Chapter 6

FLUID DYNAMICS

Learning Objectives

At the end of this chapter the students will be able to:

1. Understand that viscous forces in a fluid cause a retarding force on an object moving through it.

2. Use Stokes' law to derive an expression for terminal velocity of a spherical body falling through a viscous fluid under laminar conditions.

3. Understand the terms steady (laminar, streamline) flow, incompressible flow, non viscous flow as applied to the motion of an ideal fluid.

4. Appreciate that at a sufficiently high velocity, the flow of viscous fluid undergoes a transition from laminar to turbulence conditions.

5. Appreciate the equation of continuity Av = Constant for the flow of an ideal and incompressible fluid.

6. Appreciate that the equation of continuity is a form of the principle of conservation of mass.

7. Understand that the pressure difference can arise from different rates of flow of a fluid (Bernoulli effect).

8. Derive Bernoulli's equation in form $P + \frac{1}{2}\rho v^2 + \rho gh$ = constant.

9. Explain how Bernoulli effect is applied in the filter pump, atomizers, in the flow of air over an aerofoil, Venturimeter and in blood physics.

10. Give qualitative explanations for the swing of a spinning ball.

T he study of fluids in motion is relatively complicated, but analysis can be simplified by making a few assumptions. The analysis is further simplified by the use of two important conservation principles; the conservation of mass and the conservation of energy. The law of conservation of mass gives us the equation of continuity while the law of conservation of energy is the basis of Bernoulli's equation. The equation of continuity and the Bernoulli's equation along with their applications in aeroplane and blood circulation are discussed in this chapter.

6.1 VISCOUS DRAG AND STOKES' LAW

The frictional effect between different layers of a flowing fluid is described in terms of viscosity of the fluid. Viscosity measures, how much force is required to slide one layer of the liquid over another layer. Substances that do not flow easily, such as thick tar and honey etc; have large coefficients of viscosity, usually denoted by greek letter 'η'. Substances which flow easily, like water, have small coefficients of viscosity. Since liquids and gases have non zero viscosity, a force is required if an object is to be moved through them. Even the small viscosity of the air causes a large retarding force on a car as it travels at high speed. If you stick out your hand out of the window of a fast moving car, you can easily recognize that considerable force has to be exerted on your hand to move it through the air. These are typical examples of the following fact,

> An object moving through a fluid experiences a retarding force called a drag force. The drag force increases as the speed of the object increases.

Even in the simplest cases the exact value of the drag force is difficult to calculate. However, the case of a sphere moving through a fluid is of great importance.

The drag force F on a sphere of radius r moving slowly with speed v through a fluid of viscosity η is given by Stokes' law as under.

$$F = 6\pi\eta r v \qquad \cdots\cdots (6.1)$$

At high speeds the force is no longer simply proportional to speed.

6.2 TERMINAL VELOCITY

Consider a water droplet such as that of fog falling vertically, the air drag on the water droplet increases with speed. The droplet accelerates rapidly under the over powering force of gravity which pulls the droplet downward. However, the upward drag force on it increases as the speed of the droplet increases. The net force on the droplet is

Net force = Weight – Drag force .... (6.2)

As the speed of the droplet continues to increase, the drag force eventually approaches the weight in the magnitude. Finally, when the magnitude of the drag force becomes equal to the weight, the net force acting on the droplet is zero. Then the droplet will fall with constant speed called terminal velocity.

To find the terminal velocity v_t in this case, we use Stokes Law for the drag force. Equating it to the weight of the drop, we have

$$mg = 6\pi\eta r v_t$$

or $$v_t = \frac{mg}{6\pi\eta r}$$ .... (6.3)

The mass of the droplet is ρV,

where volume $V = \frac{4}{3}(\pi r^3)$

Substituting this value in the above equation, we get

$$v_t = \frac{2gr^2\rho}{9\eta}$$ .... (6.4)

Example 6.1: A tiny water droplet of radius 0.010 cm descends through air from a high building. Calculate its terminal velocity. Given that η for air $= 19 \times 10^{-6}$ kg m^{-1} s^{-1} and density of water $\rho = 1000$ kgm^{-3}.

Solution:

$r = 1.0 \times 10^{-4}$ m , $\rho = 1000$ kgm^{-3} , $\eta = 19 \times 10^{-6}$ kg m^{-1} s^{-1}

Putting the above values in Eq. 6.4

$$v_t = \frac{2 \times 9.8 \text{ ms}^{-2} \times (1 \times 10^{-4} \text{ m})^2 \times 1000 \text{ kgm}^{-3}}{9 \times 19 \times 10^{-6} \text{ kgm}^{-1}\text{s}^{-1}}$$
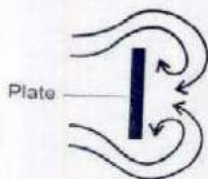
We get   Terminal velocity = 1.1 m s^{-1}

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(a) Streamlines (laminar flow)

(b) Turbulent flow

Fig. 6.1

6.3 FLUID FLOW

Moving fluids are of great importance. To learn about the behaviour of the fluid in motion, we consider their flow through the pipes. When a fluid is in motion, its flow can be either streamline or turbulent.

The flow is said to be streamline or laminar, if every particle that passes a particular point, moves along exactly the same path, as followed by particles which passed that points earlier.

In this case each particle of the fluid moves along a smooth path called a streamline as shown in Fig. 6.1 (a). The different streamlines can not cross each other. This condition is called steady flow condition. The direction of the streamlines is the same as the direction of the velocity of the fluid at that point. Above a certain velocity of the fluid flow, the motion of the fluid becomes unsteady and irregular.

Under this condition the velocity of the fluid changes abruptly as shown in Fig.6.1 (b). In this case the exact path of the particles of the fluid can not be predicted.

The irregular or unsteady flow of the fluid is called turbulent flow.

We can understand many features of the fluid in motion by considering the behaviour of a fluid which satisfies the following conditions.

1. The fluid is non-viscous i.e, there is no internal frictional force between adjacent layers of fluid.

2. The fluid is incompressible, i.e., its density is constant.

3. The fluid motion is steady.

6.4 EQUATION OF CONTINUITY

Consider a fluid flowing through a pipe of non-uniform size. The particles in the fluid move along the streamlines in a steady state flow as shown in Fig. 6.2.

In a small time Δt, the fluid at the lower end of the tube moves a distance Δx_1, with a velocity v_1. If A_1 is the area of cross section of this end, then the mass of the fluid contained in the shaded region is:

$$\Delta m_1 = \rho_1 A_1 \Delta x_1 = \rho_1 A_1 v_1 \times \Delta t$$

Where ρ_1 is the density of the fluid. Similarly the fluid that moves with velocity v_2 through the upper end of the pipe (area of cross section A_2) in the same time Δt has a mass

$$\Delta m_2 = \rho_2 A_2 v_2 \times \Delta t$$

If the fluid is incompressible and the flow is steady, the mass of the fluid is conserved. That is, the mass that flows into the bottom of the pipe through A_1 in a time Δt must be equal to mass of the liquid that flows out through A_2 in the same time. Therefore,

$$\Delta m_1 = \Delta m_2$$

or

$$\rho_1 A_1 v_1 = \rho_2 A_2 v_2$$

This equation is called the equation of continuity. Since density is constant for the steady flow of incompressible fluid, the equation of continuity becomes

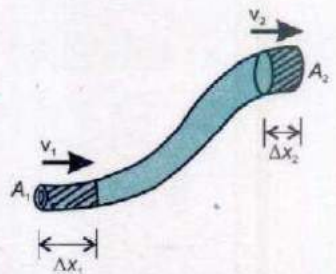$$A_1 v_1 = A_2 v_2 \qquad \dots\dots\dots \quad (6.5)$$

Fig. 6.2

The product of cross sectional area of the pipe and the fluid speed at any point along the pipe is a constant. This constant equals the volume flow per second of the fluid or simply flow rate.

Example: 6.2: A water hose with an internal diameter of 20 mm at the outlet discharges 30 kg of water in 60 s. Calculate the water speed at the outlet. Assume the density of water is 1000 kgm^{-3} and its flow is steady.

Solution:

Mass flow per second $= \dfrac{30kg}{60s} = 0.5$ kgs^{-1}

Cross sectional area $A = \pi r^2$

Tidbits

As the water falls, its speed increases and so its cross sectional area decreases as mandated by the continuity equation.

The mass of water discharging per second through area A is

$$\rho Av = \frac{\text{mass}}{\text{second}}$$

or

$$v = \frac{\text{mass}/\text{second}}{\rho A}$$

$$= \frac{0.5 \text{ kgs}^{-1}}{1000 \text{ kgm}^{-3} \times 3.14 \times (10 \times 10^{-3} \text{m})^2}$$

$$= 1.6 \text{ ms}^{-1}$$

6.5 BERNOULLI'S EQUATION

As the fluid moves through a pipe of varying cross section and height, the pressure will change along the pipe. Bernoulli's equation is the fundamental equation in fluid dynamics that relates pressure to fluid speed and height.

In deriving Bernoulli's equation, we assume that the fluid is incompressible, non viscous and flows in a steady state manner. Let us consider the flow of the fluid through the pipe in time t, as shown in Fig. 6.3.
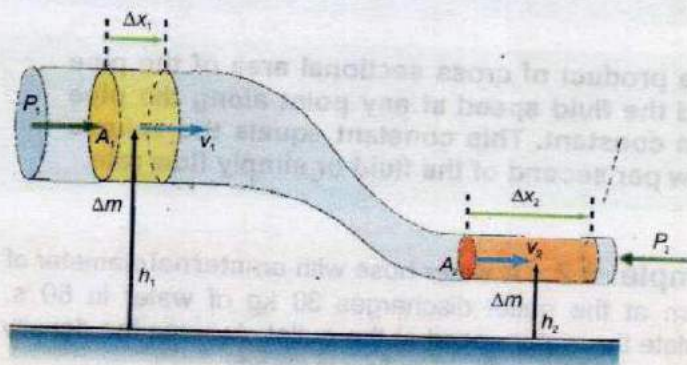

Fig. 6.3.

The force on the upper end of the fluid is $P_1 A_1$, where P_1 the pressure and A_1 is the area of cross section at the upper end. The work done on the fluid, by the fluid behind it, in moving it through a distance Δx_1, will be

$$W_1 = F_1 \Delta x_1 = P_1 A_1 \Delta x_1$$

Similarly the work done on the fluid at the lower end is

$$W_2 = -F_2\, \Delta x_2 = -P_2 A_2 \Delta x_2$$

Where P_2 is the pressure, A_2 is the area of cross section of lower end and Δx_2 is the distance moved by the fluid in the same time interval t. The work W_2 is taken to be –ive as this work is done against the fluid force.

The net work done = $W = W_1 + W_2$

or $\qquad\qquad W = P_1 A_1 \Delta x_1 - P_2 A_2 \Delta x_2 \qquad \text{.......} \quad (6.6)$

If v_1 and v_2 are the velocities at the upper and lower ends respectively, then

$$W = P_1 A_1 v_1 t - P_2 A_2 v_2 t$$

From equation of continuity (equation 6.5)

$$A_1 v_1 = A_2 v_2$$

Hence, $\quad A_1 v_1 \times t = A_2 v_2 \times t = V \quad \left(\begin{array}{l}\text{Volume of fluid}\\\text{under consideration}\end{array}\right)$

So, we have

$$W = (P_1 - P_2)\, V \qquad \text{..........} \quad (6.7)$$

If m is the mass and ρ is the density then $V = \dfrac{m}{\rho}$

So equation 6.7 becomes

$$W = (P_1 - P_2)\, \frac{m}{\rho} \qquad \text{..........} \quad (6.8)$$

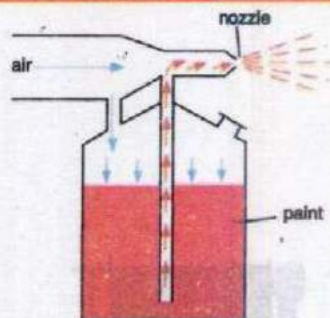Part of this work is utilized by the fluid in changing its K.E. and a part is used in changing its gravitational P.E.

Change in K.E. = $\Delta (K.E.) = \dfrac{1}{2}mv_2^2 - \dfrac{1}{2}mv_1^2 \qquad \text{....} \quad (6.9)$

Change in P.E. = $\Delta (P.E.) = mgh_2 - mgh_1 \qquad \text{......} \quad (6.10)$

Where h_1 and h_2 are the heights of the upper and lower ends respectively.
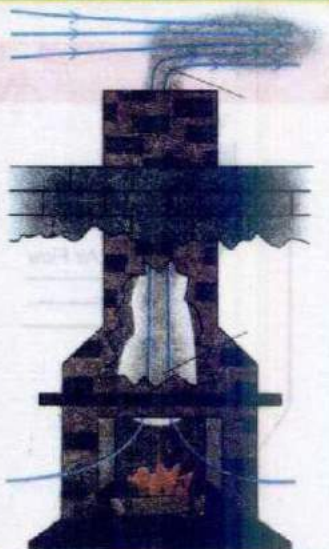
Applying, the law of conservation of energy to this volume of the fluid, we get
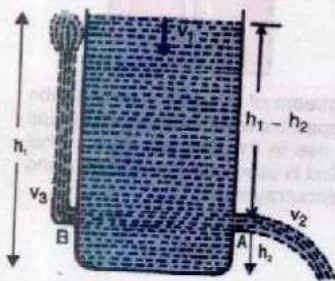
$$(P_1 - P_2)\ \frac{m}{\rho} = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2 + mgh_2 - mgh_1 \quad \text{.......} \quad (6.11)$$

rearranging the equation (6.11)

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho gh_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho gh_2$$

This is Bernoulli's equation and is often expressed as:

$$P + \frac{1}{2}\rho v^2 + \rho gh = \text{constant} \quad \text{..........} \quad (6.12)$$

6.6 APPLICATIONS OF BERNOULLI'S EQUATION

Torricelli's Theorem

A simple application of Bernoulli's equation is shown in Fig. 6.4. Suppose a large tank of fluid has two small orifices A and B on it, as shown in the figure. Let us find the speed with which the water flows from the orifice A.

Since the orifices are so small, the efflux speeds v_2 and v_3 will be much larger than the speed v_1 of the top surface of water. We can therefore, take v_1 as approximately zero. Hence, Bernoulli's equation can be written as:

$$P_1 + \rho gh_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho gh_2$$

But $P_1 = P_2 =$ atmospheric pressure

Therefore, the above equation becomes
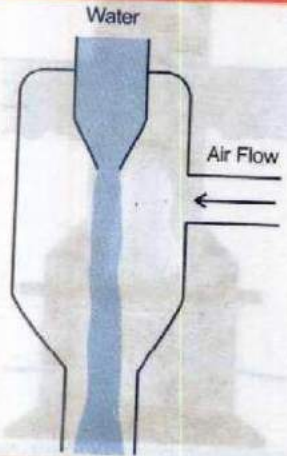
$$v_2 = \sqrt{2g(h_1 - h_2)} \quad \text{..........} \quad (6.13)$$

This is Torricelli's theorem which states that;

The speed of efflux is equal to the velocity gained by the fluid in falling through the distance $(h_1 - h_2)$ under the action of gravity.

Notice that the speed of the efflux of liquid is the same as the speed of a ball that falls through a height $(h_1 - h_2)$. The

Fig.6.4.

For your information

Water

Air Flow

A filter pump has a constriction in the centre, so that a jet of water from the tap flows faster here. This cause a drop in pressure near it and air, therefore, flows in from the side tube. The air and water together are expelled through the lower part of the pump.

top level of the tank has moved down a little and the P.E. has been transferred into K.E. of the efflux of fluid. If the orifice had been pointed upward as at B shown in Fig.6.4, this K.E. would allow the liquid to rise to the level of water tank. In practice, viscous-energy losses would alter the result to some extent.

Relation between Speed and Pressure of the Fluid

A result of the Bernoulli's equation is that the pressure will be low where the speed of the fluid is high. Suppose that water flows through a pipe system as shown in Fig. 6.5. Clearly, the water will flow faster at B than it does at A or C. Assuming the flow speed at A to be 0.20 ms^{-1} and at B to be 2.0 ms^{-1}, we compare the pressure at B with that at A.

Applying Bernoulli's equation and noting that the average P.E. is the same at both places, We have,

$$P_A + \frac{1}{2}\rho v_A^2 = P_B + \frac{1}{2}\rho v_B^2 \qquad \dots\dots\dots \quad (6.14)$$

Substituting $v_A = 0.20$ ms^{-1} , $v_B = 2.0$ ms^{-1}

And $\rho = 1000$ kgm^{-3}

We get $P_A - P_B = 1980$ Nm^{-2}

This shows that the pressure in the narrow pipe where streamlines are closer together is much smaller than in the wider pipe. Thus,

Where the speed is high, the pressure will be low.

The lift on an aeroplane is due to this effect. The flow of air around an aeroplane wing is illustrated in Fig. 6.6. The wing is designed to deflect the air so that streamlines are closer together above the wing than below it. We have seen in Fig.6.6 that where the streamlines are forced closer together, the speed is faster. Thus, air is travelling faster on the upper side of the wing than on the lower. The pressure will be lower at the top of the wing, and the wing will be forced upward.

Similarly, when a tennis ball is hit by a racket in such a way that it spins as well as moves forward, the velocity of the
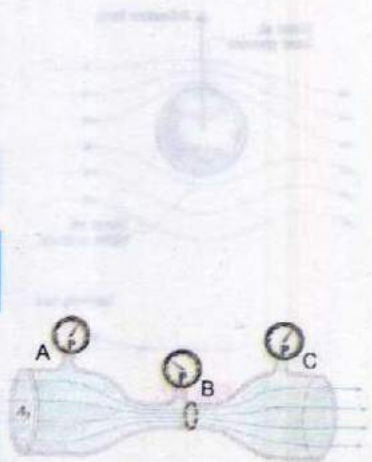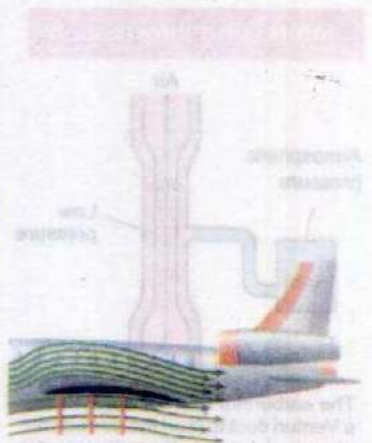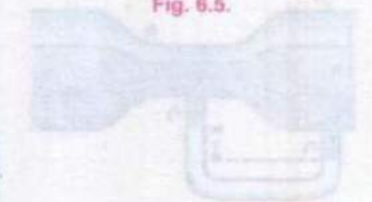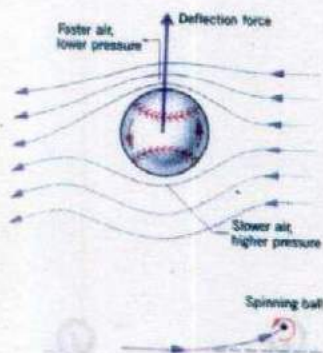

Fig. 6.5.

Fig. 6.6.

Foster air, lower pressure

Deflection force

Slower air, higher pressure

Spinning ball

Fig. 6.7.

Fig. 6.8.

Air

Atmospheric pressure

Low pressure
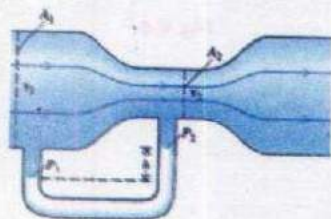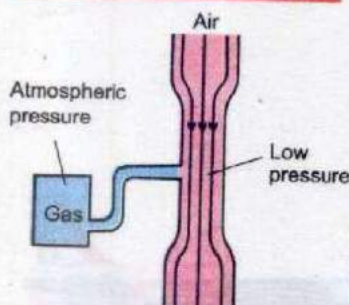
Gas

The carburetor of a car engine uses a Venturi duct to feed the correct mix of air and petrol to the cylinders. Air is drawn through the duct and along a pipe to the cylinders. A tiny inlet at the side of duct is fed with petrol. The air through the duct moves very fast, creating low pressure in the duct, which draws petrol vapour into the air stream.

air on one side of the ball increases (Fig. 6.7) due to spin and air speed in the same direction as at B and hence, the pressure decreases. This gives an extra curvature to the ball known as swing which deceives an opponent player.

Venturi Relation

If one of the pipes has a much smaller diameter than the other, as shown in Fig. 6.8, we write Bernoulli's equation in a more convenient form. It is assumed that the pipes are horizontal so that ρgh terms become equal and can, therefore, be dropped. Then

$$P_1 - P_2 = \frac{1}{2}\rho v_2^2 - \frac{1}{2}\rho v_1^2 = \frac{1}{2}\rho(v_2^2 - v_1^2) \qquad \ldots\ldots (6.15)$$

As the cross-sectional area A_2 is small as compared to the area A_1, then from equation of continuity $v_1 = (A_2/A_1) v_2$, will be small as compared to v_2. Thus for flow from a large pipe to a small pipe we can neglect v_1 on the right hand side of equation 6.15. Hence,

$$P_1 - P_2 = \frac{1}{2}\rho v_2^2 \qquad \ldots\ldots (6.16)$$

This is known as Venturi relation, which is used in Venturi-meter, a device used to measure speed of liquid flow.

Example 6.3: Water flows down hill through a closed vertical funnel. The flow speed at the top is 12.0 cms⁻¹. The flow speed at the bottom is twice the speed at the top. If the funnel is 40.0 cm long and the pressure at the top is 1.013×10^5 Nm⁻², what is the pressure at the bottom?

Solution: Using Bernoulli's equation

$$P_1 + \rho g h_1 + \frac{1}{2}\rho v_1^2 = P_2 + \rho g h_2 + \frac{1}{2}\rho v_2^2$$

Or

$$P_2 = P_1 + \rho g h + \frac{1}{2}\rho(v_1^2 - v_2^2)$$

where $h = h_1 - h_2 =$ the length of the funnel

$P_2 = (1.013 \times 10^5 \text{ Nm}^{-2}) + (1000 \text{ kgm}^{-3} \times 9.8 \text{ ms}^{-2} \times 0.4\text{m})$

$+ [\frac{1}{2}(1000\text{kgm}^{-3}) \times \{(0.12 \text{ ms}^{-1})^2 - (0.24 \text{ ms}^{-1})^2\}]$

$= 1.05 \times 10^5 \text{ N m}^{-2}$

Blood Flow

Blood is an incompressible fluid having a density nearly equal to that of water. A high concentration (~50%) of red blood cells increases its viscosity from three to five times that of water. Blood vessels are not rigid. They stretch like a rubber hose. Under normal circumstances the volume of the blood is sufficient to keep the vessels inflated at all times, even in the relaxed state between heart beats. This means there is tension in the walls of the blood vessels and consequently the pressure of blood inside is greater than the external atmospheric pressure. Fig. 6.9 shows the variation in blood pressure as the heart beats. The pressure varies from a high (systolic pressure) of 120 torr (1 torr = 133.3 Nm^{-2}) to a low diastolic pressure) of about 75-80 torr between beats in normal, healthy person. The numbers tend to increase with age, corresponding to the decrease in the flexibility of the vessel walls.

The unit torr or mm of Hg is opted instead of SI unit of pressure because of its extensive use in medical equipments.

An instrument called a sphygmomanometer measures blood pressure dynamically (Fig. 6.10).

Fig. 6.10.

Fig. 6.9

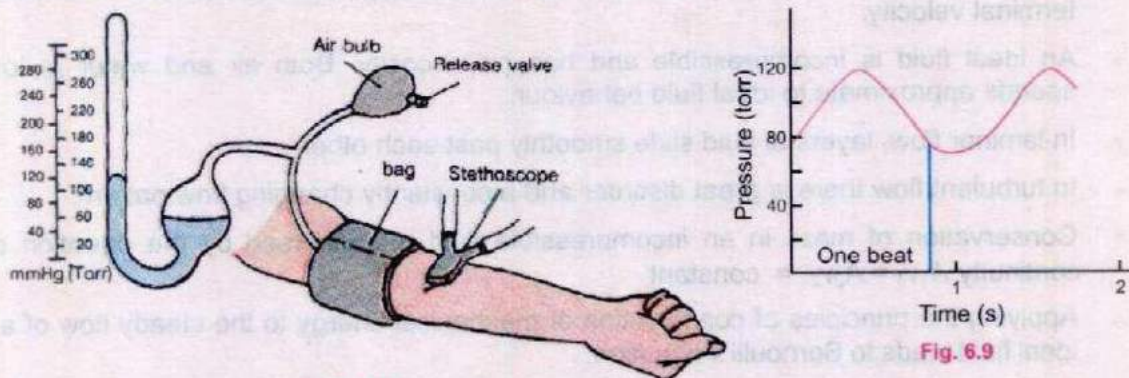An inflatable bag is wound around the arm of a patient and external pressure on the arm is increased by inflating the bag. The effect is to squeeze the arm and compress the blood vessels inside. When the external pressure applied becomes larger than the systolic pressure, the vessels collapse, cutting off the flow of the blood. Opening the release valve on the bag gradually decreases the external pres

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A stethoscope detects the instant at which the external pressure becomes equal to the systolic pressure. At this point the first surges of blood flow through the narrow stricture produces a high flow speed. As a result the flow is initially turbulent.

As the pressure drops, the external pressure eventually equals the diastolic pressure. From this point, the vessel no longer collapse during any portion of the flow cycle. The flow switches from turbulent to laminar, and the gurgle in the stethoscope disappears. This is the signal to record diastolic pressure.

SUMMARY

- An object moving through a fluid experiences a retarding force known as drag force. It increases as the speed of object increases.

- A sphere of radius r moving with speed v through a fluid of viscosity η experiences a viscous drag force F given by Stokes' law $F = 6\pi\eta r v$.

- The maximum and constant velocity of an object falling vertically downward is called terminal velocity.

- An ideal fluid is incompressible and has no viscosity. Both air and water at low speeds approximate to ideal fluid behaviour.

- In laminar flow, layers of fluid slide smoothly past each other.

- In turbulent flow there is great disorder and a constantly changing flow pattern.

- Conservation of mass in an incompressible fluid is expressed by the equation of continuity $A_1 v_1 = A_2 v_2 = $ constant

- Applying the principles of conservation of mechanical energy to the steady flow of an ideal fluid leads to Bernoulli's equation.

$$P + \frac{1}{2}\rho v^2 + \rho g h = \text{constant}$$

- The effect of the decrease in pressure with the increase in speed of the fluid in a horizontal pipe is known as Venturi effect.

6.1 Explain what do you understand by the term viscosity?

6.2 What is meant by drag force? What are the factors upon which drag force acting upon a small sphere of radius r, moving down through a liquid, depend?

6.3 Why fog droplets appear to be suspended in air?

6.4 Explain the difference between laminar flow and turbulent flow.

6.5 State Bernoulli's relation for a liquid in motion and describe some of its applications.

6.6 A person is standing near a fast moving train. Is there any danger that he will fall towards it?

6.7 Identify the correct answer. What do you infer from Bernoulli's theorem?

   (i)   Where the speed of the fluid is high the pressure will be low.

   (ii) Where the speed of the fluid is high the pressure is also high.

   (iii) This theorem is valid only for turbulent flow of the liquid.

6.8 Two row boats moving parallel in the same direction are pulled towards each other. Explain.

6.9 Explain, how the swing is produced in a fast moving cricket ball.

6.10 Explain the working of a carburetor of a motorcar using by Bernoulli's principle.

6.11 For which position will the maximum blood pressure in the body have the smallest value. (a) Standing up right (b) Sitting (c) Lying horizontally (d) Standing on one's head?

6.12 In an orbiting space station, would the blood pressure in major arteries in the leg ever be greater than the blood pressure in major arteries in the neck?

6.1 Certain globular protein particle has a density of 1246 kg m^{-3}. It falls through pure water ($\eta=8.0 \times 10^{-4}$ Nms) with a terminal speed of 3.0 cm h^{-1}. Find the radius of the particle.

(Ans: 1.6×10^{-6} m)

6.2 Water flows through a hose, whose internal diameter is 1cm at a speed of 1ms^{-1}. What should be the diameter of the nozzle if the water is to emerge at 21ms^{-1}?

(Ans: 0.2 cm)

6.3 The pipe near the lower end of a large water storage tank develops a small leak and a stream of water shoots from it. The top of water in the tank is 15m above the point of leak.

a) With what speed does the water rush from the hole?

b) If the hole has an area of 0.060 cm^2, how much water flows out in one second?

(Ans: (a) 17 m s^{-1}, (b) 102 cm^3)

6.4 Water is flowing smoothly through a closed pipe system. At one point the speed of water is 3.0 ms^{-1}, while at another point 3.0 m higher, the speed is 4.0 ms^{-1}. If the pressure is 80 kPa at the lower point, what is pressure at the upper point?

(Ans: 47 kPa)

6.5 An airplane wing is designed so that when the speed of the air across the top of the wing is 450 ms^{-1}, the speed of air below the wing is 410 ms^{-1}. What is the pressure difference between the top and bottom of the wings? (Density of air = 1.29kgm^{-3})

(Ans: 22 kPa)

6.6 The radius of the aorta is about 1.0 cm and the blood flowing through it has a speed of about 30 cms^{-1}. Calculate the average speed of the blood in the capillaries using the fact that although each capillary has a diameter of about 8 x 10^{-4} cm, there are literally millions of them so that their total cross section is about 2000cm^2.

(Ans: 5 x 10^{-4}ms^{-1})

6.7 How large must a heating duct be if air moving 3.0 ms^{-1} along it can replenish the air in a room of 300 m^3 volume every 15 min? Assume the air's density remains constant.

(Ans: Radius = 19 cm)

6.8 An airplane design calls for a "lift" due to the net force of the moving air on the wing of about 1000 Nm^{-2} of wing area. Assume that air flows past the wing of an aircraft with streamline flow. If the speed of flow past the lower wing surface is 160ms^{-1}, what is the required speed over the upper surface to give a "lift" of 1000Nm^{-2}? The density of air is 1.29 kgm^{-3} and assume maximum thickness of wing to be one metre.

(Ans: 165 ms^{-1})

6.9 What gauge pressure is required in the city mains for a stream from a fire hose connected to the mains to reach a vertical height of 15.0 m?

(Ans: 1.47 x 10^5 Pa)