Chapter 7

OSCILLATIONS

Learning Objectives

At the end of this chapter the students will be able to:

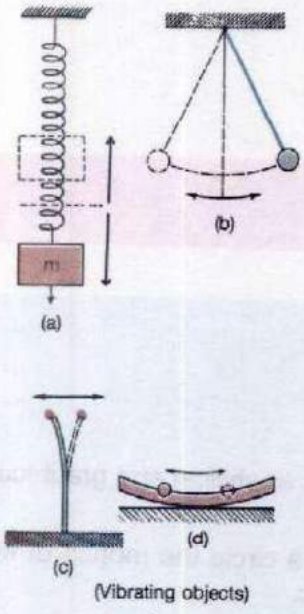
1. Investigate the motion of an oscillator using experimental, analytical and graphical methods.
2. Understand and describe that when an object moves in a circle the motion of its projection on the diameter of the circle is simple harmonic.
3. Show that the motion of mass attached to a spring is simple harmonic.
4. Understand that the motion of simple pendulum is simple harmonic and to calculate its time period.
5. Understand and use the terms amplitude, time period, frequency, angular frequency and phase difference.
6. Know and use of solutions in the form of $x = x_o \cos \omega t$ or $y = y_o \sin \omega t$.
7. Describe the interchange between kinetic and potential energies during SHM.
8. Describe practical examples of free and forced oscillations.
9. Describe practical examples of damped oscillations with particular reference to the effects of the degree of damping and the importance of critical damping in cases such as car suspension system.

Many a times, we come across a type of motion in which a body moves to and fro about a mean position. It is called oscillatory or vibratory motion. The oscillatory motion is called periodic when it repeats itself after equal intervals of time.

Some typical vibrating bodies are shown in Fig. 7.1. It is our common observation that

a) a mass, suspended from a spring, when pulled down and then released, starts oscillating (Fig. 7.1 a),

b) the bob of a simple pendulum when displaced from its rest position and released, vibrates (Fig. 7.1 b).



(Vibrating objects)

Fig. 7.1

c) a steel ruler clamped at one end to a bench oscillates when the free end is displaced sideways (Fig. 7.1 c).

d) a steel ball rolling in a curved dish, oscillates about its rest position (Fig. 7.1 d).

Thus to get oscillations, a body is pulled away from its rest or equilibrium position and then released. The body oscillates due to a restoring force. Under the action of this restoring force, the body accelerates and it overshoots the rest position due to inertia. The restoring force then pulls it back. The restoring force is always directed towards the rest position and so the acceleration is also directed towards the rest or mean position.

It is observed that the vibrating bodies produce waves. For example, a violin string produces sound waves in air. There are many phenomena in nature whose explanation requires the understanding of the concepts of vibrations and waves. Although many large structures, such as skyscrapers and bridges, appear to be rigid, they actually vibrate. The architects and the engineers who design and build them, take this fact into account.

7.1 SIMPLE HARMONIC MOTION

Let us consider a mass m attached to one end of an elastic spring which can move freely on a frictionless horizontal surface as shown in Fig. 7.2 (a). When the mass is displaced towards right through a distance **x** (Fig. 7.2 b), the force **F** at that instant is given by Hooke's law $\mathbf{F} = k\mathbf{x}$ where k is a constant known as spring constant. Due to elasticity, spring opposes the applied force which produces the displacement. This opposing force is called restoring force $\mathbf{F}_r$ which is equal and opposite to the applied force within elastic limit of the spring. Hence

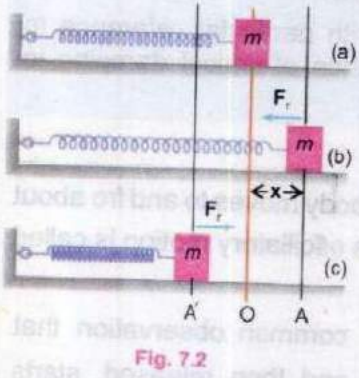


Fig. 7.2

$$\mathbf{F}_r = -k\mathbf{x} \quad \ldots\ldots\ldots \quad (7.1)$$

The negative sign indicates that $\mathbf{F}_r$ is directed opposite to **x**. i.e., towards the equilibrium position. Thus we see that in a system obeying Hooke's law, the restoring force $\mathbf{F}_r$ is directly proportional to the displacement **x** of the system from its equilibrium position and is always directed towards it. When the mass is released, it begins to oscillate about the equilibrium position (Fig. 7.2 c). The oscillatory motion taking place under the action of such a restoring force is

known as simple harmonic motion (SHM). The acceleration **a** produced in the mass *m* due to restoring force can be calculated using second law of motion

$$\mathbf{F} = m\mathbf{a}$$

Then, $$-k\mathbf{x} = m\mathbf{a}$$

or $$\mathbf{a} = -\frac{k}{m}\mathbf{x} \quad \ldots\ldots\ldots \quad (7.2)$$

or $$\mathbf{a} \propto -\mathbf{x}$$

The acceleration at any instant of a body executing SHM is proportional to displacement and is always directed towards its mean position.

We will now discuss various terms which are very often used in describing SHM.

(i) Instantaneous Displacement and Amplitude of Vibration

It can be seen in Fig. 7.2 that when a body is vibrating, its displacement from the mean position changes with time. The value of its distance from the mean position at any time is known as its instantaneous displacement. It is zero at the instant when the body is at the mean position and is maximum at the extreme positions. The maximum value of displacement is known as amplitude.

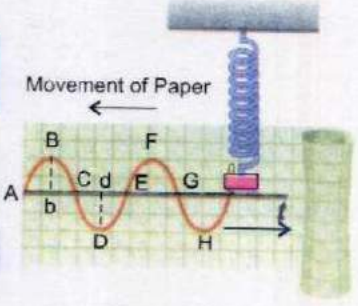


Fig. 7.3

The arrangement shown in Fig. 7.3 can be used to record the variations in displacement with time for a mass-spring system. The strip of paper is moving at a constant speed from right to left, thus providing a time scale on the strip. A pen attached with the vibrating mass records its displacement against time as shown in Fig. 7.3. It can be seen that the curve showing the variation of displacement with time is a sine curve. It is usually known as wave-form of SHM. The points B and D correspond to the extreme positions of the vibrating mass and points A, C and E show its mean position. Thus the line ACE represents the level of mean position of the mass on the strip. The amplitude of vibration is thus a measure of the line Bb or Dd in Fig. 7.3.

(ii) Vibration

A vibration means one complete round trip of the body in motion. In Fig. 7.3, it is the motion of mass from its mean position to the upper extreme position, from upper extreme position to lower extreme position and back to its mean position. In Fig. 7.3, the curve ABCDE correspond to the different positions of the pen during one complete vibration. Alternatively the vibration can also be defined as motion of the body from its one extreme position back to the same extreme position. This will correspond to the portion of curve from points B to F or from points D to H.

(iii) Time Period

It is the time T required to complete one vibration.

(iv) Frequency

Frequency f is the number of vibrations executed by a body in one second and is expressed as vibrations per second or cycles per second or hertz (Hz).

The definitions of T and f show that the two quantities are related by the equation

$$f = \frac{1}{T} \quad \ldots\ldots\ldots \quad (7.3)$$

(v) Angular Frequency

If T is the time period of a body executing SHM, its angular frequency will be

$$\omega = \frac{2\pi}{T} = 2\pi f \quad \ldots\ldots\ldots \quad (7.4)$$

Angular frequency ω is basically a characteristic of circular motion. Here it has been introduced in SHM because it provides an easy method by which the value of instantaneous displacement and instantaneous velocity of a body executing SHM can be computed.

7.2 SHM AND UNIFORM CIRCULAR MOTION

Let a mass m, attached with the end of a vertically suspended spring, vibrate simple harmonically with period T, frequency f and amplitude x_0. The motion of the mass is displayed by the pointer P_1 on the line BC with A as mean position and B, C as extreme positions (Fig. 7.4a). Assuming A as the position of the pointer at $t = 0$, it will move so that it is at B,A,C and back to A at

instants $T/4$, $T/2$, $3T/4$ and T respectively. This will complete one cycle of vibration with amplitude of vibration being x_o = AB = AC.

The concept of circular motion is introduced by considering a point P moving on a circle of radius x_o, with a uniform angular frequency $\omega = 2\pi/T$, where T is the time period of the vibration of the pointer. It may be noted that the radius of the circle is equal to the amplitude of the pointer's motion. Consider the motion of the point N, the projection of P on the diameter DE drawn parallel to the line of vibration of the pointer in Fig. 7.4 (b). Note that the level of points D and E

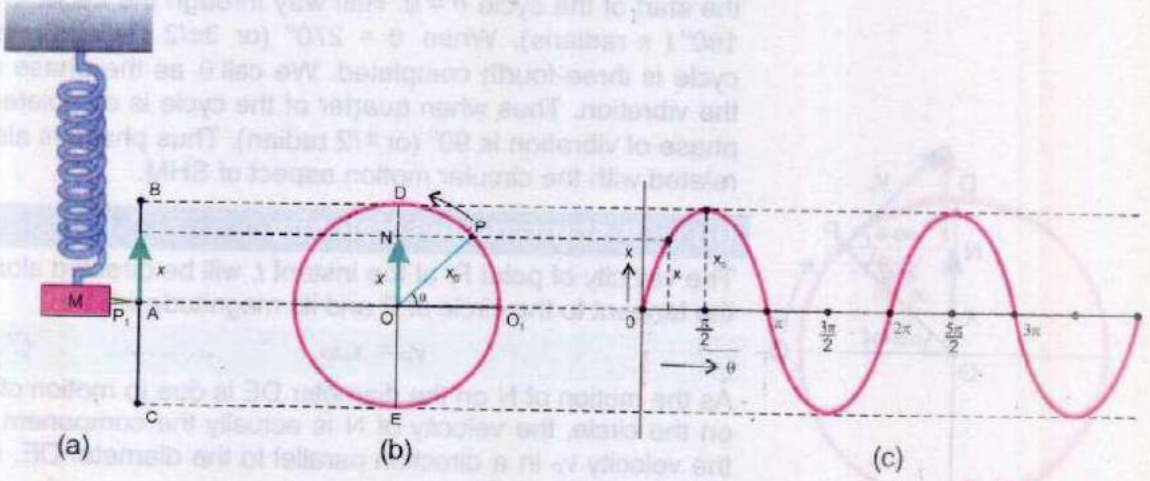


Fig. 7.4

is the same as the points B and C. As P describes uniform circular motion with a constant angular speed ω, N oscillates to and fro on the diameter DE with time period T. Assuming O_1, to be the position of P at $t = 0$, the position of the point N at the instants 0, $T/4$, $T/2$, $3T/4$ and T will be at the points O,D,O,E and O respectively. A comparison of the motion of N with that of the pointer P_1 shows that it is a replica of the pointer's motion. Thus the expressions of displacement, velocity and acceleration for the motion of N also hold good for the pointer P_1, executing SHM.

(i) Displacement

Referring to Fig. 7.4 (b), if we count the time $t = 0$ from the instant when P is passing through O_1, the angle which the radius OP sweeps out in time t is $\angle O_1OP = \theta = \omega t$. The displacement x of N at the instant t will be

$$x = ON = OP \sin \angle O_1OP$$

or $x = x_0 \sin\theta$

or $x = x_0 \sin \omega t$ ..... (7.5)

This will be also the displacement of the pointer P_1 at the instant t.

The value of x as a functions of θ is shown in Fig. 7.4 (c). This is the wave-form of SHM. In Fig. 7.3, the same wave-form was traced experimentally but here, we have traced it theoretically by linking SHM with circular motion through the concept of angular frequency. The angle θ gives the states of the system in its vibrational cycle. For example, at the start of the cycle $\theta = 0$. Half way through the cycle, is 180° (π radians). When $\theta = 270^\circ$ (or $3\pi/2$ radians), the cycle is three-fourth completed. We call θ as the phase of the vibration. Thus when quarter of the cycle is completed, phase of vibration is 90° (or $\pi/2$ radian). Thus phase is also related with the circular motion aspect of SHM.

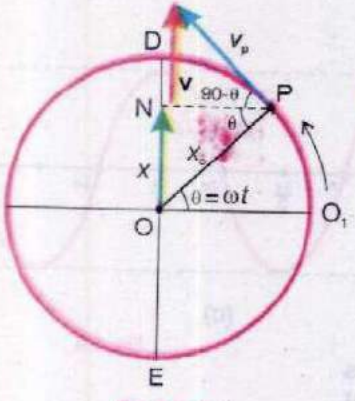


Fig. 7.5(a)

(ii) Instantaneous Velocity

The velocity of point P, at the instant t, will be directed along the tangent to the circle at P and its magnitude will be

$$v_P = x_0 \omega \quad \text{.....} \quad (7.6)$$

As the motion of N on the diameter DE is due to motion of P on the circle, the velocity of N is actually the component of the velocity v_P in a direction parallel to the diameter DE. As shown in Fig. 7.5 (a), this component is

$$v_P \sin(90^\circ - \theta) = v_P \cos\theta = x_0\, \omega \cos\theta.$$

Thus the magnitude of the velocity of N or its speed v is

$$v = x_0\, \omega \cos\theta = x_0\, \omega \cos \omega t \quad \text{.....} \quad (7.7)$$

The direction of the velocity of N depends upon the value of the phase angle θ. When θ is between 0° to 90° the direction is from O to D, for θ between 90° to 270°, its direction is from D to E. When θ is between 270° to 360°, the direction of motion is from E to D.

From Fig. 7.5, $\cos\theta = \cos \angle NPO = NP/OP = \dfrac{\sqrt{x_0^2 - x^2}}{x_0}$.

Substituting the value of $\cos\theta$ in Eq. 7.7

$$v = \frac{x_0 \omega}{x_0} \sqrt{x_0^2 - x^2} = \omega \sqrt{x_0^2 - x^2} \quad \text{.....} \quad (7.8)$$

As the motion of N on the diameter DE is just the replica of the motion of the pointer executing SHM (Fig. 7.4), so velocity of the point P or the velocity of any body executing SHM is given by equations 7.7 and 7.8 in terms of the angular frequency ω. Eq. 7.8 shows that at the mean position, where $x = 0$, the velocity is maximum and at the extreme positions where $x = x_o$, the velocity is zero.

(iii) Acceleration in Terms of ω

When the point P is moving on the circle, it has an acceleration $a_p = x_o\omega^2$, always directed towards the centre O of the circle.

At instant t, its direction will be along PO. The acceleration of the point N will be component of the acceleration a_p along the diameter DE on which N moves due to motion of P. As shown in Fig. 7.5 (b), the value of this component is

$$a_p \sin\theta = x_o\omega^2 \sin\theta.$$

Thus the acceleration a of N is $a = x_o\omega^2 \sin\theta$

and it is directed from N to O, i.e., directed towards the mean position O (Fig. 7.5 b). In this figure $\sin\theta = ON/OP = x/x_o$. Therefore,

$$a = x_o\omega^2 \times \frac{x}{x_o} = \omega^2 x$$

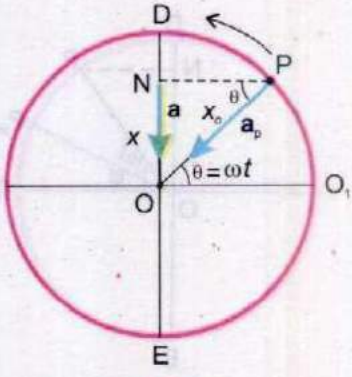


Fig. 7.5(b)

Comparison of Fig. 7.5 (b) and 7.4 (b) shows that the direction of acceleration **a** and displacement **x** are opposite. Considering the direction of **x** as reference, the acceleration **a** will be represented by

$$\mathbf{a} = -\omega^2 \mathbf{x} \qquad \ldots\ldots\ldots \qquad (7.9)$$

Eq. 7.9 shows that the acceleration is proportional to the displacement and is directed towards the mean position which is the characteristic of SHM. Thus the point N is executing SHM with the same amplitude, period and instantaneous displacement as the pointer P_1. This confirms our assertion that the motion of N is just a replica of the pointer's motion.

7.3 PHASE

Equations 7.5 and 7.7 indicate that displacement and velocity of the point executing SHM are determined by the angle $\theta = \omega t$. Note that this angle is obtained when SHM is related with circular motion. It is the angle which the rotating

radius OP makes with the reference direction OO_1 at any instant t (Fig. 7.4 b).

The angle $\theta = \omega t$ which specifies the displacement as well as the direction of motion of the point executing SHM is known as phase.

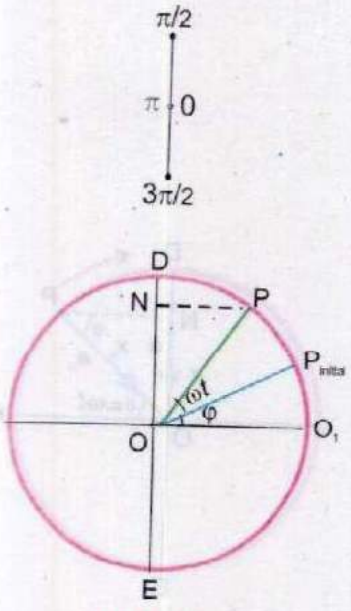


Fig. 7.6(a)

The phase determines the state of motion of the vibrating point. If a body starts its motion from mean position, its phase at this point would be 0. Similarly at the extreme positions, its phase would be $\pi/2$.

In Fig. 7.4 (b), we have assumed that to start with at $t = 0$, the position of the rotating radius OP is along OO_1 so that the point N is at its mean position and the displacement at $t = 0$, is zero. Thus it represents a special case. In general at $t = 0$, the rotating radius OP can make any angle φ with the reference OO_1 as shown in Fig. 7.6 (a). In time t, the radius will rotate by ωt. So now the radius OP would make an angle $(\omega t + \varphi)$ with OO_1 at the instant t and the displacement ON = x at instant t would be given by

$$ON = x = OP \sin(\omega t + \varphi)$$

$$= x_0 \sin(\omega t + \varphi) \quad \ldots\ldots\ldots \quad (7.10)$$

Now the phase angle is $\omega t + \varphi$ i.e.,

$$\theta = \omega t + \varphi$$

when $t = 0$, $\theta = \varphi$. So φ is the initial phase. If we take initial phase as $\pi/2$ or 90°, the displacement as given by Eq 7.10 is

$$x = x_0 \sin(\omega t + 90^\circ)$$

$$= x_0 \cos \omega t \quad \ldots\ldots\ldots \quad (7.11)$$

Thus Eq. 7.11 also gives the displacement of SHM, but in this case the point N is starting its motion from the extreme position instead of the mean position as shown in Fig. 7.6 (b).

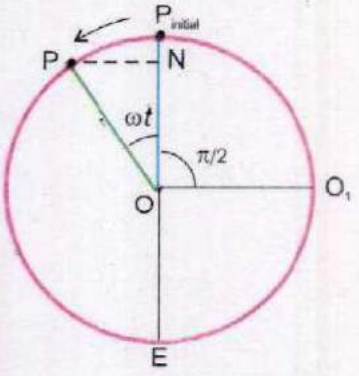


Fig. 7.6(b)

7.4 A HORIZONTAL MASS SPRING SYSTEM

Practically, for a simple harmonic system, consider again the vibrating mass attached to a spring as shown in Fig. 7.2 (a, b and c) whose acceleration at any instant is given by Eq. 7.2 which is

$$a = -\frac{k}{m}x$$

As k and m are constant, we see that the acceleration is proportional to displacement x, and its direction is towards the mean position. Thus the mass m executes SHM between A and A' with x_o as amplitude. Comparing the above equation with Eq. 7.9, the vibrational angular frequency is

$$\omega = \sqrt{\frac{k}{m}} \quad \ldots\ldots\ldots \quad (7.12)$$

The time period of the mass is

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}} \quad \ldots\ldots\ldots \quad (7.13)$$

The instantaneous displacement x of the mass as given by Eq. 7.5 is

$$x = x_o \sin \omega t$$

$$x = x_o \sin\sqrt{\frac{k}{m}}\, t \quad \ldots\ldots\ldots \quad (7.14)$$

The instantaneous velocity v of the mass m as given by Eq. 7.8 is

$$v = \omega\sqrt{x_o^2 - x^2} = \sqrt{\frac{k}{m}(x_o^2 - x^2)}$$

$$= x_o\sqrt{\frac{k}{m}\left(1 - \frac{x^2}{x_o^2}\right)} \quad \ldots\ldots\ldots \quad (7.15)$$

Eq 7.15 shows that the velocity of the mass gets maximum equal to v_o, when $x = 0$. Thus

$$v_o = x_o\sqrt{\frac{k}{m}} \quad \ldots\ldots\ldots \quad (7.16)$$

then $$v = v_o \sqrt{1 - \frac{x^2}{x_o^2}} \quad \ldots\ldots\ldots \quad (7.17)$$

The formula derived for displacement and velocity are also valid for vertically suspended mass-spring system provided air friction is not considered.

Example 7.1: A block weighing 4.0 kg extends a spring by 0.16 m from its unstretched position. The block is removed and a 0.50 kg body is hung from the same spring. If the spring is now stretched and then released, what is its period of vibration?

Solution:

Applied stretching force $F = kx$ or $k = \frac{F}{x}$

$$F = mg = 4 \text{ kg} \times 9.8 \text{ ms}^{-2} = 39.2 \text{ kgms}^{-2} = 39.2 \text{ N}$$

$$x = 0.16\text{m}, \qquad k = \frac{4 \text{ kg} \times 9.8 \text{ ms}^{-2}}{0.16 \text{ m}} = 245 \text{ kg s}^{-2}$$

Now time period $$T = 2\pi \sqrt{\frac{m}{k}}$$

or $$T = 2\pi \sqrt{\frac{0.5 \text{ kg}}{245 \text{ kgs}^{-2}}} = 0.28 \text{ s}$$

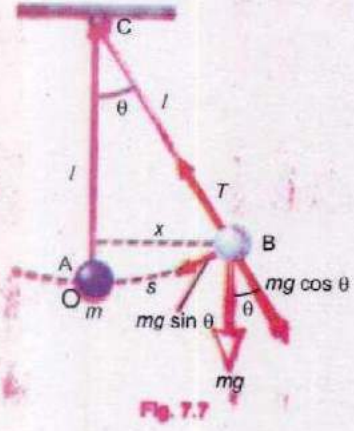


Fig. 7.7

7.5 SIMPLE PENDULUM

A simple pendulum consists of a small heavy mass m suspended by a light string of length l fixed at its upper end, as shown in Fig. 7.7. When such a pendulum is displaced from its mean position through a small angle θ to the position B and released, it starts oscillating to and fro over the same path. The weight mg of the mass can be resolved into two components; $mg \sin\theta$ along the tangent at B and $mg \cos\theta$ along CB to balance the tension of the string. The restoring force at B will be

$$F = - mg \sin\theta$$

When θ is small, $\sin\theta \approx \theta$

So $F = m\,a = -\,m\,g\theta$ ..... (7.18)

Or $a = -g\theta$

But $\theta = \dfrac{\text{Arc AB}}{l}$

When θ is small Arc AB = OB = x, hence $\theta = \dfrac{x}{l}$

Thus, $a = -\dfrac{gx}{l}$ ..... (7.19)

At a particular place 'g' is constant and for a given pendulum 'l' is also a constant.

Therefore, $\dfrac{g}{l} = k$ (a constant)

and the motion of the simple pendulum is simple harmonic. Comparing Eq. 7.19 with Eq. 7.9

$$\omega = \sqrt{\frac{g}{l}}$$

As time period $T = \dfrac{2\pi}{\omega}$

Hence $T = 2\pi\sqrt{\dfrac{l}{g}}$ ..... (7.20)

This shows that the time period depends only on the length of the pendulum and the acceleration due to gravity. It is independent of mass.

Example 7.2: What should be the length of a simple pendulum whose period is 1.0 second at a place where g = 9.8 ms^{-2}? What is the frequency of such a pendulum?

Solution:

Time period, $T = 2\pi\sqrt{\dfrac{l}{g}}$

$T = 1.0$ s , $g = 9.8\ ms^{-2}$

Squaring both sides

$$T^2 = 4\pi^2 \frac{l}{g}$$

$$l = \frac{g T^2}{4\pi^2}$$

or $$l = \frac{9.8\ \text{ms}^{-2} \times 1\text{s}^2}{4 \times 3.14 \times 3.14} = 0.25\ \text{m}$$

Frequency $$f = \frac{1}{T} = \frac{1}{1\text{s}} = 1\ \text{Hz}$$

7.6 ENERGY CONSERVATION IN SHM

Let us consider the case of a vibrating mass-spring system. When the mass m is pulled slowly, the spring is stretched by an amount x_o against the elastic restoring force F. It is assumed that stretching is done slowly so that acceleration is zero. According to Hooke's law

$$F = kx_o$$

When displacement = 0 force = 0

When displacement = x_o force = kx_o

Average force $$F = \frac{0 + kx_o}{2} = \frac{1}{2} kx_o$$

Work done in displacing the mass m through x_o is

$$W = Fd = \frac{1}{2} k\, x_o \times x_o = \frac{1}{2} k\, x_o^2$$

This work appears as elastic potential energy of the spring. Hence

$$\text{P.E.} = \frac{1}{2} k\, x_o^2 \qquad \ldots\ldots\ldots \qquad (7.21)$$

The Eq. 7.21 gives the maximum P.E. at the extreme position. Thus

$$\text{P.E.}_{\text{max}} = \frac{1}{2} k\, x_o^2$$

At any instant t, if the displacement is x, then P.E. at that instant is given by

$$\text{P.E.} = \frac{1}{2} kx^2 \quad \ldots\ldots\ldots \quad (7.22)$$

The velocity at that instant is given by Eq. 7.15 which is

$$v = x_o \sqrt{\frac{k}{m}\left(1 - \frac{x^2}{x_o^2}\right)}$$

Hence the K.E. at that instant is

$$\text{K.E. of the mass} = \frac{1}{2}mv^2 = \frac{1}{2} mx_o^2 \left(\frac{k}{m}\right)\left(1 - \frac{x^2}{x_o^2}\right)$$

$$\text{K.E.} = \frac{1}{2} kx_o^2 \left(1 - \frac{x^2}{x_o^2}\right) \quad \ldots\ldots\ldots \quad (7.23)$$

Thus, kinetic energy is maximum when $x = 0$, i.e. when the mass is at equilibrium or mean position (Fig. 7.8)

$$\text{K.E.}_{\text{max}} = \frac{1}{2} kx_o^2 \quad \ldots\ldots\ldots \quad (7.24)$$

For any displacement x, the energy is partly P.E. and partly K.E. Hence,

$$E_{\text{total}} = \text{P.E.} + \text{K.E.}$$

$$= \frac{1}{2} kx^2 + \frac{1}{2} kx_o^2 \left(1 - \frac{x^2}{x_o^2}\right)$$

$$\text{Total energy} = \frac{1}{2} kx_o^2 \quad \ldots\ldots\ldots \quad (7.25)$$

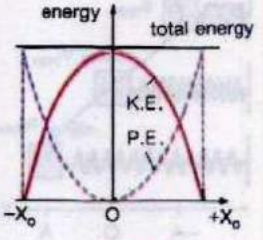


Fig. 7.8

Thus the total energy of the vibrating mass and spring is constant. When the K.E. of the mass is maximum, the P.E. of the spring is zero. Conversely, when the P.E. of the spring is maximum, the K.E. of the mass is zero. The interchange occurs continuously from one form to the other as the spring is compressed and released alternately. The variation of P.E. and K.E. with displacement is essential for maintaining oscillations. This periodic exchange of energy is a basic property of all oscillatory systems. In the case of simple pendulum gravitational P.E. of the mass, when displaced, is converted into K.E. at the

equilibrium position. The K.E. is converted into P.E. as the mass rises to the top of the swing. Because of the frictional forces, energy is dissipated and consequently, the systems do not oscillate indefinitely.

Example 7.3: A spring, whose spring constant is 80.0 Nm^{-1} vertically supports a mass of 1.0 kg in the rest position. Find the distance by which the mass must be pulled down, so that on being released, it may pass the mean position with a velocity of 1.0 ms^{-1}.

Solution:

$$k = 80.0\ Nm^{-1} \qquad m = 1.0\ kg$$

Since $\omega^2 = \frac{k}{m}$ or $\omega = \sqrt{\frac{k}{m}}$

$$= \sqrt{\frac{80\ Nm^{-1}}{1kg}} = \sqrt{\frac{80\ kgms^{-2} \times m^{-1}}{1kg}} = 8.94\ s^{-1}$$

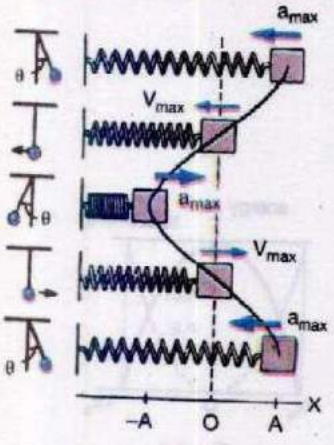
Let the amplitude of vibration be x_o

Then $v = x_o\omega$ or $x_o = \frac{v}{\omega}$

as $v = 1.0\ ms^{-1}$ and $\omega = 8.94\ s^{-1}$

Distance through which m is pulled $= x_o = \frac{1ms^{-1}}{8.94s^{-1}} = 0.11$ m

Comparison of SHMs



7.7 FREE AND FORCED OSCILLATIONS

A body is said to be executing free vibrations when it oscillates without the interference of an external force. The frequency of these free vibrations is known as its natural frequency. For example, a simple pendulum when slightly displaced from its mean position vibrates freely with its natural frequency that depends only upon the length of the pendulum.

On the other hand, if a freely oscillating system is subjected to an external periodic force, then forced vibrations will take place. Such as when the mass of a vibrating pendulum is struck repeatedly, then forced vibrations are produced.

A physical system under going forced vibrations is known as driven harmonic oscillator.

The vibrations of a vehicle body caused by the running of engine is an example of forced vibrations. Another example of forced vibration is loud music produced by sounding wooden boards of string instruments.

7.8 RESONANCE

Associated with the motion of a driven harmonic oscillator, there is a very striking phenomenon, known as resonance. It arises if the external driving force is periodic with a period comparable to the natural period of the oscillator.

In a resonance situation, the driving force may be feeble, the amplitude of the motion may become extra ordinarily large. In the case of oscillating simple pendulum, if we blow to push the pendulum whenever it comes in front of our mouth, it is found that the amplitude steadily increases.

To demonstrate this resonance effect, an apparatus is shown in Fig. 7.9. A horizontal rod AB is supported by two strings S_1 and S_2. Three pairs of pendulums aa', bb' and cc' are suspended to this rod. The length of each pair is the same but is different for different pairs. If one of these pendulums, say c, is displaced in a direction perpendicular to the plane of the paper, then its resultant oscillatory motion causes in rod AB a very slight disturbing motion, whose period is the same as that of c'. Due to this slight motion of the rod, each of the remaining pendulums (aa', bb', and cc') under go a slight periodic motion. This causes the pendulum c', whose length and, hence, period is exactly the same as that of c, to oscillate back and forth with steadily increasing amplitude. However, the amplitudes of the other pendulums remain small through out the subsequent motions of c and c', because their natural periods are not the same as that of the disturbing force due to rod AB.

The energy of the oscillation comes from the driving source. At resonance ,the transfer of energy is maximum.

Thus resonance occurs when the frequency of the applied periodic forced is equal to one of the natural frequencies of vibration of the forced or driven harmonic oscillator.

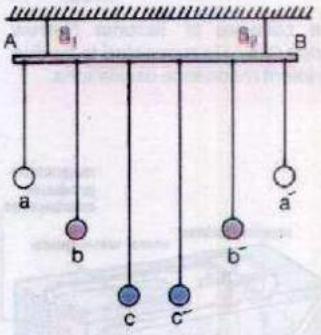


Fig. 7.9

Do You Know?

All structures are likely to resonate at one or more frequencies. This can cause problem. It is especially important to test all the components in helicopters and aeroplanes; Resonance in an aeroplane's wing or a helicopter rotor could be very dangerous.

Advantages And Disadvantages of Resonance

We come across many examples of resonance in every day life. A swing is a good example of mechanical resonance. It is like a pendulum with a single natural frequency depending on its length. If a series of regular pushes are given to the swing, its motion can be built up enormously. If pushes are given irregularly, the swing will hardly vibrate. The column of soldiers, while marching on a bridge of long span are advised to break their steps. Their rhythmic march might set up oscillations of dangerously large amplitude in the bridge structure.

Interesting Information

The collapse of Tacoma Narrow bridge (USA) is suspected to be due to violent resonance oscillations.

Tuning a radio is the best example of electrical resonance. When we turn the knob of a radio, to tune a station, we are changing the natural frequency of the electric circuit of the receiver, to make it equal to the transmission frequency of the radio station. When the two frequencies match, energy absorption is maximum and this is the only station we hear.

Another good example of resonance is the heating and cooking of food very efficiently and evenly by microwave oven (Fig. 7.10). The waves produced in this type of oven have a wavelength of 12 cm at a frequency of 2450 MHz. At this frequency the waves are absorbed due to resonance by water and fat molecules in the food, heating them up and so cooking the food.

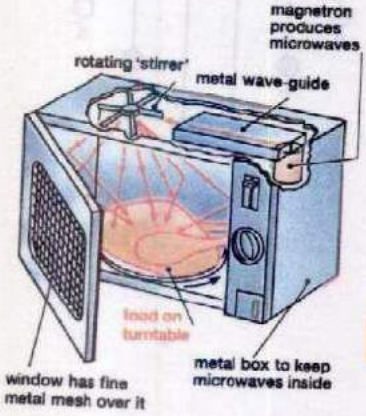


Fig. 7.10

7.9 DAMPED OSCILLATIONS

This is a common observation that the amplitude of an oscillating simple pendulum decreases gradually with time till it becomes zero. Such oscillations, in which the amplitude decreases steadily with time, are called damped oscillations.

We know from our everyday experience that the motion of any macroscopic system is accompanied by frictional effects. While describing the motion of a simple pendulum, this effect was completely ignored. As the bob of the pendulum moves to and fro, then in addition to the weight of the bob and the tension in the string, bob experiences viscous drag due to its motion through the air. Thus simple harmonic motion is an idealization (Fig. 7.11 a). In practice, the amplitude of this motion gradually becomes smaller

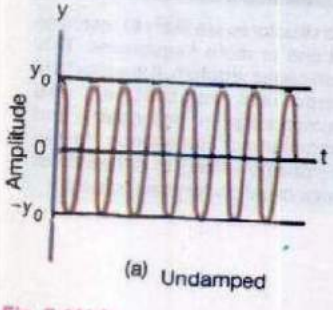


Fig. 7.11(a)
Graph between amplitude and time

and smaller because of friction and air resistance because the energy of the oscillator is used up in doing work against the resistive forces. Fig. 7.11(b) shows how the amplitude of a damped simple harmonic wave changes with time as compared with an ideal un-damped harmonic wave. Thus we see that

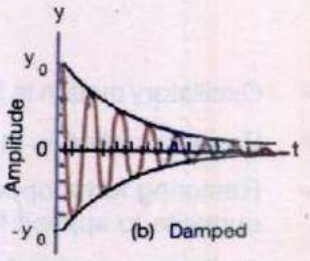


Fig. 7.11(b)
Graph between amplitude and time

Damping is the process whereby energy is dissipated from the oscillating system.

An application of damped oscillations is the shock absorber of a car which provides a damping force to prevent excessive oscillations (Fig. 7.12).

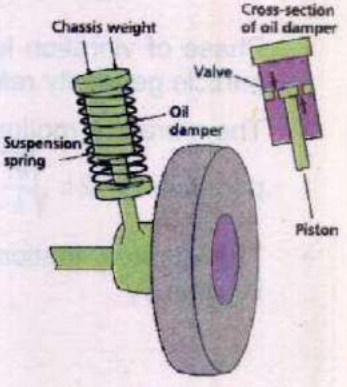


Fig. 7.12

7.10 SHARPNESS OF RESONANCE

We have seen that at resonance, the amplitude of the oscillator becomes very large. If the amplitude decreases rapidly at a frequency slightly different from the resonant frequency, the resonance will be sharp. The amplitude as well as its sharpness, both depend upon the damping. Smaller the damping, greater will be the amplitude and more sharp will be the resonance.

A heavily damped system has a fairly flat resonance curve as is shown in an amplitude frequency graph in Fig. 7.13.

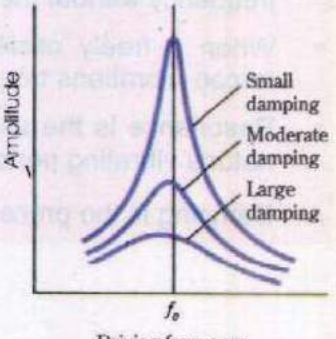


Fig. 7.13

The effect of damping can be observed by attaching a pendulum having light mass such as a pith ball as its bob and another of the same length carrying a heavy mass such as a lead bob of equal size, to a rod as shown in Fig. 7.9. They are set into vibrations by a third pendulum of equal length, attached to the same rod. It is observed that amplitude of the lead bob is much greater than that of the pith-ball. The damping effect for the pith-ball due to air resistance is much greater than for the lead bob.

SUMMARY

- Oscillatory motion is to and fro motion about a mean position.
- Periodic motion is the one that repeats itself after equal intervals of time.
- Restoring force opposes the change in shape or length of a body and is equal and opposite to applied force.
- A vibratory motion in which acceleration is directly proportional to displacement from mean position and is always directed towards the mean position is known as simple harmonic motion.
- The projection of a particle moving in a circle executes SHM. Its time period T is $\frac{2\pi}{\omega}$.
- Phase of vibration is the quantity which indicates the state of motion of a vibrating particle generally referred by the phase angle.
- The vibratory motion of a mass attached to an elastic spring is SHM and its time period is $T = 2\pi\sqrt{\frac{m}{k}}$
- The vibratory motion of the bob of simple pendulum is also SHM and its time period is given by

$$T = 2\pi\sqrt{\frac{l}{g}}$$

- In an oscillating system P.E. and K.E. interchange and total energy is conserved.
- A body is said to be executing free oscillation if it vibrates with its own natural frequency without the interference of an external force.
- When a freely oscillating system is subjected to an external periodic force, then forced vibrations take place.
- Resonance is the specific response of a system to a periodic force acting with the natural vibrating period of the system.
- Damping is the process whereby energy is dissipated from the oscillating system.

QUESTIONS

7.1 Name two characteristics of simple harmonic motion.

7.2 Does frequency depends on amplitude for harmonic oscillators?

7.3 Can we realize an ideal simple pendulum?

7.4 What is the total distance travelled by an object moving with SHM in a time equal to its period, if its amplitude is A?

7.5 What happens to the period of a simple pendulum if its length is doubled? What happens if the suspended mass is doubled?

7.6 Does the acceleration of a simple harmonic oscillator remain constant during its motion? Is the acceleration ever zero? Explain.

7.7 What is meant by phase angle? Does it define angle between maximum displacement and the driving force?

7.8 Under what conditions does the addition of two simple harmonic motions produce a resultant, which is also simple harmonic?

7.9 Show that in SHM the acceleration is zero when the velocity is greatest and the velocity is zero when the acceleration is greatest?

7.10 In relation to SHM, explain the equations;

(i) $y = A \sin(\omega t + \varphi)$

(ii) $\mathbf{a} = -\omega^2 \mathbf{x}$

7.11 Explain the relation between total energy, potential energy and kinetic energy for a body oscillating with SHM.

7.12 Describe some common phenomena in which resonance plays an important role.

7.13 If a mass spring system is hung vertically and set into oscillations, why does the motion eventually stop?

NUMERICAL PROBLEMS

7.1 A 100.0 g body hung on a spring elongates the spring by 4.0 cm. When a certain object is hung on the spring and set vibrating, its period is 0.568 s. What is the mass of the object pulling the spring?

(Ans:0.20 kg)

7.2 A load of 15.0g elongates a spring by 2.00 cm. If body of mass 294 g is attached to the spring and is set into vibration with an amplitude of 10.0 cm, what will be its (i) period (ii) spring constant (iii) maximum speed of its vibration.

[Ans: (i) 1.26s, (ii) 7.35 Nm^{-1}, (iii) 49.0 cm s^{-1}]

7.3 An 8.0 kg body executes SHM with amplitude 30 cm. The restoring force is 60 N when the displacement is 30 cm. Find

(i) Period

(ii) Acceleration, speed, kinetic energy and potential energy when the displacement is 12 cm.

[Ans: (i) 1.3 s, (ii) 3.0 ms^{-2}, 1.4 ms^{-1}, 7.6 J, 1.44 J]

7.4 A block of mass 4.0 kg is dropped from a height of 0.80 m on to a spring of spring constant k = 1960 Nm^{-1}, Find the maximum distance through which the spring will be compressed.

(Ans: 0.18 m)

7.5 A simple pendulum is 50.0 cm long. What will be its frequency of vibration at a place where g = 9.8 ms^{-2}?

(Ans: 0.70 Hz)

7.6 A block of mass 1.6 kg is attached to a spring with spring constant 1000 Nm^{-1}, as shown in Fig. 7.14. The spring is compressed through a distance of 2.0 cm and the block is released from rest. Calculate the velocity of the block as it passes through the equilibrium position, x = 0, if the surface is frictionless.

(Ans: 0.50 ms^{-1})

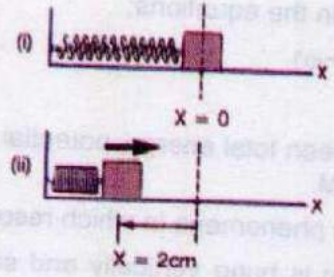


Fig. 7.14

7.7 A car of mass 1300 kg is constructed using a frame supported by four springs. Each spring has a spring constant 20,000 Nm^{-1}. If two people riding in the car have a combined mass of 160 kg, find the frequency of vibration of the car, when it is driven over a pot hole in the road. Assume the weight is evenly distributed.

(Ans: 1.18 Hz)

7.8 Find the amplitude, frequency and period of an object vibrating at the end of a spring, if the equation for its position, as a function of time, is

$$x = 0.25 \cos\left(\frac{\pi}{8}\right) t$$

What is the displacement of the object after 2.0 s?

(Ans: 0.25 m, $\frac{1}{16}$ Hz, 16 s, x = 0.18 m)