 = v_i^2 + 2aS$

- Newton's laws of motion

1st Law: The velocity of an object will be constant if net force on it is zero.

2nd Law: An object gains momentum in the direction of applied force, and the rate of change of momentum is proportional to the magnitude of the force.

3rd Law: When two objects interact, they exert equal and opposite force on each other for the same length of time, and so receive equal and opposite impulses.

- The momentum of an object is the product of its mass and velocity.
- The impulse provided by a force is the product of force and time for which it acts. It equals change in momentum of the object.
- For any isolated system, the total momentum remains constant. The momentum of all bodies in a system add upto the same total momentum at all time.
- Elastic collisions conserve both momentum and kinetic energy. In inelastic collision, some of the energy is transferred by heating and dissipative forces such as friction, air resistance and viscosity, so increasing the internal energy of nearby objects.
- Projectile motion is the motion of particle that is thrown with an initial velocity and then moves under the action of gravity.

QUESTIONS

3.1 What is the difference between uniform and variable velocity? From the explanation of variable velocity, define acceleration. Give SI units of velocity and acceleration.

3.2 An object is thrown vertically upward. Discuss the sign of acceleration due to gravity, relative to velocity, while the object is in air.

3.3 Can the velocity of an object reverse the direction when acceleration is constant? If so, give an example.

3.4 Specify the correct statements:

a. An object can have a constant velocity even its speed is changing.

b. An object can have a constant speed even its velocity is changing.

c. An object can have a zero velocity even its acceleration is not zero.

d. An object subjected to a constant acceleration can reverse its velocity.

3.5 A man standing on the top of a tower throws a ball straight up with initial velocity v_i and at the same time throws a second ball straight downward with the same speed. Which ball will have larger speed when it strikes the ground? Ignore air friction.

3.6 Explain the circumstances in which the velocity **v** and acceleration **a** of a car are

(i) Parallel (ii) Anti-parallel (iii) Perpendicular to one another

(iv) **v** is zero but **a** is not zero (v) **a** is zero but **v** is not zero

3.7 Motion with constant velocity is a special case of motion with constant acceleration. Is this statement true? Discuss.

3.8 Find the change in momentum for an object subjected to a given force for a given time and state law of motion in terms of momentum.

3.9 Define impulse and show that how it is related to linear momentum?

3.10 State the law of conservation of linear momentum, pointing out the importance of isolated system. Explain, why under certain conditions, the law is useful even though the system is not completely isolated?

3.11 Explain the difference between elastic and inelastic collisions. Explain how would a bouncing ball behave in each case? Give plausible reasons for the fact that K.E is not conserved in most cases?

3.12 Explain what is meant by projectile motion. Derive expressions for

a. the time of flight b. the range of projectile.

Show that the range of projectile is maximum when projectile is thrown at an angle of 45° with the horizontal.

3.13 At what point or points in its path does a projectile have its minimum speed, its maximum speed?

3.14 Each of the following questions is followed by four answers, one of which is correct answer. Identify that answer.

i. What is meant by a ballistic trajectory?

a. The paths followed by an un-powered and unguided projectile.

b. The path followed by the powered and unguided projectile.

c. The path followed by un-powered but guided projectile.

d. The path followed by powered and guided projectile.

ii. What happens when a system of two bodies undergoes an elastic collision?

a. The momentum of the system changes.

b. The momentum of the system does not change.

c. The bodies come to rest after collision.

d. The energy conservation law is violated.

NUMERICAL PROBLEMS

3.1 A helicopter is ascending vertically at the rate of 19.6 ms^{-1}. When it is at a height of 156.8 m above the ground, a stone is dropped. How long does the stone take to reach the ground?

(Ans:8.0s)

3.2 Using the following data, draw a velocity-time graph for a short journey on a straight road of a motorbike.

Velocity (ms^{-1})	0	10	20	20	20	20	0
Time (s)	0	30	60	90	120	150	180

Use the graph to calculate

(a) the initial acceleration

(b) the final acceleration and

(c) the total distance travelled by the motorcyclist.

[Ans: (a) 0.33 ms^{-2}, (b) -0.67 $m s^{-2}$, (c) 2.7 km]

3.3 A proton moving with speed of 1.0×10^7 ms^{-1} passes through a 0.020 cm thick sheet of paper and emerges with a speed of 2.0×10^6 ms^{-1}. Assuming uniform deceleration, find retardation and time taken to pass through the paper.

(Ans:- 2.4×10^{17} ms^{-2}, 3.3×10^{-11} s)

3.4 Two masses m_1 and m_2 are initially at rest with a spring compressed between them. What is the ratio of the magnitude of their velocities after the spring has been released?

(Ans: $\frac{v_1}{v_2} = \frac{m_2}{m_1}$)

3.5 An amoeba of mass 1.0×10^{-12} kg propels itself through water by blowing a jet of water through a tiny orifice. The amoeba ejects water with a speed of $1.0 \times 10^{-4}\ ms^{-1}$ and at a rate of $1.0 \times 10^{-13}\ kgs^{-1}$. Assume that the water is being continuously replenished so that the mass of the amoeba remains the same.

a. If there were no force on amoeba other than the reaction force caused by the emerging jet, what would be the acceleration of the amoeba?

b. If amoeba moves with constant velocity through water, what is force of surrounding water (exclusively of jet) on the amoeba?

[Ans: (a) $1.0 \times 10^{-5}\ ms^{-2}$ (b) 1.0×10^{-17} N]

3.6 A boy places a fire cracker of negligible mass in an empty can of 40 g mass. He plugs the end with a wooden block of mass 200 g. After igniting the firecracker, he throws the can straight up. It explodes at the top of its path. If the block shoots out with a speed of $3.0\ ms^{-1}$, how fast will the can be going?

(Ans: $15\ ms^{-1}$)

3.7 An electron ($m = 9.1 \times 10^{-31}$ kg) travelling at $2.0 \times 10^{7}\ ms^{-1}$ undergoes a head on collision with a hydrogen atom ($m = 1.67 \times 10^{-27}$ kg) which is initially at rest. Assuming the collision to be perfectly elastic and a motion to be along a straight line, find the velocity of hydrogen atom.

(Ans: $2.2 \times 10^{4}\ ms^{-1}$)

3.8 A truck weighing 2500 kg and moving with a velocity of $21\ ms^{-1}$ collides with stationary car weighing 1000 kg. The truck and the car move together after the impact. Calculate their common velocity.

(Ans: $15\ ms^{-1}$)

3.9 Two blocks of masses 2.0 kg and 0.50 kg are attached at the two ends of a compressed spring. The elastic potential energy stored in the spring is 10 J. Find the velocities of the blocks if the spring delivers its energy to the blocks when released.

(Ans: $1.4\ ms^{-1}$, $-5.6\ ms^{-1}$)

3.10 A foot ball is thrown upward with an angle of 30° with respect to the horizontal. To throw a 40 m pass what must be the initial speed of the ball?

(Ans: $21 ms^{-1}$)

3.11 A ball is thrown horizontally from a height of 10 m with velocity of 21 ms^{-1}. How far off it hit the ground and with what velocity?

(Ans: 30 m, 25 ms^{-1})

3.12 A bomber dropped a bomb at a height of 490 m when its velocity along the horizontal was 300 kmh^{-1}.

(a) How long was it in air?

(b) At what distance from the point vertically below the bomber at the instant the bomb was dropped, did it strike the ground?

(Ans: 10 s, 833 m)

3.13 Find the angle of projection of a projectile for which its maximum height and horizontal range are equal.

(Ans: 76°)

3.14 Prove that for angles of projection, which exceed or fall short of 45° by equal amounts, the ranges are equal.

3.15 A SLBM (submarine launched ballistic missile) is fired from a distance of 3000km. If the Earth is considered flat and the angle of launch is 45° with horizontal, find the velocity with which the missile is fired and the time taken by SLBM to hit the target.

(Ans: 5.42 kms^{-1}, 13 min)