Chapter 5

CIRCULAR MOTION

Learning Objectives

At the end of this chapter the students will be able to:

1. Describe angular motion.
2. Define angular displacement, angular velocity and angular acceleration.
3. Define radian and convert an angle from radian measure to degree and vice versa.
4. Use the equation $S = r\theta$ and $v = r\omega$.
5. Describe qualitatively motion in a curved path due to a perpendicular force and understand the centripetal acceleration in case of uniform motion in a circle.
6. Derive the equation $a_c = r\omega^2 = v^2/r$ and $F_c = m\omega^2 r = mv^2/r$
7. Understand and describe moment of inertia of a body.
8. Understand the concept of angular momentum.
9. Describe examples of conservation of angular momentum.
10. Understand and express rotational kinetic energy of a disc and a hoop on an inclined plane.
11. Describe the motion of artificial satellites.
12. Understand that the objects in satellites appear to be weightless.
13. Understand that how and why artificial gravity is produced.
14. Calculate the radius of geo-stationary orbits and orbital velocity of satellites.
15. Describe Newton's and Einstein's views of gravitation.

We have studied velocity, acceleration and the laws of motion, mostly as they are involved in rectilinear motion. However, many objects move in circular paths and their direction is continually changing. Since velocity is a vector quantity, this change of direction means that their velocities are not constant. A stone whirled around by a string, a car turning around a corner and satellites in orbits around the Earth are all examples of this kind of motion.

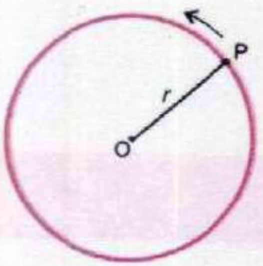


Fig. 5.1(a)

In this chapter we will study, circular motion, rotational motion, moment of inertia, angular momentum and the related topics.

5.1 ANGULAR DISPLACEMENT

Consider the motion of a single particle P of mass m in a circular path of radius r. Suppose this motion is taking place by attaching the particle P at the end of a massless rigid rod of length r whose other end is pivoted at the centre O of the circular path, as shown in Fig. 5.1 (a). As the particle is moving on the circular path, the rod OP rotates in the plane of the circle. The axis of rotation passes through the pivot O and is normal to the plane of rotation. Consider a system of axes as shown in Fig. 5.1 (b). The z-axis is taken along the axis of rotation with the pivot O as origin of coordinates. Axes x and y are taken in the plane of rotation. While OP is rotating, suppose at any instant t, its position is OP_1, making angle θ with x-axis. At later time $t + \Delta t$, let its position be OP_2 making angle $\theta + \Delta\theta$ with x-axis (Fig. 5.1c).

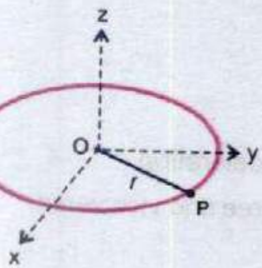


Fig. 5.1(b)

Angle $\Delta\theta$ defines the angular displacement of OP during the time interval Δt.

For very small values of $\Delta\theta$, the angular displacement is a vector quantity.

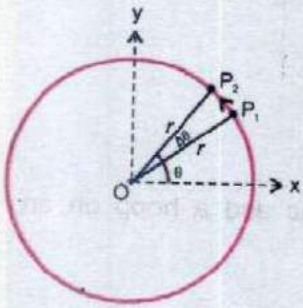


Fig. 5.1(c)

The angular displacement $\Delta\theta$ is assigned a positive sign when the sense of rotation of OP is counter clock wise.

The direction associated with $\Delta\theta$ is along the axis of rotation and is given by right hand rule which states that

Grasp the axis of rotation in right hand with fingers curling in the direction of rotation; the thumb points in the direction of angular displacement, as shown in Fig 5.1 (d).

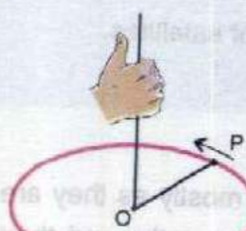


Fig. 5.1(d)

Three units are generally used to express angular displacement, namely degrees, revolution and radian. We are already familiar with the first two. As regards radian which is SI unit, consider an arc of length S of a circle of radius r (Fig 5.2) which subtends an angle θ at the centre of the circle. Its value in radians (rad) is given as

$$\theta = \frac{\text{arc length}}{\text{radius}} \text{ rad}$$

$$\theta = \frac{S}{r} \text{ rad}$$

or $S = r\theta$ (where θ is in radian) ..... (5.1)

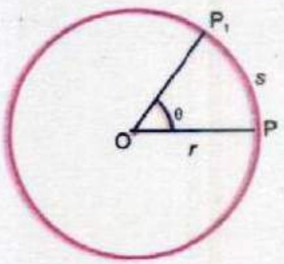


Fig. 5.2

If OP is rotating, the point P covers a distance $s = 2\pi r$ in one revolution of P. In radian it would be

$$\frac{S}{r} = \frac{2\pi r}{r} = 2\pi$$

So 1 revolution = 2π rad = 360°

Or 1 rad = $\frac{360^\circ}{2\pi} = 57.3^\circ$

5.2 ANGULAR VELOCITY

Very often we are interested in knowing how fast or how slow a body is rotating. It is determined by its angular velocity which is defined as the rate at which the angular displacement is changing with time. Referring to Fig. 5.1(c), if $\Delta\theta$ is the angular displacement during the time interval Δt, the average angular velocity ω_{av} during this interval is given by

$$\omega_{av} = \frac{\Delta\theta}{\Delta t} \quad \text{..........} \quad (5.2)$$

The instantaneous angular velocity ω is the limit of the ratio $\Delta\theta/\Delta t$ as Δt, following instant t, approaches to zero.

Thus $$\omega = \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t} \quad \text{..........} \quad (5.3)$$

In the limit when Δt approaches zero, the angular displacement would be infinitesimally small. So it would be a vector quantity and the angular velocity as defined by

Eq.5.3 would also be a vector. Its direction is along the axis of rotation and is given by right hand rule as described earlier.

Angular velocity is measured in radians per second which is its SI unit. Sometimes it is also given in terms of revolution per minute.

5.3 ANGULAR ACCELERATION

When we switch on an electric fan, we notice that its angular velocity goes on increasing. We say that it has an angular acceleration. We define angular acceleration as the rate of change of angular velocity. If ω_i and ω_f are the values of instantaneous velocity of a rotating body at instants t_i and t_f, the average angular acceleration during the interval $t_f - t_i$ is given by

$$\alpha_{av} = \frac{\omega_f - \omega_i}{t_f - t_i} = \frac{\Delta\omega}{\Delta t} \qquad \text{..........} \qquad (5.4)$$

The instantaneous angular acceleration is the limit of the ratio $\frac{\Delta\omega}{\Delta t}$ as Δt approaches zero. Therefore, instantaneous angular acceleration is given by

$$\alpha = \lim_{\Delta t \to 0} \frac{\Delta\omega}{\Delta t} \qquad \text{..........} \qquad (5.5)$$

The angular acceleration is also a vector quantity whose magnitude is given by Eq. 5.5 and whose direction is along the axis of rotation. Angular acceleration is expressed in units of $rad\,s^{-2}$.

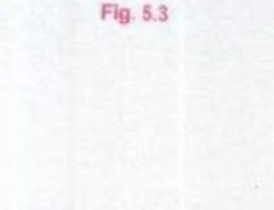


Fig. 5.3

Till now we have been considering the motion of a particle P on a circular path. The point P was fixed at the end of a rotating massless rigid rod. Now we consider the rotation of a rigid body as shown in Fig. 5.3. Imagine a point P on the rigid body. Line OP is the perpendicular dropped from P on the axis of rotation. It is usually referred as reference line. As the body rotates, line OP also rotates with it with the same angular velocity and angular acceleration. Thus the rotation of a rigid body can be described by the rotation of the reference line OP and all the terms that we defined with the help of rotating line OP are also valid for the rotational motion of a rigid body. In future while dealing with rotation of rigid body, we will replace it by its reference line OP.

5.4 RELATION BETWEEN ANGULAR AND LINEAR VELOCITIES

Consider a rigid body rotating about z-axis with an angular velocity ω as shown in Fig. 5.4 (a).

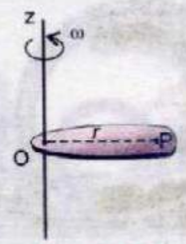


Fig. 5.4(a)

Imagine a point P in the rigid body at a perpendicular distance r from the axis of rotation. OP represents the reference line of the rigid body. As the body rotates, the point P moves along a circle of radius r with a linear velocity **v** whereas the line OP rotates with angular velocity ω as shown in Fig. 5.4 (b). We are interested in finding out the relation between ω and **v**. As the axis of rotation is fixed, so the direction of ω always remains the same and ω can be manipulated as a scalar. As regards the linear velocity of the point P, we consider its magnitude only which can also be treated as a scalar.

Suppose during the course of its motion, the point P moves through a distance $P_1P_2 = \Delta S$ in a time interval Δt during which reference line OP has an angular displacement $\Delta\theta$ radian during this interval. Δs and $\Delta\theta$ are related by Eq. 5.1.

$$\Delta S = r\Delta\theta$$

Dividing both sides by Δt

$$\frac{\Delta S}{\Delta t} = r\frac{\Delta\theta}{\Delta t} \quad \ldots\ldots\ldots\ldots \quad (5.6)$$

In the limit when $\Delta t \rightarrow 0$ the ratio $\Delta S/\Delta t$ represents v, the magnitude of the velocity with which point P is moving on the circumference of the circle. Similarly $\Delta\theta/\Delta t$ represents the angular velocity ω of the reference line OP. So equation 5.6 becomes

$$v = r\omega \quad \ldots\ldots\ldots\ldots \quad (5.7)$$

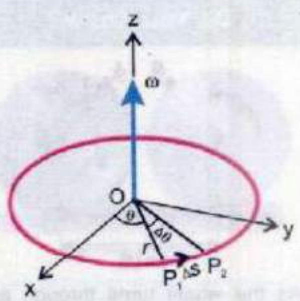


Fig. 5.4(b)

In Fig 5.4 (b), it can be seen that the point P is moving along the arc P_1P_2. In the limit when $\Delta t \rightarrow 0$, the length of arc P_1P_2 becomes very small and its direction represents the direction of tangent to the circle at point P_1. Thus the velocity with which point P is moving on the circumference of the circle has a magnitude v and its direction is always along the tangent to the circle at that point. That is why the linear velocity of the point P is also known as tangential velocity.

Similarly Eq 5.7 shows that if the reference line OP is rotating with an angular acceleration α, the point P will also have a linear or tangential acceleration a_t. Using Eq 5.7 it can be shown that the two accelerations are related by

$$a_t = r\alpha \quad \cdots\cdots\cdots \quad (5.8)$$

Eqs 5.7 and 5.8 show that on a rotating body, points that are at different distances from the axis do not have the same speed or acceleration, but all points on a rigid body rotating about a fixed axis do have the same angular displacement, angular speed and angular acceleration at any instant. Thus by the use of angular variables we can describe the motion of the entire body in a simple way.

Point to Ponder

You may feel scared at the top of roller coaster ride in the amusement parks but you never fall down even when you are upside down. Why?

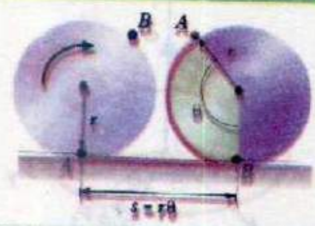
Equations Of Angular Motion

The equations (5.2, 5.3, 5.4 and 5.5) of angular motion are exactly analogous to those in linear motion except that θ, ω and α have replaced S, v and a, respectively. As the other equations of linear motion were obtained by algebraic manipulation of these equations, it follows that analogous equations will also apply to angular motion. Given below are angular equations together with their linear counterparts.

Linear	Angular	
$v_f = v_i + at$	$\omega_f = \omega_i + \alpha t$	.... (5.9)
$2aS = v_f^2 - v_i^2$	$2\alpha\theta = \omega_f^2 - \omega_i^2$	.... (5.10)
$S = v_i t + \frac{1}{2}at^2$	$\theta = \omega_i t + \frac{1}{2}\alpha t^2$	.... (5.11)

The angular equations 5.9 to 5.11 hold true only in the case when the axis of rotation is fixed, so that all the angular vectors have the same direction. Hence they can be manipulated as scalars.

Do You Know?



As the wheel turns through an angle θ, it lays out a tangential distance $S = r\theta$.

Example 5.1: An electric fan rotating at 3 rev s^{-1} is switched off. It comes to rest in 18.0 s. Assuming deceleration to be uniform, find its value. How many revolutions did it turn before coming to rest?

Solution: In this problem we have

$\omega_i = 3.0 \text{ rev s}^{-1}$, $\omega_f = 0$, $t = 18.0 \text{ s}$ and $\alpha = ?$, $\theta = ?$

From Eq. 5.4 we have

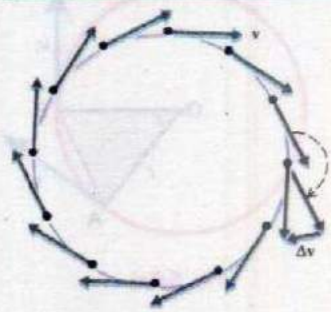
$$\alpha = \frac{\omega_f - \omega_i}{t} = \frac{(0-3.0)\text{ rev s}^{-1}}{18.0\text{ s}} = -0.167 \text{ rev s}^{-2}$$

and from Eq 5.11, we have

$$\theta = \omega_i t + \frac{1}{2}\alpha t^2$$

$$= 3.0 \text{ rev s}^{-1} \times 18.0 \text{ s} + \frac{1}{2}(-0.167 \text{ rev s}^{-2}) \times (18.0 \text{ s})^2 = 27 \text{ rev}$$

Do You Know?



Direction of motion changes continuously in circular motion.

5.5 CENTRIPETAL FORCE

The motion of a particle which is constrained to move in a circular path is quite interesting. It has direct bearing on the motion of such things as artificial and natural satellites, nuclear particles in accelerators, bodies whirling at the ends of the strings and flywheels spinning on the shafts.

We all know that a ball whirled in a horizontal circle at the end of a string would not continue in a circular path if the string is snapped. Careful observation shows at once that if the string snaps, when the ball is at the point A, in Fig. 5.5 (b), the ball will follow the straight line path AB.

The fact is that unless a string or some other mechanism pulls the ball towards the centre of the circle with a force, as shown in Fig. 5.5 (a), ball will not continue along the circular path.

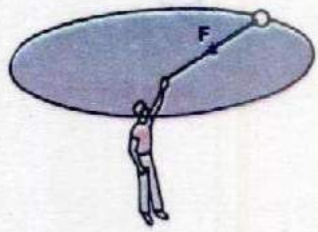


Fig. 5.5(a)

The force needed to bend the normally straight path of the particle into a circular path is called the centripetal force.

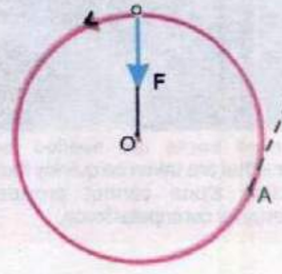


Fig. 5.5(b)

If the particle moves from A to B with uniform speed v as shown in Fig. 5.6 (a), the velocity of the particle changes its direction but not its magnitude. The change in velocity is shown in Fig. 5.6 (b). Hence, the acceleration of the particle is

$$a = \frac{\Delta v}{\Delta t}$$

where Δt is the time taken by the particle to travel from A to B. Suppose the velocities at A and B are $\mathbf{v}_1$ and $\mathbf{v}_2$ respectively. Since the speed of the particle is v, so the time taken to travel a distance s, as shown in Fig. 5.6 (a) is

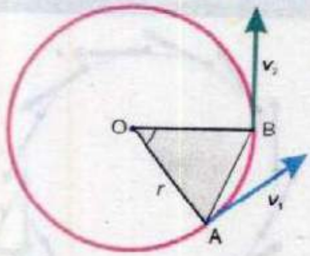


Fig. 5.6(a)

$$\Delta t = \frac{S}{v}$$

so $$a = v\frac{\Delta v}{S} \quad \text{..........} \quad (5.12)$$

Let us now draw a triangle PQR such that PQ is parallel and equal to $\boldsymbol{v}_1$ and PR is parallel and equal to $\boldsymbol{v}_2$, as shown in Fig. 5.6 (b). We know that the radius of a circle is perpendicular to its tangent, so OA is perpendicular to $\boldsymbol{v}_1$ and OB is perpendicular to v_2 (Fig. 5.6 a). Therefore, angle AOB equals the angle QPR between $\boldsymbol{v}_1$ and $\boldsymbol{v}_2$. Further, as $v_1 = v_2 = v$ and OA = OB, both triangles are isosceles. From geometry, we know "two isosceles triangles are similar, if the angles between their equal arms are equal". Hence, the triangle OAB of Fig. 5.6 (a) is similar to the triangle PQR of Fig. 5.6 (b). Hence, we can write

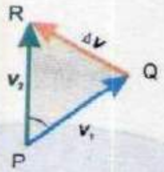


Fig. 5.6(b)

$$\frac{\Delta v}{v} = \frac{AB}{r}$$

If the point B is close to the point A on the circle, as will be the case when $\Delta t \to 0$, the arc AB is of nearly the same length as the line AB. To that approximation, we can write AB = s, and after substituting and rearranging terms, we have,

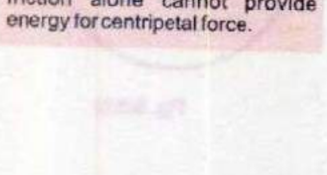
$$\Delta v = S\frac{v}{r}$$

Putting this value for Δv in the Eq. 5.12, we get

$$a = \frac{v^2}{r} \quad \text{..........} \quad (5.13)$$

where a is the instantaneous acceleration. As this acceleration is caused by the centripetal force, it is called the centripetal acceleration denoted by a_c. This acceleration is directed along the radius towards the centre of the circle. In Fig. 5.6 (a) and (b), since PQ is perpendicular to OA and PR is perpendicular to OB, so QR is perpendicular to AB. It may be noted that QR is parallel to the perpendicular bisector of AB. As the acceleration of the object moving in the circle is

Tid-bits



Banked tracks are needed for turns that are taken so quickly that friction alone cannot provide energy for centripetal force.

parallel to Δv when AB $\rightarrow 0$, so centripetal acceleration is directed along radius towards the centre of the circle. It can, therefore, be concluded that:

The instantaneous acceleration of an object travelling with uniform speed in a circle is directed towards the centre of the circle and is called centripetal acceleration.

The centripetal force has the same direction as the centripetal acceleration and its value is given by

$$F_c = ma_c = \frac{mv^2}{r} \quad \text{..........} \quad (5.14)$$

In angular measure, this equation becomes

$$F_c = mr\omega^2 \quad \text{..........} \quad (5.15)$$

Do You Know?

Curved flight at high speed requires a large centripetal force that makes the stunt dangerous even if the air planes are not so close.

Example 5.2: A 1000 kg car is turning round a corner at 10 ms^{-1} as it travels along an arc of a circle. If the radius of the circular path is 10 m, how large a force must be exerted by the pavement on the tyres to hold the car in the circular path?

Solution: The force required is the centripetal force. So

$$F_c = \frac{mv^2}{r} = \frac{1000\,\text{kg} \times 100\,\text{m}^2\text{s}^{-2}}{10\,\text{m}} = 1.0 \times 10^4\,\text{kgms}^{-2} = 1.0 \times 10^4\,\text{N}$$

This force must be supplied by the frictional force of the pavement on the wheels.

Example 5.3: A ball tied to the end of a string, is swung in a vertical circle of radius r under the action of gravity as shown in Fig. 5.7. What will be the tension in the string when the ball is at the point A of the path and its speed is v at this point?

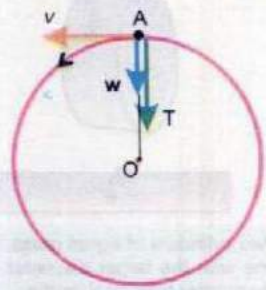


Fig. 5.7

Solution: For the ball to travel in a circle, the force acting on the ball must provide the required centripetal force. In this case, at point A, two forces act on the ball, the pull of the string and the weight w of the ball. These forces act along the radius at A, and so their vector sum must furnish the required centripetal force. We, therefore, have

$$T + w = \frac{mv^2}{r} \qquad as \quad w = mg$$

$$\therefore \qquad T = \frac{mv^2}{r} - mg = m\left(\frac{v^2}{r} - g\right)$$

If $\frac{v^2}{r} = g$, then T will be zero and the centripetal force is just equal to the weight.

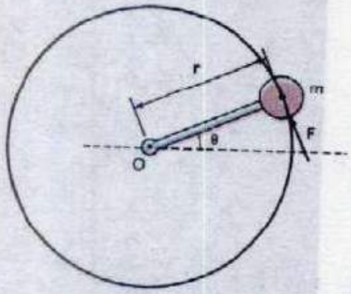


Fig. 5.8

The force F causes a torque about the axis O and gives the mass m an angular acceleration about the pivot point.

5.6 MOMENT OF INERTIA

Consider a mass m attached to the end of a massless rod as shown in Fig. 5.8. Let us assume that the bearing at the pivot point O is frictionless. Let the system be in a horizontal plane. A force F is acting on the mass perpendicular to the rod and hence, this will accelerate the mass according to

$$F = ma$$

In doing so the force will cause the mass to rotate about O. Since tangential acceleration a_t is related to angular acceleration α by the equation.

$$a_t = r\alpha$$

so,

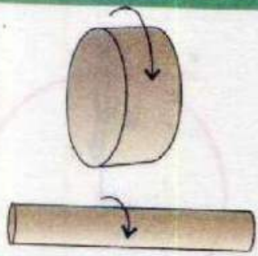
$$F = mr\alpha$$

As turning effect is produced by torque τ, it would, therefore, be better to write the equation for rotation in terms of torque. This can be done by multiplying both sides of the above equation by r. Thus

$$rF = \tau = \text{torque} = mr^2\alpha$$

which is rotational analogue of the Newton's second law of motion, $F = ma$.

Do You Know?



Two cylinders of equal mass. The one with the larger diameter has the greater rotational inertia.

Here F is replaced by τ, a by α and m by mr^2. The quantity mr^2 is known as the moment of inertia and is represented by I. The moment of inertia plays the same role in angular motion as the mass in linear motion. It may be noted that moment of inertia depends not only on mass m but also on r^2.

Most rigid bodies have different mass concentration at different distances from the axis of rotation, which means the mass distribution is not uniform. As shown in Fig. 5.9 (a), the rigid body is made up of n small pieces of masses

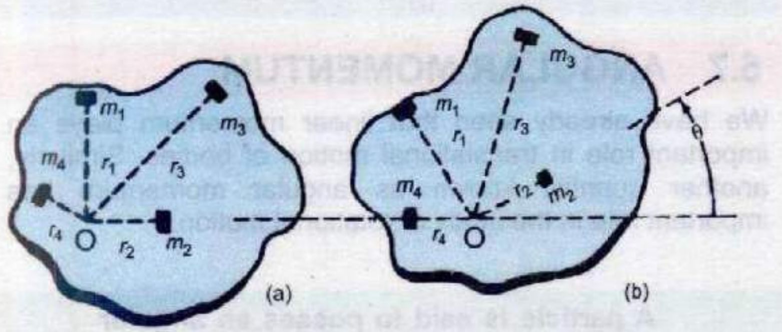


Fig. 5.9 Each small piece of mass within a large, rigid body undergoes the same angular acceleration about the pivot point.

$m_1, m_2, \ldots m_n$ at distances $r_1, r_2, \ldots r_n$ from the axis of rotation O. Let the body be rotating with the angular acceleration α so the magnitude of the torque acting on m_1 is

$$\tau_1 = m_1 r_1^2 \alpha_1$$

Similarly, the torque on m_2 is

$$\tau_2 = m_2 r_2^2 \alpha_2$$

and so on.

Since the body is rigid, so all the masses are rotating with the same angular acceleration α,

Total torque τ_{total} is then given by

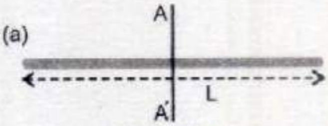
$$\tau_{total} = (m_1 r_1^2 + m_2 r_2^2 + \ldots + m_n r_n^2)\,\alpha$$

$$= \left(\sum_{i=1}^{n} m_i r_i^2\right)\alpha$$

For Your Information

Moments of Inertia of various bodies about AA'.

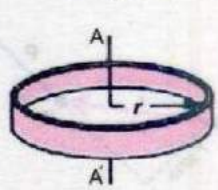
(a)



Thin Rod

$I = \frac{1}{12} mL^2$

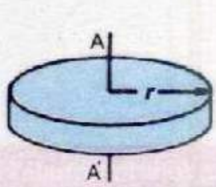
(b)



Thin ring or Hoop

$I = mr^2$

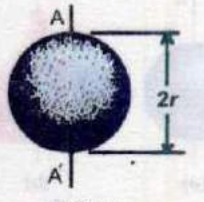
(c)



Solid disc or cylinder

$I = \frac{1}{2} mr^2$

(d)



Sphere

$I = \frac{2}{5} mr^2$

or $\tau = I\alpha$ .... (5.16)

where I is the moment of inertia of the body and is expressed as

$$I = \sum_{i=1}^{n} m_i r_i^2 \quad .... \quad (5.17)$$

5.7 ANGULAR MOMENTUM

We have already seen that linear momentum plays an important role in translational motion of bodies. Similarly, another quantity known as angular momentum has important role in the study of rotational motion.

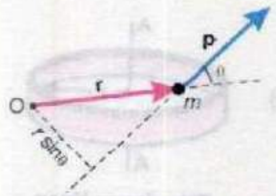


Fig. 5.10

A particle is said to posses an angular momentum about a reference axis if it so moves that its angular position changes relative to that reference axis.

The angular momentum **L** of a particle of mass m moving with velocity **v** and momentum **p** (Fig. 5.10) relative to the origin O is defined as

$$\mathbf{L} = \mathbf{r} \times \mathbf{p} \quad .... \quad (5.18)$$

where **r** is the position vector of the particle at that instant relative to the origin O. Angular momentum is a vector quantity. Its magnitude is

$$L = rp \sin\theta = m\, rv \sin\theta$$

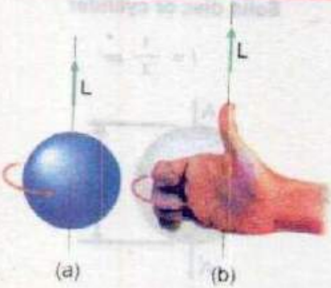
where θ is the angle between **r** and **p**. The direction of **L** is perpendicular to the plane formed by **r** and **p** and its sense is given by the right hand rule of vector product. SI unit of angular momentum is kg $m^2 s^{-1}$ or J s.

If the particle is moving in a circle of radius r with uniform angular velocity ω, then angle between r and tangential velocity is 90°. Hence

$$L = mrv \sin 90^\circ = mrv$$

But $v = r\omega$

For Your Information



The sphere in (a) is rotating in the sense given by the gold arrow. Its angular velocity and angular momentum are taken to be upward along the rotational axis, as shown by the right-hand rule in (b).

Hence $$L = m r^2 \omega$$

Now consider a symmetric rigid body rotating about a fixed axis through the centre of mass as shown in Fig 5.11. Each particle of the rigid body rotates about the same axis in a circle with an angular velocity ω. The magnitude of the angular momentum of the particle of mass m_i is $m_i v_i r_i$ about the origin O. The direction of $\mathbf{L}_i$ is the same as that of ω. Since $v_i = r_i \omega$, the angular momentum of the ith particle is $m_i r_i^2 \omega$. Summing this over all particles gives the total angular momentum of the rigid body.

$$L = \left(\sum_{i=1}^{n} m_i r_i^2\right) \omega = I\omega$$

Fig. 5.11

Where I is the moment of inertia of the rigid body about the axis of rotation.

Physicists usually make a distinction between spin angular momentum (L_s) and orbital angular momentum (L_o.) The spin angular momentum is the angular momentum of a spinning body, while orbital angular momentum is associated with the motion of a body along a circular path.

The difference is illustrated in Fig. 5.12. In the usual circumstances concerning orbital angular momentum, the orbital radius is large as compared to the size of the body, hence, the body may be considered to be a point object.

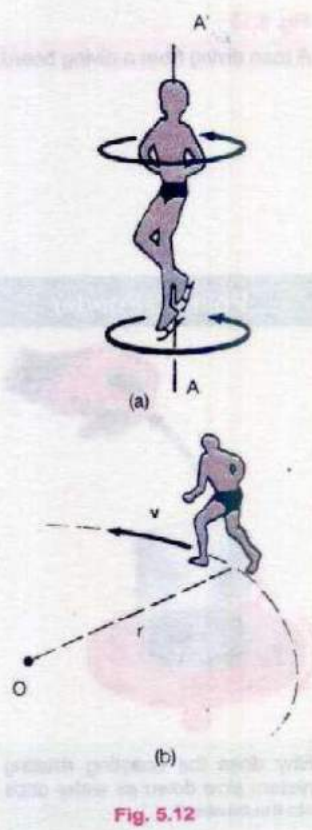


Fig. 5.12

Example 5.4: The mass of Earth is 6.00×10^{24} kg. The distance r from Earth to the Sun is 1.50×10^{11} m. As seen from the direction of the North Star, the Earth revolves counter-clockwise around the Sun. Determine the orbital angular momentum of the Earth about the Sun, assuming that it traverses a circular orbit about the Sun once a year (3.16×10^7s).

Solution: To find the Earth's orbital angular momentum we must first know its orbital speed from the given data. When the Earth moves around a circle of radius r, it travels a distance of $2\pi r$ in one year, its orbital speed v_o is thus

$$v_0 = \frac{2\pi r}{T}$$

Orbital angular momentum of the Earth = $L_o = m v_o r$

$$= \frac{2\pi r^2 m}{T}$$

$$= \frac{2\pi(1.50 \times 10^{11}\,\text{m})^2 \times (6.00 \times 10^{24}\,\text{kg})}{3.16 \times 10^7\,\text{s}}$$

$$= 2.67 \times 10^{40}\,\text{kg m}^2\,\text{s}^{-1}$$

The sign is positive because the revolution is counter clockwise.

5.8 LAW OF CONSERVATION OF ANGULAR MOMENTUM

The law of conservation of angular momentum states that if no external torque acts on a system, the total angular momentum of the system remains constant.

$$\mathbf{L}_{total} = \mathbf{L}_1 + \mathbf{L}_2 + \ldots = \text{constant}$$

The law of conservation of angular momentum is one of the fundamental principles of Physics. It has been verified from the cosmological to the submicroscopic level. The effect of the law of conservation of angular momentum is readily apparent if a single isolated spinning body alters its moment of inertia. This is illustrated by the diver in Fig.5.13. The diver pushes off the board with a small angular velocity about a horizontal axis through his centre of gravity. Upon lifting off from the board, the diver's legs and arms are fully extended which means that the diver has a large moment of inertia I_1 about this axis. The moment of inertia is considerably reduced to a new value I_2, when the legs and arms are drawn into the closed tuck position. As the angular momentum is conserved, so

$$I_1\omega_1 = I_2\omega_2$$

Hence, the diver must spin faster when moment of inertia becomes smaller to conserve angular momentum. This enables the diver to take extra somersaults.

The angular momentum is a vector quantity with direction along the axis of rotation. In the above example, we discussed the conservation of magnitude of angular momentum. The direction of angular momentum along the

Fig. 5.13
A man diving from a diving board.

Point to Ponder

Why does the coasting rotating system slow down as water drips into the beaker?

axis of rotation also remain fixed. This is illustrated by the fact given below

The axis of rotation of an object will not change its orientation unless an external torque causes it to do so.

Do You Know?

The law of conservation of angular momentum is important in many sports, particularly in diving, gymnastics and ice-skating.

This fact is of great importance for the Earth as it moves around the Sun. No other sizeable torque is experienced by the Earth, because the major force acting on it is the pull of the Sun. The Earth's axis of rotation, therefore, remains fixed in one direction with reference to the universe around us.

5.9 ROTATIONAL KINETIC ENERGY

If a body is spinning about an axis with constant angular velocity ω, each point of the body is moving in a circular path and, therefore, has some K.E. To determine the total K.E. of a spinning body, we imagine it to be composed of tiny pieces of mass $m_1, m_2, \ldots$ If a piece of mass m_i is at a distance r_i from the axis of rotation, as shown in Fig. 5.14, it is moving in a circle with speed

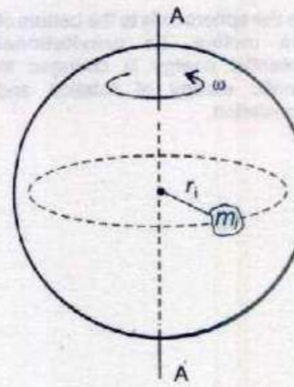


Fig. 5.14

$$v_i = r_i\,\omega$$

Thus the K.E of this piece is

$$\text{K.E}_i = \frac{1}{2}\, m_i v_i^2 = \frac{1}{2}\, m_i\,(r_i\,\omega)^2$$

$$= \frac{1}{2}\, m_i r_i^2 \omega^2$$

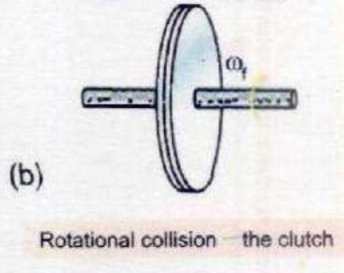
The rotational K.E of the whole body is the sum of the kinetic energies of all the parts. So we have

$$\text{K.E}_{rot} = \frac{1}{2}\,(m_1 r_1^2 \omega^2 + m_2 r_2^2 \omega^2 + \ldots\ldots\ldots)$$

$$= \frac{1}{2}\,(m_1 r_1^2 + m_2 r_2^2 + \ldots\ldots\ldots)\,\omega^2$$

We at once recognize that the quantity within the brackets is the moment of inertia I of the body. Hence, rotational kinetic energy is given by

Interesting Information



Rotational collision – the clutch

$$K.E._{rot} = \frac{1}{2} I\omega^2 \quad .... \quad (5.19)$$

If rolling or spinning bodies are present in a system, their rotational kinetic energy must be included as part of the total kinetic energy. Rotational kinetic energy is put to practical use by fly wheels, which are essential parts of many engines. A fly wheel stores energy between the power stokes of the pistons, so that the energy is distributed over the full revolution of the crankshaft and hence, the rotation remains smooth.

For Your Information

As the sphere rolls to the bottom of the incline, its gravitational potential energy is changed to kinetic energy of rotation and translation.

Rotational Kinetic Energy of a Disc and a Hoop

From equation 5.19, the rotational kinetic energy of a disc is

$$K.E_{rot} = \frac{1}{2} I\omega^2$$

From page 109, for a disc

$$I = \frac{1}{2} mr^2$$

so $$K.E_{rot} = \frac{1}{2} \times \frac{1}{2} mr^2 \omega^2$$

therefore, $$= \frac{1}{4} mr^2 \omega^2$$

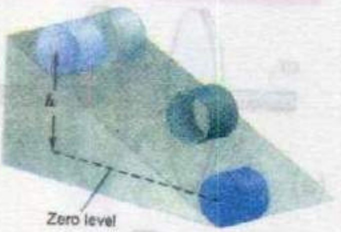
since $$r^2 \omega^2 = v^2$$

$$K.E_{rot} = \frac{1}{4} mv^2 \quad .... \quad (5.20)$$

and for a hoop, since $I = mr^2$ (page 109)

then $$K.E_{rot} = \frac{1}{2} I\omega^2 = \frac{1}{2} mr^2\omega^2 \text{ page 109}$$

$$K.E_{rot} = \frac{1}{2} mv^2 \quad .... \quad (5.21)$$



When both starts moving down an inclined plane of height h, their motion consists of both rotational and translational motions (Fig. 5.15). If no energy is lost against friction, the total kinetic energy of the disc or hoop on reaching the bottom of the incline must be equal to its potential energy at the top.

$$P.E. = K.E_{tran} + K.E_{rot}$$

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 \quad \ldots\ldots \quad (5.22)$$

For disc $$mgh = \frac{1}{2}mv^2 + \frac{1}{4}mv^2$$

or $$v = \sqrt{\frac{4gh}{3}} \quad \ldots\ldots\ldots \quad (5.23)$$

and for hoop $$mgh = \frac{1}{2}mv^2 + \frac{1}{2}mv^2$$

or $$v = \sqrt{gh} \quad \ldots\ldots\ldots \quad (5.24)$$

Example 5.5: A disc without slipping rolls down a hill of height 10.0 m. If the disc starts from rest at the top of the hill, what is its speed at the bottom?

Solution: Using Eq. 5.23

$$v = \sqrt{\frac{4gh}{3}}$$

$$= \sqrt{\frac{4 \times 9.80\,\text{ms}^{-2} \times 10.0\,\text{m}}{3}} = 11.4\ \text{ms}^{-1}$$

5.10 ARTIFICIAL SATELLITES

Satellites are objects that orbit around the Earth. They are put into orbit by rockets and are held in orbits by the gravitational pull of the Earth. The low flying Earth satellites have acceleration 9.8 ms^{-2} towards the centre of the Earth. If they do not, they would fly off in a straight line tangent to the Earth. When the satellite is moving in a circle, it has an acceleration $\frac{v^2}{r}$. In a circular orbit around the Earth, the centripetal acceleration is supplied by gravity and we have,

$$g = \frac{v^2}{R} \quad \ldots\ldots\ldots \quad (5.25)$$

Tid bits

As the wheel rolls, it has both rotational and translational kinetic energies.

For Your Information

Satellites Orbits

Where v is the orbital velocity and R is the radius of the Earth (6400 km). From Eq. 5.25 we get,

$$v = \sqrt{gR}$$

$$= \sqrt{9.8\ \text{ms}^{-2} \times 6.4 \times 10^{6}\ \text{m}}$$

$$= 7.9\ \text{kms}^{-1}$$

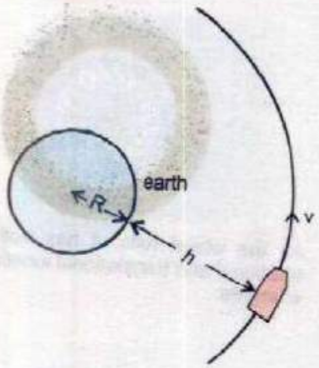


Fig. 5.16

This is the minimum velocity necessary to put a satellite into the orbit and is called critical velocity. The period T is given by

$$T = \frac{2\pi R}{v} = 2 \times 3.14 \times \frac{6400\ \text{km}}{7.9\ \text{km s}^{-1}}$$

$$= 5060\text{s} = 84\ \text{min approx.}$$

If, however, a satellite in a circular orbit is at an appreciable distance h above the Earth's surface, we must take into account the experimental fact that the gravitational acceleration decreases inversely as the square of the distance from the centre of the Earth (Fig. 5.16).

The higher the satellite, the slower will the required speed and longer it will take to complete one revolution around the Earth.

Close orbiting satellites orbit the Earth at a height of about 400 km. Twenty four such satellites form the Global Positioning System. An airline pilot, sailor or any other person can now use a pocket size instrument or mobile phone to find his position on the Earth's surface to within 10m accuracy.

Tidbits

The moment you switch on your mobile phone, your location can be tracked immediately by global positioning system.

5.11 REAL AND APPARENT WEIGHT

We often hear that objects appear to be weightless in a spaceship circling round the Earth. In order to examine the effect in some detail, let us first define, what do we mean by the weight? The real weight of an object is the gravitational pull of the Earth on the object. Similarly the weight of an object on the surface of the Moon is taken to be the gravitational pull of the Moon on the object.

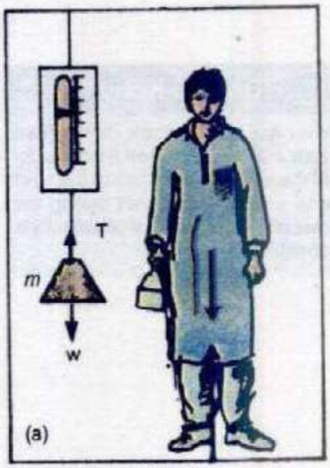
Generally the weight of an object is measured by a spring balance. The force exerted by the object on the scale is equal to the pull due to gravity on the object, i.e., the weight of the object. This is not always true, as will be explained a little later, so we call the reading of the scale as apparent weight.

To illustrate this point, let us consider the apparent weight of an object of mass m, suspended by a string and spring balance, in a lift as shown in Fig. 5.17 (a). When the lift is at rest, Newton's second law tells us that the acceleration of the object is zero, the resultant force on it is also zero. If w is the gravitational force acting on it and T is the tension in the string then we have,

$$T - w = ma$$

As $a = 0$

hence, $T = w$ .... (5.26)



at rest
$a = 0$
$T = w$
Fig. 5.17(a)

This situation will remain so long as $a = 0$. The scale thus shows the real weight of the object. The weight of the object seems to a person in the lift to vary, depending on its motion.

When the lift is moving upwards with an acceleration a, then

$$T - w = ma$$

or $T = w + ma$ .... (5.27)

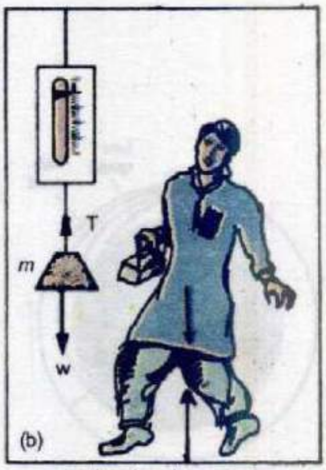
the object will then weigh more than its real weight by an amount ma.

Now suppose, the lift and hence, the object is moving downwards with an acceleration a (Fig. 5.17 b), then we have

$$w - T = ma$$

which shows that

$T = w - ma$ .... (5.28)



acceleration downward
$w - T = ma$
$T = w - ma$
Fig. 5.17(b)

The tension in the string, which is the scale reading, is less than w by an amount ma. To a person in the accelerating lift, the object appears to weigh less than w. Its apparent weight is then $(w - ma)$.

Do You Know?

Your apparent weight differs from your true weight when the velocity of the elevator changes at the start and end of a ride, not during the rest of the ride when that velocity is constant.

Let us now consider that the lift is falling freely under gravity. Then $a = g$, and hence,

$$T = w - mg$$

As the weight w of the body is equal to mg so

$$T = mg - mg = 0$$

The apparent weight of the object will be shown by the scale to be zero.

It is understood from these considerations that apparent weight of the object is not equal to its true weight in an accelerating system. It is equal and opposite to the force required to stop it from falling in that frame of reference.

5.12 WEIGHTLESSNESS IN SATELLITES AND GRAVITY FREE SYSTEM

When a satellite is falling freely in space, everything within this freely falling system will appear to be weightless. It does not matter where the object is, whether it is falling under the force of attraction of the Earth, the Sun, or some distant star.

An Earth's satellite is freely falling object. The statement may be surprising at first, but it is easily seen to be correct. Consider the behaviour of a projectile shot parallel to the horizontal surface of the Earth in the absence of air friction. If the projectile is thrown at successively larger speeds, then during its free fall to the Earth, the curvature of the path decreases with increasing horizontal speeds. If the object is thrown fast enough parallel to the Earth, the curvature of its path will match the curvature of the Earth as shown in Fig. 5.18. In this case the space ship will simply circle round the Earth.

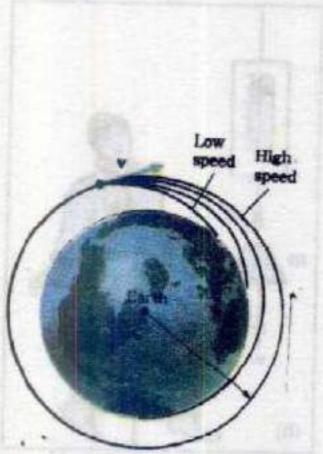


Fig. 5.18

The space ship is accelerating towards the centre of the Earth at all times since it circles round the Earth. Its radial acceleration is simply g, the free fall acceleration. In fact, the space ship is falling towards the centre of the Earth at all the times but due to spherical shape of the Earth, it never strikes the surface of the Earth. Since the space ship is in free fall, all the objects within it appear to be weightless. Thus no force is required to hold an object falling in the frame of reference of the space craft or satellite. Such a system is called gravity free system.

5.13 ORBITAL VELOCITY

The Earth and some other planets revolve round the Sun in nearly circular paths. The artificial satellites launched by men also adopt nearly circular course around the Earth. This type of motion is called orbital motion.

Fig. 5.19 shows a satellite going round the Earth in a circular path. The mass of the satellite is m_s and v is its orbital speed. The mass of the Earth is M and r represents the radius of the orbit. A centripetal force $m_s v^2/r$ is required to hold the satellite in orbit. This force is provided by the gravitational force of attraction between the Earth and the satellite. Equating the gravitational force to the required centripetal force, gives

$$\frac{Gm_sM}{r^2} = \frac{m_sv^2}{r}$$

or $$v = \sqrt{\frac{GM}{r}} \quad .... \quad (5.29)$$

This shows that the mass of the satellite is unimportant in describing the satellite's orbit. Thus any satellite orbiting at distance r from Earth's centre must have the orbital speed given by Eq. 5.29. Any speed less than this will bring the satellite tumbling back to the Earth.

Example 5.6: An Earth satellite is in circular orbit at a distance of 384,000 km from the Earth's surface. What is its period of one revolution in days? Take mass of the Earth $M = 6.0 \times 10^{24}$kg and its radius $R = 6400$ km.

Solution:

As $\quad r = R + h = (6400 + 384000) = 390400$ km

Using $$v = \sqrt{\frac{GM}{r}} = \sqrt{\frac{6.67 \times 10^{-11}\,\text{Nm}^2\text{kg}^{-2} \times 6 \times 10^{24}\,\text{kg}}{390400\text{km}}}$$

$$= 1.01\ \text{kms}^{-1}$$

Also

$$T = \frac{2\pi r}{v} = 2 \times 3.14 \times 390400\,\text{km} \times \frac{1}{1.01\text{kms}^{-1}} \times \frac{1\,\text{day}}{60 \times 60 \times 24\text{s}}$$

$$= 27.5\ \text{days}$$

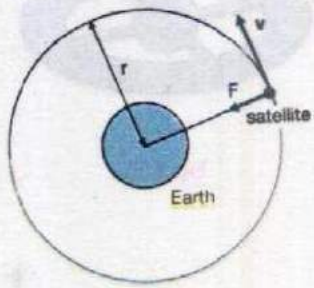
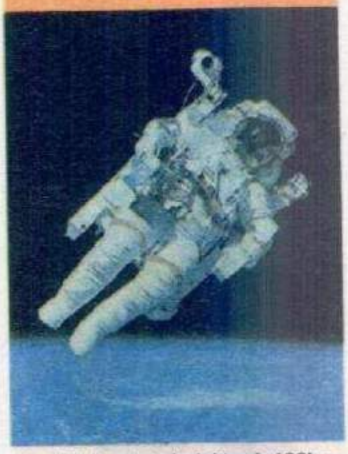


Fig. 5.19

Tid-bits



In 1984, at a height of 100km above Hawaii island with a speed of 29000kmh^{-1} Bruce McCandless stepped into space from a space shuttle and became the first human satellite of the Earth.

5.14 ARTIFICIAL GRAVITY

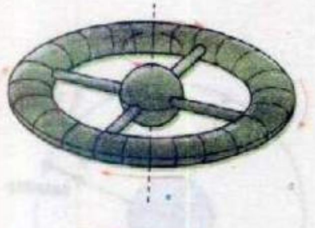


Fig. 5.20

In a gravity free space satellite there will be no force that will force any body to any side of the spacecraft. If this satellite is to stay in orbit over an extended period of time, this weightlessness may affect the performance of the astronauts present in that spacecraft. To over come this difficulty, an artificial gravity is created in the spacecraft. This could enable the crew of the space ships to function in an almost normal manner. For this situation to prevail, the space ship is set into rotation around its own axis. The astronaut then is pressed towards the outer rim and exerts a force on the 'floor' of the spaceship in much the same way as on the Earth.

Consider a spacecraft of the shape as shown in Fig. 5.20. The outer radius of the spaceship is R and it rotates around its own central axis with angular speed ω. then its angular acceleration a_c is

$$a_c = R\omega^2$$

But $\omega = \dfrac{2\pi}{T}$ where T is the period of revolution of spaceship

Hence
$$a_c = R\frac{(2\pi)^2}{T^2} = R\frac{4\pi^2}{T^2}$$

As frequency $f = 1/T$, therefore $a_c = R\,4\,\pi^2 f^2$

or
$$f^2 = \frac{a_c}{4\pi^2 R} \quad \text{or} \quad f = \frac{1}{2\pi}\sqrt{\frac{a_c}{R}}$$

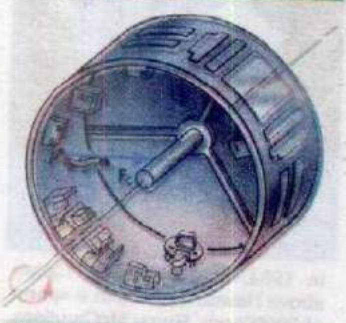
The frequency f is increased to such an extent that a_c equals to g. Therefore,

$$a_c = g.$$

and
$$f = \frac{1}{2\pi}\sqrt{\frac{g}{R}} \qquad \text{..........} \qquad (5.30)$$

When the space ship rotates with this frequency, the artificial gravity like Earth is provided to the inhabitants of the space ship.

Do You Know?



The surface of the rotating space ship pushes on an object with which it is in contact and thereby provides the centripetal force needed to keep the object moving on a circular path.

5.15 GEOSTATIONARY ORBITS

An interesting and useful example of satellite motion is the geo-synchronous or geo-stationary satellite. This type of satellite is the one whose orbital motion is synchronized with the rotation of the Earth. In this way the synchronous satellite remains always over the same point on the equator as the Earth spins on its axis. Such a satellite is very useful for worldwide communication, weather observations, navigation, and other military uses.

What should the orbital radius of such a satellite be so that it could stay over the same point on the Earth surface? The speed necessary for the circular orbit, given by Eq. 5.29, is

$$v = \sqrt{\frac{GM}{r}}$$

but this speed must be equal to the average speed of the satellite in one day, i.e.,

$$v = \frac{s}{t} = \frac{2\pi r}{T}$$

where T is the period of revolution of the satellite, that is equal to one day. This means that the satellite must move in one complete orbit in a time of exactly one day. As the Earth rotates in one day and the satellite will revolve around the Earth in one day, the satellite at A will always stay over the same point A on the Earth, as shown in Fig. 5.21. Equating the above two equations, we get

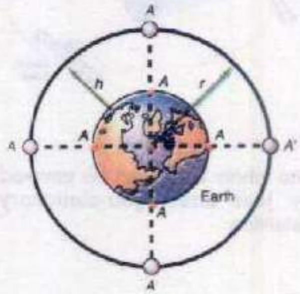


Fig. 5.21

$$\frac{2\pi r}{t} = \sqrt{\frac{GM}{r}}$$

Squaring both sides

$$\frac{4\pi^2 r^2}{t^2} = \frac{GM}{r}$$

or $$r^3 = \frac{GMT^2}{4\pi^2}$$

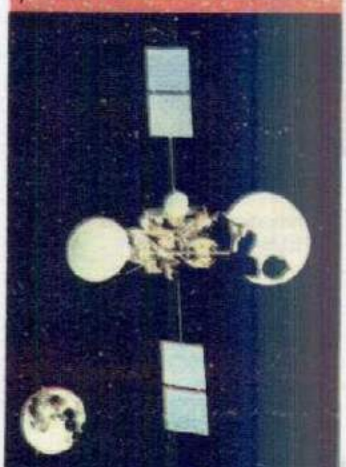
From this we get the orbital radius

$$r = \left(\frac{GMT^2}{4\pi^2}\right)^{\frac{1}{3}} \quad \text{..........} \quad (5.31)$$

Substituting the values for the Earth into Eq. 5.31 we get

$$r = 4.23 \times 10^4 \text{ km}$$

For Your Information



A geostationary satellite orbits the Earth once per day over the equator so it appears to be stationary. It is used now for international communications.

which is the orbital radius measured from the centre of the Earth, for a geostationary satellite. A satellite at this height will always stay directly above a particular point on the surface of the Earth. This height above the equator comes to be 36000 km.

5.16 COMMUNICATION SATELLITES

Fig. 5.22

The whole Earth can be covered by just three geo-stationary satellites.

A satellite communication system can be set up by placing several geostationary satellites in orbit over different points on the surface of the Earth. One such satellite covers 120° of longitude, so that whole of the populated Earth's surface can be covered by three correctly positioned satellites as shown in Fig. 5.22. Since these geostationary satellites seem to hover over one place on the Earth, continuous communication with any place on the surface of the Earth can be made. Microwaves are used because they travel in a narrow beam, in a straight line and pass easily through the atmosphere of the Earth. The energy needed to amplify and retransmit the signals is provided by large solar cell panels fitted on the satellites. There are over 200 Earth stations which transmit signals to satellites and receive signals via satellites from other countries. You can also pick up the signal from the satellite using a dish antenna on your house. The largest satellite system is managed by 126 countries, International Telecommunication Satellite Organization (INTELSAT). An INTELSAT VI satellite is shown in the Fig.5.23. It operates at microwave frequencies of 4, 6,11 and 14 GHz and has a capacity of 30, 000 two way telephone circuits plus three TV channels.

Fig. 5.23

Communications satellite INTELSAT VI

Do You Know?

1GHz = 10^9 Hz

Example 5.7: Radio and TV signals bounce from a synchronous satellite. This satellite circles the Earth once in 24 hours. So if the satellite circles eastward above the equator, it stays over the same spot on the Earth because the Earth is rotating at the same rate. (a) What is the orbital radius for a synchronous satellite? (b) What is its speed?

Solution:

From Eq. 5.31, $r = \left(\frac{GMT^2}{4\pi^2}\right)^{\frac{1}{3}}$

where $G = 6.67 \times 10^{-11}\ \text{N m}^2\ \text{kg}^{-2}$, $M = 6.0 \times 10^{24}\ \text{kg}$

and $T = 24 \times 60 \times 60\text{s}$.

Therefore, on substitution; we get

$$\text{a)} \quad r = \left(\frac{6.67 \times 10^{-11}\,\text{N m}^2\,\text{kg}^{-2} \times 6.0 \times 10^{24}\,\text{kg} \times (24 \times 60 \times 60\text{s})^2}{4\,(3.14)^2} \right)^{1/3}$$

$$= 4.23 \times 10^7 \text{m}$$

b) Substituting the value of r in equation $v = \frac{2\pi r}{T}$.

we get,

$$v = \frac{2\pi(4.23 \times 10^7\,\text{m})}{86400\text{s}} = 3.1\ \text{kms}^{-1}$$

5.17 NEWTON'S AND EINSTEIN'S VIEWS OF GRAVITATION

According to Newton, the gravitation is the intrinsic property of matter that every particle of matter attracts every other particle with a force that is directly proportional to the product of their masses and is inversely proportional to the square of the distance between them.

According to Einstein's theory, space time is curved, especially locally near massive bodies. To visualize this, we might think of space as a thin rubber sheet; if a heavy weight is hung from it, it curves as shown in Fig 5.24. The weight corresponds to a huge mass that causes space itself to curve. Thus, in Einstein's theory we do not speak of the force of gravity acting on bodies; instead we say that bodies and light rays move along geodesics (equivalent to straight lines in plane geometry) in curved space time. Thus, a body at rest or moving slowly near the great mass of Fig. 5.24 would follow a geodesic toward that body.

Einstein's theory gives us a physical picture of how gravity works; Newton discovered the inverse square law of gravity; but explicitly said that he offered no explanation of why gravity should follow an inverse square law. Einstein's theory also says that gravity follows an inverse square law (except in strong gravitational fields), but it tells us why this should be so. That is why Einstein's theory is better than Newton's, even though it includes Newton's theory within itself and

Do You Know?

The gravity can bend light. The gravity of a star could be used to focus light from stars.

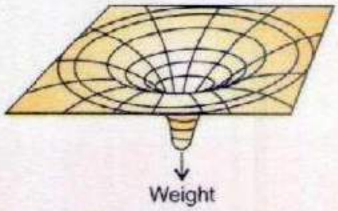


Fig. 5.24
Rubber sheet analogy for curved space-time.

Interesting Information

Bending of starlight by the Sun. Light from the star A is deflected as it passes close to the Sun on its way to Earth. We see the star in the apparent direction B, shifted by the angle ϕ. Einstein predicted that $\phi = 1.745$ seconds of angle which was found to be the same during the solar eclipse of 1919.

gives the same answers as Newton's theory everywhere except where the gravitational field is very strong.

Einstein inferred that if gravitational acceleration and inertial acceleration are precisely equivalent, gravity must bend light, by a precise amount that could be calculated. This was not entirely a startling suggestion: Newton's theory, based on the idea of light as a stream of tiny particles, also suggested that a light beam would be deflected by gravity. But in Einstein's theory, the deflection of light is predicted to be exactly twice as great as it is according to Newton's theory. When the bending of starlight caused by the gravity of the Sun was measured during a solar eclipse in 1919, and found to match Einstein's prediction rather than Newton's, then Einstein's theory was hailed as a scientific triumph.

SUMMARY

- Angular displacement is the angle subtended at the centre of a circle by a particle moving along the circumference in a given time.
- SI unit of angular measurement is radian.
- Angular acceleration is the rate of change of angular velocity.
- Relationship between angular and tangential or linear quantities.

  i. $s = r\theta$ ii. $v_T = r\omega$ iii. $a_T = r\alpha$
- The force needed to move a body around a circular path is called centripetal force and is calculated by the expression $F_c = mr\omega^2 = \frac{mv^2}{r}$
- Moment of inertia is the rotational analogue of mass in linear motion. It depends on the mass and the distribution of mass from the axis of rotation.
- Angular momentum is the analogue of linear momentum and is defined as the product of moment of inertia and angular velocity.
- Total angular momentum of all the bodies in a system remains constant in the absence of an external torque.
- Artificial satellites are the objects that orbit around the Earth due to gravity.
- Orbital velocity is the tangential velocity to put a satellite in orbit around the Earth.
- Artificial gravity is the gravity like effect produced in an orbiting spaceship to overcome weightlessness by spinning the spaceship about its own axis.
- Geo-stationary satellite is the one whose orbital motion is synchronized with the rotation of the Earth.
- Albert Einstein viewed gravitation as a space-time curvature around an object.

QUESTIONS

5.1 Explain the difference between tangential velocity and the angular velocity. If one of these is given for a wheel of known radius, how will you find the other?

5.2 Explain what is meant by centripetal force and why it must be furnished to an object if the object is to follow a circular path?

5.3 What is meant by moment of inertia? Explain its significance.

5.4 What is meant by angular momentum? Explain the law of conservation of angular momentum.

5.5 Show that orbital angular momentum $L_o = mvr$.

5.6 Describe what should be the minimum velocity, for a satellite, to orbit close to the Earth around it.

5.7 State the direction of the following vectors in simple situations; angular momentum and angular velocity.

5.8 Explain why an object, orbiting the Earth, is said to be freely falling. Use your explanation to point out why objects appear weightless under certain circumstances.

5.9 When mud flies off the tyre of a moving bicycle, in what direction does it fly? Explain.

5.10 A disc and a hoop start moving down from the top of an inclined plane at the same time. Which one will be moving faster on reaching the bottom?

5.11 Why does a diver change his body positions before and after diving in the pool?

5.12 A student holds two dumb-bells with stretched arms while sitting on a turn table. He is given a push until he is rotating at certain angular velocity. The student then pulls the dumb-bells towards his chest (Fig. 5.25). What will be the effect on rate of rotation?

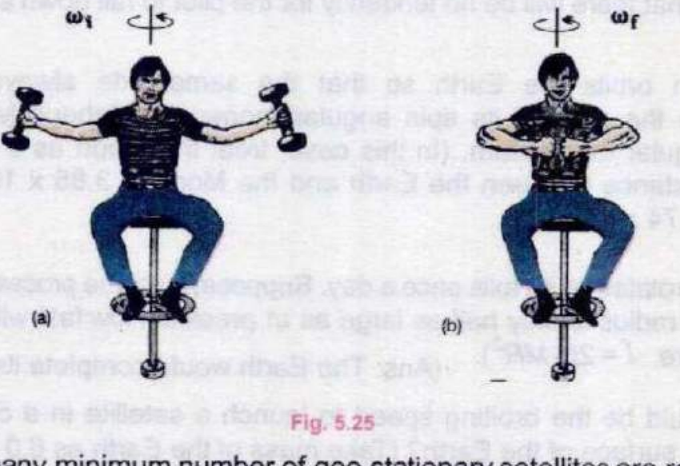


Fig. 5.25

5.13 Explain how many minimum number of geo-stationary satellites are required for global coverage of T.V transmission.

NUMERICAL PROBLEMS

5.1 A tiny laser beam is directed from the Earth to the Moon. If the beam is to have a diameter of 2.50 m at the Moon, how small must divergence angle be for the beam? The distance of Moon from the Earth is 3.8×10^8 m.

(Ans: 6.6×10^{-9} rad)

5.2 A gramophone record turntable accelerates from rest to an angular velocity of 45.0 rev min^{-1} in 1.60s. What is its average angular acceleration?

(Ans: 2.95 rad s^{-2})

5.3 A body of moment of inertia I = 0.80 kg m^2 about a fixed axis, rotates with a constant angular velocity of 100 rad s^{-1}. Calculate its angular momentum L and the torque to sustain this motion.

(Ans: 80 Js, 0)

5.4 Consider the rotating cylinder shown in Fig. 5.26. Suppose that m = 5.0 kg, F = 0.60 N and r = 0.20 m. Calculate (a) the torque acting on the cylinder, (b) the angular acceleration of the cylinder. (Moment of inertia of cylinder = $\frac{1}{2}mr^2$)

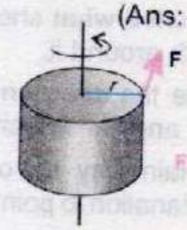


Fig. 5.26

(Ans: 0.12 Nm, 1.2 rad s^{-2})

5.5 Calculate the angular momentum of a star of mass 2.0×10^{30} kg and radius 7.0×10^5 km. If it makes one complete rotation about its axis once in 20 days, what is its kinetic energy?

(Ans: 1.4×10^{42} J s, 2.5×10^{36} J)

5.6 A 1000 kg car travelling with a speed of 144 km h^{-1} round a curve of radius 100 m. Find the necessary centripetal force.

(Ans: 1.60×10^4 N)

5.7 What is the least speed at which an aeroplane can execute a vertical loop of 1.0 km radius so that there will be no tendency for the pilot to fall down at the highest point?

(Ans: 99 ms^{-1})

5.8 The Moon orbits the Earth so that the same side always faces the Earth. Determine the ratio of its spin angular momentum (about its own axis) and its orbital angular momentum. (In this case, treat the Moon as a particle orbiting the Earth). Distance between the Earth and the Moon is 3.85×10^8 m. Radius of the Moon is 1.74×10^6 m.

(Ans: 8.2×10^{-6})

5.9 The Earth rotates on its axis once a day. Suppose, by some process the Earth contracts so that its radius is only half as large as at present. How fast will it be rotating then? (For sphere $I = 2/5\,MR^2$). (Ans: The Earth would complete its rotation in 6 hours)

5.10 What should be the orbiting speed to launch a satellite in a circular orbit 900 km above the surface of the Earth? (Take mass of the Earth as 6.0×10^{24} and its radius as 6400 km).

(Ans: 7.4 km s^{-1})