

Chapter 11

HEAT AND THERMODYNAMICS

Learning Objectives

At the end of this chapter the students will be able to:

1. State the basic postulates of Kinetic theory of gases.
2. Explain how molecular movement causes the pressure exerted by a gas and derive the equation $P = \frac{2}{3} N_0 \langle \frac{1}{2} m v^2 \rangle$, where N_0 is the number of molecules per unit volume of the gas.
3. Deduce that the average translational kinetic energy of molecules is proportional to temperature of the gas.
4. Derive gas laws on the basis of Kinetic theory.
5. Describe that the internal energy of an ideal gas is due to kinetic energy of its molecules.
6. Understand and use the terms work and heat in thermodynamics.
7. Differentiate between isothermal and adiabatic processes.
8. Explain the molar specific heats of a gas.
9. Apply first law of thermodynamics to derive $C_p - C_v = R$.
10. Explain the second law of thermodynamics and its meaning in terms of entropy.
11. Understand the concept of reversible and irreversible processes.
12. Define the term heat engine.
13. Understand and describe Carnot theorem.
14. Describe the thermodynamic scale of temperature.
15. Describe the working of petrol and diesel engines.
16. Explain the term entropy.
17. Explain that change in entropy $\Delta S = \pm \frac{\Delta Q}{T}$
18. Appreciate environmental crisis as an entropy crisis.

Thermodynamics deals with various phenomena of energy and related properties of matter, especially the transformation of heat into other forms of energy. An example of such transformation is the process converting heat into mechanical work. Thermodynamics thus plays central role in technology, since almost all the raw energy available for our use is liberated in the form of heat. In this chapter we shall study the behaviour of gases and laws of thermodynamics, their significance and applications.

11.1 KINETIC THEORY OF GASES

The behavior of gases is well accounted for by the kinetic theory based on microscopic approach. Evidence in favour of the theory is exhibited in diffusion of gases and Brownian motion of smoke particles etc.

The following postulates help to formulate a mathematical model of gases.

- i. A finite volume of gas consists of very large number of molecules.
- ii. The size of the molecules is much smaller than the separation between molecules.
- iii. The gas molecules are in random motion and may change their direction of motion after every collision.
- iv. Collision between gas molecules themselves and with walls of container are assumed to be perfectly elastic.
- v. Molecules do not exert force on each other except during a collision.

Pressure of Gas

According to kinetic theory, the pressure exerted by a gas is merely the momentum transferred to the walls of the container per second per unit area due to the continuous collisions of molecules of the gas. An expression for the pressure exerted by a gas can, therefore, be obtained as follows:



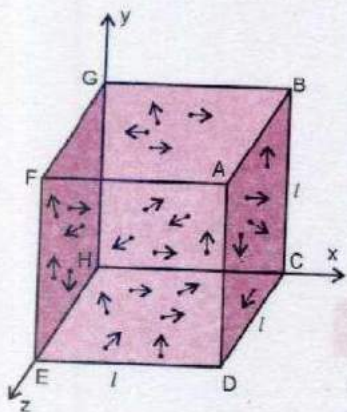


Fig. 11.1

Let a cubical vessel of side l , contains N molecules, each of mass m (Fig. 11.1). The velocity \mathbf{v} , of any one of these molecules can be resolved into three rectangular components v_{1x}, v_{1y}, v_{1z} parallel to three co-ordinate axes x, y and z .

Initial momentum of the molecule striking the face ABCDA is then mv_{1x} . If the collision is assumed perfectly elastic, the molecule will rebound from the face ABCDA with the same speed. Thus each collision produces a change in momentum, which is equal to

Final momentum - Initial momentum

or change in momentum = $-mv_{1x} - mv_{1x}$

$$\text{Change in momentum} = -2mv_{1x} \dots\dots\dots (11.1)$$

After recoil the molecule travels to opposite face EFGHE and collides with it, rebounds and travels back to the face ABCDA after covering a distance $2l$. The time Δt between two successive collisions with face ABCDA is

$$\Delta t = \frac{2l}{v_{1x}} \dots\dots\dots (11.2)$$

So the number of collisions per second that the molecule will make with this face is $\frac{v_{1x}}{2l}$

Thus the rate of change of momentum of the molecule due

$$\text{to collisions with face ABCDA} = -2mv_{1x} \times \frac{v_{1x}}{2l} = \frac{-mv_{1x}^2}{l}$$

The rate of change of momentum of the molecule is equal to the force applied by the wall. According to Newton's third law of motion, force F_{1x} exerted by the molecule on face ABCDA is equal but opposite, so

$$F_{1x} = \frac{-(-mv_{1x}^2)}{l} = \frac{mv_{1x}^2}{l}$$

Similarly the forces due to all other molecules can be determined. Thus the total x -directed force F_x due to N

number of molecules of the gas moving with velocities $v_1, v_2, v_3, \dots, v_N$ is

$$F_x = F_{1x} + F_{2x} + F_{3x} + \dots + F_{Nx}$$

or
$$F_x = \frac{mv_{1x}^2}{l} + \frac{mv_{2x}^2}{l} + \frac{mv_{3x}^2}{l} + \dots + \frac{mv_{Nx}^2}{l}$$

As pressure is normal force per unit area, hence pressure P_x on the face perpendicular to x-axis is

$$\begin{aligned} P_x &= \frac{F_x}{A} = \frac{F_x}{l^2} \\ &= \frac{1}{l^2} \left(\frac{mv_{1x}^2}{l} + \frac{mv_{2x}^2}{l} + \frac{mv_{3x}^2}{l} + \dots + \frac{mv_{Nx}^2}{l} \right) \\ &= \frac{m}{l^3} (v_{1x}^2 + v_{2x}^2 + v_{3x}^2 + \dots + v_{Nx}^2) \dots \dots \dots (11.3) \end{aligned}$$

As the mass of single molecule is m , the mass of N molecules will be mN .

Since density $\rho = \frac{\text{Mass}}{\text{Volume}} = \frac{mN}{l^3}$

Hence,
$$\frac{m}{l^3} = \frac{\rho}{N}$$

Substituting the value of $\frac{m}{l^3}$ in equation (11.3)

we get

$$P_x = \frac{\rho}{N} (v_{1x}^2 + v_{2x}^2 + v_{3x}^2 + \dots + v_{Nx}^2)$$

or
$$P_x = \rho \left(\frac{v_{1x}^2 + v_{2x}^2 + v_{3x}^2 + \dots + v_{Nx}^2}{N} \right)$$

where $\left(\frac{v_{1x}^2 + v_{2x}^2 + v_{3x}^2 + \dots + v_{Nx}^2}{N} \right)$ is called the mean of squared velocities of the molecules moving along x direction, known as mean square velocity, represented by

$\langle v_x^2 \rangle$. Substituting $\langle v_x^2 \rangle$ in parenthesis of pressure expression

$$P_x = \rho \langle v_x^2 \rangle \quad \dots \quad (11.4)$$

Similarly pressure on the faces perpendicular to y and z axes will be $P_y = \rho \langle v_y^2 \rangle$ and $P_z = \rho \langle v_z^2 \rangle$

As there is no preference to one direction or another and molecules are supposed to be moving randomly, the mean square of all the component velocities will be equal. Hence

$$\langle v_x^2 \rangle = \langle v_y^2 \rangle = \langle v_z^2 \rangle$$

and from vector addition $\langle v^2 \rangle = \langle v_x^2 \rangle + \langle v_y^2 \rangle + \langle v_z^2 \rangle$

thus, $\langle v^2 \rangle = 3 \langle v_x^2 \rangle$

or
$$\langle v_x^2 \rangle = \frac{1}{3} \langle v^2 \rangle$$

putting this value of $\langle v_x^2 \rangle$ in equation 11.4

$$P_x = \frac{\rho}{3} \langle v^2 \rangle$$

We have considered the pressure on the face perpendicular to x-axis.

By Pascal's Law the pressure on the other sides and everywhere inside the vessel will be the same provided the gas is of uniform density. So

$$P_x = P_y = P_z = \frac{\rho}{3} \langle v^2 \rangle$$

Thus in general

$$P = \frac{1}{3} \rho \langle v^2 \rangle$$

Since density $\rho = \frac{mN}{V}$

Hence
$$P = \frac{mN}{3V} \langle v^2 \rangle$$

or
$$P = \frac{2}{3} \frac{N}{V} \left\langle \frac{1}{2} mv^2 \right\rangle \dots\dots\dots (11.5)$$

$$P = \frac{2}{3} N_0 \left\langle \frac{1}{2} mv^2 \right\rangle$$

where N_0 is the number of molecules per unit volume.

Thus,
$$P = \text{Constant} \langle \text{K.E.} \rangle$$

or
$$P \propto \langle \text{K.E.} \rangle$$

While deriving the equation for pressure we have not accounted rotational and vibrational motion of molecules except the linear motion.

Hence pressure exerted by the gas is directly proportional to the average translational kinetic energy of the gas molecules.

Interpretation of Temperature

From experimental data the ideal gas law is deduced to be

$$PV = nRT \dots\dots\dots (11.6)$$

Where n is the number of moles of the gas contained in volume V at absolute temperature T and R is called universal gas constant. Its value is $8.314 \text{ J mol}^{-1} \text{ K}^{-1}$.

If N_A is the Avogadro number, then the above equation can be written as

$$PV = \frac{N}{N_A} RT$$

or
$$PV = NkT \dots\dots\dots (11.7)$$

where $k = R/N_A$ is the Boltzman constant. It is the gas constant per molecule and has the value $= 1.38 \times 10^{-23} \text{ J K}^{-1}$.
Comparing equations 11.5 and 11.7

$$NkT = \frac{2}{3} N \left\langle \frac{1}{2} mv^2 \right\rangle$$

or $T = \frac{2}{3k} \left\langle \frac{1}{2} mv^2 \right\rangle \dots\dots\dots (11.8)$

or $T = \text{constant} \langle \text{K.E.} \rangle$

so $T \propto \langle \text{K.E.} \rangle$

This relation shows that Absolute temperature of an ideal gas is directly proportional to the average translational kinetic energy of gas molecules.

We can, therefore, also say that average translational kinetic energy of the gas molecules shows itself macroscopically in the form of temperature.

Derivation of Gas Laws

(i) Boyle's Law

From kinetic theory of gases (Eq. 11.5)

$$PV = \frac{2}{3} N \left\langle \frac{1}{2} mv^2 \right\rangle$$

If we keep the temperature constant, average K.E. i.e., $\left\langle \frac{1}{2} mv^2 \right\rangle$ remains constant, so the right hand side of the equation is constant.

Hence $PV = \text{Constant}$

or $P \propto \frac{1}{V}$

Thus pressure P is inversely proportional to volume V at constant temperature of the gas which is Boyle's law.

(ii) Charles' Law

Equation 11.5 can be written as

$$V = \frac{2}{3} \frac{N}{P} \left\langle \frac{1}{2} mv^2 \right\rangle$$

If pressure is kept constant

$$V \propto \left\langle \frac{1}{2} mv^2 \right\rangle$$

As $\left\langle \frac{1}{2} mv^2 \right\rangle \propto T$

Hence $V \propto T$

Thus volume is directly proportional to absolute temperature of the gas provided pressure is kept constant. This is known as Charles' law.

Example 11.1: What is the average translational Kinetic energy of molecules in a gas at temperature 27°C ?

Solution:

Using Eq. 11.8 $T = \frac{2}{3k} \langle \text{K.E.} \rangle$

or $\langle \text{K.E.} \rangle = \frac{3kT}{2}$

where $T = 27 + 273 = 300 \text{ K}$

$$k = 1.38 \times 10^{-23} \text{ JK}^{-1}$$

so $\langle \text{K.E.} \rangle = \frac{3}{2} \times 1.38 \times 10^{-23} \text{ JK}^{-1} \times 300 \text{ K}$

$$= 6.21 \times 10^{-21} \text{ J}$$

Example 11.2: Find the average speed of oxygen molecule in the air at S.T.P.

Solution: Under standard conditions

Temperature $T = 0^\circ\text{C} = 273 \text{ K}$

From Eq. 11.8

$$T = \frac{2}{3k} \left\langle \frac{1}{2} mv^2 \right\rangle$$

or $\langle v^2 \rangle = \frac{3kT}{m}$

Using Avogadro's number $N_A = 6.022 \times 10^{23}$, the mass m of one molecule of oxygen is

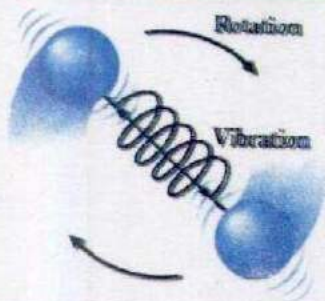
$$m = \frac{\text{molecular mass}}{N_A} = \frac{32 \text{ g}}{6.022 \times 10^{23}} = \frac{32 \text{ kg}}{6.022 \times 10^{26}}$$

Substituting the values of k , T and m , we get

$$\langle v^2 \rangle = \frac{3 \times 1.38 \times 10^{-23} \text{ JK}^{-1} \times 273 \text{ K} \times 6.022 \times 10^{26}}{32 \text{ kg}} = 212693 \text{ m}^2 \text{ s}^{-2}$$

or $\langle v \rangle = 461 \text{ ms}^{-1}$

Do You Know?



A diatomic gas molecule has both translational and rotational energy. It also has vibrational energy associated with the spring like bond between its atoms.

11.2 INTERNAL ENERGY

The sum of all forms of molecular energies (kinetic and potential) of a substance is termed as its internal energy. In the study of thermodynamics, usually ideal gas is considered as a working substance. The molecules of an ideal gas are mere mass points which exert no forces on one another. So the internal energy of an ideal gas system is generally the translational K.E. of its molecules. Since the temperature of a system is defined as the average K.E. of its molecules, thus for an ideal gas system, the internal energy is directly proportional to its temperature.

When we heat a substance, energy associated with its atoms or molecules is increased i.e., heat is converted to internal energy.

It is important to note that energy can be added to a system even though no heat transfer takes place. For example, when two objects are rubbed together, their internal energy increases because of mechanical work. The increase in temperature of the object is an indication of increase in the internal energy. Similarly, when an object slides over any surface and comes to rest because of frictional forces, the mechanical work done on or by the system is partially converted into internal energy.

In thermodynamics, internal energy is a function of state. Consequently, it does not depend on path but depends on initial and final states of the system. Consider a system which undergoes a pressure and volume change from P_a and V_a to P_b and V_b respectively, regardless of the process by which

the system changes from initial to final state. By experiment it has been seen that the change in internal energy is always the same and is independent of paths C_1 and C_2 as shown in the Fig. 11.2.

Thus internal energy is similar to the gravitational P.E. So like the potential energy, it is the change in internal energy and not its absolute value, which is important.

11.3 WORK AND HEAT

We know that both heat and work correspond to transfer of energy by some means. The idea was first applied to the steam engine where it was natural to pump heat in and get work out. Consequently it made a sense to define both heat in and work out as positive quantities. Hence work done by the system on its environment is considered +ive while work done on the system by the environment is taken as -ive. If an amount of heat Q enters the system it could manifest itself as either an increase in internal energy or as a resulting quantity of work performed by the system on the surrounding or both.

We can express the work in terms of directly measurable variables. Consider the gas enclosed in the cylinder with a moveable, frictionless piston of cross-sectional area A (Fig. 11.3 a). In equilibrium the system occupies volume V , and exerts a pressure P on the walls of the cylinder and its piston. The force F exerted by the gas on the piston is PA .

We assume that the gas expands through ΔV very slowly, so that it remains in equilibrium (Fig. 11.3 b). As the piston moves up through a small distance Δy , the work (W) done by the gas is

$$W = F\Delta y = PA\Delta y$$

Since $A\Delta y = \Delta V$ (Change in volume)

$$\text{Hence } W = P\Delta V \quad \dots\dots\dots (11.9)$$

The work done can also be calculated by area of the curve under P - V graph as shown in Fig.11.4.

Knowing the details of the change in internal energy and the mechanical work done, we are in a position to describe the general principles which deal with heat energy and its

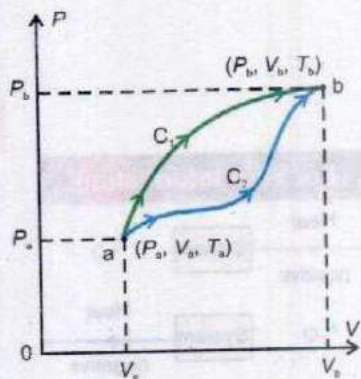


Fig.11.2

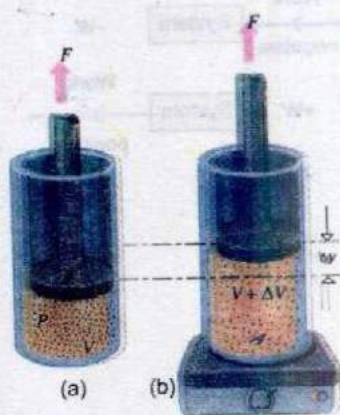


Fig. 11.3

A gas sealed in a cylinder by a weightless, frictionless piston. The constant downward applied force F equals PA , and when the piston is displaced, downward work is done on the gas.

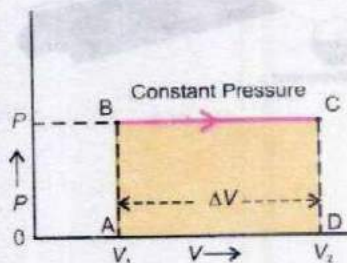


Fig.11.4

transformation into mechanical energy. These principles are known as laws of thermodynamics.

11.4 FIRST LAW OF THERMODYNAMICS

When heat is added to a system there is an increase in the internal energy due to the rise in temperature, an increase in pressure or change in the state. If at the same time, a substance is allowed to do work on its environment by expansion, the heat Q required will be the heat necessary to change the internal energy of the substance from U_1 in the first state to U_2 in the second state plus the work W done on the environment.

Thus

$$Q = (U_2 - U_1) + W$$

or

$$Q = \Delta U + W \quad \dots \dots \dots (11.10)$$

Thus the change in internal energy $\Delta U = U_2 - U_1$ is defined as $Q - W$. Since it is the same for all processes concerning the state, the first law of thermodynamics, thus can be stated as,

In any thermodynamic process, when heat Q is added to a system, this energy appears as an increase in the internal energy ΔU stored in the system plus the work W done by the system on its surroundings.

A bicycle pump provides a good example. When we pump on the handle rapidly, it becomes hot due to mechanical work done on the gas, raising thereby its internal energy. One such simple arrangement is shown in Fig. 11.5. It consists of a bicycle pump with a blocked outlet. A thermocouple connected through the blocked outlet allows the air temperature to be monitored. When piston is rapidly pushed, thermometer shows a temperature rise due to increase of internal energy of the air. The push force does work on the air, thereby, increasing its internal energy, which is shown, by the increase in temperature of the air.

Human metabolism also provides an example of energy conservation. Human beings and other animals do work

For Your Information

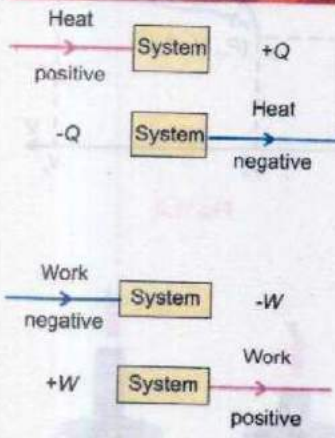


Fig. 11.5

when they walk, run, or move heavy objects. Work requires energy. Energy is also needed for growth to make new cells and to replace old cells that have died. Energy transforming processes that occur within an organism are named as metabolism. We can apply the first law of thermodynamics,

$$\Delta U = Q - W$$

to an organism of the human body. Work (W) done will result in the decrease in internal energy of the body. Consequently the body temperature or in other words internal energy is maintained by the food we eat.

Example 11.3: A gas is enclosed in a container fitted with a piston of cross-sectional area 0.10 m^2 . The pressure of the gas is maintained at 8000 Nm^{-2} . When heat is slowly transferred, the piston is pushed up through a distance of 4.0 cm . If 42 J heat is transferred to the system during the expansion, what is the change in internal energy of the system?

Solution:

The work done by the gas is

$$\begin{aligned} W &= P\Delta V = P A \Delta y = 8000 \text{ Nm}^{-2} \times 0.10 \text{ m}^2 \times 4.0 \times 10^{-2} \text{ m} \\ &= 32 \text{ Nm} = 32 \text{ J} \end{aligned}$$

The change in internal energy is found from first law of thermodynamics,

$$\Delta U = Q - W = 42 \text{ J} - 32 \text{ J} = 10 \text{ J}$$

Isothermal Process

It is a process which is carried out at constant temperature and hence the condition for the application of Boyle's Law on the gas is fulfilled. Therefore, when gas expands or compresses isothermally, the product of its pressure and volume during the process remains constant. If P_1, V_1 are initial pressure and volume where as P_2, V_2 are pressure and volume after the isothermal change takes place (Fig.11.6 a), then

$$P_1 V_1 = P_2 V_2$$

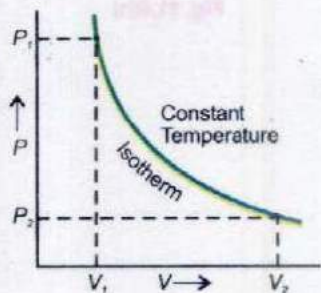
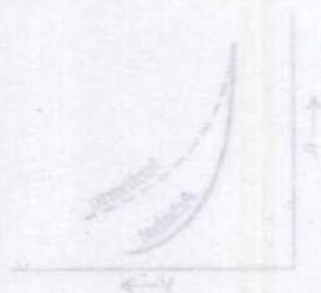


Fig.11.6(a)

In case of an ideal gas, the P.E. associated with its molecules is zero, hence, the internal energy of an ideal gas depends only on its temperature, which in this case remains constant, therefore, $\Delta U = 0$. Hence, the first law of thermodynamics reduces to

$$Q = W$$

Thus if gas expands and does external work W , an amount of heat Q has to be supplied to the gas in order to produce an isothermal change. Since transfer of heat from one place to another requires time, hence, to keep the temperature of the gas constant, the expansion or compression must take place slowly. The curve representing an isothermal process is called an isotherm (Fig. 11.6a).

Adiabatic Process

An adiabatic process is the one in which no heat enters or leaves the system. Therefore, $Q = 0$ and the first law of thermodynamics gives

$$W = -\Delta U$$

Thus if the gas expands and does external work, it is done at the expense of the internal energy of its molecules and, hence, the temperature of the gas falls. Conversely an adiabatic compression causes the temperature of the gas to rise because of the work done on the gas.

Adiabatic change occurs when the gas expands or is compressed rapidly, particularly when the gas is contained in an insulated cylinder. The examples of adiabatic processes are

- (i) The rapid escape of air from a burst tyre.
- (ii) The rapid expansion and compression of air through which a sound wave is passing.
- (iii) Cloud formation in the atmosphere.

In case of adiabatic changes it has been seen that

$$PV^\gamma = \text{Constant}$$

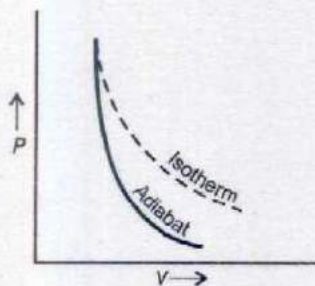


Fig. 11.6(b)

where, γ is the ratio of the molar specific heat of the gas at constant pressure to molar specific heat at constant volume. The curve representing an adiabatic process is called an adiabat (Fig. 11.6 b).

11.5 MOLAR SPECIFIC HEATS OF A GAS

One kilogram of different substances contain different number of molecules. Sometimes it is preferred to consider a quantity called a mole, since one mole of any substance contains the same number of molecules. The molar specific heat of the substance is defined as the heat required to raise the temperature of one mole of the substance through 1 K. In case of solids and liquids the change of volume and hence work done against external pressure during a change of temperature is negligibly small. But same can not be said about gases which suffer variation in pressure as well as in volume with the rise in temperature. Hence, to study the effect of heating the gases, either pressure or volume is kept constant. Thus, it is customary to define the molar specific heats of a gas in two ways:

- (i) The molar specific heat at constant volume is the amount of heat transfer required to raise the temperature of one mole of the gas through 1 K at constant volume and is symbolized by C_v .

If 1 mole of an ideal gas is heated at constant volume so that its temperature rises by ΔT , the heat transferred Q_v must be equal to $C_v \Delta T$. Because $\Delta V = 0$, no work is done (Fig 11.7. a). Applying first law of thermodynamics,

$$Q_v = \Delta U + W$$

Hence, $C_v \Delta T = \Delta U + 0$

or $\Delta U = C_v \Delta T$ (11.11)

- (ii) The molar specific heat at constant pressure is the amount of heat transfer required to raise the temperature of one mole of the gas through 1 K at constant pressure and it is represented by symbol C_p . To raise the temperature of 1 mole of the gas by ΔT at constant pressure, the heat transfer Q_p must be equal to $C_p \Delta T$ (Fig 11.7 b). Thus,

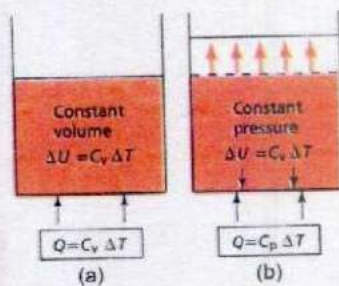


Fig. 11.7

$$Q_p = C_p \Delta T \quad \dots\dots\dots (11.12)$$

Derivation of $C_p - C_v = R$

When one mole of a gas is heated at constant pressure, the internal energy increases by the same amount as at constant volume for the same rise in temperature ΔT . Thus from Eq. 11.11

$$\Delta U = C_v \Delta T$$

Since the gas expands to keep the pressure constant, so it does work $W = P \Delta V$, where ΔV is the increase in volume.

Substituting the values of heat transfer Q_p , internal energy ΔU and the work done W in Eq. 11.10, we get

$$C_p \Delta T = C_v \Delta T + P \Delta V \quad \dots\dots\dots (11.13)$$

Using equation 11.6 for one mole of an ideal gas,

$$PV = RT \quad \dots\dots\dots (11.14)$$

At constant pressure P , amount of work done by one mole of a gas due to expansion ΔV (Fig. 11.7 b) caused by the rise in temperature ΔT is given by Eq. 11.14

$$P \Delta V = R \Delta T$$

Substituting for $P \Delta V$ in Eq. 11.13

$$C_p \Delta T = C_v \Delta T + R \Delta T$$

or $C_p = C_v + R$

or $C_p - C_v = R \quad \dots\dots\dots (11.15)$

It is obvious from Eq. 11.15 that $C_p > C_v$ by an amount equal to universal gas constant R .

11.6 REVERSIBLE AND IRREVERSIBLE PROCESSES

A reversible process is one which can be retraced in exactly reverse order, without producing any change in the surroundings. In the reverse process, the working substance passes through the same stages as in the direct process but thermal and mechanical effects at each stage are exactly reversed. If heat is absorbed in the direct

process, it will be given out in the reverse process and if work is done by the substance in the direct process, work will be done on the substance in the reverse process. Hence, the working substance is restored to its original conditions.

A succession of events which bring the system back to its initial condition is called a cycle. A reversible cycle is the one in which all the changes are reversible.

Although no actual change is completely reversible but the processes of liquefaction and evaporation of a substance, performed slowly, are practically reversible. Similarly the slow compression of a gas in a cylinder is reversible process as the compression can be changed to expansion by slowly decreasing the pressure on the piston to reverse the operation.

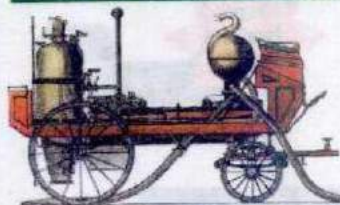
If a process can not be retraced in the backward direction by reversing the controlling factors, it is an irreversible process.

All changes which occur suddenly or which involve friction or dissipation of energy through conduction, convection or radiation are irreversible. An example of highly irreversible process is an explosion.

11.7 HEAT ENGINE

A heat engine converts some thermal energy to mechanical work. Usually the heat comes from the burning of a fuel. The earliest heat engine was the steam engine. It was developed on the fact that when water is boiled in a vessel covered with a lid, the steam inside tries to push the lid off showing the ability to do work. This observation helped to develop a steam engine.

Do You Know?



The steam engine is a thermodynamics system.

Basically a heat engine (Fig. 11.8) consists of hot reservoir or source which can supply heat at high temperature and a cold reservoir or sink into which heat is rejected at a lower temperature. A working substance is needed which can absorb heat Q_1 from source, converts some of it into work W by its expansion and rejects the rest heat Q_2 to the cold reservoir or sink. A heat engine is made cyclic to provide a continuous supply of work.

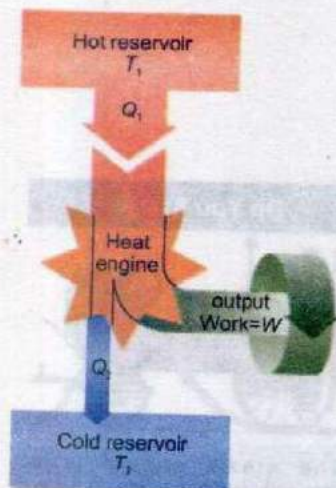


Fig. 11.8

Schematic representation of a heat engine. The engine absorbs heat Q_1 from the hot reservoir, expels heat Q_2 to the cold reservoir and does work W .

11.8 SECOND LAW OF THERMODYNAMICS

First law of thermodynamics tells us that heat energy can be converted into equivalent amount of work, but it is silent about the conditions under which this conversion takes place. The second law is concerned with the circumstances in which heat can be converted into work and direction of flow of heat.

Before initiating the discussion on formal statement of the second law of thermodynamics, let us analyze briefly the factual operation of an engine. The engine or the system represented by the block diagram Fig. 11.8 absorbs a quantity of heat Q_1 from the heat source at temperature T_1 . It does work W and expels heat Q_2 to low temperature reservoir at temperature T_2 . As the working substance goes through a cyclic process, in which the substance eventually returns to its initial state, the change in internal energy is zero. Hence from the first law of thermodynamics, net work done should be equal to the net heat absorbed.

$$W = Q_1 - Q_2$$

In practice, the petrol engine of a motor car extracts heat from the burning fuel and converts a fraction of this energy to mechanical energy or work and expels the rest to atmosphere. It has been observed that petrol engines convert roughly 25% and diesel engines 35 to 40% available heat energy into work.

The second law of thermodynamics is a formal statement based on these observations. It can be stated in a number of different ways.

According to Lord Kelvin's statement based on the working of a heat engine

It is impossible to devise a process which may convert heat, extracted from a single reservoir, entirely into work without leaving any change in the working system.

This means that a single heat reservoir, no matter how much energy it contains, can not be made to perform any work. This is true for oceans and our atmosphere which contain a large amount of heat energy but can not be converted into useful mechanical work. As a consequence of second law of thermodynamics, two bodies at different temperatures are essential for the conversion of heat into work. Hence for the working of heat engine there must be a source of heat at a high temperature and a sink at low temperature to which heat may be expelled. The reason for our inability to utilize the heat contents of oceans and atmosphere is that there is no reservoir at a temperature lower than any one of the two.

11.9 CARNOT ENGINE AND CARNOT'S THEOREM

Sadi Carnot in 1840 described an ideal engine using only isothermal and adiabatic processes. He showed that a heat engine operating in an ideal reversible cycle between two heat reservoirs at different temperatures, would be the most efficient engine. A Carnot cycle using an ideal gas as the working substance is shown on PV diagram (Fig. 11.9). It consists of following four steps.

1. The gas is allowed to expand isothermally at temperature T_1 , absorbing heat Q_1 from the hot reservoir. The process is represented by curve AB.
2. The gas is then allowed to expand adiabatically until its temperature drops to T_2 . The process is represented by curve BC.
3. The gas at this stage is compressed isothermally at temperature T_2 rejecting heat Q_2 to the cold reservoir. The process is represented by curve CD.



According to the Kelvin statement of the second law of thermodynamics, the process pictured here is impossible. Heat from a source at a single temperature cannot be converted entirely into work.

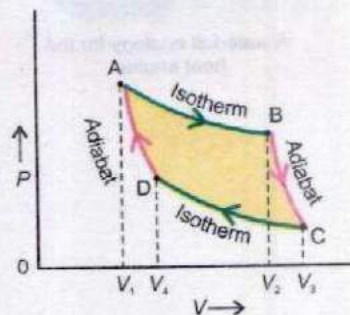
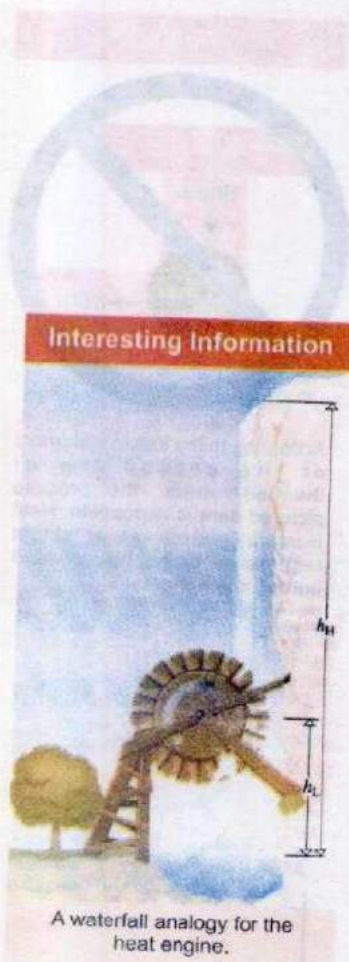


Fig.11.9



4. Finally the gas is compressed adiabatically to restore its initial state at temperature T_1 . The process is represented by curve DA.

Thermal and mechanical equilibrium is maintained all the time so that each process is perfectly reversible. As the working substance returns to the initial state, there is no change in its internal energy i.e. $\Delta U = 0$.

The net work done during one cycle equals to the area enclosed by the path ABCDA of the PV diagram. It can also be estimated from net heat Q absorbed in one cycle.

$$Q = Q_1 - Q_2$$

From 1st law of thermodynamics

$$Q = \Delta U + W$$

$$W = Q_1 - Q_2$$

The efficiency η of the heat engine is defined as

$$\eta = \frac{\text{Output (Work)}}{\text{Input (Energy)}}$$

thus,
$$\eta = \frac{Q_1 - Q_2}{Q_1} = 1 - \frac{Q_2}{Q_1} \dots\dots\dots (11.16)$$

The energy transfer in an isothermal expansion or compression turns out to be proportional to Kelvin temperature. So Q_1 and Q_2 are proportional to Kelvin temperatures T_1 and T_2 respectively and hence,

$$\eta = \frac{T_1 - T_2}{T_1} = 1 - \frac{T_2}{T_1} \dots\dots\dots (11.17)$$

The efficiency is usually taken in percentage, in that case,

$$\text{percentage efficiency} = \left(1 - \frac{T_2}{T_1}\right) 100$$

Thus the efficiency of Carnot engine depends on the temperature of hot and cold reservoirs. It is independent of the nature of working substance. The larger the

temperature difference of two reservoirs, the greater is the efficiency. But it can never be one or 100% unless cold reservoir is at absolute zero temperature ($T_2 = 0$ K).

Such reservoirs are not available and hence the maximum efficiency is always less than one. Nevertheless the Carnot cycle establishes an upper limit on the efficiency of all heat engines. No practical heat engine can be perfectly reversible and also energy dissipation is inevitable. This fact is stated in Carnot's theorem

No heat engine can be more efficient than a Carnot engine operating between the same two temperatures.

The Carnot's theorem can be extended to state that,

All Carnot's engines operating between the same two temperatures have the same efficiency, irrespective of the nature of working substance.

In most practical cases, the cold reservoir is nearly at room temperature. So the efficiency can only be increased by raising the temperature of hot reservoir. All real heat engines are less efficient than Carnot engine due to friction and other heat losses.

Example 11.4: The turbine in a steam power plant takes steam from a boiler at 427°C and exhausts into a low temperature reservoir at 77°C . What is the maximum possible efficiency?

Solution:

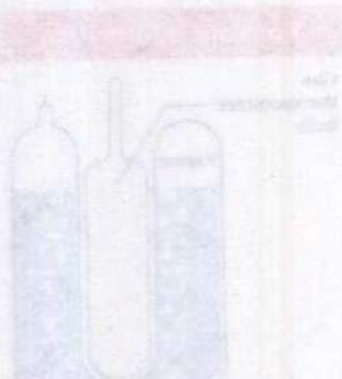
Maximum efficiency for any engine operating between temperatures T_1 and T_2 is

$$\eta = \frac{T_1 - T_2}{T_1}$$

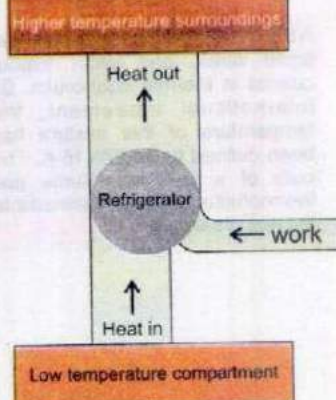
where
and

$$T_1 = 427 + 273 = 700 \text{ K}$$

$$T_2 = 77 + 273 = 350 \text{ K}$$



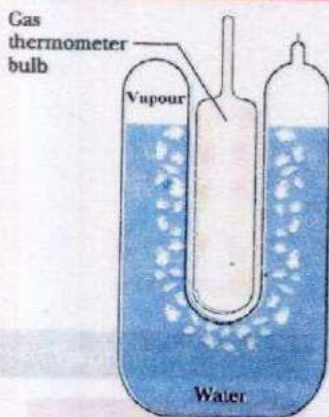
Do You Know?



A refrigerator transfers heat from a low-temperature compartment to higher-temperature surroundings with the help of external work. It is a heat engine operating in reverse order.



For Your Information



A triple-point cell, in which solid ice, liquid water, and water vapour coexist in thermal equilibrium. By international agreement, the temperature of this mixture has been defined to be 273.16 K. The bulb of a constant-volume gas thermometer is shown inserted into the well of the cell.

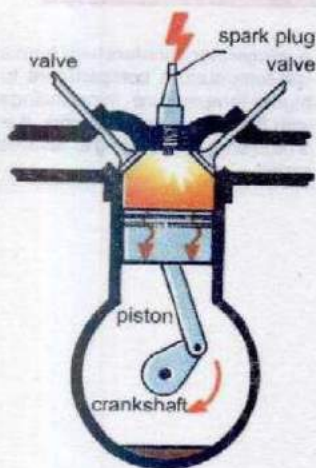


Fig. 11.10(a)

so

$$\eta = \frac{700 \text{ K} - 350 \text{ K}}{700 \text{ K}} = \frac{350 \text{ K}}{700 \text{ K}} = \frac{1}{2} = 0.5$$

or

$$\eta = 50\%$$

11.10 THERMODYNAMIC SCALE OF TEMPERATURE

Generally a temperature scale is established by two fixed points using certain physical properties of a material which varies linearly with temperature. The Carnot cycle provides us the basis to define a temperature scale that is independent of material properties. According to it, the ratio Q_2/Q_1 depends only on the temperature of two heat reservoirs. The ratio of the two temperatures T_2/T_1 can be found by operating a reversible Carnot cycle between these two temperatures and carefully measuring the heat transfers Q_2 and Q_1 . The thermodynamic scale of temperature is defined by choosing 273.16 K as the absolute temperature of the triple point of water as one fixed point and absolute zero, as the other. The unit of thermodynamic scale is kelvin. 1 K is defined as $1/273.16$ of the thermodynamic temperature of the triple point of water. It is a state in which ice, water and vapour coexists in equilibrium and it occurs uniquely at one particular pressure and temperature. If heat Q is absorbed or rejected by the system at corresponding temperature T when the system is taken through a Carnot cycle and Q_3 is the heat absorbed or rejected by the system when it is at the temperature of triple point of water, then unknown temperature T in kelvin is given by

$$T = 273.16 \frac{Q}{Q_3} \dots\dots\dots (11.18)$$

Since this scale is independent of the property of the working substance, hence, can be applied at very low temperature.

11.11 PETROL ENGINE

Although different engines may differ in their construction technology but they are based on the principle of a Carnot cycle. A typical four stroke petrol engine (Fig. 11.10 a) also undergoes four successive processes in each cycle.

1. The cycle starts on the intake stroke in which piston moves outward and petrol air mixture is drawn through an inlet valve into the cylinder from the carburetor at atmospheric pressure.
2. On the compression stroke, the inlet valve is closed and the mixture is compressed adiabatically by inward movement of the piston.
3. On the power stroke, a spark fires the mixture causing a rapid increase in pressure and temperature. The burning mixture expands adiabatically and forces the piston to move outward. This is the stroke which delivers power to crank shaft to drive the flywheels.
4. On the exhaust stroke, the outlet valves opens. The residual gases are expelled and piston moves inward.

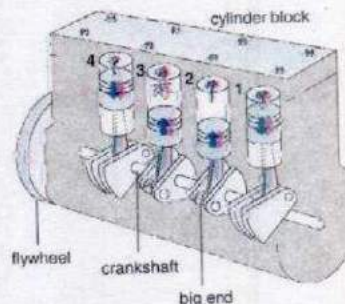


Fig. 11.10(b)

The cycle then begins again. Most motorbikes have one cylinder engine but cars usually have four cylinders on the same crankshaft (Fig 11.10 b). The cylinders are timed to fire turn by turn in succession for a smooth running of the car. The actual efficiency of properly tuned engine is usually not more than 25% to 30% because of friction and other heat losses.

Diesel Engine

No spark plug is needed in the diesel engine (Fig. 11.11). Diesel is sprayed into the cylinder at maximum compression. Because air is at very high temperature immediately after compression, the fuel mixture ignites on contact with the air in the cylinder and pushes the piston outward. The efficiency of diesel engine is about 35% to 40%.

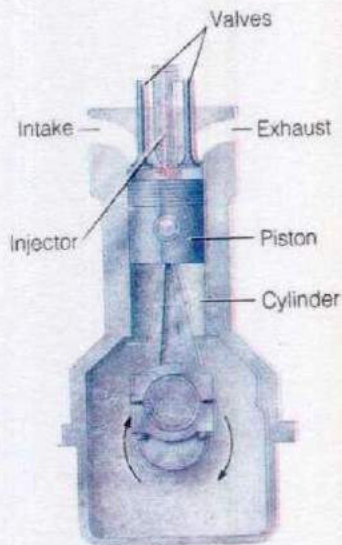


Fig. 11.11

11.12 ENTROPY

The concept of entropy was introduced into the study of thermodynamics by Rudolph Clausius in 1856 to give a quantitative basis for the second law. It provides another variable to describe the state of a system to go along with pressure, volume, temperature and internal energy. If a system undergoes a reversible process during which it absorbs a quantity of heat ΔQ at absolute temperature T , then the increase in the state variable called entropy S of the system is given by

$$\Delta S = \frac{\Delta Q}{T} \quad \dots\dots\dots (11.19)$$

Like potential energy or internal energy, it is the change in entropy of the system which is important.

Change in entropy is positive when heat is added and negative when heat is removed from the system. Suppose, an amount of heat Q flows from a reservoir at temperature T_1 through a conducting rod to a reservoir at temperature T_2 when $T_1 > T_2$. The change in entropy of the reservoir, at temperature T_1 , which loses heat, decreases by Q/T_1 and of the reservoir at temperature T_2 , which gains heat, increases by Q/T_2 . As $T_1 > T_2$ so Q/T_2 will be greater than Q/T_1 i.e. $Q/T_2 > Q/T_1$.

Hence, net change in entropy = $\frac{Q}{T_2} - \frac{Q}{T_1}$ is positive.

It follows that in all natural processes where heat flows from one system to another, there is always a net increase in entropy. This is another statement of 2nd law of thermodynamics. According to this law

If a system undergoes a natural process, it will go in the direction that causes the entropy of the system plus the environment to increase.

It is observed that a natural process tends to proceed towards a state of greater disorder. Thus, there is a relation between entropy and molecular disorder. For example an irreversible heat flow from a hot to a cold substance of a system increases disorder because the molecules are initially sorted out in hotter and cooler regions. This order is lost when the system comes to thermal equilibrium. Addition of heat to a system increases its disorder because of increase in average molecular speeds and therefore, the randomness of molecular motion. Similarly, free expansion of gas increases its disorder because the molecules have greater randomness of position after expansion than before. Thus in both examples, entropy is said to be increased.

We can conclude that only those processes are probable for which entropy of the system increases or remains constant. The process for which entropy remains constant is a reversible process; whereas for all irreversible processes, entropy of the system increases.

Every time entropy increases, the opportunity to convert some heat into work is lost. For example there is an increase in entropy when hot and cold waters are mixed. Then warm water which results cannot be separated into a hot layer and a cold layer. There has been no loss of energy but some of the energy is no longer available for conversion into work. Therefore, increase in entropy means degradation of energy from a higher level where more work can be extracted to a lower level at which less or no useful work can be done. The energy in a sense is degraded, going from more orderly form to less orderly form, eventually ending up as thermal energy.

In all real processes where heat transfer occurs, the energy available for doing useful work decreases. In other words the entropy increases. Even if the temperature of some system decreases, thereby decreasing the entropy, it is at the expense of net increase in entropy for some other system. When all the systems are taken together as the universe, the entropy of the universe always increases.

Example 11.5: Calculate the entropy change when 1.0 kg ice at 0°C melts into water at 0°C. Latent heat of fusion of ice $L_f = 3.36 \times 10^5 \text{ J kg}^{-1}$.

Solution:

$$m = 1 \text{ kg}$$

$$T = 0^\circ\text{C} = 273 \text{ K}$$

$$L_f = 3.36 \times 10^5 \text{ J kg}^{-1}$$

$$\Delta S = \frac{\Delta Q}{T}$$

where

$$\Delta Q = mL_f$$

$$\Delta S = \frac{mL_f}{T}$$

$$\Delta S = \frac{1.00 \text{ kg} \times 3.36 \times 10^5 \text{ J kg}^{-1}}{273 \text{ K}}$$

$$\Delta S = 1.23 \times 10^3 \text{ J K}^{-1}$$

Thus entropy increases as it changes to water. The increase in entropy in this case is a measure of increase in the disorder of water molecules that change from solid to liquid state.

Do You Know?

Approximate efficiencies of various devices

Device	Efficiency (%)
Electric generator	70-99
Electric motor	50-93
Dry cell battery	90
Domestic gas furnace	70-85
Storage battery	72
Hydrogen-oxygen fuel cell	60
Liquid fuel rocket	47
Steam turbine	35-46
Fossil-fuel power plant	30-40
Nuclear power plant	30-35
Nuclear reactor	39
Aircraft gas turbine engine	36
Solid-state laser	30
Internal combustion gasoline engine	20-30
Gallium arsenide solar cells	>20
Fluorescent lamp	20
Silicon solar cell	12-16
Steam locomotive	8
Incandescent lamp	5
Watt's steam engine	1

11.13 ENVIRONMENTAL CRISIS AS ENTROPY CRISIS

The second law of thermodynamics provides us the key for both understanding our environmental crisis, and for understanding how we must deal with this crisis.

From a human standpoint the environmental crisis results from our attempts to order nature for our comforts and greed. From a physical standpoint, however, the environmental crisis is an entropy or disorder crisis resulting from our futile efforts to ignore the second law of thermodynamics. According to which, any increase in the order in a system will produce an even greater increase in entropy or disorder in the environment. An individual impact may not have a major consequence but an impact of large number of all individuals disorder producing activities can affect the overall life support system.

The energy processes we use are not very efficient. As a result most of the energy is lost as heat to the environment. Although we can improve the efficiency but 2nd law eventually imposes an upper limit on efficiency improvement. Thermal pollution is an inevitable consequence of 2nd law of thermodynamics and the heat is the ultimate death of any form of energy. The increase in thermal pollution of the environment means increase in the entropy and that causes great concern. Even small temperature changes in the environment can have significant effects on metabolic rates in plants and animals. This can cause serious disruption of the overall ecological balance.

In addition to thermal pollution, the most energy transformation processes such as heat engines used for transportation and for power generation cause air pollution. In effect, all forms of energy production have some undesirable effects and in some cases all problems can not be anticipated in advance.

The imperative from thermodynamics is that whenever you do anything, be sure to take into account its present and possible future impact on your environment. This is an ecological imperative that we must consider now if we are to prevent a drastic degradation of life on our beautiful but fragile Earth.

For Your Information



The jet engines on this aircraft convert thermal energy to work, but the visible exhaust clearly shows that a considerable amount of thermal energy is lost as waste heat.

SUMMARY

- From the Kinetic theory of gases $P = \frac{1}{3} \rho \langle v^2 \rangle$.
- The first law of thermodynamics states that energy is conserved.
- The sum of all forms of molecular energy present in a thermodynamic system is called its internal energy.
- Isothermal process is the process in which Boyle's law holds good.
- Adiabatic process is the one in which no thermal energy is added or extracted from the system.
- Molar specific heat at constant volume is the amount of heat required to raise the temperature of one mole of the gas through 1 K keeping volume constant.
- Molar specific heat at constant pressure is the amount of heat required to raise the temperature of one mole of the gas through 1 K keeping pressure constant.
- A heat engine is a device which converts a part of thermal energy into useful work.
- Efficiency of Carnot engine is $1 - \frac{T_2}{T_1}$.
- The second law of thermodynamics can be stated as
 - (i) There is no perpetual motion machine that can convert the given amount of heat completely into work.
 - (ii) The total entropy of any system plus that of its environment increases as a result of any natural process.
- Entropy change ΔS due to heat transfer ΔQ at absolute temperature T is given by
$$\Delta S = \pm \frac{\Delta Q}{T}$$
- Thermal pollution is an inevitable consequence of 2nd law of thermodynamics.

QUESTIONS

- 11.1 Why is the average velocity of the molecules in a gas zero but the average of the square of velocities is not zero?
- 11.2 Why does the pressure of a gas in a car tyre increase when it is driven through some distance?

- 11.3 A system undergoes from state P_1V_1 to state P_2V_2 as shown in Fig 11.12. What will be the change in internal energy?

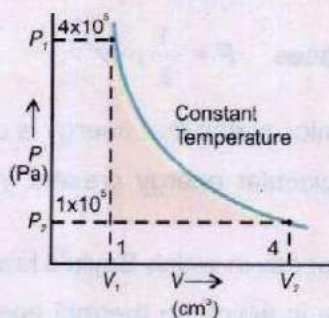


Fig.11.12

- 11.4 Variation of volume by pressure is given in Fig 11.13. A gas is taken along the paths ABCDA, ABCA and A to A. What will be the change in internal energy?

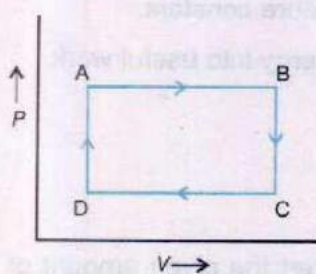


Fig.11.13(a)

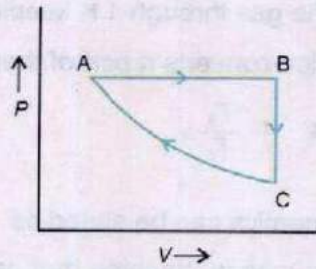


Fig.11.13(b)

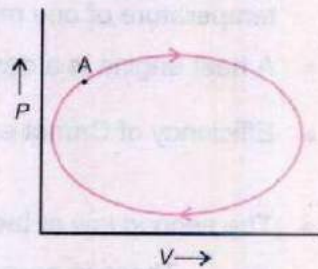


Fig.11.13(c)

- 11.5 Specific heat of a gas at constant pressure is greater than specific heat at constant volume. Why?
- 11.6 Give an example of a process in which no heat is transferred to or from the system but the temperature of the system changes.
- 11.7 Is it possible to convert internal energy into mechanical energy? Explain with an example.
- 11.8 Is it possible to construct a heat engine that will not expel heat into the atmosphere?
- 11.9 A thermos flask containing milk as a system is shaken rapidly. Does the temperature of milk rise?
- 11.10 What happens to the temperature of the room, when an airconditioner is left running on a table in the middle of the room?

- 11.11 Can the mechanical energy be converted completely into heat energy? If so give an example.
- 11.12 Does entropy of a system increase or decrease due to friction?
- 11.13 Give an example of a natural process that involves an increase in entropy.
- 11.14 An adiabatic change is the one in which
- No heat is added to or taken out of a system
 - No change of temperature takes place
 - Boyle's law is applicable
 - Pressure and volume remains constant
- 11.15 Which one of the following process is irreversible?
- Slow compressions of an elastic spring
 - Slow evaporation of a substance in an isolated vessel
 - Slow compression of a gas
 - A chemical explosion
- 11.16 An ideal reversible heat engine has
- 100% efficiency
 - Highest efficiency
 - An efficiency which depends on the nature of working substance
 - None of these

NUMERICAL PROBLEMS

- 11.1 Estimate the average speed of nitrogen molecules in air under standard conditions of pressure and temperature.
- (Ans: 493 ms^{-1})
- 11.2 Show that ratio of the root mean square speeds of molecules of two different gases at a certain temperature is equal to the square root of the inverse ratio of their masses.
- 11.3 A sample of gas is compressed to one half of its initial volume at constant pressure of $1.25 \times 10^5 \text{ Nm}^{-2}$. During the compression, 100 J of work is done on the gas. Determine the final volume of the gas.

(Ans: $8 \times 10^{-4} \text{ m}^3$)

- 11.4 A thermodynamic system undergoes a process in which its internal energy decreases by 300 J. If at the same time 120 J of work is done on the system, find the heat lost by the system.
(Ans: - 420 J)
- 11.5 A Carnot engine utilises an ideal gas. The source temperature is 227°C and the sink temperature is 127°C . Find the efficiency of the engine. Also find the heat input from the source and heat rejected to the sink when 10000 J of work is done.
(Ans: 20%, $5.00 \times 10^4 \text{ J}$, $4.00 \times 10^4 \text{ J}$)
- 11.6 A reversible engine works between two temperatures whose difference is 100°C . If it absorbs 746 J of heat from the source and rejects 546 J to the sink, calculate the temperature of the source and the sink.
(Ans: 100°C , 0°C)
- 11.7 A mechanical engineer develops an engine, working between 327°C and 27°C and claims to have an efficiency of 52%. Does he claim correctly? Explain.
(Ans: No)
- 11.8 A heat engine performs 100 J of work and at the same time rejects 400 J of heat energy to the cold reservoirs. What is the efficiency of the engine?
(Ans: 20%)
- 11.9 A Carnot engine whose low temperature reservoir is at 7°C has an efficiency of 50%. It is desired to increase the efficiency to 70%. By how many degrees the temperature of the source be increased?
(Ans: 373°C)
- 11.10 A steam engine has a boiler that operates at 450 K. The heat changes water to steam, which drives the piston. The exhaust temperature of the outside air is about 300 K. What is maximum efficiency of this steam engine?
(Ans: 33%)
- 11.11 336 J of energy is required to melt 1 g of ice at 0°C . What is the change in entropy of 30 g of water at 0°C as it is changed to ice at 0°C by a refrigerator?
(Ans: -36.8 J K^{-1})

Appendix

1

Standard Definitions of Base Units

Metre: The unit of length is named as metre. Before 1960 it was defined as the distance between two lines marked on the bar of an alloy of platinum (90%) and iridium (10%) kept under controlled conditions at the International Bureau of Weights and Measures in France. The 11th General Conference on Weights and Measures (1960) redefined the standard metre as follows: One metre is a length equal to 1,650,763.73 wave lengths in vacuum of the orange red radiation emitted by the Krypton 86-atom. However, in 1983 the metre was redefined to be the distance traveled by light in vacuum during a time of $1/299,792,458$ second. In effect, this latest definition establishes that the speed of light in vacuum is $299,792,458 \text{ ms}^{-1}$.

Kilogram: The unit of mass is known as kilogram. It is defined as the mass of a platinum (90%) and iridium (10%) alloy cylinder, 3.9 cm in diameter and 3.9 cm in height, kept at the International Bureau of Weights and Measures in France. This mass standard was established in 1901.

Second: The unit of time is termed as second. It is defined as $1/86400$ part of an average day of the year 1900 A.D. The recent time standard is based on the spinning motion of electrons in atoms. This is since 1967 when the International Committee on Weights and Measures adopted a new definition of second, making one second equal to the duration in which the outer most electron of the cesium-133 atom makes 9,192,631,770 vibrations.

Kelvin: Temperature is regarded as a thermodynamic quantity, because its equality determines the thermal equilibrium between two systems. The unit of temperature is kelvin. It is the fraction $1/273.16$ of the thermodynamic temperature of the triple point of water. It should be noted that the triple point of a substance means the temperature at which solid, liquid and vapour phases are in equilibrium. The triple point of water is taken as 273.16 K. This standard was adopted in 1967.

Ampere: The unit of electric current is ampere. It is that constant current which if maintained in two straight parallel conductors of infinite length, of negligible circular cross-section and placed a metre apart in vacuum, would produce between these conductors a force equal to 2×10^{-7} newton per metre of length. This unit was established in 1971.

Candela: The unit of luminous intensity is candela. It is defined as the luminous intensity in the perpendicular direction of a surface of $1/600000$ square metre of a black body radiator at the solidification temperature of platinum under standard atmospheric pressure. This definition was adopted by the 13th General Conference of Weights and measures in 1967.

Mole: The mole is the amount of substance of a system which contains as many elementary entities as there are atoms in 0.012 kg of carbon 12 (adopted in 1971). When this unit i.e. mole is used, the elementary entities must be specified; these may be atoms, molecules, ions, electrons, other particles or specified groups of such particles. One mole of any substance contains 6.0225×10^{23} entities.

Appendix 2

Possible Error in A Compound Quantity

(i) ERROR IN THE COMPOUND QUANTITY $z = x + y$

If the errors in the quantities x and y are Δx and Δy respectively, the possible sum is then;

$$x \pm \Delta x + y \pm \Delta y$$

The maximum possible error is when we have

$$x + \Delta x + y + \Delta y$$

or

$$x - \Delta x + y - \Delta y$$

Hence, the quantity can be expressed as $x + y \pm (\Delta x + \Delta y)$

i.e., the errors are added.

Hence, $\text{error in } z = \text{error in } x + \text{error in } y$ (A 2.1)

(ii) ERROR IN THE COMPOUND QUANTITY $z = xy$

If the errors in the quantities x and y are Δx and Δy respectively, the compound quantity could be as large as $(x + \Delta x)(y + \Delta y)$ or as small as $(x - \Delta x)(y - \Delta y)$. The product is thus between about $xy + x\Delta y + y\Delta x + \Delta x\Delta y$ and $xy - x\Delta y - y\Delta x + \Delta x\Delta y$. If we neglect $\Delta x\Delta y$, as being small, then the error is between

$$x\Delta y + y\Delta x \quad \text{and} \quad -(x\Delta y + y\Delta x)$$

or

$$\pm (x\Delta y + y\Delta x)$$

The possible fractional error is thus

$$= \frac{\pm(x\Delta y + y\Delta x)}{xy} = \pm \left(\frac{\Delta y}{y} + \frac{\Delta x}{x} \right)$$

which is the sum of possible fractional errors. Since the fractional error is generally written as percentage error, hence the possible percentage error is the sum of the percentage errors for the product of the two physical quantities.

i.e., $\% \text{ error in } z = \% \text{ error in } x + \% \text{ error in } y$ (A 2.2)

(iii) ERROR IN THE COMPOUND QUANTITY $z = k x^a y^b$

Let z , x and y be the numerical values of the physical quantities and k be a constant. Taking log of both sides;

$$\log z = \log k + a \log x + b \log y$$

Differentiating:
$$\frac{dz}{z} = 0 + a \frac{dx}{x} + b \frac{dy}{y}$$

Multiply by 100

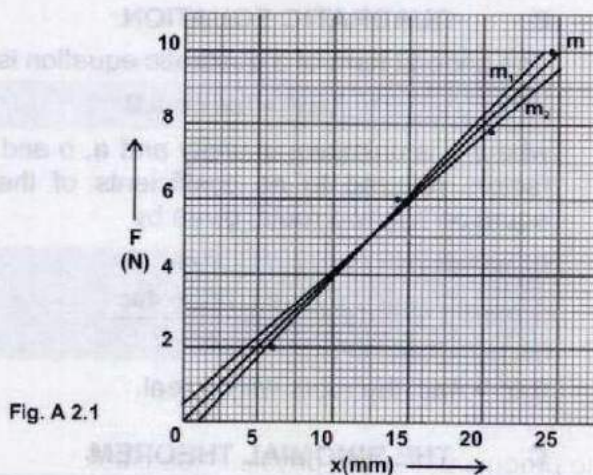
$$\left(\frac{dz}{z}\right) 100 = a \left(\frac{dx}{x}\right) 100 + b \left(\frac{dy}{y}\right) 100$$

If dx , dy and dz represent the errors in the quantities x , y , and z respectively, then

$$\% \text{ error in } z = a (\% \text{ error in } x) + b (\% \text{ error in } y) \quad (\text{A 2.3})$$

(iv) Error or Uncertainty from Graphs

To find uncertainty in an average value obtained by plotting graphs, the first step is to draw best straight line through the plotted points using a transparent ruler. The best straight line passes through as many of plotted points as possible or which leaves almost an equal distribution of points on either side of the line. The second step is to pivot a transparent ruler about the centre of best straight line to draw greatest and least possible slopes. If slope of best straight line is m and greatest and least slopes are m_1 and m_2 as illustrated in Fig. A 2.1, then evaluate $m_1 - m$ and $m_2 - m$ which ever of these is



greater is the maximum possible uncertainty in the slope. If the intercept on a particular axis is required, the similar procedure can be followed.

Mathematical Review

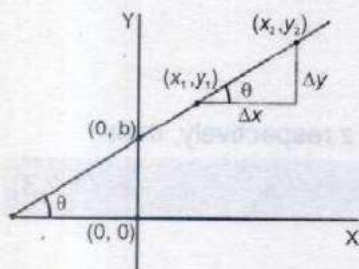


Fig. A 3.1

A. LINEAR EQUATION

A linear equation has the general form

$$y = ax + b \quad (\text{A 3.1})$$

Where a and b are constants. This equation is referred to as being linear because the graph of y versus x is a straight line, as shown in Fig. A3.1. The constant b , called the intercept, represents the value of y at which the straight line intersects the Y -axis. The constant a is equal to the slope of the straight line and is also equal to the tangent of the angle that the line makes with the X -axis. If any two points on the straight line are specified by the coordinates (x_1, y_1) and (x_2, y_2) , as in Fig. A 3.1, then the slope of the straight line can be expressed

$$\text{Slope } a = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\Delta y}{\Delta x} = \tan \theta \quad (\text{A 3.2})$$

Note that a and b can be either positive or negative.

B. QUADRATIC EQUATION

The general form of a quadratic equation is

$$ax^2 + bx + c = 0 \quad (\text{A 3.3})$$

where x is unknown quantity and a , b and c are numerical factors referred to as coefficients of the equation. This equation has two roots, given by

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad (\text{A 3.4})$$

If $b^2 > 4ac$, the roots will be real.

C. THE BINOMIAL THEOREM

$$(i) \quad (a + b)^n = a^n + na^{n-1}b + \frac{n(n-1)}{2 \times 1} a^{n-2}b^2 + \frac{n(n-1)(n-2)}{3 \times 2 \times 1} a^{n-3}b^3 + \dots \quad (\text{A 3.5})$$

(ii) If $x < 1$, then

$$(1 + x)^n = 1 + nx + \text{negligible terms} \quad (\text{A 3.6})$$

D. GEOMETRY

(i) Areas and volumes of some geometrical shapes are given in Table A3.1.

(ii) TABLE A 3.2

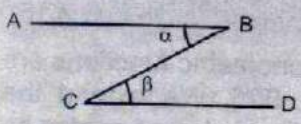
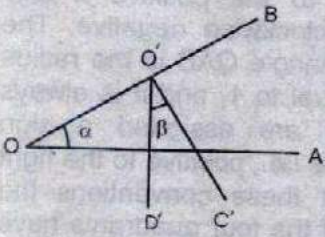
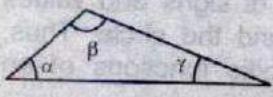
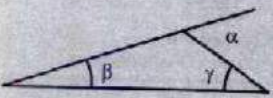
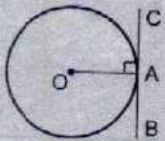
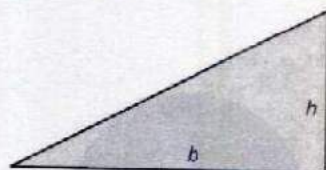
Condition	Theorem
	1. If AB is parallel to CD, then $\alpha = \beta$
	2. If $O'C'$ is perpendicular to OB and $O'D'$ is perpendicular to OA, then $\alpha = \beta$
	3. $\alpha + \beta + \gamma = 180^\circ$
	4. $\alpha = \beta + \gamma$
	5. The radius OA is perpendicular to the tangent line BC.

Table A 3.1

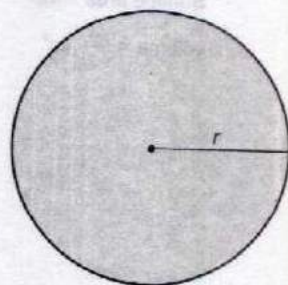
Areas and volumes of some geometrical shapes.



(i) Rectangle
Area = lw



(ii) Triangle
Area = $\frac{1}{2}bh$



(iii) circle
Area = πr^2
(Circumference = $2\pi r$)



(iv) Cylinder
volume = $\pi r^2 l$



(v) Rectangular box
volume = lwh



(vi) Sphere
Surface area = $4\pi r^2$
volume = $\frac{4}{3}\pi r^3$

E. TRIGONOMETRY

$$\sin^2 \theta + \cos^2 \theta = 1 \quad (\text{A 3.7})$$

$$\cos(90^\circ + \theta) = \sin \theta \quad (\text{A 3.8})$$

$$\sin(90^\circ + \theta) = \cos \theta \quad (\text{A 3.9})$$

$$\sin(180^\circ - \theta) = \sin \theta \quad (\text{A 3.10})$$

$$\cos(180^\circ - \theta) = -\cos \theta \quad (\text{A 3.11})$$

$$\sin(\theta \pm \phi) = \sin \theta \cos \phi \pm \cos \theta \sin \phi \quad (\text{A 3.12})$$

$$\cos(\theta \pm \phi) = \cos \theta \cos \phi \mp \sin \theta \sin \phi \quad (\text{A 3.13})$$

$$\sin 2\theta = 2 \sin \theta \cos \theta \quad (\text{A 3.14})$$

$$\cos 2\theta = \cos^2 \theta - \sin^2 \theta \quad (\text{A 3.15})$$

According to our definitions, the trigonometric functions are limited to angles in the range $[0, 90^\circ]$. We extend the meaning of these functions to negative or larger angles by a circle of unit radius, the unit circle (Fig. A 3.2). The angle is always measured with respect to the positive x axis counter clockwise positive and clockwise negative. The hypotenuse of the right angled triangle OAB is the radius of the unit circle. Its length is equal to 1, and it is always positive. The other two sides are assigned a sign according to the usual conventions i.e., positive to the right of the x-axis, and so on. With these conventions the trigonometric functions in each of the four quadrants have the signs listed in Table A 3.3.

If θ exceeds 360° , the whole pattern of signs and values repeats itself on the next pass around the circle. Thus, sine, cosine, and tangent are periodic functions of an angle with period 360° .

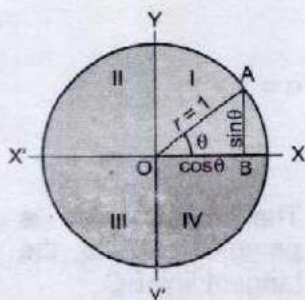


Fig. A 3.2

Table A 3.3

The Signs of the Trigonometric Functions

Quadrant	$\sin \theta$	$\cos \theta$	$\tan \theta$
I	+	+	+
II	+	-	-
III	-	-	+
IV	-	+	-

GLOSSARY

- ▶ **Adiabatic process** A completely isolated process in which no heat transfer can take place.
- ▶ **Angular acceleration** The rate of change of angular velocity with time.
- ▶ **Angular displacement** Angle subtended at the centre of a circle by a particle moving along the circumference in a given time.
- ▶ **Angular momentum** The cross product of position vector and linear momentum.
- ▶ **Angular velocity** Angular displacement per second.
- ▶ **Antinode** The point of maximum displacement on a stationary wave.
- ▶ **Artificial gravity** The gravity like effect produced in orbiting space ship to overcome weightlessness.
- ▶ **Average acceleration** Ratio of the change in velocity, that occurs within a time interval, to that time interval.
- ▶ **Average velocity** Average rate at which displacement vector changes with time.
- ▶ **Base quantities** Certain physical quantities such as length, mass and time.
- ▶ **Blue shift** The shift of received wavelength from a star into the shorter region.
- ▶ **Bulk modulus** Ratio of volumetric stress to volumetric strain.
- ▶ **Centre of mass** The point at which all the mass of the body is assumed to be concentrated.
- ▶ **Centripetal force** The force needed to move a body around a circular path.
- ▶ **Cladding** A layer of lower refractive index (less density) over the central core of high refractive index (high density).
- ▶ **Compression** The region of maximum density of a wave.
- ▶ **Conservative field** The field in which work done along a closed path is zero.
- ▶ **Constructive interference** When two waves meet each other in the same phase.
- ▶ **Core** The central part of optical fibre which has relatively high refractive index (high density).
- ▶ **Crest** The portion of a wave above the mean level.
- ▶ **Critical angle** The angle of incidence for which the angle of refraction is 90° .
- ▶ **CRO** A device used to display input signal into waveform.

- ▶ **Damping**
A process whereby energy is dissipated from the oscillatory system.
- ▶ **Denser medium**
The medium which has greater density.
- ▶ **Derived quantities**
The physical quantities defined in terms of base quantities.
- ▶ **Destructive interference**
When two waves overlap each other in opposite phases.
- ▶ **Diffraction**
Bending of light around obstacles.
- ▶ **Dimension**
One of the basic measurable physical property such as length, mass and time.
- ▶ **Displacement**
The change in the position of a body from its initial position to its final position.
- ▶ **Doppler shift**
The apparent change in the frequency due to relative motion of source and observer.
- ▶ **Drag force**
A retarding force experienced by an object moving through a fluid.
- ▶ **Elastic collision**
The interaction in which both momentum and kinetic energy conserve.
- ▶ **Energy**
Capacity to do work.
- ▶ **Entropy**
Measure of increase in disorder of a thermodynamic system or degradation of energy.
- ▶ **Escape velocity**
The initial velocity of a body to escape from Earth's gravitational field.
- ▶ **Forced oscillations**
The oscillations of a body subjected to an external force.
- ▶ **Free oscillations**
Oscillations of a body at its own frequency without the interference of an external force.
- ▶ **Freely falling body**
A body moving under the action of gravity only.
- ▶ **Fundamental mode**
Stationary wave setup with minimum frequency.
- ▶ **Geo-stationary satellite**
The satellite whose orbital motion is synchronized with the rotation of the Earth.
- ▶ **Harmonics**
Stationary waves setup with integral multiples of the fundamental frequency.
- ▶ **Heat engine**
A device that converts a part of input heat energy into mechanical work.
- ▶ **Ideal fluid**
An incompressible fluid having no viscosity.
- ▶ **Impulse**
The product of force and time for which it acts on a body.
- ▶ **Inelastic collision**
The interaction in which kinetic energy does not conserve.

- ▶ **Instantaneous acceleration** Acceleration at a particular instant of time.
- ▶ **Instantaneous velocity** Velocity at a particular instant of time.
- ▶ **Internal energy** The sum of all forms of molecular energies in a thermodynamic system.
- ▶ **Isothermal process** A process in which Boyle's law is applicable.
- ▶ **Kinetic energy** Energy possessed by a body due to its motion.
- ▶ **Laminar flow** Smooth sliding of layers of fluid past each other.
- ▶ **Least distance of distinct vision** The minimum distance from the eye at which an object can be seen distinctly.
- ▶ **Line spectrum** Set of discrete wavelengths.
- ▶ **Longitudinal wave** The wave in which the particles of the medium vibrate parallel to the propagation of the wave.
- ▶ **Magnification** The ratio of the angle subtended by the image as seen through the optical device to that subtended by the object at the unaided eye.
- ▶ **Modulus of elasticity** Ratio of stress and the strain.
- ▶ **Molar specific heat at constant pressure** Amount of heat needed to change the temperature of one mole of a gas through 1K keeping pressure constant.
- ▶ **Molar specific heat at constant volume** Amount of heat needed to change the temperature of one mole of a gas through 1K keeping volume constant.
- ▶ **Moment Arm** Perpendicular distance between the axis of rotation and line of action of the force.
- ▶ **Moment of inertia** The rotational analogue of mass in linear motion.
- ▶ **Momentum** The product of mass and velocity of an object.
- ▶ **Multi-mode graded index fibre** An optical fibre in which the central core has high refractive index which gradually decreases towards its periphery.
- ▶ **Node** The point of zero displacement.
- ▶ **Null vector** A vector of magnitude zero without any specific direction.
- ▶ **Orbital velocity** The tangential velocity to put a satellite in orbit around the Earth.
- ▶ **Oscillatory motion** To and fro motion of a body about its mean position.
- ▶ **Periodic motion** The motion which repeats itself after equal intervals of time.
- ▶ **Phase** A quantity which indicates the state and direction of motion of a vibrating particle.

- ▶ **Pitch** The characteristics of sound by which a shrill sound can be distinguished from the grave sound.
- ▶ **Plane wavefront** A disturbance lying in a plane surface.
- ▶ **Polarization** The orientation of vibration along a particular direction.
- ▶ **Position vector** A vector that describes the location of a point.
- ▶ **Potential energy** Energy possessed by a body due to its position.
- ▶ **Power** The rate of doing work.
- ▶ **Progressive wave** The wave which transfers energy away from the source.
- ▶ **Projectile** An object moving under the action of gravity and moving horizontally at the same time.
- ▶ **Radar speed trap** An instrument used to detect the speed of moving object on the basis of Doppler shift.
- ▶ **Random error** Error due to fluctuations in the measured quantity.
- ▶ **Range of a projectile** The horizontal distance from the point where the projectile is launched to the point it returns to its launching height.
- ▶ **Rarefaction** The region of minimum density.
- ▶ **Rarer medium** The medium which has relatively less density.
- ▶ **Rays** Radial lines leaving the point source in all directions.
- ▶ **Red shift** The shift in the wavelength of light from a star towards longer wavelength region.
- ▶ **Resolving power** The ability of an instrument to reveal the minor details of the object under examination.
- ▶ **Resonance** A specific response of vibrating system to a periodic force acting with the natural period of the system.
- ▶ **Restoring force** The force that brings the body back to its equilibrium position.
- ▶ **Resultant vector** The sum vector of two or more vectors.
- ▶ **Root mean square velocity** Square root of the average of the square of molecular velocities.
- ▶ **Rotational equilibrium** A body having zero angular acceleration.
- ▶ **Scalar quantity** A physical quantity that has magnitude only.
- ▶ **Scalar product** The product of two vectors that results into a scalar quantity.
- ▶ **Significant figures** The measured or calculated digits for a quantity which are reasonably reliable.

- ▶ **Simple harmonic motion** A motion in which acceleration is directly proportional to displacement from mean position and is always directed towards the mean position.
- ▶ **Slinky spring** A loose spring which has small initial length but a relatively large extended length.
- ▶ **Space time curvature** Einstein's view of gravitation.
- ▶ **Spherical wavefront** When the disturbance is propagated in all directions from a point source.
- ▶ **Stationary wave** The resultant wave arising due to the interference of two identical but oppositely directed waves.
- ▶ **System international (SI)** The internationally agreed system of units used almost world over.
- ▶ **Systematic error** Error due to incorrect design or calibration of the measuring device.
- ▶ **Terminal velocity** Maximum constant velocity of an object falling vertically downward.
- ▶ **Torque** The turning effect of a force.
- ▶ **Total internal reflection** When the angle of incidence increases by the critical angle, then the incident light is reflected back in the same material.
- ▶ **Trajectory** The path through space followed by a projectile.
- ▶ **Translational equilibrium** A body having zero linear acceleration.
- ▶ **Transverse wave** The wave in which the particles of the medium vibrate perpendicular to the propagation of wave.
- ▶ **Trough** The lower portion of a wave below the mean level.
- ▶ **Turbulent flow** Disorderly and changing flow pattern of fluids.
- ▶ **Unit vector** A vector of magnitude one used to denote direction.
- ▶ **Vector quantity** A physical quantity that has both magnitude and direction.
- ▶ **Vector product** The product of two vectors that results into another vector.
- ▶ **Wavefront** A surface passing through all the points undergoing a similar disturbance (i.e., having the same phase) at a given instant.
- ▶ **Wavelength** The distance between two consecutive wavefronts.
- ▶ **Work** The product of magnitude of force and that of displacement in the direction of force.

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