Chapter 12

ELECTROSTATICS

Learning Objectives

At the end of this chapter the students will be able to:

1. Understand and describe Coulomb's law.
2. Describe that a charge has a field of force around it.
3. Understand fields of like and unlike charges.
4. Appreciate the principle of inkjet printers and photostat copier as an application of electrostatic phenomena.
5. Explain the electric intensity in a free space and in other media.
6. State and prove Gauss's law.
7. Appreciate the applications of Gauss's law.
8. Explain electric potential at a point in terms of work done in bringing a unit positive charge from infinity to that point.
9. Relate electric field strength and potential gradient.
10. Find expression for potential at a point due to a point charge.
11. Describe and derive the value of electric charge by Millikan's method.
12. Calculate the capacitance of parallel plate capacitor.
13. Recognize the effect of dielectric on the capacitance of parallel plate capacitor.
14. Understand and describe electric polarization of dielectric.
15. Know the process of charging and discharging of a capacitor through a resistance and calculate the time constant.
16. Find energy expression of a charged capacitor.

The study of electric charges at rest under the action of electric forces is known as electrostatics. An electric force is the force which holds the positive and negative charges that make up atoms and molecules. The human body is composed entirely of atoms and molecules, thus we owe our existence to the electric force.

A lightning flash

12.1 COULOMB'S LAW

We know that there are two kinds of charges, namely, positive and negative charges. The charge on an electron is assumed to be negative and charge on a proton is positive. Moreover, we also learnt that like charges repel each other and unlike charges attract each other. Now we investigate the quantitative nature of these forces. The first measurement of the force between electric charges was made in 1874 AD by Charles Coulomb, a French military engineer. On the basis of these measurements, he deduced a law known as Coulomb's law. It states that

The force between two point charges is directly proportional to the product of the magnitudes of charges and inversely proportional to the square of the distance between them. It is mathematically expressed as

$$F \propto \frac{q_1 q_2}{r^2} \quad \text{or} \quad F = k\frac{q_1 q_2}{r^2} \quad .... \quad (12.1)$$

where F is the magnitude of the mutual force that acts on each of the two point charges q_1, q_2 and r is the distance between them. The force F always acts along the line joining the two point charges (Fig. 12.1), k is the constant of proportionality. Its value depends upon the nature of medium between the two charges and system of units in which F, q and r are measured. If the medium between the two point charges is free space and the system of units is SI, then k is represented as

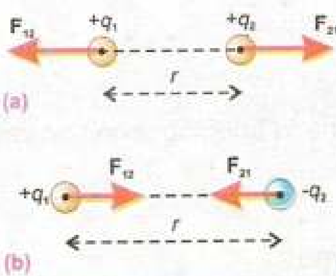


Fig 12.1 (a) Repulsive forces between like charges and (b) attractive forces between unlike charges

$$k = \frac{1}{4\pi\varepsilon_o} \quad .... \quad (12.2)$$

where ε_o is an electrical constant, known as permittivity of free space. In SI units, its value is 8.85×10^{-12} $Nm^{-2}C^{-2}$. Substituting the value of ε_o the constant

$$k = \frac{1}{4\pi\varepsilon_o} = 9 \times 10^9 \ Nm^2 C^{-2}$$

Thus Coulomb's force in free space is

$$F = \frac{1}{4\pi\varepsilon_o}\frac{q_1 q_2}{r^2} \quad .... \quad (12.3)$$

As stated earlier, Coulombs' force is mutual force, it means that if q_1 exerts a force on q_2, then q_2 also exerts an equal and opposite force on q_1. If we denote the force exerted on q_2 by q_1 as $\mathbf{F}_{21}$ and that on charge q_1 due to q_2 as $\mathbf{F}_{12}$, then

$$\mathbf{F}_{12} = -\mathbf{F}_{21} \qquad .... \qquad (12.4)$$

The magnitude of both these two forces is the same and is given by Eq. 12.3. To represent the direction of these forces we introduce unit vectors. If $\hat{\mathbf{r}}_{21}$ is the unit vector directed from q_1 to q_2 and $\hat{\mathbf{r}}_{12}$ is the unit vector directed from q_2 to q_1, then

$$\mathbf{F}_{21} = \frac{1}{4\pi\varepsilon_o}\frac{q_1q_2}{r^2}\hat{\mathbf{r}}_{21} \qquad .... \qquad 12.5\,(a)$$

and

$$\mathbf{F}_{12} = \frac{1}{4\pi\varepsilon_o}\frac{q_1q_2}{r^2}\hat{\mathbf{r}}_{12} \qquad .... \qquad 12.5\,(b)$$

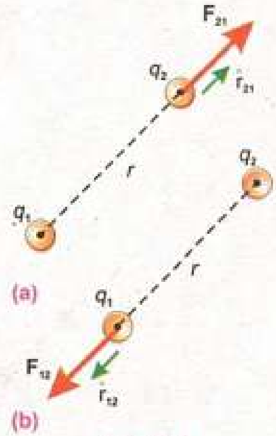


Fig. 12.2

The forces $\mathbf{F}_{21}$ and $\mathbf{F}_{12}$ are shown in Fig.12.2 (a & b). It can be seen that $\hat{\mathbf{r}}_{21} = -\hat{\mathbf{r}}_{12}$, so Eqs. 12.5 (a & b) show that

$$\mathbf{F}_{21} = -\mathbf{F}_{12}$$

The sign of the charges in Eqs. 12.5 (a & b) determine whether the forces are attractive or repulsive.

We shall now consider the effect of medium between the two charges upon the Coulomb's force. If the medium is an insulator, it is usually referred as dielectric. It has been found that the presence of a dielectric always reduces the electrostatic force as compared with that in free space by a certain factor which is a constant for the given dielectric. This constant is known as relative permittivity and is represented by ε_r. The values of relative permittivity of different dielectrics are given in Table 12.1.

Table 2.1

Material	ε_r
Vacuum	1
Air (1 atm)	1.0006
Ammonia (liquid)	22–25
Bakelite	5–18
Benzene	2.284
Germanium	16
Glass	4.8–10
Mica	3–7.5
Paraffined paper	2
Plexiglas	3.40
Rubber	2.94
Teflon	2.1
Transformer oil	2.1
Water (distilled)	78.5

Thus the Coulomb's force in a medium of relative permittivity ε_r is given by

$$F = \frac{1}{4\pi\varepsilon_o\varepsilon_r}\frac{q_1q_2}{r^2} \qquad .... \qquad (12.6)$$

It can be seen in the table that ε_r for air is 1.0006. This value is so close to one that with negligible error, the Eq. 12.3 gives the electric force in air.

Example 12.1: Charges q_1 = 100 µC and q_2 = 50 µC are located in xy-plane at positions $\mathbf{r}_1 = 3.0\hat{j}$ and $\mathbf{r}_2 = 4.0\hat{i}$ respectively, where the distances are measured in metres. Calculate the force on q_2 (Fig. 12.3).

Solution: $q_1 = 100\,\mu C$, $q_2 = 50\,\mu C$

Position vector of q_2 relative to q_1

$$= \mathbf{r}_{21} = \mathbf{r}_2 - \mathbf{r}_1 = 4\hat{i} - 3\hat{j}$$

$$r = \text{magnitude of } \mathbf{r}_{21} = \sqrt{(4\text{m})^2 + (-3\text{m})^2} = 5\text{m}$$

$$\hat{\mathbf{r}}_{21} = \frac{4\hat{i} - 3\hat{j}}{5}$$

$$\mathbf{F}_{21} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}\hat{\mathbf{r}}_{21}$$

$$= \frac{9\times10^9\ \text{Nm}^2\text{C}^{-2} \times 100\times10^{-6}\ \text{C} \times 50\times10^{-6}\ \text{C}}{(5\,\text{m})^2} \times \frac{4\hat{i} - 3\hat{j}}{5}$$

$$= 1.44\hat{i} - 1.08\hat{j}$$

Magnitude of $\mathbf{F}_{21} = F = \sqrt{(1.44)^2 + (-1.08)^2} = 1.8\ \text{N}$

Direction of $\mathbf{F}_{21} = \tan^{-1}\left(\frac{-1.08}{1.44}\right) = -37°$ with x - axis

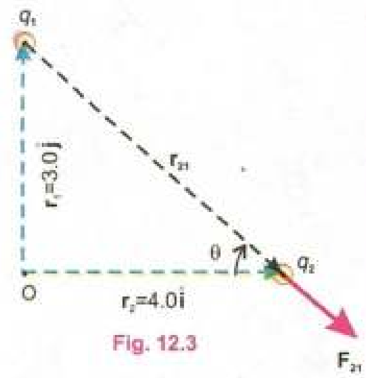


Fig. 12.3

12.2 FIELDS OF FORCE

Newton's universal gravitational law and Coulomb's law enable us to calculate the magnitude as well as the directions of the gravitational and electric forces, respectively. However one may question, (a) What are the origins of these forces? (b) How are these forces transmitted from one mass to another or from one charge to another?

The answer to (a) is still unknown; the existence of these forces is accepted as it is. That is why they are called basic forces of nature.

To describe the mechanism by which electric force is transmitted, Michael Faraday (1791-1867) introduced the concept of an electric field. According to his theory, it is the intrinsic property of nature that an electric field exists in the space around an electric charge. This electric field is considered to be a force field that exerts a force on other charges placed in that field. For example, a charge q produces an electric field in the space surrounding it. This field exists whether the other charges are present in space or not. However, the presence of field cannot be tested until another charge q_o is brought into the field. Thus the field of charge q interacts with q_o to produce an electrical force. The interaction between q and q_o is accomplished in two steps: (a) the charge q produces a field and (b) the field interacts with charge q_o to produce a force **F** on q_o. These two steps are illustrated in Fig. 12.4.

In this figure the density of dots is proportional to the strength of the field at the various points. We may define electric field strength or electric field intensity **E** at any point in the field as

$$\mathbf{E} = \frac{\mathbf{F}}{q_o} \quad \text{..........} \quad (12.7)$$

where **F** is the force experienced by a positive test charge q_o placed at the point. The test charge q_o has to be very small so that it may not distort the field which it has to measure.

Since electric field intensity is force per unit charge, it is measured in newton per coulomb in SI units. It is a vector quantity and its direction is the same as that of the force **F**.

Fig. 12.4 (a) Dots surrounding the positive charge indicate the presence of the electric field. The density of the dots is proportional to the strength of the electric field at different points. (b) Interaction of the field with the charge q_o

The force experienced by a test charge q_o placed in the field of a charge q in vacuum is given by Eq. (12.3). Eq. 12.7 can be used to evaluate electric intensity due to a point charge q at a point distant r from it. Place a positive test charge q_o at this point. The Coulomb's force that this charge will experience due to q is

$$\mathbf{F} = \frac{1}{4\pi\varepsilon_o}\frac{qq_o}{r^2}\hat{\mathbf{r}} \quad \text{..........} \quad (12.8)$$

where $\hat{\mathbf{r}}$ is a unit vector directed from the point charge q to the test point where q_o has been placed, i.e., the point where the electric intensity is to be evaluated. By Eq. 12.7

$$\mathbf{E} = \frac{\mathbf{F}}{q_o} = \frac{1}{4\pi\varepsilon_o}\frac{qq_o}{r^2}\hat{\mathbf{r}} \times \frac{1}{q_o} = \frac{1}{4\pi\varepsilon_o}\frac{q}{r^2}\hat{\mathbf{r}} \quad \text{..........} \quad (12.9)$$

Example 12.2: Two positive point charges q_1 = 16.0 μC and q_2 = 4.0 μC are separated by a distance of 3.0 m, as shown in Fig. 12.5. Find the spot on the line joining the two charges where electric field is zero.

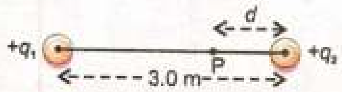


Fig. 12.5

Solution: Between the charges, the two field contribution has opposite directions, and electric field would be zero at a point P, where the magnitude of $\mathbf{E}_1$ equals that of $\mathbf{E}_2$. In the Fig. 12.5, let the distance of P from q_2 be d.

At P, $E_1 = E_2$, which implies that,

$$\frac{1}{4\pi\varepsilon_o}\frac{q_1}{(3.0-d)^2} = \frac{1}{4\pi\varepsilon_o}\frac{q_2}{d^2}$$

or

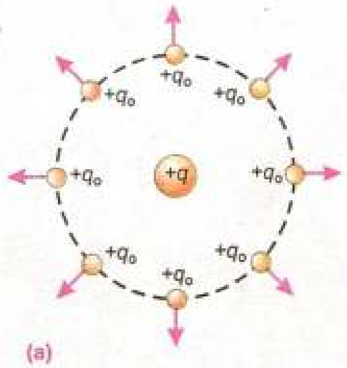
$$\frac{16.0 \times 10^{-6}\,\text{C}}{9+d^2-6d} = \frac{4.0 \times 10^{-6}\,\text{C}}{d^2}$$

or

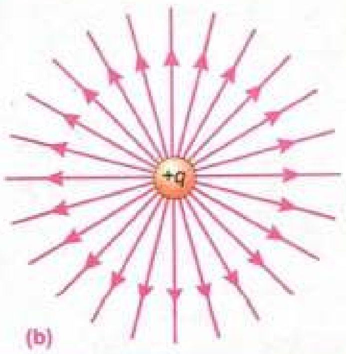
$$d^2 + 2d - 3 = 0, \quad \text{which gives} \quad d = +1\,\text{m}, -3\,\text{m}$$

There are two possible values of d, the negative value corresponds to a location off to the right of both the charges where magnitudes of $\mathbf{E}_1$ and $\mathbf{E}_2$ are equal but directions are same. In this case $\mathbf{E}_1$ and $\mathbf{E}_2$ do not cancel at this spot. The positive value corresponds to the location shown in figure and is the zero field location, hence, $d = +1.0$ m.

12.3 ELECTRIC FIELD LINES



(a)



(b)

Fig. 12.6 (a) A positive test charge $+q_o$ placed anywhere in the vicinity of a positive point charge $+q$, experiences a repulsive force directed radially outward. (b) the electric field lines are directed radially outward from the positive point charge $+q$.

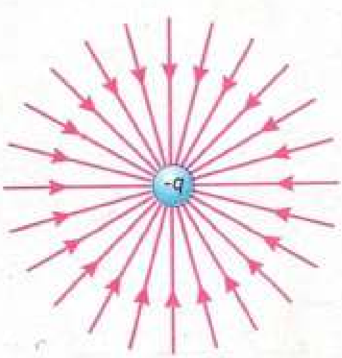


Fig. 12.7 The electric field lines are directed radially inward towards a negative point charge $-q$.

A visual representation of the electric field can be obtained in terms of electric field lines; an idea proposed by Michael Faraday. Electric field lines can be thought of a "map" that provides information about the direction and strength of the electric field at various places. As electric field lines provide information about the electric force exerted on a charge, the lines are commonly called "lines of force".

To introduce electric field lines, we place positive test charges each of magnitude q_o at different places but at equal distances from a positive charge $+q$ as shown in the figure. Each test charge will experience a repulsive force, as indicated by arrows in Fig. 12.6(a). Therefore, the electric field created by the charge $+q$ is directed radially outward. Fig. 12.6 (b) shows corresponding field lines which show the field direction. Fig. 12.7 shows the electric field lines in the vicinity of a negative charge $-q$. In this case the lines are directed radially "inward", because the force on a positive test charge is now of attraction, indicating the electric field points inward.

Figures 12.6 and 12.7 represent two dimensional pictures of the field lines. However, electric field lines emerge from the charges in three dimensions, and an infinite number of lines could be drawn.

The electric field lines "map" also provides information about the strength of the electric field. As we notice in Figs. 12.6 and 12.7 that field lines are closer to each other near the charges where the field is strong while they continuously spread out indicating a continuous decrease in the field strength.

"The number of lines per unit area passing perpendicularly through an area is proportional to the magnitude of the electric field".

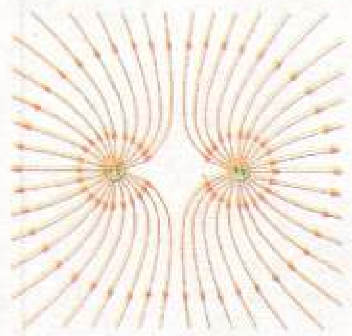


Fig. 12.8 The electric field lines for two identical positive point charges.

The electric field lines are curved in case of two identical separated charges. Fig.12.8 shows the pattern of lines associated with two identical positive point charges of equal magnitude. It reveals that the lines in the region between two like charges seem to repel each other. The behaviour of two identical negatively charges will be exactly the same. The middle region shows the presence of a zero field spot or neutral zone.

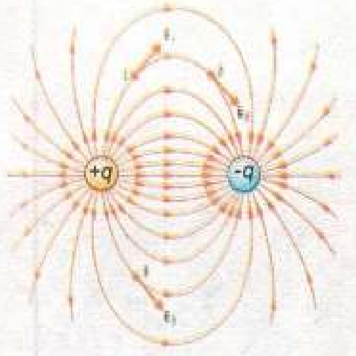


Fig. 12.9 Attractive forces between unlike charges

The Fig.12.9 shows the electric field pattern of two opposite charges of same magnitudes. The field lines start from positive charge and end on a negative charge. The electric field at points such as 1, 2, 3 is the resultant of fields created by the two charges at these points. The directions of the resultant intensities is given by the tangents drawn to the field lines at these points.

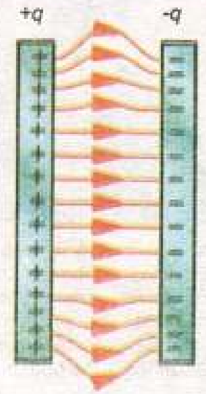


Fig. 12.10 In the central region of a parallel plate capacitor the electric field lines are parallel and evenly spaced, indicating that the electric field there has the same magnitude and direction at all points.

In the regions where the field lines are parallel and equally spaced, the same number of lines pass per unit area and therefore, field is uniform on all points. Fig.12.10 shows the field lines between the plates of a parallel plate capacitor. The field is uniform in the middle region where field lines are equally spaced.

We are now in a position to summarize the properties of electric field lines.

1) Electric field lines originate from positive charges and end on negative charges.
2) The tangent to a field line at any point gives the direction of the electric field at that point.
3) The lines are closer where the field is strong and the lines are farther apart where the field is weak.
4) No two lines cross each other. This is because **E** has only one direction at any given point. If the lines cross, **E** could have more than one direction.

12.4 APPLICATIONS OF ELECTROSTATICS

(i) Xerography (Photocopier)

Fig.12.11 illustrates a photocopy machine. The copying

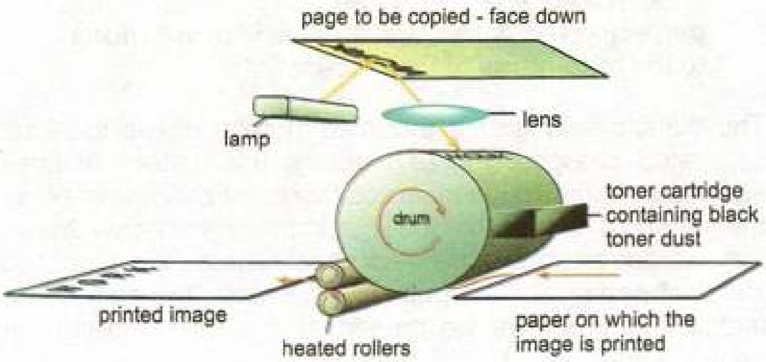


Fig. 12.11 The basics of photocopying. The lamp transfers an image of the page to the drum, which leaves a static charge. The drum collects toner dust and transfers it to the paper. The toner is melted onto the page.

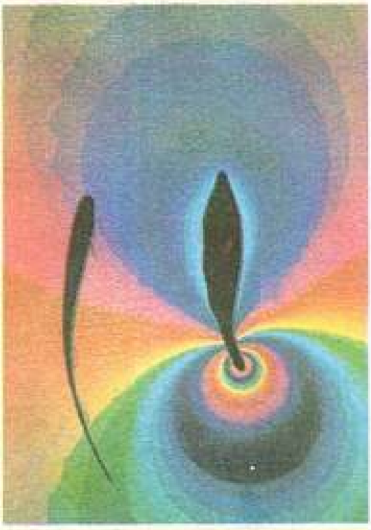
process is called xerography, from the Greek word "xeros" and "graphos", meaning "dry writing". The heart of machine is a drum which is an aluminium cylinder coated with a layer of selenium. Aluminium is an excellent conductor. On the other hand, selenium is an insulator in the dark and becomes a conductor when exposed to light; it is a photoconductor. As a result, if a positive charge is sprinkled over the selenium it will remain there as long as it remains in dark. If the drum is exposed to light, the electrons from aluminium pass through the conducting selenium and neutralize the positive charge.

If the drum is exposed to an image of the document to be copied, the dark and light areas of the document produce corresponding areas on the drum. The dark areas retain their positive charge, but light areas become conducting, lose their positive charge and become neutral.

In this way, a positive charge image of the document remains on the selenium surface. Then a special dry, black powder called "toner" is given a negative charge and spread over the drum, where it sticks to the positive charged areas.

The toner from the drum is transferred on to a sheet of paper on which the document is to be copied. Heated pressure

For Your Information



This computer image shows the electric field lines generated by the fish at the top of the picture. Through the electric field, the presence of other fish can be detected, such as the one silhouetted at the bottom.

rollers then melt the toner into the paper which is also given an excess positive charge to produce the permanent impression of the document.

(ii) Inkjet Printers

An inkjet printer (Fig. 12.12 a) is a type of printer which uses electric charge in its operation. While shuttling back and forth across the paper, the inkjet printer "ejects" a thin stream of ink. The ink is forced out of a small nozzle and breaks up into extremely small droplets. During their flight, the droplets pass through two electrical components, a "charging electrode" and the "deflection plates" (a parallel plate capacitor). When the printhead moves over regions of the paper which are not to be inked, the charging electrode is left on and gives the ink droplets a net charge. The deflection plates divert such charged drops into a gutter and in this way such drops are not able to reach the paper. Whenever ink is to be placed on the paper, the charging control, responding to computer, turns off the charging electrode. The uncharged droplets fly straight through the deflection plates and strike the paper. Schematic diagram of such a printer is shown by Fig. 12.12 (b).

Fig. 12.12 (a) An inkjet printer.

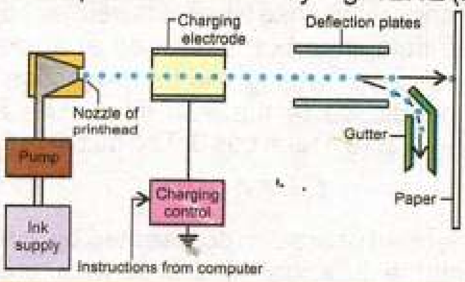


Fig. 12.12 (b) An inkjet printhead ejects a steady flow of ink droplets. The charging electrodes are used to charge the droplets that are not needed on the paper. Charged droplets are deflected into a gutter by the deflection plates, while uncharged droplets fly straight onto the paper.

Inkjet printers can also produce coloured copies.

12.5 ELECTRIC FLUX

When we place an element of area in an electric field, some of the lines of force pass through it (Fig. 12.13 a).

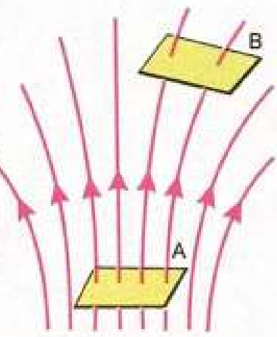


Fig. 12.13 (a) Electric flux through a surface normal to E.

The number of the field lines passing through a certain element of area is known as electric flux through that area. It is usually denoted by Greek letter Φ. For example the flux Φ through the area A in Fig. 12.13 (a) is 4 while the flux through B is 2.

In order to give a quantitative meaning to flux, the field lines are drawn such that the number of field lines passing through

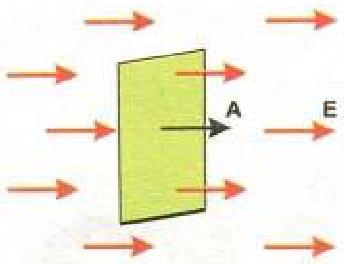


Fig. 12.13 (b) Maximum

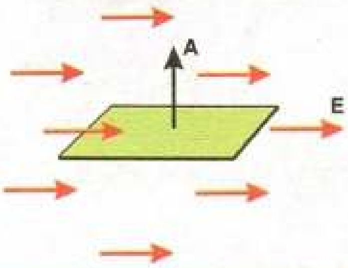


Fig. 12.13 (c) Minimum

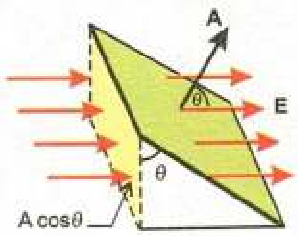


Fig.12.13 (d)

a unit area held perpendicular to field lines at a point represent the intensity **E** of the field at that point. Suppose at a given point the value of **E** is $4NC^{-1}$. This means that if $1m^2$ area is held perpendicular to field lines at this point, 4 field lines will pass through it. In order to establish relation between electric flux Φ, electric intensity **E** and area **A** we consider the Fig.12.13 (b,c,d) which shows the three dimensional representation of the electric field lines due to a uniform electric field of intensity **E**.

In Fig.12.13 (b), area is held perpendicular to the field lines, then $EA_{\perp}$ lines pass through it. The flux Φ_e in this case is

$$\Phi_e = EA_{\perp} \quad .... \quad (12.10)$$

where $A_{\perp}$ denotes that the area is held perpendicular to field lines. In Fig.12.13 (c), area A is held parallel to field lines and, as is obvious no lines cross this area, so that flux Φ_e in this case is

$$\Phi_e = EA_{\parallel} = 0 \quad .... \quad (12.11)$$

where $A_{\parallel}$ indicates that A is held parallel to the field lines. Fig.12.13 (d) shows the case when A is neither perpendicular nor parallel to field lines but is inclined at angle θ with the lines. In this case we have to find the projection of the area which is perpendicular to the field lines. The area of this projection, (Fig. 12.13 d) is $A\cos\theta$. The flux Φ in this case is

$$\Phi_e = EA\cos\theta$$

Usually the element of area is represented by a vector area **A** whose magnitude is equal to the surface area A of the element and whose direction is direction of normal to the area. The electric flux Φ_e through a patch of flat surface in terms of **E** and **A** is then given by

$$\Phi_e = EA\cos\theta = \mathbf{E.A} \quad .... \quad (12.12)$$

where θ is the angle between the field lines and the normal to the area.

Electric flux, being a scalar product, is a scalar quantity. Its SI unit is Nm^2C^{-1}.

12.6 ELECTRIC FLUX THROUGH A SURFACE ENCLOSING A CHARGE

Let us calculate the electric flux through a closed surface, in shape of a sphere of radius r due to a point charge q

placed at the centre of sphere as shown in Fig. 12.14. To apply the formula $\Phi_e = \mathbf{E}.\mathbf{A}$ for the computation of electric flux, the surface area should be flat. For this reason the total surface area of the sphere is divided into n small patches with areas of magnitudes $\Delta A_1, \Delta A_2, \Delta A_3, \ldots\ldots \Delta A_n$. If n is very large, each patch would be a flat element of area. The corresponding vector areas are $\Delta\mathbf{A}_1, \Delta\mathbf{A}_2, \Delta\mathbf{A}_3, \ldots\ldots \Delta\mathbf{A}_n$ respectively. The direction of each vector area is along perpendicular drawn outward to the corresponding patch. The electric intensities at the centres of vector areas $\Delta\mathbf{A}_1, \Delta\mathbf{A}_2, \ldots\ldots \Delta\mathbf{A}_n$ are $\mathbf{E}_1, \mathbf{E}_2, \ldots\ldots \mathbf{E}_n$ respectively.

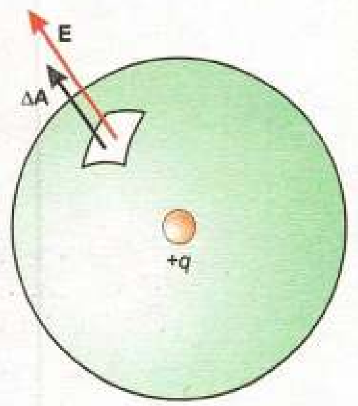


Fig. 12.14 The total electric flux through the surface of the sphere due to a charge q at its centre is q/ε_o.

According to Eq.12.12, the total flux passing through the closed surface is

$$\Phi_e = \mathbf{E}_1.\Delta\mathbf{A}_1 + \mathbf{E}_2.\Delta\mathbf{A}_2 + \mathbf{E}_3.\Delta\mathbf{A}_3 + \ldots\ldots + \mathbf{E}_n.\Delta\mathbf{A}_n \quad \ldots\ldots \quad (12.13)$$

The direction of electric intensity and vector area is same at each patch. Moreover, because of spherical symmetry, at the surface of sphere,

$$|\mathbf{E}_1| = |\mathbf{E}_2| = |\mathbf{E}_3| = \ldots\ldots = |\mathbf{E}_n| = E = \frac{1}{4\pi\varepsilon_o}\frac{q}{r^2} \quad \ldots\ldots \quad (12.14)$$

$$\Phi_e = E\Delta A_1 + E\Delta A_2 + E\Delta A_3 + \ldots\ldots + E\Delta A_n$$
$$= E \times (\Delta A_1 + \Delta A_2 + \Delta A_3 + \ldots\ldots + \Delta A_n)$$
$$= E \times (\text{total spherical surface area})$$
$$= \frac{1}{4\pi\varepsilon_o}\frac{q}{r^2} \times 4\pi r^2$$

$$\Phi_e = \frac{q}{\varepsilon_o} \quad \ldots\ldots\ldots \quad (12.15)$$

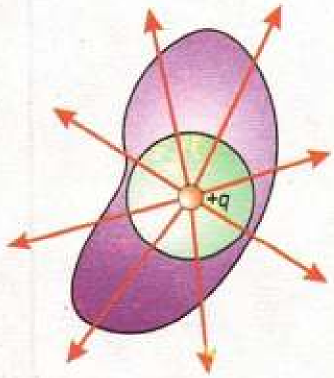


Fig. 12.15

Now imagine that a closed surface S is enclosing this sphere. It can be seen in Fig.12.15 that the flux through the closed surface S is the same as that through the sphere. So we can conclude that total flux through a closed surface does not depend upon the shape or geometry of the closed surface. It depends upon the medium and the charge enclosed.

12.7 GAUSS'S LAW

Suppose point charges $q_1, q_2, q_3, \ldots\ldots, q_n$ are arbitrarily distributed in an arbitrary shaped closed surface as shown in Fig. 12.16. Using idea given in previous section, the electric flux passing through the closed surface is

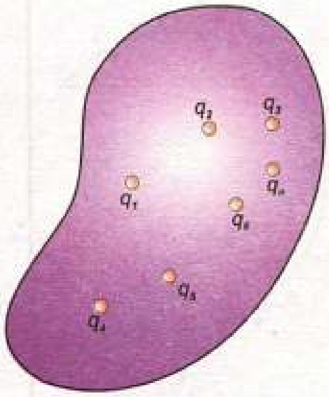


Fig. 12.16

$$\Phi_e = \frac{q_1}{\varepsilon_o} + \frac{q_2}{\varepsilon_o} + \frac{q_3}{\varepsilon_o} + ... + \frac{q_n}{\varepsilon_o}$$

$$\Phi_e = \frac{1}{\varepsilon_o} x (q_1 + q_2 + q_3 + ... + q_n)$$

$$\Phi_e = \frac{1}{\varepsilon_o} \text{ x (total charge enclosed by closed surface)}$$

$$\Phi_e = \frac{1}{\varepsilon_o} x Q \quad .... \quad (12.16)$$

where $Q = q_1 + q_2 + q_3 + ... + q_n$, is the total charge enclosed by closed surface. Eq.12.16 is mathematical expression of Gauss's law which can be stated as,

"The flux through any closed surface is $1/\varepsilon_o$ times the total charge enclosed in it".

12.8 APPLICATIONS OF GAUSS'S LAW

Gauss's law is applied to calculate the electric intensity due to different charge configurations. In all such cases, an imaginary closed surface is considered which passes through the point at which the electric intensity is to be evaluated. This closed surface is known as Gaussian surface. Its choice is such that the flux through it can be easily evaluated. Next the charge enclosed by Gaussian surface is calculated and finally the electric intensity is computed by applying Gauss's law Eq.12.16. We will illustrate this procedure by considering some examples.

(a) Intensity of Field Inside a Hollow Charged Sphere

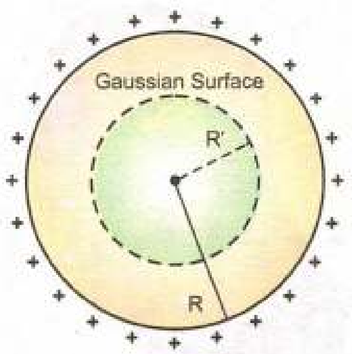


Fig. 12.17

Suppose that a hollow conducting sphere of radius R is given a positive charge q. We wish to calculate the field intensity first at a point inside the sphere.

Now imagine a sphere of radius $R' < R$ to be inscribed within the hollow charged sphere as shown in Fig. 12.17. The surface of this sphere is the Gaussian surface. Let Φ be flux through this closed surface. It can be seen in the figure that the charge enclosed by the Gaussian surfaces is zero. Applying Gaussian law, we have

$$\Phi_e = \frac{q}{\varepsilon_o} = 0$$

Since $\Phi_e = \mathbf{E.A} = 0$ as $\mathbf{A} \neq 0$, therefore, $\mathbf{E} = 0$

Thus the interior of a hollow charged metal sphere is a field free region. As a consequence, any apparatus placed within a metal enclosure is "shielded" from electric fields.

Do You Know?

To eliminate stray electric field interference, circuits of sensitive electronics devices such as T.V and Computers are often enclosed within metal boxes.

(b) Electric Intensity Due to an Infinite Sheet of Charge

Suppose we have a plane sheet of infinite extent on which positive charges are uniformly distributed. The uniform surface charge density is, say, σ. A finite part of this sheet is shown in Fig. 12.18. To calculate the electric intensity E at a point P, close to the sheet, imagine a closed Gaussian surface in the form of a cylinder passing through the sheet, whose one flat face contains point P. From symmetry we can conclude that **E** points at right angle to the end faces and away from the plane. Since **E** is parallel to the curved surface of the cylinder, so there is no contribution to flux from the curved wall of the cylinder. While it will be, $EA + EA = 2\,EA$, through the two flat end faces of the closed cylindrical surface, where A is the surface area of the flat faces (Fig. 12.18). As the charge enclosed by the closed surface is σA, therefore, according to Gauss's law,

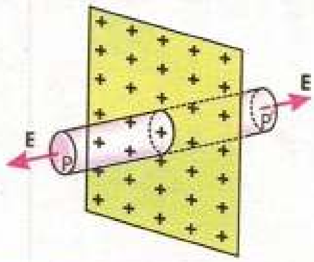


Fig. 12.18 The closed surface is in the form of a cylinder whose one face contains the point P at which electric intensity has to be determined.

$$\Phi_e = \frac{1}{\varepsilon_o} \text{ x charge enclosed by closed surface}$$

$$\Phi_e = \frac{1}{\varepsilon_o} \times \sigma A \quad \ldots\ldots\ldots \quad (12.17)$$

Therefore, $$2EA = \frac{1}{\varepsilon_o} \times \sigma A$$

or $$E = \frac{\sigma}{2\varepsilon_o} \quad \ldots\ldots\ldots \quad (12.18)$$

In vector form, $$\mathbf{E} = \frac{\sigma}{2\varepsilon_o}\hat{\mathbf{r}} \quad \ldots\ldots\ldots \quad (12.19)$$

where $\hat{\mathbf{r}}$ is a unit vector normal to the sheet directed away from it.

(c) Electric Intensity Between Two Oppositely Charged Parallel Plates

Suppose that two parallel and closely spaced metal plates of infinite extent separated by vacuum are given opposite charges. Under these conditions the charges are essentially concentrated on the inner surfaces of the plates. The field lines which originate on positive charges on the inner face of one plate, terminate on negative charges on the inner face of the other plate (Fig. 12.19). Thus the charges

Fig. 12.19 The lines of force between the plates are normal to the plates and are directed from the positive plate towards the negative one.

are uniformly distributed on the inner surface of the plate in a form of sheet of charges of surface density $\sigma = q/A$, where A is the area of plate and q is the amount of charge on either of the plates.

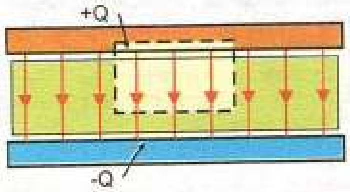


Fig. 12.20 Dotted rectangle represents the cross section of Gaussian box with its top inside the upper metal plate and its bottom in the dielectric between the plates.

Imagine now, a Gaussian surface in the form of a hollow box with its top inside the upper metal plate and its bottom in the space between the plates as shown in Fig.12.20. As the field lines are parallel to the sides of the box, therefore, the flux through the sides is zero. The field lines are uniformly distributed on the lower bottom face and are directed normally to it. If A is the area of this face and **E** the electric intensity at its site, the flux through it would be EA. There is no flux through the upper end of the box because there is no field inside the metal plate. Thus the total flux Φ_e through the Gaussian surface is EA. The charge enclosed by the Gaussian surface is σA. Applying Gauss's law

$$\Phi_e = \frac{1}{\varepsilon_o} \times \sigma A$$

or

$$EA = \frac{1}{\varepsilon_o} \times \sigma A$$

or

$$E = \frac{\sigma}{\varepsilon_o} \quad \text{..........} \quad (12.20)$$

The field intensity is the same at all points between the plates. The direction of field is from positive to negative plate because a unit positive charge anywhere between the plates would be repelled from positive and attracted to negative plate and these forces are in the same direction. In vector form

$$\mathbf{E} = \frac{\sigma}{\varepsilon_o}\hat{\mathbf{r}} \quad \text{..........} \quad (12.21)$$

where $\hat{\mathbf{r}}$ is a unit vector directed from positive to negative plate.

12.9 ELECTRIC POTENTIAL

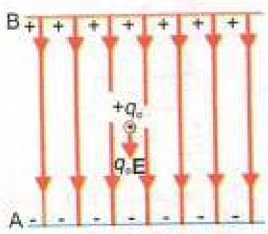


Fig. 12.21 (a)

Let us consider a positive charge q_o which is allowed to move in an electric field produced between two oppositely charged parallel plates as shown in Fig. 12.21 (a). The positive charge will move from plate B to A and will gain K.E. If it is to be moved from A to B, an external force is needed to make the charge move against the electric field and will gain P.E. Let us impose a condition that as the charge is moved from A to B, it is moved keeping electrostatic equilibrium, i.e., it moves with uniform velocity. This condition could be achieved by applying a force **F** equal and opposite to $q_o\mathbf{E}$ at every point along its path as shown in Fig. 12.21 (b). The work done by the external force against the electric field increases electrical potential energy of the charge that is moved.

Let W_{AB} be the work done by the force in carrying the positive charge q_o from A to B while keeping the charge in equilibrium. The change in its potential energy $\Delta U = W_{AB}$

or $$U_B - U_A = W_{AB} \qquad \text{..........} \qquad (12.22)$$

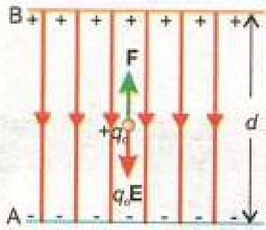


Fig. 12.21 (b)

where U_A and U_B, are defined to be the potential energies at points A and B, respectively.

To describe electric field we introduce the idea of electric potential difference. The potential difference between two points A and B in an electric field is defined as the work done in carrying a unit positive charge from A to B while keeping the charge in equilibrium, that is,

$$\Delta V = V_B - V_A = \frac{W_{AB}}{q_o} = \frac{\Delta U}{q_o} \qquad \text{..........} \qquad (12.23)$$

where V_A and V_B are defined electric potentials at point A and B respectively. Electric potential energy difference and electric potential difference between the points A and B are related as

$$\Delta U = q_o \Delta V = W_{AB} \qquad \text{..........} \qquad (12.24)$$

Thus the potential difference between the two points can be defined as the difference of the potential energy per unit charge.

As the unit of P.E. is joule, Eq.12.23 shows that the unit of potential difference is joule per coulomb. It is called volt such that,

$$1\,\text{volt} = \frac{1\,\text{joule}}{1\,\text{coulomb}} \qquad \text{..........} \qquad (12.25)$$

That is, a potential difference of 1 volt exists between two points if work done in moving a unit positive charge from one point to other, keeping equilibrium, is one joule.

In order to give a concept of electric potential at a point in an electric field, we must have a reference to which we assign zero electric potential. This point is usually taken at infinity. Thus in Eq. 12.23, if we take A to be at infinity and choose V_A=0, the electric potential at B will be $V_B = W_{\infty B}/q_o$ or dropping the subscripts.

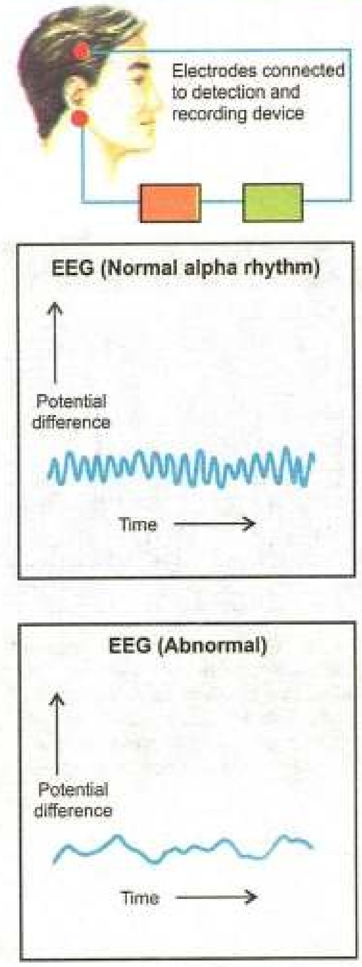
$$V = \frac{W}{q_o} \qquad \text{..........} \qquad (12.26)$$

Do You Know?

An ECG records the "voltage" between points on human skin generated by electrical process in the heart. This ECG is made in running position providing information about the heart's performance under stress.

which states that the electric potential at any point in an electric field is equal to work done in bringing a unit positive charge from infinity to that point keeping it in equilibrium. It is to be noted that potential at a point is still potential difference between the potential at that point and potential at infinity. Both potential and potential differences are scalar quantities because both W and q_o are scalars.

Do You Know?



In electroencephalography the potential differences created by the electrical activity of the brain are used for diagnosing abnormal behaviour.

Electric Field as Potential Gradient

In this section we will establish a relation between electric intensity and potential difference. As a special case, let us consider the situation shown in Fig. 12.21 (b). The electric field between the two charged plates is uniform, let its value be E. The potential difference between A and B is given by the equation

$$V_B - V_A = \frac{W_{AB}}{q_o} \quad .... \quad (12.27)$$

where $W_{AB} = Fd = -q_oEd$ (the negative sign is needed because F must be applied opposite to q_oE so as to keep it in equilibrium). With this, Eq. 12.27 becomes

$$V_B - V_A = -\frac{q_oEd}{q_o} = -Ed$$

or

$$E = -\frac{(V_B - V_A)}{d} = -\frac{\Delta V}{d} \quad .... \quad (12.28)$$

If the plates A & B are separated by infinitesimally small distance Δr, the Eq. 12.28 is modified as

$$E = -\frac{\Delta V}{\Delta r} \quad .... \quad (12.29)$$

The quantity $\frac{\Delta V}{\Delta r}$ gives the maximum value of the rate of change of potential with distance because the charge has been moved along a field line along which the distance Δr between the two plates is minimum. It is known as potential gradient. Thus the electric intensity is equal to the negative of the gradient of potential. The negative sign indicates that the direction of E is along the decreasing potential.

The unit of electric intensity from Eq. 12.29 is volt/metre which is equal to NC^{-1} as shown below:

$$1\frac{\text{volt}}{\text{metre}} = 1\frac{\text{joule/coulomb}}{\text{metre}} = 1\frac{\text{newton x metre}}{\text{metre x coulomb}} = 1\frac{\text{newton}}{\text{coulomb}}$$

Electric Potential at a Point due to a Point Charge

Let us derive an expression for the potential at a certain point in the field of a positive point charge q. This can be accomplished by bringing a unit positive charge from infinity to that point keeping the charge in equilibrium. The target can be achieved using Eq. 12.29 in the form $\Delta V = -E\,\Delta r$, provided electric intensity E remains constant. However in this case E varies inversely as square of distance from the point charge, it no more remains constant so we use basic principles to compute the electric potential at a point. The field is radial as shown in Fig. 12.22.

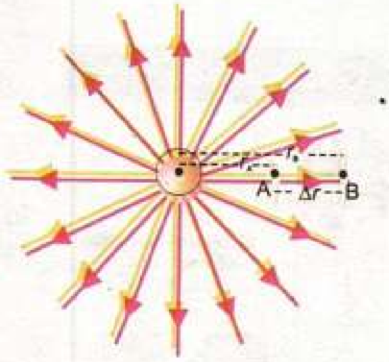


Fig. 12.22

Let us take two points A and B, infinitesimally close to each other, so that E remains almost constant between them. The distance of points A and B from q are r_A and r_B respectively and distance of midpoint of space interval between A and B is r from q. Then according to Fig. 12:22,

$$r_B = r_A + \Delta r \quad \text{..........} \quad (12.30)$$

$$\Delta r = r_B - r_A \quad \text{..........} \quad (12.31)$$

As r represents mid point of interval between A and B so

$$r = \frac{r_A + r_B}{2} \quad \text{..........} \quad (12.32)$$

The magnitude of electric intensity at this point is,

$$E = \frac{1}{4\pi\varepsilon_o}\frac{q}{r^2} \quad \text{..........} \quad (12.33)$$

As the points A and B are very close then, as a first approximation, we can take the arithmetic mean to be equal to geometric mean which gives

$$\frac{r}{r_A} = \frac{r_B}{r}$$

Therefore, $$r^2 = r_A r_B \quad \text{..........} \quad (12.34)$$

Thus, Eq.12.33 can be written as

$$E = \frac{1}{4\pi\varepsilon_o}\frac{q}{r_A r_B} \quad \text{..........} \quad (12.35)$$

Now, if a unit positive charge is moved from B to A, the work done is equal to the potential difference between A and B,

$$V_A - V_B = -E(r_A - r_B)$$

Do You Know?

Fish and other sea creatures produce electric fields in a variety of ways. Sharks have special organs, called the ampullae of Lorenzini, that are very sensitive to electric field and can detect potential difference of the order of nanovolt and can locate their prey very precisely.

$$V_A - V_B = E(r_B - r_A) \quad .... \quad (12.36)$$

Substituting value of E from Eq.12.35,

$$V_A - V_B = \frac{q}{4\pi\varepsilon_o}\left(\frac{r_B - r_A}{r_A r_B}\right) \quad .... \quad (12.37)$$

$$V_A - V_B = \frac{q}{4\pi\varepsilon_o}\left(\frac{1}{r_A} - \frac{1}{r_B}\right) \quad .... \quad (12.38)$$

To calculate absolute potential or potential at A, point B is assumed to be infinity point so that $V_B = 0$ and hence

$$\frac{1}{r_B} = \frac{1}{r_\infty} = \frac{1}{\infty} = 0$$

This gives, $$V_A = \frac{1}{4\pi\varepsilon_o}\frac{q}{r_A} \quad .... \quad (12.39)$$

The general expression for electric potential V_r, at a distance r from q is,

$$V_r = \frac{1}{4\pi\varepsilon_o}\frac{q}{r} \quad .... \quad (12.40)$$

Example 12.3: Two opposite point charges, each of magnitude q are separated by a distance $2d$. What is the electric potential at a point P mid-way between them?

Solution:

$$V^+ = \frac{1}{4\pi\varepsilon_o}\frac{q}{d} \quad \& \quad V^- = -\frac{1}{4\pi\varepsilon_o}\frac{q}{d}$$

$$V = V^+ + V^- = \frac{1}{4\pi\varepsilon_o}\frac{q}{d} - \frac{1}{4\pi\varepsilon_o}\frac{q}{d} = 0$$

So potential at P due to opposite charges is zero.

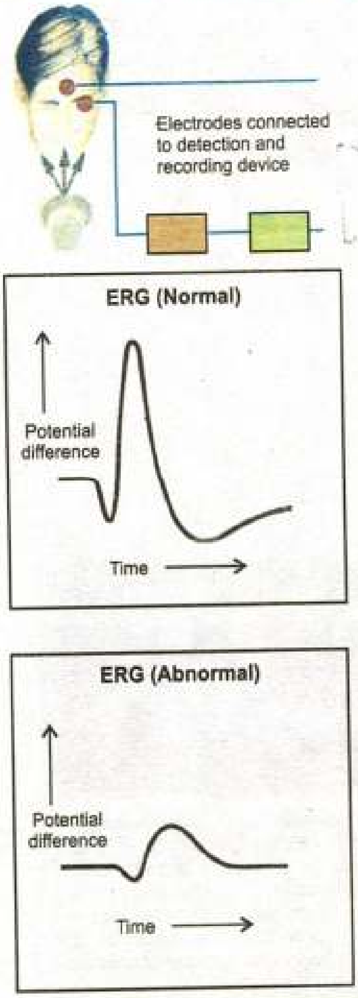
12.10 ELECTRON VOLT

Referring Fig. 12.21, we know that when a particle of charge q moves from point A with potential V_A to a point B with potential V_B, keeping electrostatic equilibrium, the change in potential energy ΔU of particle is,

$$\Delta U = q(V_B - V_A) = q\Delta V \quad .... \quad (12.41)$$

If no external force acts on the charge to maintain equilibrium, this change in P.E. appears in the form of change in K.E. Suppose charge carried by the particle is $q = e = 1.6 \times 10^{-19}$ C.

Do You Know?



The electrical activity of the retina of the eye generates the potential differences used in electroretinography.

Thus, in this case, the energy acquired by the charge will be

$$\Delta K.E = q\Delta V = e\Delta V = (1.6 \times 10^{-19}C)(\Delta V)$$

Moreover, assume that $\Delta V = 1$ volt, hence

$$\Delta K.E = q\Delta V = (1.6 \times 10^{-19}C) \times (1 \text{ volt})$$

$$\Delta K.E = (1.6 \times 10^{-19}) \times (C \times V) = 1.6 \times 10^{-19} J$$

The amount of energy equal to 1.6×10^{-19} J is called one electron-volt and is denoted by 1eV. It is defined as "the amount of energy acquired or lost by an electron as it traverses a potential difference of one volt". Thus,

$$1 eV = 1.6 \times 10^{-19} J \quad .... \quad (12.42)$$

Example 12.4: A particle carrying a charge of 2e falls through a potential difference of 3.0V. Calculate the energy acquired by it.

Solution: $q = 2e$, $\Delta V = 3.0 V$

The energy acquired by the particle is

$$\Delta K.E = q\Delta V = (2e)(3.0 V) = 6.0 eV$$

$$= 6.0 \times 1.6 \times 10^{-19} J = 9.6 \times 10^{-19} J$$

12.11 ELECTRIC AND GRAVITATIONAL FORCES (A COMPARISON)

In chapter 4, we pointed out that gravitational force is a conservative force, that is, work done in such a field is independent of path. It can also be proved that Coulomb's electrostatic force is also conservative force. The electric force between two charges $F = \frac{q_1 q_2}{4\pi\varepsilon_o r^2}$, is similar in form to the gravitational force between the two point masses, $F = G\frac{m_1 m_2}{r^2}$. Both forces vary inversely with the square of the distance between the two charges or the two masses. However, the value of gravitational constant G is very small as compared to electrical constant $\frac{1}{4\pi\varepsilon_o}$. It is because of this fact that the gravitational force is a very weak force as compared to electrostatic force. As regards their qualitative aspect, the electrostatic force could be attractive or repulsive while, on the other hand, gravitational force is only attractive. Another difference to be noted is that the electrostatic force is medium dependant and can be shielded while gravitational force lacks this property.

12.12 CHARGE ON AN ELECTRON BY MILLIKAN'S METHOD

In 1909, R.A Millikan devised a technique that resulted in precise measurement of the charge on an electron.

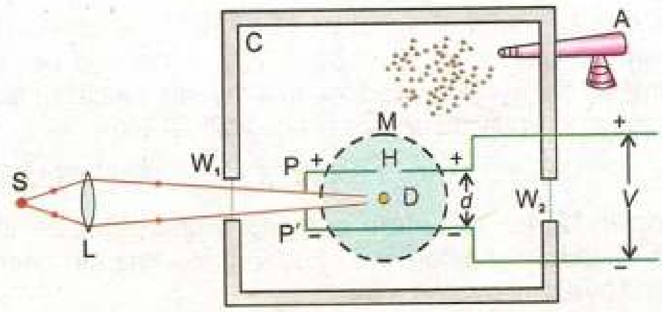


Fig. 12.23 (a)

A schematic diagram of the Millikan oil drop experiment is shown in Fig. 12.23 (a). Two parallel plates PP′ are placed inside a container C, to avoid disturbances due to air currents. The separation between the plates is *d*. The upper plate P has a small hole H, as shown in the figure. A voltage *V* is applied to the plates due to which the electric field *E* is setup between the plates. The magnitude of its value is $E = V/d$. An atomizer A is used for spraying oil drops into the container through a nozzle. The oil drop gets charged because of friction between walls of atomizer and oil drops. These oil drops are very small, and are actually in the form of mist. Some of these drops happen to pass through the hole in the upper plate. The space between the plates is illuminated by the light coming from the source S through the lens L and window W_1. The path of motion of these drops can be carefully observed by a microscope M.

Fig. 12.23 (b) Oil drop balanced by the gravitational force and the Coulomb force.

A given droplet between the two plates could be suspended in air if the gravitational force $F_g = mg$ acting on the drop is equal to the electrical force $F_e = qE$, as shown in Fig. 12.23(b). The F_e can be adjusted equal to F_g by adjusting the voltage. In this case, we can write,

$$F_e = F_g \quad \ldots\ldots \quad (12.43)$$

or $$qE = mg$$

If *V* is the value of p. d. between the plates for this setting, then

$$E = \frac{V}{d} \quad , \qquad \text{we may write} \quad q\frac{V}{d} = mg$$

$$q = \frac{mgd}{V} \qquad \text{..........} \qquad (12.44)$$

In order to determine the mass m of the droplet, the electric field between the plates is switched off. The droplet falls under the action of gravity through air. It attains terminal speed v_t almost at the instant the electric field is switched off. Its terminal speed v_t is determined by timing the fall of the droplet over a measured distance. Since the drag force **F** due to air acting upon the droplet when it is falling with constant terminal speed is equal to its weight. Hence, using Stokes's law

$$F = 6\pi\eta r v_t = mg$$

where r is the radius of the droplet and η is the coefficient of viscosity for air. If ρ is the density of the droplet, then

$$m = \frac{4}{3}\pi r^3 \rho \qquad \text{..........} \qquad (12.45)$$

Hence, $\quad \frac{4}{3}\pi r^3 \rho g = 6\pi\eta r v_t$

or $\quad r^2 = \frac{9\eta v_t}{2\rho g}$

Knowing the value of r, the mass m can be calculated by using Eq. (12.45). This value of m is substituted in Eq.12.44 to get the value of charge q on the droplet.

Millikan measured the charge on many drops and found that each charge was an integral multiple of a minimum value of charge equal to 1.6×10^{-19} C. He, therefore, concluded that this minimum value of the charge is the charge on an electron.

Example 12.5: In Millikan oil drop experiment, an oil drop of mass 4.9×10^{-15} kg is balanced and held stationary by the electric field between two parallel plates. If the potential difference between the plates is 750 V and the spacing between them is 5.0 mm, calculate the charge on the droplet. Assume $g = 9.8\ \text{ms}^{-2}$.

Solution:

Mass of drop = $m = 4.9 \times 10^{-15}$ kg

Potential difference = $V = 750$ V

Spacing between plates = $d = 5.0$ mm $= 5.0 \times 10^{-3}$ m

For the droplet of the charge q, we have

$$q = \frac{mgd}{V} = \frac{4.9 \times 10^{-15}\ \text{kg} \times 9.8\ \text{ms}^{-2} \times 5.0 \times 10^{-3}\ \text{m}}{750\ \text{V}}$$

$$q = 3.2 \times 10^{-19}\ \text{C}$$

12.13 CAPACITOR

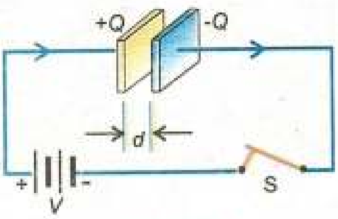


Fig. 12.24

A capacitor is a device that can store charge. It consists of two conductors placed near one another separated by vacuum, air or any other insulator, known as dielectric. Usually the conductors are in the form of parallel plates, and the capacitor is known as parallel plate capacitor. When the plates of such a capacitor are connected to a battery of voltage V (Fig.12.24), it establishes a potential difference of V volts between the two plates and the battery places a charge $+Q$ on the plate connected with its positive terminal and a charge $-Q$ on the other plate, connected to its negative terminal. let Q be the magnitude of the charge on either of the plates. It is found that

$$Q \propto V \quad \text{or} \quad Q = CV \quad \text{or} \quad C = \frac{Q}{V} \qquad \ldots\ldots \quad (12.46)$$

The proportionality constant C is called the capacitance of the capacitor. As we shall see later, it depends upon the geometry of the plates and the medium between them. It is a measure of the ability of capacitor to store charge. The capacitance of a capacitor can be defined as the amount of charge on one plate necessary to raise the potential of that plate by one volt with respect to the other. The SI unit of capacitance is coulomb per volt, which because of its frequent use, is commonly called farad (F), after the famous English scientist Faraday.

"The capacitance of a capacitor is one farad if a charge of one coulomb, given to one of the plates of a parallel plate capacitor, produces a potential difference of one volt between them".

For Your Information

One farad is an enormous amount of capacitance. For practical purposes its sub-multiple units are used which are given below,

1 micro-farad = 1 μF = 10^{-6} farad

1 pico-farad = 1 pF = 10^{-12} farad

12.14 CAPACITANCE OF A PARALLEL PLATE CAPACITOR

Consider a parallel plate capacitor consisting of two plane metal plates, each of area A, separated by a distance d as shown in Fig. 12.24. The distance d is small so that the electric field E between the plates is uniform and confined almost entirely in the region between the plates. Let initially the medium between the plates be air or vacuum. Then according to Eq.12.46,

$$C_{vac} = \frac{Q}{V} \quad \text{..........} \quad (12.47)$$

where Q is the charge on the capacitor and V is the potential difference between the parallel plates. The magnitude E of electric intensity is related with the distance d by Eq.12.28 as

$$E = \frac{V}{d} \quad \text{..........} \quad (12.48)$$

As Q is the charge on either of the plates of area A, the surface density of charge on the plates is as

$$\sigma = \frac{Q}{A}$$

As already shown in section 12.8, the electric intensity between two oppositely charged plates is given by $E = \frac{\sigma}{\varepsilon_o}$. Substituting the value of σ, we have

$$\frac{V}{d} = \frac{Q}{A\varepsilon_o} \quad \text{..........} \quad (12.49)$$

It gives $$C_{vac} = \frac{Q}{V} = \frac{A\varepsilon_o}{d} \quad \text{..........} \quad (12.50)$$

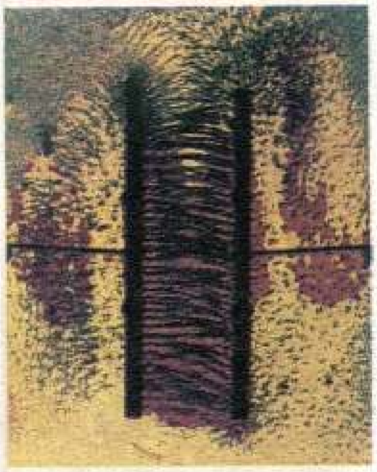
If an insulating material, called dielectric, of relative permittivity ε_r is introduced between the plates, the capacitance of capacitor is enhanced by the factor ε_r. Capacitors commonly have some dielectric medium, thereby ε_r is also called as dielectric constant.

Following experiment gives the effect of insertion of dielectric between the plates of a capacitor.

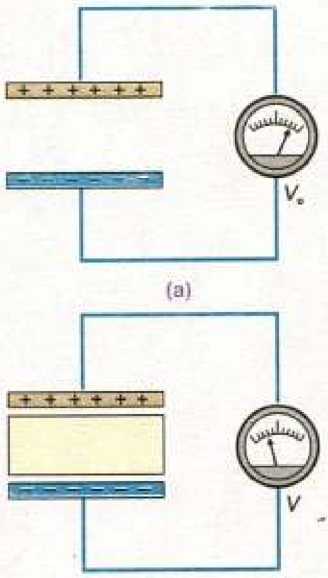
Consider a charged capacitor whose plates are connected to a voltmeter (Fig. 12.25 a). The deflection of the meter is a measure of the potential difference between the plates. When a dielectric material is inserted between the plates, reading drops indicating a decrease in the potential difference between the plates (Fig. 12.25 b). From the definition, $C = Q/V$, since V decreases while Q remains constant, the value of C increases. Then Eq.12.50 becomes,

$$C_{med} = \frac{A\varepsilon_o\varepsilon_r}{d} \quad \text{..........} \quad (12.51)$$

For Your Information



The electric field lines between the plates of a parallel-plate capacitor. Small bits of thread are suspended in oil and become aligned with the electric field. Note that the lines are equally spaced, indicating that the electric field there is uniform.



$V < V_o$

Fig. 12.25 Effect of a dielectric on the capacitance of a capacitor.

Eq.12.50 shows the dependence of a capacitor upon the area of plates, the separation between the plates and medium between them.

Dividing Eq.12.51 by Eq.12.50 we get expression for dielectric constant as,

$$\varepsilon_r = \frac{C_{med}}{C_{vac}} \qquad .... \qquad (12.52)$$

From Eq.12.52 dielectric co-efficient or dielectric constant is defined as

"The ratio of the capacitance of a parallel plate capacitor with an insulating substance as medium between the plates to its capacitance with vacuum (or air) as medium between them".

For Your Information

A collection of capacitors used in various appliances.

12.15 ELECTRIC POLARIZATION OF DIELECTRICS

The increase in the capacity of a capacitor due to presence of dielectric is due to electric polarization of dielectric.

The dielectric consists of atoms and molecules which are electrically neutral on the average, i.e, they contain equal amounts of negative and positive charges. The distribution of these charges in the atoms and molecules is such that the centre of the positive charge coincides with the centre of negative charge. When the molecules of dielectric are subjected to an electric field between the plates of a capacitor, the negative charges (electrons) are attracted towards the positively charged plate of the capacitor and the positive charges (nuclei) towards the negatively charged plate. The electrons in the dielectric (insulator) are not free to move but it is possible that the electrons and nuclei can undergo slight displacement when subjected to an electric field. As a result of this displacement the centre of positive and negative charges now no longer coincide with each other and one end of molecules shows a negative charge and the other end, an equal amount of positive charge but the molecule as a whole is still neutral. Two equal and opposite charges separated by a small distance are said to constitute a dipole. Thus the molecules of the dielectric under the action of electric field become dipoles and the dielectric is said to be polarized.

The effect of the polarization of dielectric is shown in

Fig. 12.26. The positively charged plate attracts the negative end of the molecular dipoles and the negatively charged plate attracts the positive end. Thus the surface of the dielectric which is in contact with the positively charged plate places a layer of negative charges on the plate. Similarly the surface of the dielectric in contact with the negatively charged plate places a layer of positive charges. It effectively decreases the surface density of the charge σ on the plates. As the electric intensity E between the plates is $\frac{\sigma}{\varepsilon_o}$, so E decreases due to polarization of the dielectric. This results into a decrease of potential difference between the plates due to presence of dielectric as demonstrated by the experiment described in the previous section.

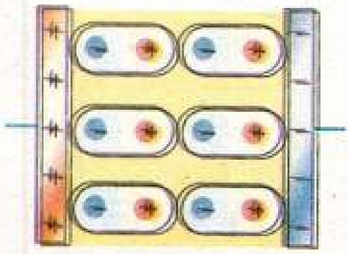


Fig. 12.26

12.16 ENERGY STORED IN A CAPACITOR

A capacitor is a device to store charge. Alternatively, it is possible to think of a capacitor as a device for storing electrical energy. After all, the charge on the plate possesses electrical potential energy which arises because work is to be done to deposit charge on the plates. This is due to the fact that with each small increment of charge being deposited during the charging process, the potential difference between the plates increases, and a larger amount of work is needed to bring up next increment of charge.

Initially when the capacitor is uncharged, the potential difference between plates is zero and finally it becomes V when q charge is deposited on each plate. Thus, the average potential difference is $\frac{0+V}{2} = \frac{1}{2}V$

Therefore $\quad$ P.E. = Energy = $\frac{1}{2} q V$

Using the relation $q = CV$ for capacitor we get

$$\text{Energy} = \frac{1}{2} CV^2 \quad .... \quad (12.53)$$

It is also possible to regard the energy as being stored in electric field between the plates, rather than the potential energy of the charges on the plates. Such a view point is useful when electric field strength between the plates instead of charges on the plates causing field is to be considered. This relation can be obtained by substituting $V = Ed$ and $C = A\varepsilon_r\varepsilon_o/d$ in Eq.12.53,

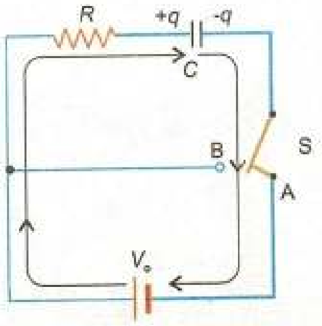


Fig. 12.27 Charging a capacitor

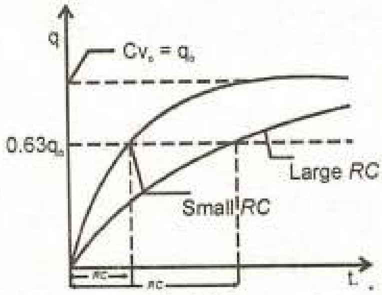


Fig. 12.28

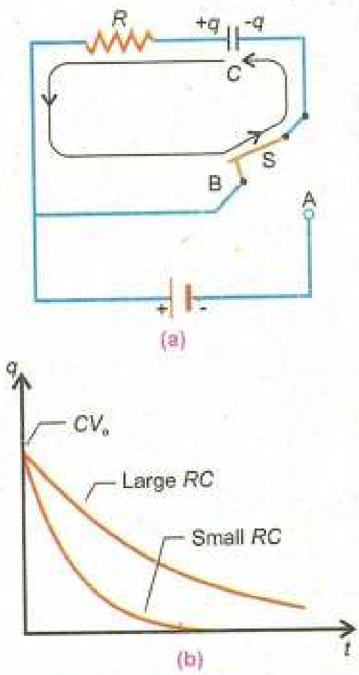


Fig. 12.29 Discharging a capacitor

$$\text{Energy} = \frac{1}{2}\left(\frac{A\varepsilon_r\varepsilon_o}{d}\right)(Ed)^2$$

$$= \frac{1}{2}\,\varepsilon_r\varepsilon_o E^2 \times (Ad)$$

As (Ad) is volume between the plates so

$$\text{Energy density} = \frac{\text{Energy}}{\text{Volume}} = \frac{1}{2}\,\varepsilon_r\varepsilon_o E^2 \quad \ldots\ldots. \quad (12.54)$$

This equation is valid for any electric field strength.

12.17 CHARGING AND DISCHARGING A CAPACITOR

Many electric circuits consist of both capacitors and resistors. Fig.12.27 shows a resistor-capacitor circuit called R-C-circuit. When the switch S is set at terminal A, the R-C combination is connected to a battery of voltage V_o which starts charging the capacitor through the resistor R.

The capacitor is not charged immediately, rather charges build up gradually to the equilibrium value of $q_o = CV_o$. The growth of charge with time for different resistances is shown in Fig.12.28. According to this graph $q = 0$ at $t = 0$ and increases gradually with time till it reaches its equilibrium value $q_o = CV_o$. The voltage V across capacitor at any instant can be obtained by dividing q by C, as $V = q/C$.

How fast or how slow the capacitor is charging or discharging, depends upon the product of the resistance R and the capacitance C used in the circuit. As the unit of product RC is that of time, so this product is known as time constant and is defined as the time required by the capacitor to deposit 0.63 times the equilibrium charge q_0. The graphs of Fig.12.28 show that the charge reaches its equilibrium value sooner when the time constant is small.

Fig. 12.29(a) illustrates the discharging of a capacitor through a resistor. In this figure, the switch S is set at point B, so the charge $+q$ on the left plate can flow anti-clockwise through the resistance and neutralize the charge $-q$ on the right plate.

The graphs in Fig. 12.29(b) shows that discharging begins at $t = 0$ when $q = CV_o$ and decreases gradually to zero. Smaller values of time constant RC lead to a more rapid discharge.

Example 12.6: The time constant of a series RC circuit is $t = RC$. Verify that an ohm times farad is equivalent to second.

Solution: Ohm's law in terms of potential difference V, current I and resistance R can be written as,

$$V = IR$$

Putting $I = q/t$, this equation transforms into the equation,

$$V = \frac{q}{t} R$$

or

$$R = \frac{V \times t}{q}$$

According to equation $q = CV$, $C = q/V$

Multiplying this equation with above equation gives,

$$RC = \frac{V \times t}{q} \times \frac{q}{V} = t$$

Hence 1 ohm x 1 farad = 1 second

where ohm is the unit of resistance R.

Interesting Application

The charging/discharging of a capacitor enables some windshield wipers of cars to be used intermittently during a light drizzle. In this mode of operation the wipers remain off for a while and then turn on briefly. The timing of the on-off cycle is determined by the time constant of a resistor-capacitor combination.

SUMMARY

- The Coulomb's law states that the force between two point charges is directly proportional to the product of the magnitudes of charges and inversely proportional to the square of the distance between them.
- Electric field force per unit charge at a point is called electric field strength or electric field intensity at that point.
- The number of the field lines passing through a certain element of area is known as electric flux through that area, denoted by Φ.
- The electric flux Φ through a vector area **A**, in the electric field of intensity **E** is given by $\Phi = \mathbf{E}.\mathbf{A} = EA\cos\theta$, where θ is the angle between the field lines and the normal to surface area.
- Gauss's law is stated as "the flux through any closed surface is $1/\varepsilon_o$ times the total charge enclosed in it.
- The interior of a hollow charged metal sphere is a field free region.
- The electric intensity between two oppositely charged parallel plates is $E = \frac{\sigma}{\varepsilon_o}$.
- The amount of work done in bringing a unit positive charge from infinity to a point against electric field is the electric potential at that point.

- Capacitance of a capacitor is a measure of the ability of a capacitor to store charge.
- The capacitance of a parallel plate capacitor is $C_{vac} = \frac{Q}{V} = \frac{A\varepsilon_o}{d}$.
- The increase in the capacitance of a capacitor due to presence of dielectric is due to electric polarization of the dielectric.

QUESTIONS

12.1 The potential is constant throughout a given region of space. Is the electrical field zero or non-zero in this region? Explain.

12.2 Suppose that you follow an electric field line due to a positive point charge. Do electric field and the potential increase or decrease?

12.3 How can you identify that which plate of a capacitor is positively charged?

12.4 Describe the force or forces on a positive point charge when placed between parallel plates

(a) with similar and equal charges (b) with opposite and equal charges

12.5 Electric lines of force never cross. Why?

12.6 If a point charge q of mass m is released in a non-uniform electric field with field lines pointing in the same direction, will it make a rectilinear motion?

12.7 Is **E** necessarily zero inside a charged rubber balloon if balloon is spherical? Assume that charge is distributed uniformly over the surface.

12.8 Is it true that Gauss's law states that the total number of lines of forces crossing any closed surface in the outward direction is proportional to the net positive charge enclosed within surface?

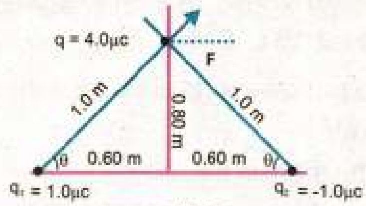
12.9 Do electrons tend to go to region of high potential or of low potential?

PROBLEMS

12.1 Compare magnitudes of electrical and gravitational forces exerted on an object (mass = 10.0 g, charge = 20.0 μC) by an identical object that is placed 10.0 cm from the first. ($G = 6.67 \times 10^{-11}\ Nm^2kg^{-2}$)

(**Ans:** $\frac{F_e}{F_g} = 5.4 \times 10^{14}$)

12.2 Calculate vectorially the net electrostatic force on q as shown in the figure.



(**Ans:** **F** = 0.058 $\hat{i}$N)

12.3 A point charge q = - 8.0 x 10^{-8} C is placed at the origin. Calculate electric field at a point 2.0 m from the origin on the z-axis. [**Ans:** (-1.8 x10^2 $\hat{k}$)NC^{-1}]

12.4 Determine the electric field at the position **r** = (4 $\hat{i}$ + 3 $\hat{j}$)m caused by a point charge q = 5.0 x 10^{-6} C placed at origin. [**Ans:** (1440 $\hat{i}$ + 1080 $\hat{j}$)N C^{-1}]

12.5 Two point charges, q_1 = -1.0 x 10^{-6} C and q_2 = + 4.0 x 10^{-6} C, are separated by a distance of 3.0 m. Find and justify the zero-field location. (**Ans:** 3.0 m)

12.6 Find the electric field strength required to hold suspended a particle of mass 1.0 x10^{-6}kg and charge 1.0μc between two plates 10.0 cm apart. (**Ans:** 9.8 vm^{-1})

12.7 A particle having a charge of 20 electrons on it falls through a potential difference of 100 volts. Calculate the energy acquired by it in electron volts (eV). [**Ans:** (2.0 x 10^3 eV)]

12.8 In Millikan's experiment, oil droplets are introduced into the space between two flat horizontal plates, 5.00 mm apart. The plate voltage is adjusted to exactly 780V so that the droplet is held stationary. The plate voltage is switched off and the selected droplet is observed to fall a measured distance of 1.50 mm in 11.2 s. Given that the density of the oil used is 900 kg m^{-3}, and the viscosity of air at laboratory temperature is 1.80 x 10^{-5} Nm^{-2}s, calculate

a) The mass, and b) The charge on the droplet (Assume g = 9.8 ms^{-2})

[**Ans:** (a) 5.14 x 10^{-15} kg, (b) 3.20 x 10^{-19} C]

12.9 A proton placed in a uniform electric field of 5000 NC^{-1} directed to right is allowed to go a distance of 10.0 cm from A to B. Calculate

(a) Potential difference between the two points
(b) Work done by the field
(c) The change in P.E. of proton
(d) The change in K.E. of the proton
(e) Its velocity (mass of proton is 1.67 x 10^{-27}kg)

(**Ans:** -500 V, 500 eV, -500 eV, 500 eV, 3.097 x 10^5 ms^{-1})

12.10 Using zero reference point at infinity, determine the amount by which a point charge of 4.0 x 10^{-8} C alters the electric potential at a point 1.2 m away, when

(a) Charge is positive (b) Charge is negative

(**Ans:** +3.0 x 10^2 V, -3.0 x 10^2 V)

12.11 In Bohr's atomic model of hydrogen atom, the electron is in an orbit around the nuclear proton at a distance of 5.29×10^{-11} m with a speed of 2.18×10^{6} ms^{-1}. ($e = 1.60 \times 10^{-19}$C, mass of electron $= 9.10 \times 10^{-31}$kg). Find

(a) The electric potential that a proton exerts at this distance

(b) Total energy of the atom in eV

(c) The ionization energy for the atom in eV

(**Ans:** +27.20 V, -13.6 eV, +13.60 eV)

12.12 The electronic flash attachment for a camera contains a capacitor for storing the energy used to produce the flash. In one such unit, the potential difference between the plates of a 750 μF capacitor is 330 V. Determine the energy that is used to produce the flash. (**Ans:** 40.8 J)

12.13 A capacitor has a capacitance of 2.5×10^{-8} F. In the charging process, electrons are removed from one plate and placed on the other one. When the potential difference between the plates is 450 V, how many electrons have been transferred? ($e = 1.60 \times 10^{-19}$C) (**Ans:** 7.0×10^{13} electrons)