Chapter 4

WORK AND ENERGY

Learning Objectives

At the end of this chapter the students will be able to:

1. Understand the concept of work in terms of the product of a force and displacement in the direction of the force.
2. Understand and derive the formula Work = **wd** = *mgh* for work done in a gravitational field near Earth's surface.
3. Understand that work can be calculated from area under the force-displacement graph.
4. Relate power to work done.
5. Define power as the product of force and velocity.
6. Quote examples of power from everyday life.
7. Explain the two types of mechanical energy.
8. Understand the work-energy principle.
9. Derive an expression for absolute potential energy.
10. Define escape velocity.
11. Understand that in a resistive medium loss of potential energy of a body is equal to gain in kinetic energy of the body plus work done by the body against friction.
12. Give examples of conservation of energies from everyday life.
13. Describe some non-conventional sources of energy.

Work is often thought in terms of physical or mental effort. In Physics, however, the term work involves two things (i) force (ii) displacement. We shall begin with a simple situation in which work is done by a constant force.

4.1 WORK DONE BY A CONSTANT FORCE

Let us consider an object which is being pulled by a constant force **F** at an angle θ to the direction of motion. The force displaces the object from position A to B through a displacement d (Fig. 4.1).

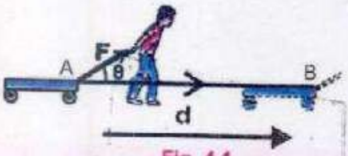

Fig. 4.1

We define work W done by the force **F** as the scalar product of **F** and **d**.

$$W = \mathbf{F}.\mathbf{d} = Fd\cos\theta \qquad \ldots\ldots\ldots \qquad (4.1)$$

$$= (F\cos\theta)\, d$$

The quantity ($F\cos\theta$) is the component of the force in the direction of the displacement **d**.

Thus, the work done on a body by a constant force is defined as the product of the magnitudes of the displacement and the component of the force in the direction of the displacement.

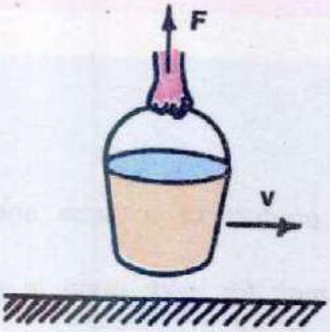

Fig. 4.2(a)

Can you tell how much work is being done?

(i) On the pail when a person holding the pail by the force **F** is moving forward (Fig. 4.2 a).

(ii) On the wall (Fig. 4.2 b)?

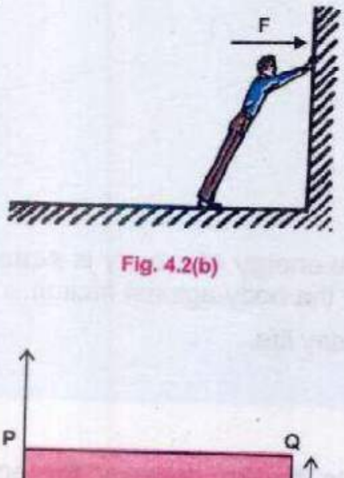

Fig. 4.2(b)

When a constant force acts through a distance d, the event can be plotted on a simple graph (Fig. 4.3). The distance is normally plotted along x-axis and the force along y-axis. In this case as the force does not vary, the graph will be a horizontal straight line. If the constant force **F** (newton) and the displacement **d** (metre) are in the same direction then the work done is Fd (joule). Clearly shaded area in Fig. 4.3 is also Fd. Hence the area under a force- displacement curve can be taken to represent the work done by the force. In case the force **F** is not in the direction of displacement, the graph is plotted between $F\cos\theta$ and d.

From the definition of work, we find that:

(i) Work is a scalar quantity.

(ii) If $\theta < 90^{\circ}$, work is done and it is said to be positive work.

(iii) If $\theta = 90^{\circ}$, no work is done.

(iv) If $\theta > 90^{\circ}$, the work done is said to be negative.

(v) SI unit of work is N m known as joule (J).

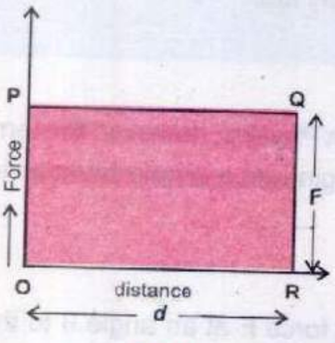

Fig. 4.3

4.2 WORK DONE BY A VARIABLE FORCE

In many cases the force does not remain constant during the process of doing work. For example, as a rocket moves away from the Earth, work is done against the force of gravity, which varies as the inverse square of the distance from the Earth's centre. Similarly, the force exerted by a spring increases with the amount of stretch. How can we calculate the work done in such a situation?

Fig. 4.4 shows the path of a particle in the x-y plane as it moves from point a to point b. The path has been divided into n short intervals of displacements $\Delta \mathbf{d_1}$, $\Delta \mathbf{d_2}$, ..., $\Delta \mathbf{d_n}$ and $\mathbf{F_1}$, $\mathbf{F_2}$, ..., $\mathbf{F_n}$ are the forces acting during these intervals.

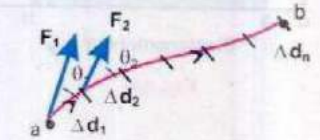

Fig. 4.4

A particle acted upon by a variable force, moves along the path shown from point a to point b.

During each small interval, the force is supposed to be approximately constant. So the work done for the first interval can then be written as

$$\Delta W_1 = \mathbf{F_1} \cdot \Delta \mathbf{d_1} = F_1 \cos\theta_1 \Delta d_1$$

and in the second interval

$$\Delta W_2 = \mathbf{F_2} \cdot \Delta \mathbf{d_2} = F_2 \cos\theta_2 \Delta d_2$$

and so on. The total work done in moving the object can be calculated by adding all these terms.

$$W_{total} = \Delta W_1 + \Delta W_2 + ... + \Delta W_n$$

$$= F_1 cos\theta_1 \Delta d_1 + F_2 \; cos\theta_2 \Delta d_2 + ... + F_n \; cos\theta_n \; d_n$$

$$W_{total} = \sum_{i=1}^{n} F_i \; cos\theta_i \Delta d_i \qquad .... \qquad (4.2)$$

We can examine this graphically by plotting $F \cos\theta$ verses d as shown in Fig. 4.5. The displacement d has been subdivided into n equal intervals. The value of $F\cos\theta$ at the beginning of each interval is indicated in the figure.

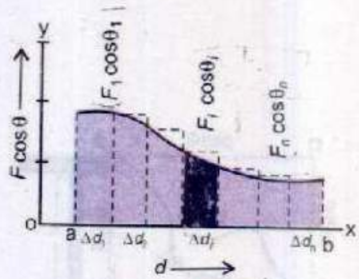

Fig. 4.5

Now the ith shaded rectangle has an area $F_i \; cos\theta_i \Delta d_i$ which is the work done during the ith interval. Thus, the work done given by Eq. 4.2 equals the sum of the areas of all the rectangles. If we subdivide the distance into a large number of intervals so that each Δd becomes very small, the work done given by Eq. 4.2 becomes more accurate. If we let each Δd to approach zero then we obtain an exact result for the work done, such as

$$W_{total} = \underset{\Delta d \to 0}{Limit} \sum_{i=1}^{n} F_i \cos\theta_i \Delta d_i \quad \ldots\ldots\ldots \quad (4.3)$$

In this limit Δd approaches zero, the total area of the rectangles (Fig. 4.5) approaches the area between the $F\cos\theta$ curve and d-axis from a to b as shown shaded in Fig. 4.6.

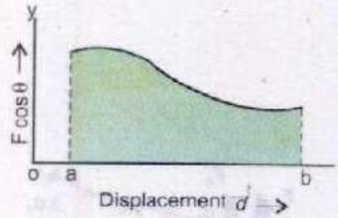

Fig. 4.6

Thus, the work done by a variable force in moving a particle between two points is equal to the area under the $F\cos\theta$ verses d curve between the two points a and b as shown in Fig. 4.6.

Example 4.1: A force F acting on an object varies with distance x as shown in Fig. 4.7. Calculate the work done by the force as the object moves from $x = 0$ to $x = 6$ m.

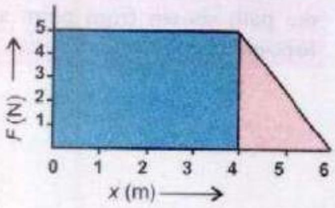

Fig. 4.7

Solution: The work done by the force is equal to the total area under the curve from $x = 0$ to $x = 6$ m. This area is equal to the area of the rectangular section from $x = 0$ to $x = 4$ m, plus the area of triangular section from $x = 4$ m to $x = 6$ m. Hence

Work done represented by the area of rectangle = 4 m x 5 N

$= 20$ N m $= 20$ J

Work done represented by the area of triangle = $\frac{1}{2}$ x 2 m x 5 N

$= 5$ N m$= 5$ J

Therefore, the total work done = 20 J + 5 J = 25 J

4.3 WORK DONE BY GRAVITATIONAL FIELD

The space around the Earth in which its gravitational force acts on a body is called the gravitational field. When an object is moved in the gravitational field, the work is done by the gravitational force. If displacement is in the direction of gravitational force, the work is positive. If the displacement is against the gravitational force, the work is negative.

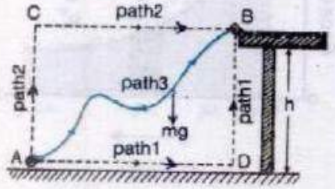

Fig. 4.8

Let us consider an object of mass m being displaced with constant velocity from point A to B along various paths in the presence of a gravitational force (Fig. 4.8). In this case the gravitational force is equal to the weight mg of the object.

The work done by the gravitational force along the path ADB can be split into two parts. The work done along AD is zero, because the weight *mg* is perpendicular to this path, the work done along DB is (–mgh) because the direction of *mg* is opposite to that of the displacement i.e. $\theta = 180°$. Hence, the work done in displacing a body from A to B through path 1 is

$$W_{ADB} = 0 + (-mgh) = -mgh$$

If we consider the path ACB, the work done along AC is also (*-mgh*). Since the work done along CB is zero, therefore,

$$W_{ACB} = -mgh + 0 = -mgh$$

Let us now consider path 3, i.e. a curved one. Imagine the curved path, to be broken down into a series of horizontal and vertical steps as shown in Fig. 4.9. There is no work done along the horizontal steps, because *mg* is perpendicular to the displacement for these steps. Work is done by the force of gravity only along the vertical displacements.

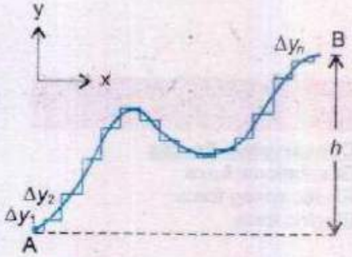

Fig. 4.9

A smooth path may be replaced by a series of infinitesimal *x* and *y* displacements. Work is done only during the y displacements.

$$W_{AB} = -mg(\Delta y_1 + \Delta y_2 + \Delta y_3 + \dots\dots + \Delta y_n)$$

as $\quad (\Delta y_1 + \Delta y_2 + \Delta y_3 + \dots\dots + \Delta y_n) = h$

Hence, $\quad W_{AB} = -mgh$

The net amount of work done along AB path is still *(-mgh)*.

We conclude from the above discussion that

Work done in the Earth's gravitational field is independent of the path followed.

Can you prove that the work done along a closed path such as ACBA or ADBA (Fig. 4.8), in a gravitational field is zero?

The field in which the work done be independent of the path followed or work done in a closed path be zero, is called a conservative field.

The frictional force is a non-conservative force, because if an object is moved over a rough surface between two points along different paths, the work done against the frictional force certainly depends on the path followed.

4.4 POWER

In the definition of work, it is not clear, whether the same amount of work is done in one second or in one hour. The rate, at which work is done, is often of interest in practical applications.

Power is the measure of the rate at which work is being done.

If work ΔW is done in a time interval Δt, then the average power P_{av} during the interval Δt is defined as

$$P_{av} = \frac{\Delta W}{\Delta t} \quad \text{..........} \quad (4.4)$$

If work is expressed as a function of time, the instantaneous power P at any instant is defined as

$$P = \underset{\Delta t \to 0}{Limit} \frac{\Delta W}{\Delta t} \quad \text{..........} \quad (4.5)$$

Where ΔW is the work done in short interval of time Δt following the instant t.

For Your Information

Conservative Forces
Gravitational force
Elastic spring force
Electric force

Non Conservative forces
Frictional force
Air resistance
Tension in a string
Normal force
Propulsion force of a rocket
Propulsion force of a motor

Power and Velocity

It is, sometimes, convenient to express power in terms of a constant force **F** acting on an object moving at constant velocity **v**. For example, when the propeller of a motor boat causes the water to exert a constant force **F** on the boat, it moves with a constant velocity **v**. The power delivered by the motor at any instant is, then, given by

$$P = \underset{\Delta t \to 0}{Limit} \frac{\Delta W}{\Delta t}$$

we know $\Delta W = \mathbf{F}.\Delta \mathbf{d}$

so
$$P = \underset{\Delta t \to 0}{Limit} \frac{\mathbf{F}.\Delta \mathbf{d}}{\Delta t}$$

Since $\quad \lim\limits_{\Delta t \to 0} \dfrac{\Delta \mathbf{d}}{\Delta t} = \mathbf{v}$

Hence, $\quad P = \mathbf{F}.\mathbf{v} \quad \ldots\ldots\ldots \quad (4.6)$

The SI unit of power is watt, defined as one joule of work done in one second.

Sometimes, for example, in the electrical measurements, the unit of work is expressed as watt second. However, a commercial unit of electrical energy is kilowatt-hour.

One kilowatt hour is the work done in one hour by an agency whose power is one kilowatt.

Therefore, $\quad 1\ \text{kWh} = 1000\ \text{W} \times 3600\ \text{s}.$

or $\quad 1\ \text{kWh} = 3.6 \times 10^6\ \text{J} = 3.6\ \text{MJ}$

Example 4.2: A 70 kg man runs up a long flight of stairs in 4.0 s. The vertical height of the stairs is 4.5 m. Calculate his power output in watts.

Solution: Work done = mgh

$$\text{Power} = \frac{mgh}{t}$$

$$P = \frac{70\ \text{kg} \times 9.8\ \text{ms}^{-2} \times 4.5\ \text{m}}{4\ \text{s}}$$

$$P = 7.7 \times 10^2\ \text{kgm}^2\text{s}^{-3} = 7.7 \times 10^2\ \text{W}$$

For Your Information

Approximate Powers

Device	Power (W)
Jumbo Jet Aircraft	$1.3 \times 10^{[illegible]}$
Car at 90 km h^{-1}	1.1×10^{5}
Electric heater	2×10^{3}
Colour Tv	120
Flash light (two cells)	1.5
Pocket calculator	7.5×10^{-4}

4.5 ENERGY

Energy of a body is its capacity to do work. There are two basic forms of energy.

(i) Kinetic energy (ii) Potential energy

The kinetic energy is possessed by a body due to its motion and is given by the formula

$$\text{K.E.} = \frac{1}{2} mv^2 \quad \ldots\ldots\ldots \quad (4.7)$$

Do You Know?

It takes about 9×10^9 J to make a car and the car then uses about 1×10^{12} J of energy from petrol in its life time.

where m is the mass of the body moving with velocity **v**.

The potential energy is possessed by a body because of its position in a force field e.g. gravitational field or because of its constrained state. The potential energy due to gravitational field near the surface of the Earth at a height h is given by the formula

$$\text{P.E.} = mgh \qquad \ldots\ldots\ldots\ldots \qquad (4.8)$$

This is called gravitational potential energy. The gravitational P.E. is always determined relative to some arbitrary position which is assigned the value of zero P.E. In the present case, this reference level is the surface of the Earth as position of zero P.E. In some cases a point at infinity from the Earth can also be chosen as zero reference level.

For Your Information

Approximate Energy Values

Source	Energy (J)
Burning 1 ton coal	30×10^{9}
Burning 1 litre petrol	5×10^{7}
K.E. of a car at 90 km h^{-1}	1×10^{6}
Running Person at 10 km h^{-1}	3×10^{2}
Fission of one atom of uranium	1.8×10^{-11}
K.E. of a molecule of air	6×10^{-21}

The energy stored in a compressed spring is the potential energy possessed by the spring due to its compressed or stretched state. This form of energy is called the elastic potential energy.

Work-Energy Principle

Whenever work is done on a body, it increases its energy. For example a body of mass m is moving with velocity v_i. A force F acting through a distance d increases the velocity to v_f, then from equation of motion

$$2ad = v_f^2 - v_i^2$$

or
$$d = \frac{1}{2a}(v_f^2 - v_i^2) \qquad \ldots\ldots\ldots\ldots \qquad (4.9)$$

From second law of motion

$$F = ma \qquad \ldots\ldots\ldots\ldots \qquad (4.10)$$

Multiplying equations 4.9 and 4.10, we have

$$Fd = \frac{1}{2}m(v_f^2 - v_i^2)$$

or
$$Fd = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 \qquad \ldots\ldots\ldots\ldots \qquad (4.11)$$

Tid-bits

All the food you eat in one day has about the same energy as 1/3 litre of petrol.

where the left hand side of the above equation gives the work done on the body and right hand side gives the increase or change in kinetic energy of the body. Thus

Work done on the body equals the change in its kinetic energy.

This is known as work-energy principle. If a body is raised up from the Earth's surface, the work done changes the gravitational potential energy. Similarly, if a spring is compressed, the work done on it equals the increase in its elastic potential energy.

Absolute Potential Energy

The absolute gravitational potential energy of an object at a certain position is the work done by the gravitational force in displacing the object from that position to infinity where the force of gravity becomes zero. The relation for the calculation of the work done by the gravitational force or potential energy = *mgh*, is true only near the surface of the Earth where the gravitational force is nearly constant. But if the body is displaced through a large distance in space from, let, point 1 to N (Fig. 4.10) in the gravitational field, then the gravitational force will not remain constant, since it varies inversely to the square of the distance.

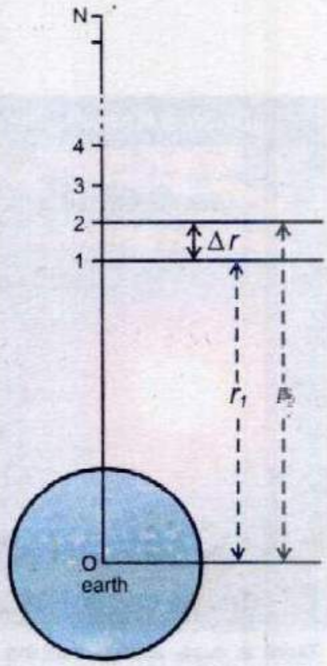

Fig. 4.10

In order to overcome this difficulty, we divide the distance between points 1 and N into small steps each of length Δr so that the value of the force remains constant for each small step. Hence, the total work done can be calculated by adding the work done during all these steps. If r_1 and r_2 are the distances of points 1 and 2 respectively, from the centre O of the Earth (Fig. 4.10.), the work done during the first step i.e., displacing a body from point 1 to point 2 can be calculated as below.

The distance between the centre of this step and the centre of the Earth will be

$$r = \frac{r_1 + r_2}{2}$$

if $\quad r_2 - r_1 = \Delta r \quad$ then $\quad r_2 = r_1 + \Delta r$

Hence, $r = \frac{r_1 + r_1 + \Delta r}{2} = r_1 + \frac{\Delta r}{2}$ .... (4.12)

The gravitational force F at the centre of this step is

$$F = G\frac{Mm}{r^2} \quad \text{..........} \quad (4.13)$$

where m = mass of an object , M = mass of the Earth

and G = Gravitational constant.

Squaring Eq. 4.12

$$r^2 = \left(r_1 + \frac{\Delta r}{2}\right)^2$$

$$r^2 = r_1^2 + 2r_1\frac{\Delta r}{2} + \left(\frac{\Delta r}{2}\right)^2$$

As $(\Delta r)^2 << r_1^2$, so this term can be neglected as compared to r_1^2

Hence $r^2 = r_1^2 + r_1 \Delta r$

Substituting the value of Δr

$$r^2 = r_1^2 + r_1(r_2 - r_1) = r_1 r_2$$

Hence, Eq. 4.13 becomes

$$F = G\frac{Mm}{r_1 r_2} \quad \text{..........} \quad (4.14)$$

Do You Know?

There is more energy reaching Earth in 10 days of sunlight than in all the fossil fuels on the Earth.

As this force is assumed to be constant during the interval Δr, so the work done is

$$W_{1\to 2} = \mathbf{F}.\Delta \mathbf{r} = F\Delta r \cos 180° = - GMm\frac{\Delta r}{r_1 r_2}$$

The negative sign indicates that the work has to be done on the body from point 1 to 2 because displacement is opposite to gravitational force. Putting the value of Δr, we get

$$W_{1\to 2} = - GMm\frac{r_2 - r_1}{r_1 r_2}$$

or $$W_{1\to 2} = -GMm\left(\frac{1}{r_1}-\frac{1}{r_2}\right)$$

Similarly the work done during the second step in which the body is displaced from point 2 to 3 is

$$W_{2\to 3} = -GMm\left(\frac{1}{r_2}-\frac{1}{r_3}\right)$$

and the work done in the last step is

$$W_{N-1\to N} = -GMm\left(\frac{1}{r_{N-1}}-\frac{1}{r_N}\right)$$

Hence, the total work done in displacing a body from point1 to N is calculated by adding up the work done during all these steps.

$$W_{total} = W_{1\to 2} + W_{2\to 3} + \ldots\ldots\ldots + W_{N-1\to N}$$

$$= -G\,Mm\left[\left(\frac{1}{r_1}-\frac{1}{r_2}\right)+\left(\frac{1}{r_2}-\frac{1}{r_3}\right)+\ldots\ldots+\left(\frac{1}{r_{N-1}}-\frac{1}{r_N}\right)\right]$$

On simplification, we get

$$W_{total} = -GMm\left(\frac{1}{r_1}-\frac{1}{r_N}\right)$$

If the point N is situated at an infinite distance from the Earth, so

$r_N = \infty$, then $\frac{1}{r_N} = \frac{1}{\infty} = 0$

Hence, $W_{total} = \frac{-GMm}{r_1}$

Therefore, the general expression for the gravitational potential energy of a body situated at distance r from the centre of Earth is

$$U = \frac{-GMm}{r}$$

This is also known as the absolute value of gravitational potential energy of a body at a distance r from the centre of the Earth.

Tid-bits

More coal has been used since 1945 than was used in the whole of history before that.

Note that when r increases, U becomes less negative i.e., U increases. It means when we raise a body above the surface of the Earth its P.E. increases. The choice of zero point is arbitrary and only the difference of P.E. From one point to another is significant, wether we consider the surface of the Earth or the point at infinity as zero P.E. reference, the change in P.E. as we move a body above the surface of the Earth, will always be positive.

Now the absolute potential energy on the surface of the Earth is found by putting $r = R$ (Radius of the Earth)

$$\text{Absolute potential energy} = U_g = -\frac{GMm}{R} \quad \ldots\ldots \quad (4.15)$$

The negative sign shows that the Earth's gravitational field for mass m is attractive. The above expression gives the work or the energy required to take the body out of the Earth's gravitational field, where its potential energy with respect to Earth is zero.

For Your Information

Some Escape speeds (kms^{-1})

Heavenly body	Escape speed
Moon	2.4
Mercury	4.3
Mars	5.0
Venus	10.4
Earth	11.2
Uranus	22.4
Neptune	25.4
Saturn	37.0
Jupiter	61

Escape Velocity

It is our daily life experience that an object projected upward comes back to the ground after rising to a certain height. This is due to the force of gravity acting downward. With increased initial velocity, the object rises to the greater height before coming back. If we go on increasing the initial velocity of the object, a stage comes when it will not return to the ground. It will escape out of the influence of gravity. The initial velocity of an object with which it goes out of the Earth's gravitational field, is known as escape velocity.

The escape velocity corresponds to the initial kinetic energy gained by the body, which carries it to an infinite distance from the surface of Earth.

$$\text{Initial K.E.} = \frac{1}{2} m v_{esc}^2$$

We know that the work done in lifting a body from Earth's surface to an infinite distance is equal to the increase in its potential energy

$$\text{Incerase in P.E.} = 0-(-G\frac{Mm}{R}) = G\frac{Mm}{R}$$

where M and R are the mass and radius of the Earth respectively. The body will escape out of the gravitational field if the initial K.E. of the body is equal to the increase in P.E. of the body in lifting it up to infinity. Then

$$\frac{1}{2}mv_{esc}^2 = G\frac{Mm}{R}$$

or $$v_{esc} = \sqrt{\frac{2GM}{R}} \quad \ldots\ldots\ldots \quad (4.16)$$

As $$g = \frac{GM}{R^2}$$

Hence, $$v_{esc} = \sqrt{2gR} \quad \ldots\ldots\ldots \quad (4.17)$$

The value of v_{esc} comes out to be approximately 11 kms^{-1}

4.6 INTERCONVERSION OF POTENTIAL ENERGY AND KINETIC ENERGY

Consider a body of mass m at rest, at a height h above the surface of the Earth as shown in Fig. 4.11. At position A, the body has P.E. = mgh and K.E. = 0. We release the body and as it falls, we can examine how kinetic and potential energies associated with it interchange.

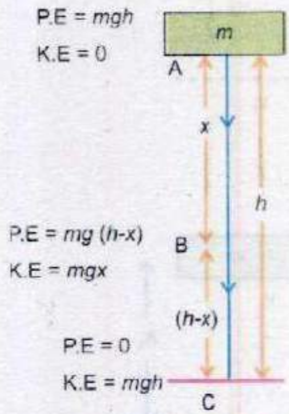

Fig. 4.11

Let us calculate P.E. and K.E. at position B when the body has fallen through a distance x, ignoring air friction.

$$\text{P.E.} = mg(h-x)$$

and $$\text{K.E.} = \frac{1}{2}mv_B^2$$

Velocity v_B, at B, can be calculated from the relation,

$$v_f^2 = v_i^2 + 2gS$$

$$v_f = v_B \quad , \quad v_i = 0 \quad , \quad S = x$$

$$v_B^2 = 0 + 2gx = 2gx$$

$$K.E. = \frac{1}{2} m\,(2gx) = mgx$$

Total energy at B = P.E. + K.E.

$$= mg\,(h - x) + mgx = mgh \quad \ldots\ldots \quad (4.18)$$

At position C, just before the body strikes the Earth, P.E. = 0 and K.E. = $\frac{1}{2} m v_C^2$, where v_C can be found out by the following expression.

$$v_C^2 = v_i^2 + 2gh = 2gh \quad \text{as} \quad v_i = 0$$

i.e.

$$K.E. = \frac{1}{2} m v_C^2 = \frac{1}{2} m \times 2gh = mgh$$

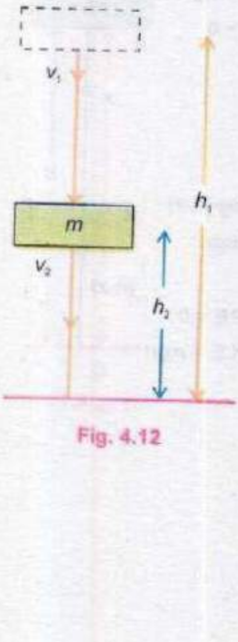

Fig. 4.12

Thus at point C, kinetic energy is equal to the original value of the potential energy of the body. Actually when a body falls, its velocity increases i.e., the body is being accelerated under the action of gravity. The increase in velocity results in the increase in its kinetic energy. On the other hand, as the body falls, its height decreases and hence, its potential energy also decreases. Thus we see (Fig. 4.12) that,

Loss in P.E. = Gain in K.E.

$$mg\,(h_1 - h_2) = \frac{1}{2} m\,(v_2^2 - v_1^2) \quad \ldots\ldots\ldots \quad (4.19)$$

Where v_1 and v_2 are velocities of the body at heights h_1 and h_2 respectively. This result is true only when frictional force is not considered.

If we assume that a frictional force f is present during the downward motion, then a part of P.E. is used in doing work against friction equal to $f\,h$. The remaining P.E. = $mgh - f\,h$ is converted into K.E.

Hence, $$mgh - f\,h = \frac{1}{2} mv^2$$

or $$mgh = \frac{1}{2} mv^2 + f\,h \quad \ldots\ldots\ldots \quad (4.20)$$

Thus,

Loss in P.E. = Gain in K.E. + Work done against friction.

4.7 CONSERVATION OF ENERGY

The kinetic and potential energies are both different forms of the same basic quantity, i.e. mechanical energy. Total mechanical energy of a body is the sum of the kinetic energy and potential energy. In our previous discussion of a falling body, potential energy may change into kinetic energy and vice versa, but the total energy remains constant. Mathematically,

Total Energy = P.E. + K.E. = Constant

This is a special case of the law of conservation of energy which states that:

Energy cannot be destroyed. It can be transformed from one kind into another, but the total amount of energy remains constant.

This is one of the basic laws of physics. We daily observe many energy transformations from one form to another. Some forms, such as electrical and chemical energy, are more easily transferred than others, such as heat. Ultimately all energy transfers result in heating of the environment and energy is wasted. For example, the P.E. of the falling object changes to K.E., but on striking the ground, the K.E. changes into heat and sound. If it seems in an energy transfer that some energy has disappeared, the lost energy is often converted into heat. This appears to be the fate of all available energies and is one reason why new sources of useful energy have to be developed.

For Your Information

Source of energy	Original source
Solar	Sun
Bio mass	Sun
Fossil fuels	Sun
Wind	Sun
Waves	Sun
Hydro electric	Sun
Tides	Moon
Geothermal	Earth

Energy Sources

Renewable	Nonrenewable
Hydroelectric	Coal
Wind	Natural gas
Tides	Oil
Geothermal *	Uranium
Biomass	Oil shale
Sunlight	Tar sands
Ethanol/Methanol**	

* Individual fields may run off
**Renewable when made from bio mass

Example 4.3: A brick of mass 2.0 kg is dropped from a rest position 5.0 m above the ground. What is its velocity at a height of 3.0 m above the ground?

Solution: Using Eq. 4.19

$$mg\,(h_1 - h_2) = \frac{1}{2} m\,(v_2^2 - v_1^2)$$

As $v_1 = 0$ and $v_2 = v$

Hence $v = \sqrt{2g(h_1 - h_2)}$

or $v = \sqrt{2 \times 9.8\,\text{ms}^{-2} \times 2.0\text{m}} = 6.3\ \text{m s}^{-1}$

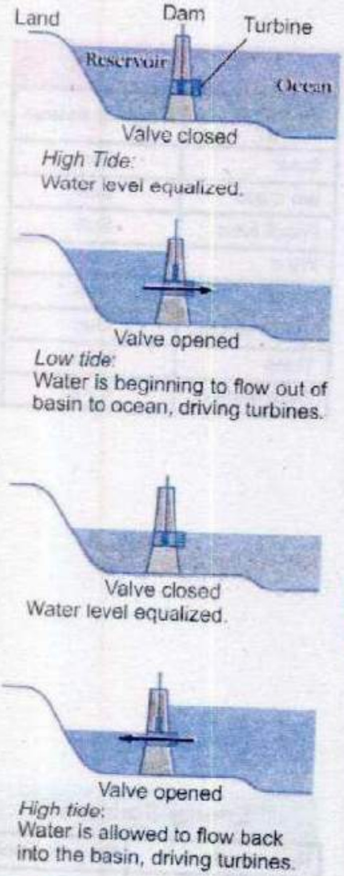

Fig. 4.13

Tidal power plant. Turbines are located inside the dam.

Do You Know?

The pull of the Moon does not only pull the sea up and down. This tidal effect can also distort the continents pulling land up and down by as much as 25cm.

4.8 NON CONVENTIONAL ENERGY SOURCES

These are the energy sources which are not very common these days. However, it is expected that these sources will contribute substantially to the energy demand of the future. Some of these are introduced briefly here.

Energy from Tides

One very novel example of obtaining energy from gravitational field is the energy obtained from tides. Gravitational force of the moon gives rise to tides in the sea. The tides raise the water in the sea roughly twice a day. If the water at the high tide is trapped in a basin by constructing a dam, then it is possible to use this as a source of energy. The dam is filled at high tide and water is released in a controlled way at low tide to drive the turbines. At the next high tide the dam is filled again and the in rushing water also drives turbines and generates electricity as shown systematically in the Fig. 4.13.

Energy from Waves

The tidal movement and the winds blowing across the surface of the ocean produce strong water waves. Their energy can be utilized to generate electricity. A method of harnessing wave energy is to use large floats which move up and down with the waves. One such device invented by Professor Salter is known Salter's duck (Fig. 4.14). It consists of two parts (i) Duck float. (ii) Balance float.

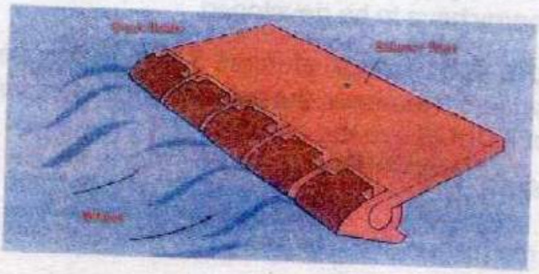

Fig. 4.14

The wave energy makes duck float move relative to the balance float. The relative motion of the duck float is then used to run electricity generators.

Solar Energy

The Earth receives huge amount of energy directly from the Sun each day. Solar energy at normal incidence outside the Earth's atmosphere is about 1.4 kWm^{-2} which is referred as solar constant. While passing through the atmosphere, the total energy is reduced due to reflection, scattering and absorption by dust particles, water vapours and other gases. On a clear day at noon, the intensity of solar energy reaching the Earth's surface is about 1kWm^{-2}. This energy can be used directly to heat water with the help of large solar reflectors and thermal absorbers. It can also be converted to electricity. In one method the flat plate collectors are used for heating water. A typical collector is shown in Fig. 4.15 (a). It has a blackened surface which absorbs energy directly from solar radiation. Cold water passes over the surface and is heated upto about 70°C.

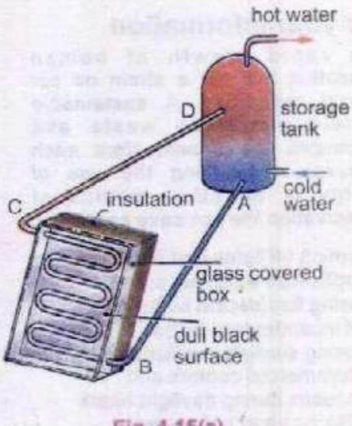

Fig. 4.15(a)

Much higher temperature can be achieved by concentrating solar radiation on to a small surface area by using huge reflectors (mirrors) or lenses to produced steam for running a turbine.

The other method is the direct conversion of sunlight into electricity through the use of semi conductor devices called solar cells also known as photo voltaic cells. Solar cells are thin wafers made from silicon. Electrons in the silicon gain energy from sunlight to create a voltage. The voltage produced by a single voltaic cell is very low. In order to get sufficient high voltage for practical use, a large number of such cells are connected in series forming a solar cell panel.

For cloudy days or nights, electric energy can be stored during the Sun light in Nickel cadmium batteries by connecting them to solar panels. These batteries can then provide power to electrical appliances at nights or on cloudy days.

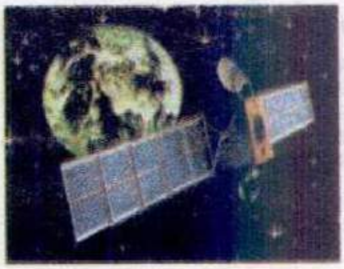
Fig. 4.15(b)

Solar cells, although, are expensive but last a long time and have low running cost. Solar cells are used to power satellites having large solar panels which are kept facing the Sun (Fig. 4.15 b). Other examples of the use of solar cells are remote ground based weather stations and rain forest communication systems. Solar calculators and watches are also in use now-a-days.

For your information

The rapid growth of human population has put a strain on our natural resources. A sustainable society minimizes waste and maximizes the benefit from each resource. Minimizing the use of energy is an other method of conservation.We can save energy by,

(i) turning off lights and electrical appliances when not in use.
(ii) using fluorescent bulbs instead of incandescent bulbs
(iii) using sunlight in offices, commercial centers and houses during daylight hours
(iv) Taking short hot shawers.

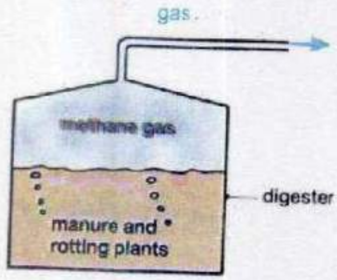

Fig. 4.16

Do you know ?

Pollution can be reduced if
(i) People use mass transportation
(ii) Use geothermal, solar, hydroelectrical and wind energy as alternative forms of energy.

Energy From Biomass

Biomass is a potential source of renewable energy. This includes all the organic materials such as crop residue, natural vegetation, trees, animal dung and sewage. Biomass energy or bio conversion refers to the use of this material as fuel or its conversion into fuels.

There are many methods used for the conversion of biomass into fuels. But the most common are

1. Direct combustion 2. Fermentation

Direct combustion method is usually applied to get energy from waste products commonly known as solid waste. It will be discussed in the next section.

Biofuel such as ethanol (alcohol) is a replacement of gasoline. It is obtained by fermentation of biomass using enzymes and by decomposition through bacterial action in the absence of air (oxygen).

The rotting of biomass in a closed tank called a digester produces Biogas which can be piped out to use for cooking and heating (Fig. 4.16).

The waste material of the process is a good organic fertilizer. Thus, production of biogas provides us energy source and also solves the problem of organic waste disposal.

Energy from Waste Products

Waste products like wood waste, crop residue, and particularly municipal solid waste can be used to get energy by direct combustion. It is probably the most commonly used conversion process in which waste material is burnt in a confined container. Heat produced in this way is directly utilized in the boiler to produce steam that can run turbine generator.

Geothermal Energy

This is the heat energy extracted from inside the Earth in the form of hot water or steam. Heat within the Earth is generated by the following processes.

1. Radioactive Decay

The energy, heating the rocks, is constantly being released by the decay of radioactive elements.

2. Residual Heat of the Earth

At some places hot igneous rocks, usually within 10 km of the Earth's surface, are in a molten and partly molten state. They conduct heat energy from the Earth's interior which is still very hot. The temperature of these rocks is about 200°C or more.

3. Compression of Material

The compression of material deep inside the Earth also causes generation of heat energy.

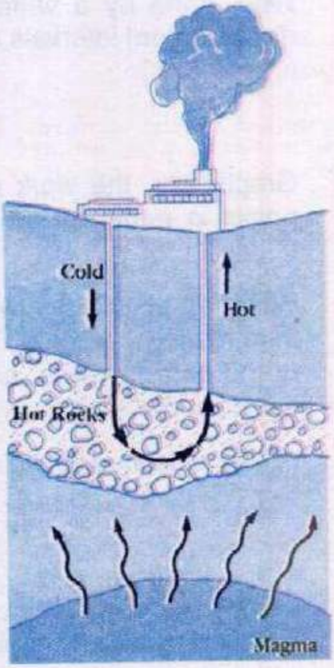

Fig. 4.17

In some place water beneath the ground is in contact with hot rocks and is raised to high temperature and pressure. It comes to the surface as hot springs, geysers, or steam vents. The steam can be directed to turn turbines of electric generators.

At places water is not present and hot rocks are not very deep, the water is pumped down through them to get steam (Fig. 4.17). The steam then can be used to drive turbines or for direct heating.

An interesting phenomenon of geothermal energy is a geyser. It is a hot spring that discharges steam and hot water, intermittently releasing an explosive column into the air (Fig. 4.18). Most geysers erupt at irregular intervals. They usually occur in volcanic regions. Extraction of geothermal heat energy often occurs closer to geyser sights. This extraction seriously disturbs geyser system by reducing heat flow and aquifer pressure. Aquifer is a layer of rock holding water that allows water to percolate through it with pressure.


Fig. 4.18

SUMMARY

- The work done on a body by a constant force is defined as the product of the magnitude of the displacement and the component of the force in the direction of the displacement.

$$W = \mathbf{F}.\mathbf{d} = Fd\cos\theta$$

- Work done by a variable force is computed by dividing the path into very small displacement intervals and then taking the sum of works done for all such intervals.

$$W = \sum_{i=1}^{n} F_i \cos\theta_i \ \Delta d_i$$

- Graphically, the work done by a variable force in moving a particle between two points is equal to the area under the $F \cos\theta$ verses d curve between these two points.
- When an object is moved in the gravitational field of the Earth, the work is done by the gravitational force. The work done in the Earth's gravitational field is independent of the path followed, and the work done along a closed path is zero. Such a force field is called a conservative field.
- Power is defined as the rate of doing work and is expressed as

$$P = \frac{\Delta W}{\Delta t} \quad \text{or} \quad P = \mathbf{F.v}$$

- Energy of a body is its capacity to do work. The kinetic energy is the energy possessed by a body due to its motion.
- The potential energy is possessed by a body because of its position in a force field.
- The absolute P.E of a body on the surface of Earth is

$$U_g = \frac{-GMm}{R}$$

- The initial velocity of a body with which it should be projected upward so that it does not come back, is called escape velocity.

$$v_{esc} = \sqrt{\frac{2GM}{R}} = \sqrt{2gR}$$

- Some of the non conventional energy sources are
  1. Energy from the tides
  2. Energy from waves
  3. Solar energy
  4. Energy from biomass
  5. Energy from waste products
  6. Geothermal energy

QUESTIONS

4.1 A person holds a bag of groceries while standing still, talking to a friend. A car is stationary with its engine running. From the stand point of work, how are these two situations similar?

4.2 Calculate the work done in kilo joules in lifting a mass of 10 kg (at a steady velocity) through a vertical height of 10 m.

4.3 A force *F* acts through a distance *L*. The force is then increased to 3 *F*, and then acts through a further distance of 2 *L*. Draw the work diagram to scale.

4.4 In which case is more work done? When a 50 kg bag of books is lifted through 50 cm, or when a 50 kg crate is pushed through 2m across the floor with a force of 50 N?

4.5 An object has 1 J of potential energy. Explain what does it mean?

4.6 A ball of mass *m* is held at a height h_1 above a table. The table top is at a height h_2 above the floor. One student says that the ball has potential energy mgh_1 but another says that it is $mg(h_1 + h_2)$. Who is correct?

4.7 When a rocket re-enters the atmosphere, its nose cone becomes very hot. Where does this heat energy come from?

4.8 What sort of energy is in the following:

a) Compressed spring

b) Water in a high dam

c) A moving car

4.9 A girl drops a cup from a certain height, which breaks into pieces. What energy changes are involved?

4.10 A boy uses a catapult to throw a stone which accidentally smashes a green house window. List the possible energy changes.

NUMERICAL PROBLEMS

4.1 A man pushes a lawn mower with a 40 N force directed at an angle of 20° downward from the horizontal. Find the work done by the man as he cuts a strip of grass 20 m long.

(Ans: 7.5×10^2 J)

4.2 A rain drop ($m = 3.35 \times 10^{-5}$ kg) falls vertically at a constant speed under the influence of the forces of gravity and friction. In falling through 100 m, how much work is done by (a) gravity and (b) friction.

[Ans: (a) 0.0328 J (b) – 0.0328 J]

4.3 Ten bricks, each 6.0 cm thick and mass 1.5 kg, lie flat on a table. How much work is required to stack them one on the top of another?

(Ans: 40 J)

4.4 A car of mass 800 kg travelling at 54 kmh^{-1} is brought to rest in 60 metres. Find the average retarding force on the car. What has happened to original kinetic energy?

(Ans: 1500 N)

4.5 A 1000 kg automobile at the top of an incline 10 metre high and 100 m long is released and rolls down the hill. What is its speed at the bottom of the incline if the average retarding force due to friction is 480 N?

(Ans: 10 ms^{-1})

4.6 100 m^3 of water is pumped from a reservoir into a tank, 10 m higher than the reservoir, in 20 minutes. If density of water is 1000 kg m^{-3}, find

(a) the increase in P.E.

(b) the power delivered by the pump.

[Ans: (a) 9.8×10^6 J (b) 8.2 kW]

4.7 A force (thrust) of 400 N is required to overcome road friction and air resistance in propelling an automobile at 80 kmh^{-1}. What power (kW) must the engine develop?

(Ans: 8.9 kW)

4.8 How large a force is required to accelerate an electron ($m = 9.1 \times 10^{-31}$ kg) from rest to a speed of $2.0 \times 10^7 ms^{-1}$ through a distance of 5.0 cm?

(Ans: 3.6×10^{-15} N)

4.9 A diver weighing 750 N dives from a board 10 m above the surface of a pool of water. Use the conservation of mechanical energy to find his speed at a point 5.0 m above the water surface, neglecting air friction.

(Ans: 9.9 ms^{-1})

4.10. A child starts from rest at the top of a slide of height 4.0 m. (a) What is his speed at the bottom if the slide is frictionless? (b) if he reaches the bottom, with a speed of 6 ms^{-1}, what percentage of his total energy at the top of the slide is lost as a result of friction?

[Ans: (a) 8.8 ms^{-1} (b) 54%]