Chapter 13

CURRENT ELECTRICITY

Learning Objectives

At the end of this chapter the students will be able to:

1. Understand the concept of steady current.
2. Describe some sources of current.
3. Recognize effects of current.
4. Understand and describe Ohm's law.
5. Sketch and explain the current-voltage characteristics of a metallic conductor at constant temperature, diode and filament lamp.
6. Understand resistivity and explain its dependence upon temperature.
7. Understand and elaborate conductance and conductivity of conductor.
8. Solve problems relating the variation of resistance with temperature for one dimension current flow.
9. Know the value of resistance by reading colour code on it.
10. Know the working and use of rheostat in the potential divider circuit.
11. Describe the characteristics of thermistor.
12. Use the energy considerations to distinguish between emf and p.d.
13. Understand the internal resistance of sources and its consequences for external circuits.
14. Describe the conditions for maximum power transfer.
15. Know and use the application of Kirchhoff's first law as conservation of charge.
16. Know and use the application of Kirchhoff's second law as conservation of energy.
17. Describe the function of Wheatstone Bridge to measure the unknown resistance.
18. Describe the function of potentiometer to measure and compare potentials without drawing any current from the circuit.

Most practical applications of electricity involve charges in motion or the electric current. A light bulb glows due to the flow of electric current. The current that flows through the coil of a motor causes its shaft to rotate. Most of the devices in the industry and in our homes operate with current. The electric current has become a necessity of our life.

13.1 ELECTRIC CURRENT

An electric current is caused by the motion of electric charge. If a net charge ΔQ passes through any cross section of a conductor in time Δt, we say that an electric current I has been established through the conductor where

$$I = \frac{\Delta Q}{\Delta t} \qquad \text{..........} \qquad (13.1)$$

The SI unit of current is ampere and it is a current due to flow of charge at the rate of one coulomb per second.

Motion of electric charge which causes an electric current is due to the flow of charge carriers. In case of metallic conductors, the charge carriers are electrons. The charge carriers in electrolyte are positive and negative ions e.g., in a $CuSO_4$ solution the charge carriers are Cu^{++} and SO_4^{--} ions. In gases, the charge carriers are electrons and ions.

Current Direction

Interesting Information

When eel senses danger, it turns itself into a living battery. Any one who attacks this fish is likely to get a shock. The potential difference between the head and tail of an electric eel can be up to 600 V.

Early scientists regarded an electric current as a flow of positive charge from positive to negative terminal of the battery through an external circuit. Later on, it was found that a current in metallic conductors is actually due to the flow of negative charge carriers called electrons moving in the opposite direction i.e., from negative to positive terminal of the battery, but it is a convention to take the direction of current as the direction in which positive charges flow. This current is referred as conventional current. The reason is that it has been found experimentally that positive charge moving in one direction is equivalent in all external effects to a negative charge moving in the opposite direction. As the current is measured by its external effects so a current due to motion of negative charges, after reversing its direction of flow can be substituted by an equivalent current due to flow of positive charges. Thus

The conventional current in a circuit is defined as that equivalent current which passes from a point at higher potential to a point at a lower potential as if it represented a movement of positive charges.

While analyzing the electric circuit, we use the direction of the current according to the above mentioned convention.

If we wish to refer to the motion of electrons, we use the term electronic current (Fig. 13.1).

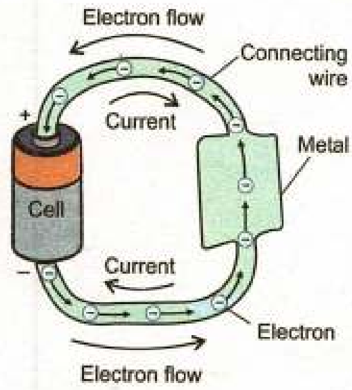
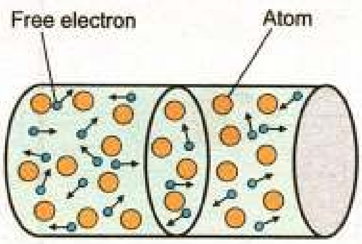


Fig. 13.1

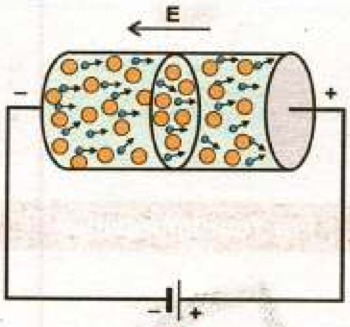
Current Through a Metallic Conductor

In a metal, the valence electrons are not attached to individual atoms but are free to move about within the body. These electrons are known as free electrons. The free electrons are in random motion just like the molecules of a gas in a container and they act as charge carriers in metals. The speed of randomly moving electrons depends upon temperature.

If we consider any section of metallic wire, the rate at which the free electrons pass through it from right to left is the same as the rate at which they pass from left to right (Fig. 13.2 a). As a result the current through the wire is zero. If the ends of the wire are connected to a battery, an electric field **E** will be set up at every point within the wire (Fig. 13.2 b). The free electrons will now experience a force in the direction opposite to **E**. As a result of this force the free electrons acquire a motion in the direction of -**E**. It may be noted that the force experienced by the free electrons does not produce a net acceleration because the electrons keep on colliding with the atoms of the conductor. The overall effect of these collisions is to transfer the energy of accelerating electrons to the lattice with the result that the electrons acquire an average velocity, called the drift velocity in the direction of -**E** (Fig. 13.2 b). The drift velocity is of the order of $10^{-3}ms^{-1}$, whereas the velocity of free electrons at room temperature due to their thermal motion is several hundred kilometres per second.



(a)



(b)
Fig. 13.2

Thus, when an electric field is established in a conductor, the free electrons modify their random motion in such a way that they drift slowly in a direction opposite to the field. In other words the electrons, in addition to their violent thermal motion, acquire a constant drift velocity due to which a net directed motion of charges takes place along the wire and a current begins to flow through it. A steady current is established in a wire when a constant potential difference is maintained across it which generates the requisite electric field **E** along the wire.

Example 13.1: 1.0×10^7 electrons pass through a conductor in 1.0 μs. Find the current in ampere flowing through the conductor. Electronic charge is 1.6×10^{-19}C.

Solution: Number of electrons = $n = 1.0 \times 10^7$

Charge on an electron = e = 1.6×10^{-19} C

Time = Δt = 1.0μs

Current I through the conductor is given by

$$I = \frac{\Delta Q}{\Delta t} = \frac{ne}{\Delta t}$$

$$I = \frac{1.0 \times 10^{7} \times 1.6 \times 10^{-19}\ C}{1.0 \times 10^{-6}\ s} = 1.6 \times 10^{-6}\ Cs^{-1} = 1.6 \times 10^{-6}\ A$$

13.2 SOURCE OF CURRENT

Fig. 13.3 Conventional current flows from higher to lower potential through a wire.

When two conductors at different potentials are joined by a metallic wire, current will flow through the wire. The current continues to flow from higher potential to the lower potential until both are at the same potential (Fig. 13.3). After this the current ceases to flow. Thus the current through the wire decreases from a maximum value to zero. In order to have a constant current the potential difference across the conductors or the ends of the wire should be maintained constant. This is achieved by connecting the ends of the wire to the terminals of a device called a source of current (Fig. 13.4).

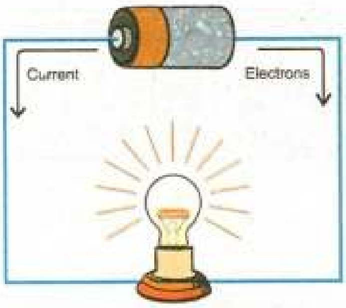


Fig. 13.4 A source of current such as battery maintains a nearly constant potential difference between ends of a conductor.

Every source of current converts some non electrical energy such as chemical, mechanical, heat or solar energy into electrical energy. There are many types of sources of current. A few examples are mentioned below:

(i) Cells (primary as well as secondary) which convert chemical energy into electrical energy.

(ii) Electric generators which convert mechanical energy into electrical energy.

(iii) Thermo-couples which convert heat energy into electrical energy.

(iv) Solar cells which convert sunlight directly into electrical energy.

For Your Information

Heating effect of current is used in electric kettle.

13.3 EFFECTS OF CURRENT

The presence of electric current can be detected by the various effects which it produces. The obvious effects of the current are:

(i) Heating effect
(ii) Magnetic effect
(iii) Chemical effect

Heating Effect

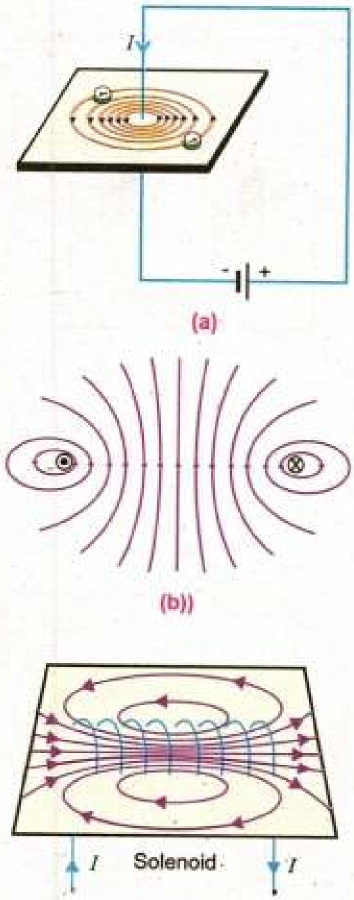
Current flows through a metallic wire due to motion of free electrons. During the course of their motion, they collide frequently with the atoms of the metal. At each collision, they lose some of their kinetic energy and give it to atoms with which they collide. Thus as the current flows through the wire, it increases the kinetic energy of the vibrations of the metal atoms. i.e., it generates heat in the wire. It is found that the heat H produced by a current I in the wire of resistance R during a time interval t is given by

$$H = I^2Rt$$

The heating effect of current is utilized in electric heaters, kettles, toasters and electric irons etc.

Magnetic Effect

The passage of current is always accompanied by a magnetic field in the surrounding space. The strength of the field depends upon the value of current and the distance from the current element. The pattern of the field produced by a current carrying straight wire, a coil and a solenoid is shown in Fig. 13.5 (a, b & c). Magnetic effect is utilized in the detection and measurement of current. All the machines involving electric motors also use the magnetic effect of current.



(a) (b)) (c)

Fig. 13.5

Chemical Effect

Certain liquids such as dilute sulphuric acid or copper sulphate solution conduct electricity due to some chemical reactions that take place within them. The study of this process is known as electrolysis. The chemical changes produced during the electrolysis of a liquid are due to chemical effects of the current. It depends upon the nature of the liquid and the quantity of electricity passed through the liquid.

The liquid which conducts current is known as electrolyte. The material in the form of wire or rod or plate which leads the current into or out of the electrolyte is known as electrode. The electrode connected with the positive terminal of the current source is called anode and that connected with negative terminal is known as cathode. The vessel containing the two electrodes and the liquid is known as voltameter. As an example we will consider the electrolysis of copper sulphate solution. The voltameter contains dilute solution of copper sulphate. The anode and cathode are both copper plates

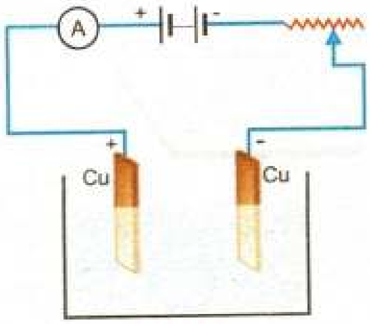


Fig.13.6

(Fig. 13.6). When copper sulphate is dissolved in water, it dissociates into Cu^{++} and SO^{-}_{4} ions. On passing current through the voltameter. Cu^{++} moves towards the cathode and the following reaction takes place.

$$Cu^{++} + 2e^{-} \longrightarrow Cu$$

The copper atoms thus formed are deposited at cathode plate. While copper is being deposited at the cathode, the SO^{-}_{4} ions move towards the anode. Copper atoms from the anode go into the solution as copper ions which combine with sulphate ions to form copper sulphate.

$$Cu^{++} + SO_4^{--} \longrightarrow CuSO_4$$

As the electrolysis proceeds, copper is continuously deposited on the cathode while an equal amount of copper from the anode is dissolved into the solution and the density of copper sulphate solution remains unaltered.

This example also illustrates the basic principle of electroplating - a process of coating a thin layer of some expensive metal (gold, silver etc.) on an article of some cheap metal.

13.4 OHM'S LAW

We have seen that when a battery is connected across a conductor, an electric current begins to flow through it. How much current flows through the conductor when a certain potential difference is set up across its ends? The answer to this question was given by a German Physicist George Simon Ohm. He showed by elaborate experiments that the current through a metallic conductor is directly proportional to the potential difference across its ends. This fact is known as Ohms' law which states that

"The current flowing through a conductor is directly proportional to the potential difference across its ends provided the physical state such as temperature etc. of the conductor remains constant".

Symbolically Ohm's law can be written as

$$I \propto V$$

It implies that $V = RI$ .... (13.2)

where R, the constant of proportionality is called the resistance of the conductor. The value of the resistance depends upon the nature, dimensions and the physical state of the conductor. In fact the resistance is a measure of the opposition to the motion of electrons due to their continuous bumping with the atoms of the lattice. The unit of resistance is ohm. A conductor has a resistance of 1 ohm if a current of 1 ampere flows through it when a potential difference of 1 volt is applied across its ends. The symbol of ohm is Ω. If I is measured in amperes, V in volts, then R is measured in ohms i.e.,

$$R\,(\text{ohms}) = \frac{V\,(\text{volts})}{I\,(\text{amperes})} \quad .... \quad (13.3)$$

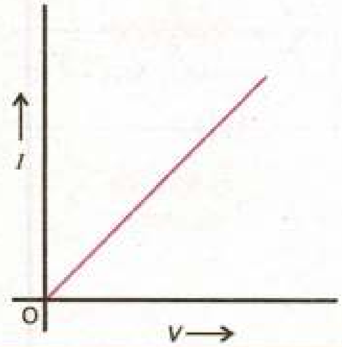


Fig. 13.7 Current - voltage graph of an ohmic material.

A sample of a conductor is said to obey Ohm's law if its resistance R remains constant that is, the graph of its V versus I is exactly a straight line (Fig. 13.7). A conductor which strictly obeys Ohm's law is called ohmic. However, there are devices, which do not obey Ohms' law i.e., they are non ohmic. The examples of non ohmic devices are filament bulbs and semiconductor diodes.

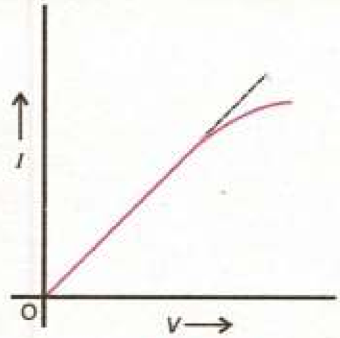


Fig.13.8

Let us apply a certain potential difference across the terminals of a filament lamp and measure the resulting current passing through it. If we repeat the measurement for different values of potential difference and draw a graph of voltage V versus current I, it will be seen that the graph is not a straight line (Fig. 13.8). It means that a filament is a non ohmic device. This deviation of I - V graph from straight line is due to the increase in the resistance of the filament with temperature - a topic which is discussed in the next section. As the current passing through the filament is increased from zero, the graph is a straight line in the initial stage because the change in the resistance of the filament with temperature due to small current is not appreciable. As the current is further increased, the resistance of the filament continues to increase due to rise in its temperature.

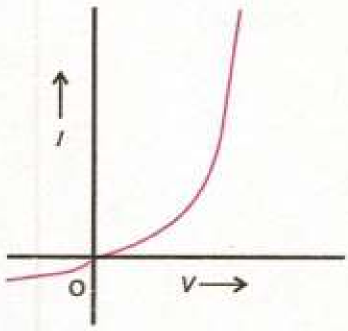


Fig.13.9

Another example of non ohmic device is a semiconductor diode. The current - voltage plot of such a diode is shown in Fig. 13.9. The graph is not a straight line so semi conductor is also a non ohmic device.

Review of Series and Parallel Combinations of Resistors

In an electrical circuit, usually, a number of resistors are connected together. There are two arrangements in which resistors can be connected with each other ., one is known as series arrangement and other one as parallel arrangement.

$R_e = R_1 + R_2 + R_3$

Fig.13.10 (a)

If the resistors are connected end to end such that the same current passes through all of them, they are said to be connected in series as shown in Fig. 13.10(a). There

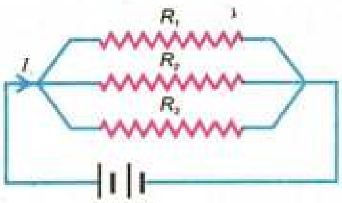


Fig. 13.10 (b)

equivalent resistance R_e is given by

$$R_e = R_1 + R_2 + R_3 + \text{------} \quad .... \quad (13.4)$$

In parallel arrangement a number of resistors are connected side by side with their ends joined together at two common points as shown in Fig. 13.10(b). The equivalent resistance R_e of this arrangement is given by

$$\frac{1}{R_e} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \text{------} \quad .... \quad (13.5)$$

13.5 RESISTIVITY AND ITS DEPENDENCE UPON TEMPERATURE

It has been experimentally seen that the resistance R of a wire is directly proportional to its length L and inversely proportional to its cross sectional area A. Expressing mathematically we have

$$R \propto \frac{L}{A}$$

or

$$R = \rho \frac{L}{A} \quad .... \quad (13.6)$$

where ρ is a constant of proportionality known as resistivity of specific resistance of the material of the wire. It may be noted that resistance is the characteristic of a particular wire whereas the resistivity is the property of the material of which the wire is made. From Eq. 13.6 we have

$$\rho = \frac{RA}{L} \quad .... \quad (13.7)$$

The above equation gives the definition of resistivity as the resistance of a metre cube of a material. The SI unit of resistivity is ohm-metre (Ω m).

Conductance is another quantity used to describe the electrical properties of materials. In fact conductance is the reciprocal of resistance i.e.,

$$\text{Conductance} = \frac{1}{\text{resistance } (R)}$$

The SI unit of conductance is mho or siemen.

Likewise conductivity, σ is the reciprocal of resistivity i.e.,

$$\sigma = \frac{1}{\rho} \quad .... \quad (13.8)$$

The SI unit of conductivity is $ohm^{-1}m^{-1}$ or mh m^{-1}. Resistivity of various materials are given in Table 13.1.

It may be noted from Table 13.1 that silver and copper are two best conductors. That is the reason that most electric wires are made of copper.

The resistivity of a substance depends upon the temperature also. It can be explained by recalling that the resistance offered by a conductor to the flow of electric current is due to collisions, which the free electrons encounter with atoms of the lattice. As the temperature of the conductor rises, the amplitude of vibration of the atoms in the lattice increases and hence, the probability of their collision with free electrons also increases. One may say that the atoms then offer a bigger target, that is, the collision cross-section of the atoms increases with temperature. This makes the collisions between free electrons and the atoms in the lattice more frequent and hence, the resistance of the conductor increases.

Experimentally the change in resistance of a metallic conductor with temperature is found to be nearly linear over a considerable range of temperature above and below 0 °C (Fig. 13.11). Over such a range the fractional change in resistance per kelvin is known as the temperature coefficient of resistance i.e.,

$$\alpha = \frac{R_t - R_o}{R_o t} \quad .... \quad (13.9)$$

where R_o and R_t are resistances at temperature 0 °C and t °C. As resistivity ρ depends upon the temperature, Eq. 13.6 gives

$R_t = \rho_t L/A$ and $R_o = \rho_o L/A$

Substituting the values of R_t and R_o in Eq. 13.9, we get as

$$\alpha = \frac{\rho_t - \rho_o}{\rho_o t} \quad .... \quad (13.10)$$

where ρ_o is the resistivity of a conductor at 0 °C and ρ_t is the resistivity at t °C. Values of temperature co-efficients of resistivity of some substances are also listed in Table 13.1.

There are some substances like germanium, silicon etc., whose resistance decreases with increase in temperature. i.e., these substances have negative temperature coefficients.

Example 13.2: 0.75 A current flows through an iron wire when a battery of 1.5 V is connected across its ends. The

Table 13.1

Substance	ρ (Ωm)	α (K^{-1})
Silver	1.52×10^{-8}	0.00380
Copper	1.54×10^{-8}	0.00390
Gold	2.27×10^{-8}	0.00340
Aluminium	2.63×10^{-8}	0.00390
Tungsten	5.00×10^{-8}	0.00460
Iron	11.00×10^{-8}	0.00520
Platinum	11.00×10^{-8}	0.00520
Constanton	49.00×10^{-8}	0.00001
Mercury	94.00×10^{-8}	0.00091
Nichrome	100.0×0^{-8}	0.00020
Carbon	3.5×10^{-5}	-0.0005
Germanium	0.5	-0.05
Silicon	20-2300	-0.07

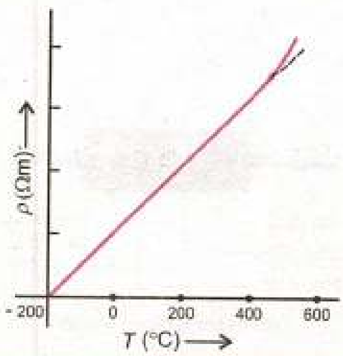


Fig. 13.11 Variation of resistivity of Cu with temperature.

Interesting Information

Inspectors can easily check the reliability of a concrete bridge made with carbon fibers. The fibers conduct electricity. If sensors show that electrical resistance is increasing over time the fibers are separating because of cracks.

length of the wire is 5.0 m and its cross sectional area is $2.5 \times 10^{-7}\ m^2$. Compute the resistivity of iron.

Solution:

The resistance R of the wire can be calculated by Eq. 13.2 i.e.,

$$R = \frac{V}{I} = \frac{1.5\ V}{0.75\ A} = 2.0\ VA^{-1} = 2.0\ \Omega$$

The resistivity ρ of iron of which the wire is made is given by

$$\rho = R\frac{A}{L} = \frac{2.0\ \Omega \times 2.5 \times 10^{-7} m^2}{5.0\ m} = 1.0 \times 10^{-7}\ \Omega m$$

Example 13.3: A platinum wire has resistance of 10 Ω at 0°C and 20 Ω at 273 °C. Find the value of temperature coefficient of resistance of platinum.

Solution:

$$R_o = 10\Omega,\ R_t = 20\Omega,\ t = 546\,K - 273\,K = 273\,K$$

Temperature coefficient of resistance can be found by

$$\alpha = \frac{R_t - R_o}{R_o t} = \frac{20\,\Omega - 10\,\Omega}{10\,\Omega \times 273\,K} = \frac{1}{273\,K} = 3.66 \times 10^{-3}\ K^{-1}$$

Table 13.2 The Colour Code

Colour	Value
Black	0
Brown	1
Red	2
Orange	3
Yellow	4
Green	5
Blue	6
Violet	7
Gray	8
White	9

13.6 COLOUR CODE FOR CARBON RESISTANCES

Carbon resistors are most common in electronic equipment. They consist of a high-grade ceramic rod or cone (called the substrate) on which is deposited a thin resistive film of carbon. The numerical value of their resistance is indicated by a colour code which consists of bands of different colours printed on the body of the resistor. The colour used in this code and the digits represented by them are given in Table 13.2.

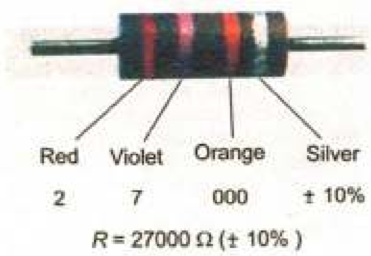
Resistor Colour Code

Fig. 13.12

Usually the code consists of four bands (Fig. 13.12). Starting from left to right, the colour bands are interpreted as follows:

1. The first band indicates the first digit in the numerical value of the resistance.
2. The second band gives the second digit.
3. The third band is decimal multiplier i.e., it gives the number of zeros after the first two digits.
4. The fourth band gives resistance tolerance. Its colour is either silver or gold. Silver band indicates a tolerance of ± 10%, a gold band shows a tolerance of

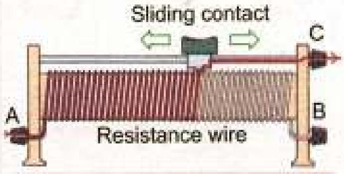
For Your Information



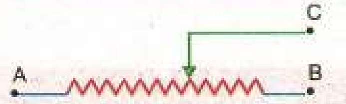
± 5 %. If there is no fourth band, tolerance is understood to be + 20%. By tolerance, we mean the possible variation from the marked value. For example, a 1000 Ω resistor with a tolerance of ± 10% will have an actual resistance anywhere between 900Ω and 1100 Ω.

Rheostat

It is a wire wound variable resistance. It consists of a bare manganin wire wound over an insulating cylinder. The ends of the wire are connected to two fixed terminals A and B (Fig. 13.13 a). A third terminal C is attached to a sliding contact which can also be moved over the wire.



(a) A Rheostat



(b) Its use as variable resistor

Fig. 13.13

A rheostat can be used as a variable resistor as well as a potential divider. To use it as a variable resistor one of the fixed terminal say A and the sliding terminal C are inserted in the circuit (Fig. 13.13 b). In this way the resistance of the wire between A and the sliding contact C is used. If the sliding contact is shifted away from the terminal A, the length and hence the resistance included in the circuit increases and if the sliding contact is moved towards A, the resistance decreases. A rheostat can also be used as a potential divider.

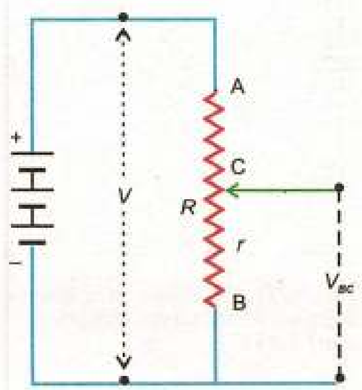
This is illustrated in Fig. 13.14. A potential difference V is applied across the ends A and B of the rheostat with the help of a battery. If R is the resistance of wire AB, the current I passing through it is given by $I = V/R$.

The potential difference between the portion BC of the wire AB is given by

$$V_{BC} = \text{current} \times \text{resistance}$$

$$= \frac{V}{R} \times r = \frac{r}{R} V \qquad \text{..........} \qquad (13.11)$$

where r is the resistance of the portion BC of the wire. The circuit shown in Fig. 13.14 is known as potential divider. Eq.13.11 shows that this circuit can provide at its output terminals a potential difference varying from zero to the full potential difference of the battery depending on the position of the sliding contact. As the sliding contact C is moved towards the end B, the length and hence the resistance r of the portion BC of the wire decreases which according to Eq. 13.11, decreases V_{BC}. On the other hand if the sliding contact C is moved towards the end A, the output voltage V_{BC} increases.



Use of rheostat as potential divider

Fig.13.14

Thermistors

A thermistor is a heat sensitive resistor. Most thermistors have negative temperature coefficient of resistance i.e., the resistance of such thermistors decreases when their temperature is increased. Thermistors with positive temperature coefficient are also available.

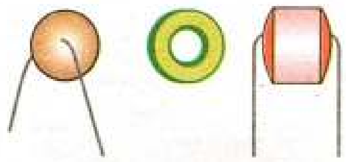


Fig. 13.15 Thermistors of different shapes.

Thermistors are made by heating under high pressure semiconductor ceramic made from mixtures of metallic oxides of manganese, nickel, cobalt, copper, iron etc. These are pressed into desired shapes and then baked at high temperature. Different types of thermistors are shown in Fig.13.15. They may be in the form of beads, rods or washers.

Thermistors with high negative temperature coefficient are very accurate for measuring low temperatures especially near 10 K. The higher resistance at low temperature enables more accurate measurement possible.

For Your Information

A zero-ohm resistor is indicated by a single black colour band around the body of the resistor.

Thermistors have wide applications as temperature sensors i.e., they convert changes of temperature into electrical voltage which is duly processed.

13.7 ELECTRICAL POWER AND POWER DISSIPATION IN RESISTORS

Consider a circuit consisting of a battery E connected in series with a resistance R (Fig. 13.16). A steady current I flows through the circuit and a steady potential difference V exists between the terminals A and B of the resistor R. Terminal A, connected to the positive pole of the battery, is at a higher potential than the terminal B. In this circuit the battery is continuously lifting charge uphill through the potential difference V. Using the meaning of potential difference, the work done in moving a charge ΔQ up through the potential difference V is given by

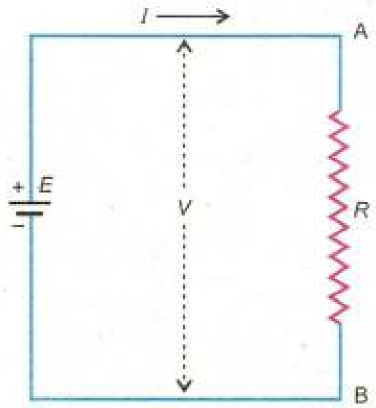


Fig. 13.16 The power of a battery appears as the power dissipated in the resistor R.

$$\text{Work done} = \Delta W = V \times \Delta Q \text{..........(13.12)}$$

This is the energy supplied by the battery. The rate at which the battery is supplying electrical energy is the power output or electrical power of the battery. Using the definition of power we have

$$\text{Electrical power} = \frac{\text{Energy supplied}}{\text{Time taken}} = V\frac{\Delta Q}{\Delta t}$$

Since $I = \frac{\Delta Q}{\Delta t}$, so

$$\text{Electrical power} = V \times I \quad .... \quad (13.12a)$$

Eq. 13.12a is a general relation for power delivered from a source of current I operating on a voltage V. In the circuit shown in Fig.13.16 the power supplied by the battery is expended or dissipated in the resistor R. The principle of conservation of energy tells us that the power dissipated in the resistor is also given by Eq. 13.12.a

$$\text{Power dissipated } (P) = V \times I \quad .... \quad (13.13)$$

Alternative equation for calculating power can be found by substituting $V = IR$, $I = V/R$ in turn in Eq. 13.13

$$P = V \times I = IR \times I = I^2R$$

$$P = V \times I = V \times \frac{V}{R} = \frac{V^2}{R}$$

Thus we have three equations for calculating the power dissipated in a resistor.

$$P = V \times I, \quad P = I^2R, \quad P = \frac{V^2}{R} \quad ...... \quad (13.14)$$

If V is expressed in volts and I in amperes, the power is expressed in watts.

13.8 ELECTROMOTIVE FORCE (EMF) AND POTENTIAL DIFFERENCE

We know that a source of electrical energy, say a cell or a battery, when connected across a resistance maintains a steady current through it (Fig. 13.17). The cell continuously supplies energy which is dissipated in the resistance of the circuit. Suppose when a steady current has been established in the circuit, a charge ΔQ passes through any cross section of the circuit in time Δt. During the course of motion, this charge enters the cell at its low potential end and leaves at its high potential end. The source must supply energy ΔW to the positive charge to force it to go to the point of high potential. The emf E of the source is defined as the energy supplied to unit charge by the cell.

Fig. 13.17 Electromotive force of a cell.

i.e $$E = \frac{\Delta W}{\Delta Q} \quad .... \quad (13.15)$$

It may be noted that electromotive force is not a force and we do not measure it in newtons. The unit of emf is joule/coulomb which is volt (V).

The energy supplied by the cell to the charge carriers is derived from the conversion of chemical energy into electrical energy inside the cell.

Like other components in a circuit a cell also offers some resistance. This resistance is due to the electrolyte present between the two electrodes of the cell and is called the internal resistance r of the cell. Thus a cell of emf E having an internal resistance r is equivalent to a source of pure emf E with a resistance r in series as shown in Fig. 13.18.

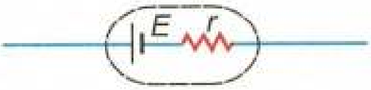


Fig. 13.18 An equivalent circuit of a cell of emf E and internal resistance r.

Let us consider the performance of a cell of emf E and internal resistance r as shown in Fig. 13.19. A voltmeter of infinite resistance measures the potential difference across the external resistance R or the potential difference V across the terminals of the cell. The current I flowing through the circuit is given by

$$I = \frac{E}{R + r}$$

or $$E = IR + Ir \qquad \text{..........} \qquad (13.16)$$

Here $IR = V$ is the terminal potential difference of the cell in the presence of current I. When the switch S is open, no current passes through the resistance. In this case the voltmeter reads the emf E as terminal voltage. Thus terminal voltage in the presence of the current (switch on) would be less than the emf E by Ir.

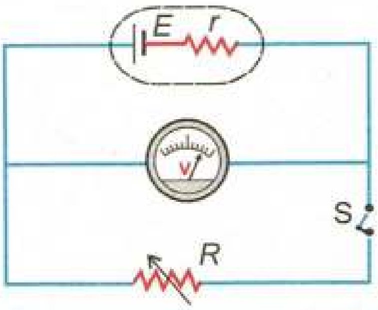


Fig. 13.19 The terminal potential difference V of a cell is $E\ I\ \text{-}r$.

Let us interpret the Eq. 13.16 on energy considerations. The left side of this equation is the emf E of the cell which is equal to energy gained by unit charge as it passes through the cell from its negative to positive terminal. The right side of the equation gives an account of the utilization of this energy as the current passes the circuit. It states that, as a unit charge passes through the circuit, a part of this energy equal to Ir is dissipated into the cell and the rest of the energy is dissipated into the external resistance R. It is given by potential drop IR. Thus the emf gives the energy supplied to unit charge by the cell and the potential drop across the various elements account for the dissipation of this energy into other forms as the unit charge passes through these elements.

The emf is the "cause" and potential difference is its "effect". The emf is always present even when no current is drawn through the battery or the cell, but the potential difference across the conductor is zero when no current flows through it.

Example 13.4: The potential difference between the terminals of a battery in open circuit is 2.2 V. When it is connected across a resistance of 5.0 Ω, the potential falls to 1.8 V. Calculate the current and the internal resistance of the battery.

Solution:

Given $E = 2.2\,V$, $R = 5.0\,\Omega$, $V = 1.8\,V$

We are to calculate I and r.

We have $V = IR$

or $$I = \frac{V}{R} = \frac{1.8\,V}{5.0\,\Omega} = 0.36\,A$$

Internal resistance r can be calculated by using

$$E = V + Ir$$

or $$2.2\,V = 1.8\,V + 0.36\,A \times r$$

or $$r = 1.11\,VA^{-1} = 1.11\,\Omega$$

Maximum Power Output

In the circuit of Fig. 13.19, as the current I flows through the resistance R, the charges flow from a point of higher potential to a point of lower potential and as such, they loose potential energy. If V is the potential difference across R, the loss of potential energy per second is VI. This loss of energy per second appears in other forms of energy and is known as power delivered to R by current I.

$\therefore$ Power delivered to $R = P_{out} = VI$

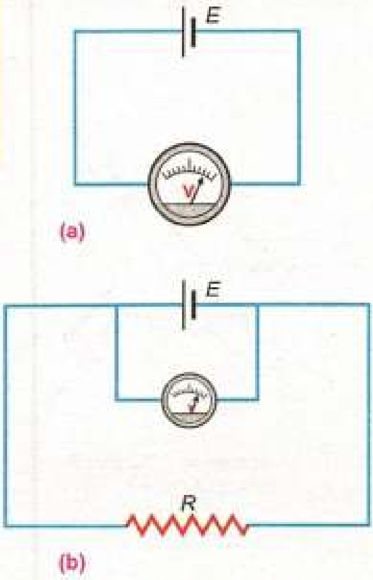
$$= I^2R \qquad (\because \quad V = IR)$$

As $$I = \frac{E}{R+r}$$

$$P_{out} = \frac{E^2R}{(R+r)^2} = \frac{E^2R}{(R-r)^2 + 4Rr} \quad .... \quad (13.17)$$

when $R = r$, the denominator of the expression of P_{out} is least and so P_{out} is then a maximum. Thus we see that maximum power is delivered to a resistance (load), when the internal resistance of the source equals the load resistance. The

Do You Know?



A voltmeter connected across the terminals of a cell measures (a) the emf of the cell on open circuit, (b) the terminal potential difference on a closed circuit.

value of this maximum output power as given by Eq. 13.17 is $\frac{E^2}{4R}$.

13.9 KIRCHHOFF'S RULES

Ohm's law and rules of series and parallel combination of resistance are quite useful to analyze simple electrical circuits consisting of more than one resistance. However such a method fails in the case of complex networks consisting of a number of resistors, and a number of voltage sources. Problems of such networks can be solved by a system of analysis, which is based upon two rules, known as Kirchhoff's rules.

Kirchhoff's First Rule

It states that the sum of all the currents meeting at a point in the circuit is zero i.e.,

$$\Sigma I = 0 \qquad .... \qquad (13.18)$$

It is a convention that a current flowing towards a point is taken as positive and that flowing away from a point is taken as negative.

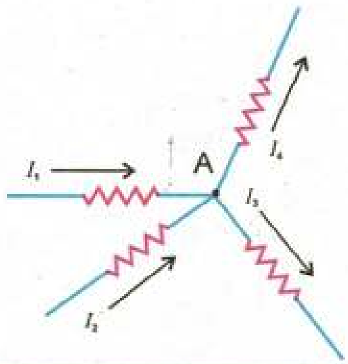


Fig. 13.20 According to Kirchhoff's 1st rule $I_1 + I_2 = I_3 + I_4$.

Consider a situation where four wires meet at a point A (Fig. 13.20). The currents flowing into the point A are I_1 and I_2 and currents flowing away from the point are I_3 and I_4. According to the convention currents I_1 and I_2 are positive and currents I_3 and I_4 are negative. Applying Eq. 13.18 we have

$$I_1 + I_2 + (-I_3) + (-I_4) = 0$$

or

$$I_1 + I_2 = I_3 + I_4 \qquad .... \qquad (13.19)$$

Using Eq. 13.19 Kirchhoff's first rule can be stated in other words as

The sum of all the currents flowing towards a point is equal to the sum of all the currents flowing away from the point.

Kirchhoff's first rule which is also known as Kirchhoff's point rule is a manifestation of law of conservation of charge. If there is no sink or source of charge at a point, the total charge flowing towards the point must be equal to the total charge flowing away from it.

Kirchhoff's Second Rule

It states that the algebraic sum of voltage changes in a closed circuit or a loop must be equal to zero. Consider a closed circuit shown in Fig. 13.21. The direction of the current I flowing through the circuit depends on the cell having the greater emf. Suppose E_1 is greater than E_2, so the current flows in counter clockwise direction (Fig. 13.21). We know that a steady current is equivalent to a continuous flow of positive charges through the circuit. We also know that a voltage change or potential difference is equal to the work done on a unit positive charge or energy gained or lost by it in moving from one point to the other. Thus when a positive charge ΔQ due to the current I in the closed circuit (Fig. 13.21), passes through the cell E_1 from low (-ve) to high potential (+ve), it gains energy because work is done on it. Using Eq. 13.12 the energy gain is $E_1 \Delta Q$. When the current passes through the cell E_2 it loses energy equal to $-E_2 \Delta Q$ because here the charge passes from high to low potential. In going through the resistor R_1, the charge ΔQ loses energy equal to $-IR_1 \Delta Q$ where IR_1 is potential difference across R_1. The minus sign shows that the charge is passing from high to low potential. Similarly the loss of energy while passing through the resistor R_2 is $-IR_2\Delta Q$. Finally the charge reaches the negative terminal of the cell E_1 from where we started. According to the law of conservation of energy the total change in energy of our system is zero. Therefore, we can write

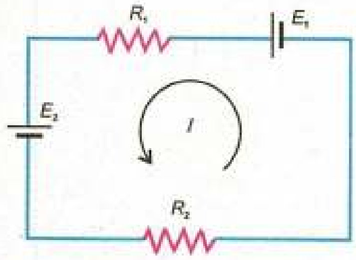


Fig. 13.21 According to Kirchhoff's 2nd rule $E_1 - IR_1 - E_2 - IR_2 = 0$.

$$E_1\Delta Q - IR_1\Delta Q - E_2\Delta Q - IR_2\Delta Q = 0$$

or $$E_1 - IR_1 - E_2 - IR_2 = 0 \qquad \text{..........} \qquad (13.20)$$

which is Kirchhoff's second rule and it states that

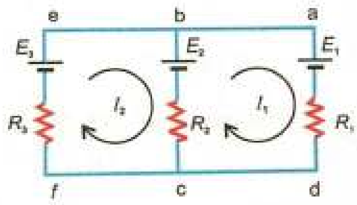
The algebraic sum of potential changes in a closed circuit is zero.

We have seen that this rule is simply a particular way of stating the law of conservation of energy in electrical problems.

Before applying this rule for the analysis of complex network it is worthwhile to thoroughly understand the rules for finding the potential changes.

(i) If a source of emf is traversed from negative to positive terminal, the potential change is positive, it is negative in the opposite direction.

(ii) If a resistor is traversed in the direction of current, the change in potential is negative, it is positive in the opposite direction.



R_1=10 Ω, R_2 = 30 Ω, R_3 = 15 Ω

E_1=40 V, E_2 = 60 V, E_3 = 50 V

Fig. 13.22

Example 13.6: Calculate the currents in the three resistances of the circuit shown in Fig. 13.22.

Solution:

First we select two loops abcda and ebcfe. The choice of loops is quite arbitrary, but it should be such that each resistance is included at least once in the selected loops.

After selecting the loops, suppose a current I_1 is flowing in the first loop and I_2 in the second loop, all flowing in the same sense. These currents are called loop currents. The actual currents will be calculated with their help. It should be noted that the sense of the current flowing in all loops should essentially be the same. It may be clockwise or anticlockwise. Here we have assumed it to be clockwise (Fig. 13.22).

We now apply Kirchhoff's second rule to obtain the equations required to calculate the currents through the resistances. We first consider the loop abcda. Starting at a point 'a' we follow the loop clockwise. The voltage change while crossing the battery E_1 is - E_1 because the current flows through it from positive to negative. The voltage change across R_1 is - I_1R_1. The resistance R_2 is common to both the loops I_1 and I_2 therefore, the currents I_1 and I_2 simultaneously flow through it. The directions of currents I_1 and I_2 as flowing through R_2 are opposite, so we have to decide that which of these currents is to be assigned a positive sign. The convention regarding the sign of the current is that if we are applying the Kirchhoffs second rule in the first loop, then the current of this loop i.e, I_1 will be assigned a positive sign and all currents, flowing opposite to I_1 have a negative sign. Similarly, while applying Kirchhoffs second rule in the second loop, the current I_2 will be considered as positive and I_1 as negative. Using this convention the current flowing through R_2 is (I_1 - I_2) and the voltage change across is - (I_1 - I_2) R_2. The voltage change across the battery E_2 is E_2. Thus the Kirchhoffs second rule as applied to the loop abcda gives

$$-E_1 - I_1R_1 - (I_1 - I_2)R_2 + E_2 = 0$$

Substituting the values, we have

$$-40\text{ V} - I_1 \times 10\ \Omega - (I_1 - I_2) \times 30\ \Omega + 60\text{ V} = 0$$

$$20\text{ V} - 10\ \Omega \times [I_1 + 3(I_1 - I_2)] = 0$$

or $$4I_1 - 3I_2 = 2\text{ V}\Omega^{-1} = 2\text{ A} \quad .... \quad (13.21)$$

Similarly applying Kirchhoff's second rule to the loop ebcfe, we get

$$-E_2 - (I_2 - I_1)R_2 - I_2R_3 + E_3 = 0$$

Substituting the values, we have

$$-60\text{ V} - (I_2 - I_1) \times 30\ \Omega - I_2 \times 15\ \Omega + 50\text{ V} = 0$$

$$-10\text{ V} - 15\ \Omega \times [I_2 + 2(I_2 - I_1)] = 0$$

or $$6I_1 - 9I_2 = 2\text{ V}\Omega^{-1} = 2\text{ A} \quad .... \quad (13.22)$$

Solving Eq. 13.21 and Eq. 13.22 for I_1 and I_2, we get

$$I_1 = \frac{2}{3}\text{ A} \quad \text{and} \quad I_2 = \frac{2}{9}\text{ A}$$

Knowing the values of loop currents I_1 and I_2 the actual current flowing through each resistance of the circuit can be determined. Fig. 13.22 shows that I_1 and I_2 are the actual currents through the resistances R_1 and R_3. The actual current through R_2 is the difference of I_1 and I_2 and its direction is along the larger current. Thus

The current through $R_1 = I_1 = \frac{2}{3}\text{ A} = 0.66\text{ A}$ flowing in the direction of I_1 i.e., from a to d.

The current through $R_2 = I_1 - I_2 = \frac{2}{3}\text{ A} - \frac{2}{9}\text{ A} = 0.44\text{ A}$ flowing in the direction of I_1 i.e., from c to b.

The current through $R_3 = I_2 = \frac{2}{9}\text{A} = 0.22\text{ A}$ flowing in the direction of I_1 i.e., from f to e.

Procedure of Solution of Circuit Problems

After solving the above problem we are in a position to apply the same procedure to analyse other direct current complex networks. While using Kirchhoff's rules in other problems, it is worthwhile to follow the approach given below:

(i) Draw the circuit diagram.

(ii) The choice of loops should be such that each resistance is included at least once in the selected loops.

(iii) Assume a loop current in each loop, all the loop currents should be in the same sense. It may be either clockwise or anticlockwise.

(iv) Write the loop equations for all the selected loops. For writing each loop equation the voltage change across any component is positive if traversed from low to high potential and it is negative if traversed from high to low potential.

(v) Solve these equations for the unknown quantities.

13.9 WHEATSTONE BRIDGE

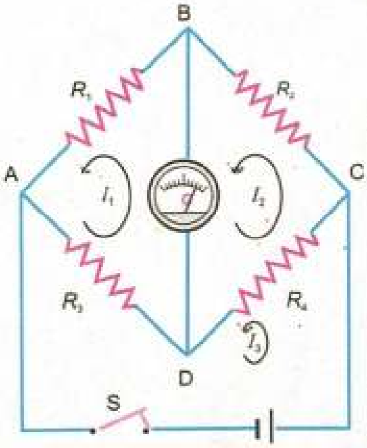


Fig. 13.23 Wheatstone bridge circuit

The Wheatstone bridge circuit shown in Fig. 13.23 consists of four resistances R_1, R_2, R_3 and R_4 connected in such a way so as to form a mesh ABCDA. A battery is connected between points A and C. A sensitive galvanometer of resistance R_g is connected between points B and D. If the switch S is closed, a current will flow through the galvanometer. We are to determine the condition under which no current flows through the galvanometer even after the switch is closed. For this purpose we analyse this circuit using Kirchhoff's second rule. We consider the loops ABDA, BCDB and ADCA and assume anticlockwise loop currents I_1, I_2 and I_3 through the loops respectively. The Kirchhoff's second rule as applied to loop ABDA gives

$$-I_1R_1-(I_1-I_2)R_g-(I_1-I_3)R_3=0 \quad \text{..........} \quad (13.23)$$

Similarly by applying the Kirchhoff's second rule to loop BCDB we have

$$-I_2R_2-(I_2-I_3)R_4-(I_2-I_1)R_g=0 \quad \text{..........} \quad (13.24)$$

The current flowing through the galvanometer will be zero if, $I_1 - I_2 = 0$ *or* $I_1 = I_2$. With this condition Eq. 13.23 and Eq. 13.24 reduce to

$$-I_1R_1=(I_1-I_3)R_3 \quad \text{..........} \quad (13.25)$$

$$-I_1R_2=(I_2-I_3)R_4 \quad \text{..........} \quad (13.26)$$

Dividing Eq. 13.25 by Eq. 13.26

$$\frac{R_1}{R_2}=\frac{R_3}{R_4} \quad \text{..........} \quad (13.27)$$

Thus whenever the condition of Eq. 13.27 is satisfied, no current flows through the galvanometer and it shows no deflection, or conversely when the galvanometer in the Wheatstone bridge circuit shows no deflection, Eq. 13.27 is satisfied.

If we connect three resistances R_1, R_2 and R_3 of known adjustable values and a fourth resistance R_4 of unknown value and the resistances R_1, R_2 and R_3 are so adjusted that the galvanometer shows no deflection then, from the known resistances R_1, R_2 and R_3 the unknown resistance R_4 can be determined by using Eq. 13.27.

13.10 POTENTIOMETER

Potential difference is usually measured by an instrument called a voltmeter. The voltmeter is connected across the two points in a circuit between which potential difference is to be measured. It is necessary that the resistance of the voltmeter be large compared to the circuit resistance across which the voltmeter is connected. Otherwise an appreciable current will flow through the voltmeter which will alter the circuit current and the potential difference to be measured. Thus the voltmeter can read the correct potential difference only when it does not draw any current from the circuit across which it is connected. An ideal voltmeter would have an infinite resistance.

However, there are some potential measuring instruments such as digital voltmeter and cathode ray oscilloscope which practically do not draw any current from the circuit because of their large resistance and are thus very accurate potential measuring instruments. But these instruments are very expensive and are difficult to use. A very simple instrument which can measure and compare potential differences accurately is a potentiometer.

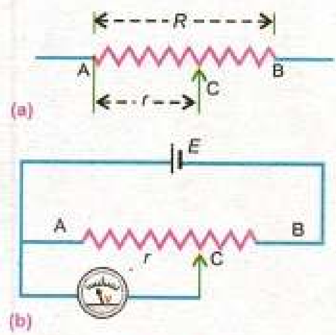


Fig. 13.24

A potentiometer consists of a resistor R in the form of a wire on which a terminal C can slide (Fig. 13.24 a). The resistance between A and C can be varied from 0 to R as the sliding contact C is moved from A to B. If a battery of emf E is connected across R (Fig. 13.24 b), the current flowing through it is $I = E/R$. If we represent the resistance between A and C by r, the potential drop between these points will be $rI = r\ E/R$. Thus as C is moved from A to B, r varies from 0 to R and the potential drop between A and C changes from 0 to E. Such an arrangement also known as potential divider can be used to measure the unknown emf of a source by using the circuit shown in Fig. 13.25. Here R is in the form of a straight wire of uniform area of cross section. A source of potential, say a cell whose emf E_x is to be measured, is connected between A and the sliding contact C through a galvanometer G. It should be noted that the positive terminal of E_x and that of the potential divider are connected to the same point A. If, in

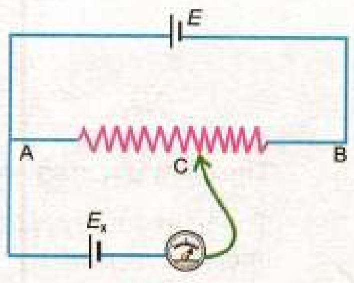


Fig. 13.25

the loop AGCA, the point C and the negative terminal of E_x are at the same potential then the two terminals of the galvanometer will be at the same potential and no current will flow through the galvanometer. Therefore, to measure the potential E_x, the position of C is so adjusted that the galvanometer shows no deflection. Under this condition, the emf E_x of the cell is equal to the potential difference between A and C whose value Er/R is known. In case of a wire of uniform cross section, the resistance is proportional to the length of the wire. Therefore, the unknown emf is also given by

$$E_x = E\frac{r}{R} = E\frac{\ell}{L} \quad \text{..........} \quad (13.28)$$

where L is the total length of the wire AB and ℓ is its length from A to C, after C has been adjusted for no deflection. As the maximum potential that can be obtained between A and C is E, so the unknown emf E_x should not exceed this value, otherwise the null condition will not be obtained. It can be seen that the unknown emf E_x is determined when no current is drawn from it and therefore, potentiometer is one of the most accurate methods for measuring potential.

The method for measuring the emf of a cell as described above can be used to compare the emfs E_1 and E_2 of two cells. The balancing lengths ℓ_1 and ℓ_2 are found separately for the two cells. Then,

$$E_1 = E\frac{\ell_1}{L} \quad \text{and} \quad E_2 = E\frac{\ell_2}{L}$$

Dividing these two equations, we get

$$\frac{E_1}{E_2} = \frac{\ell_1}{\ell_2} \quad \text{..........} \quad (13.29)$$

So the ratio of the emfs is equal to ratio of the balancing lengths.

SUMMARY

- The electric current is said to be caused by the motion of electric charge.
- The heat energy H produced by a current I in the wire of resistance R during a time interval t is given by $H = I^2Rt$
- The passage of current is always accompanied by a magnetic field in the surrounding space.

- Certain liquids conduct electricity due to some chemical reaction that takes place within them. The study of this process is known as electrolysis.
- The potential difference V across the ends of a conductor is directly proportional to the current I flowing through it provided the physical state such as temperature etc. of the conductor remains constant.
- The fractional change in resistance per kelvin is known as temperature coefficient of resistance.
- A thermistor is a heat sensitive resistor. Most thermistors have negative temperature coefficient of resistance.
- Electrical power $P = VI = I^2R = \frac{V^2}{R}$
- The emf E of the source is the energy supplied to unit charge by the cell.
- The sum of all the currents meeting at a point in a circuit is zero is the Kirchhoff's first rule.
- The algebraic sum of potential changes in a closed circuit is zero is known as Kirchhoff's second rule.

QUESTIONS

13.1 A potential difference is applied across the ends of a copper wire. What is the effect on the drift velocity of free electrons by

(i) increasing the potential difference

(ii) decreasing the length and the temperature of the wire

13.2 Do bends in a wire affect its electrical resistance? Explain.

13.3 What are the resistances of the resistors given in the figures A and B? What is the tolerance of each? Explain what is meant by the tolerance?

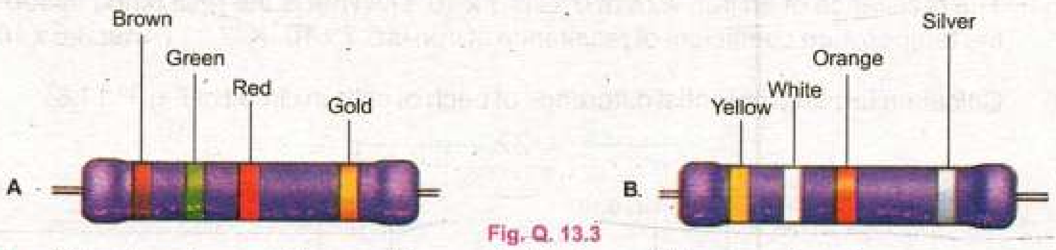


Fig. Q. 13.3

13.4 Why does the resistance of a conductor rise with temperature?

13.5 What are the difficulties in testing whether the filament of a lighted bulb obeys Ohm's law?

13.6 Is the filament resistance lower or higher in a 500 W, 220 V light bulb than in a 100 W, 220 V bulb?

13.7 Describe a circuit which will give a continuously varying potential.

13.8 Explain why the terminal potential difference of a battery decreases when the current drawn from it is increased?

13.9 What is Wheatstone bridge? How can it be used to determine an unknown resistance?

PROBLEMS

13.1 How many electrons pass through an electric bulb in one minute if the 300 mA current is passing through it? **(Ans:** 1.12×10^{20})

13.2 A charge of 90 C passes through a wire in 1 hour and 15 minutes. What is the current in the wire? **(Ans:** 20 mA)

13.3 Find the equivalent resistance of the circuit (Fig.P.13.3), total current drawn from the source and the current through each resistor.

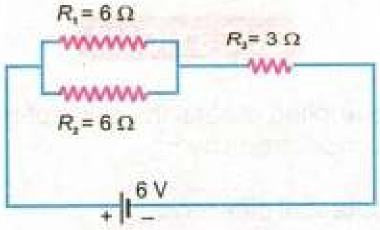


Fig. P. 13.3

(Ans: 6.0 Ω, 1.0 A, 0.5 A, 0.5 A, 1.0 A)

13.4 A rectangular bar of iron is 2.0 cm by 2.0 cm in cross section and 40 cm long. Calculate its resistance if the resistivity of iron is $11 \times 10^{-8}\ \Omega m$. **(Ans:** $1.1 \times 10^{-4}\ \Omega$)

13.5 The resistance of an iron wire at 0 °C is $1 \times 10^{4}\ \Omega$. What is the resistance at 500 °C if the temperature coefficient of resistance of iron is $5.2 \times 10^{-3}\ K^{-1}$? **(Ans:** $3.6 \times 10^{4}\ \Omega$)

13.6 Calculate terminal potential difference of each of cells in circuit of Fig. P.13.6.

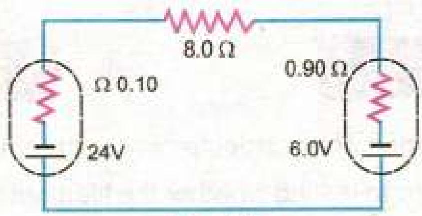


Fig.P. 13.6

(Ans: 23.8 V, 7.8 V)

13.7 Find the current which flows in all the resistances of the circuit of Fig. P.13.7.

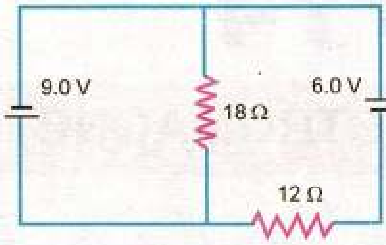


Fig. P. 13.7

(**Ans:** 1.25 A, 0.5 A)

13.8 Find the current and power dissipated in each resistance of the circuit, shown in Fig. P.13.8.

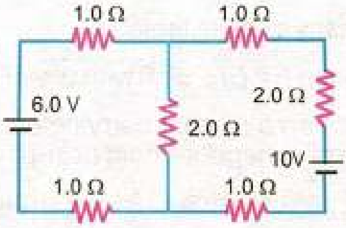


Fig.P. 13.8

(**Ans:** 0.8 A, 1.4 A, 2.2 A, 0.64 W, 1.96 W, 3.92 W, & 9.68 W)