Chapter 14

ELECTROMAGNETISM

Learning Objectives

At the end of this chapter the students will be able to:

1. Appreciate that a force might act on a current carrying conductor placed in a magnetic field.
2. Define magnetic flux density and the tesla.
3. Derive and use the equation $F = BIL \sin\theta$ with directions.
4. Understand how the force on a current carrying conductor can be used to measure the magnetic flux density of a magnetic field using a current balance.
5. Describe and sketch flux patterns due to a long straight wire.
6. Define magnetic flux and the weber.
7. Derive and use the relation $\Phi = \mathbf{B}.\mathbf{A}$.
8. Understand and describe Ampere's law.
9. Appreciate the use of Ampere's law to find magnetic flux density inside a solenoid.
10. Appreciate that there acts a force on a charged particle when it moves in a uniform magnetic field and in electric field.
11. Describe the deflection of beams of charged particles moving in a uniform magnetic field.
12. Understand and describe method to measure e/m.
13. Know the basic principle of cathode ray oscilloscope and appreciate its use.
14. Derive the expression of torque due to couple acting on a coil.
15. Know the principle, construction and working of a galvanometer.
16. Know how a galvanometer is converted into a voltmeter and an ammeter.
17. Describe and appreciate the use of AVO meter/multimeter.
18. Read through analogue scale and digital display on electrical meters.

Electric current generates magnetic field. At the same time, a changing magnetic field produces electric current. This interplay of electricity and magnetism is widely used in a number of electrical devices and appliances in modern age technology.

14.1 MAGNETIC FIELD DUE TO CURRENT IN A LONG STRAIGHT WIRE

Take a straight, thick copper wire and pass it vertically through a hole in a horizontal piece of cardboard. Place small compass needles on the cardboard along a circle with the centre at the wire. All the compass needles will point in the direction of N - S. Now pass a heavy current through the wire. It will be seen that the needles will rotate and will set themselves tangential to the circle (Fig. 14.1 a). On reversing the direction of current, the direction of needles is also reversed. As the current through the wire is stopped, all the needles again point along the N - S direction.

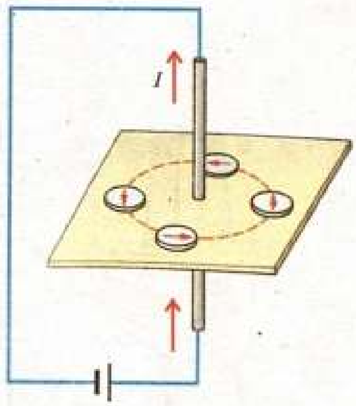


Fig. 14.1 (a)

Following conclusions can be drawn from the above mentioned experiment:

(i) A magnetic field is set up in the region surrounding a current carrying wire.

(ii) The lines of force are circular and their direction depends upon the direction of current.

(iii) The magnetic field lasts only as long as the current is flowing through the wire.

The direction of the lines of force can be found by a rule concluded directly from the above experiment which is stated as follows:

If the wire is grasped in fist of right hand with the thumb pointing in the direction of the current, the fingers of the hand will circle the wire in the direction of the magnetic field.

This is known as right hand rule and is illustrated in Fig. 14.1 (b).

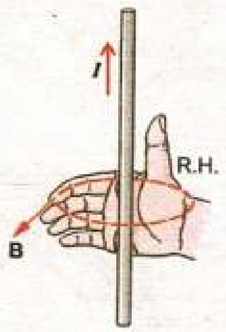


Fig. 14.1 (b)

14.2 FORCE ON A CURRENT CARRYING CONDUCTOR IN A UNIFORM MAGNETIC FIELD

We have seen that a current carrying conductor sets up its own magnetic field. If such a conductor is placed in an external magnetic field, the magnetic field of the conductor will interact with the external magnetic field, as a result of which the conductor may experience a force. To demonstrate

Do You Know?

If the middle finger of the right hand points in the direction of the magnetic field, the thumb in the direction of current, the force on the conductor will be normal to the palm towards the reader.

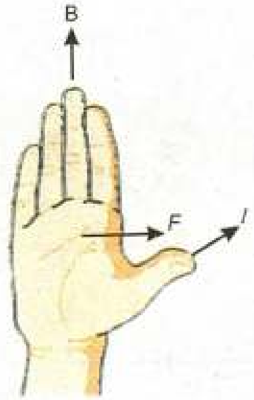


Fig. 14.3

this effect, consider a rod of copper, capable of moving on a pair of copper rails. The whole arrangement is placed in between the pole pieces of a horseshoe magnet so that the copper rod is subjected to a magnetic field directed vertically upwards (Fig. 14.2).

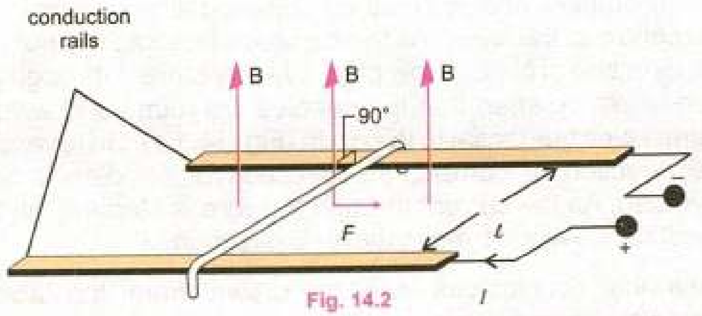


Fig. 14.2

When a current is passed through the copper rod from a battery, the rod moves on the rails. The relative directions of the current, magnetic field and the motion of the conductor are shown in Fig. 14.3. It can be seen that the force on a conductor is always at right angles to the plane which contains the rod and the direction of the magnetic field. The magnitude of the force depends upon the following factors:

(i) The force F is directly proportional to $\sin\alpha$ where α is the angle between the conductor and the field. From this, it follows that the force is zero if the rod is placed parallel to the field and is maximum when the conductor is placed at right angles to the field.

$$F \propto \sin\alpha$$

(ii) The force F is directly proportional to the current I flowing through the conductor. The more the current, greater is the force.

$$F \propto I$$

(iii) The force F is directly proportional to the length L of the conductor inside the magnetic field.

$$F \propto L$$

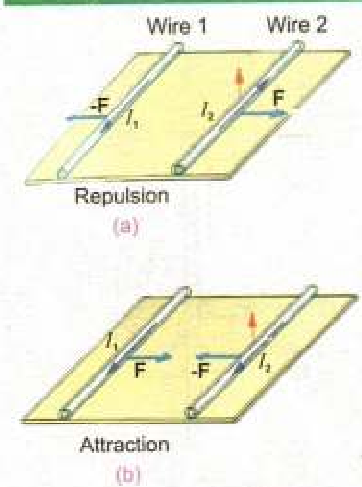
(iv) The force F is directly proportional to the strength of the applied magnetic field. The stronger the field, the greater is the force. If we represent the strength of the field by B, then

$$F \propto B$$

Combining all these factors,

$$F \propto I L B \sin\alpha$$

Do You Know?



(a) Two long parallel wires carrying currents I_1 and I_2 in opposite direction repel each other. (b) The wires attract each other when the currents are in the same direction.

or $$F = k\,I\,L\,B\sin\alpha$$

where k is constant of proportionality. If we follow SI units, the value of k is 1. Thus in SI units

$$F = ILB\sin\alpha \qquad \text{..........} \qquad (14.1)$$

Eq.14.1 provides a definition for the strength of magnetic field. If $I = 1$ A, $L = 1$ m and $\alpha = 90^\circ$, then $F = B$. Thus B, the strength of magnetic field which is also known as magnetic induction is defined as the force acting on one metre length of the conductor placed at right angle to the magnetic field when 1 A current is passing through it. In SI units the unit of B is tesla. A magnetic field is said to have a strength of one tesla if it exerts a force of one newton on one metre length of the conductor placed at right angles to the field when a current of one ampere passes through the conductor. Thus

$$1\,T = 1\,NA^{-1}m^{-1}$$

It can be seen that the force on a current carrying conductor is given both in magnitude and direction by the following equation:

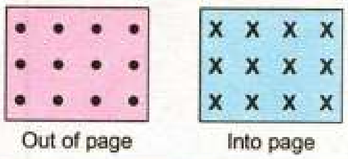
$$\mathbf{F} = I\,\mathbf{L} \times \mathbf{B} \qquad \text{..........} \qquad (14.2)$$

where the vector **L** is in the direction of current flow. The magnitude of the vector $I\,\mathbf{L} \times \mathbf{B}$ is $I\,L\,B\sin\alpha$, where α is the angle between the vector **L** and **B**. This gives the magnitude of the force. The direction of the force **F** (Fig. 14.3) is also correctly given by the right hand rule of the cross product of vectors of **L** and **B** i.e., rotate **L** to coincide with **B** through the smaller angle. Curl the fingers of right hand in the direction of rotation. The thumb points in the direction of force. In some situations the direction of the force is conveniently determined by applying the following rule:

Consider a straight current carrying conductor held at right angle to a magnetic field such that the current flows out of the plane of paper i.e., towards the reader as shown in Fig. 14.4. It is customary to represent a current flowing towards the reader by a symbol dot (•) and a current flowing away from him by a cross (x).

In order to find the direction of force, consider the lines of force (Fig. 14.4). The two fields tend to reinforce each other on left hand side of the conductor and cancel each other on the right side of it. The conductor tends to move towards the weaker part of the field i.e., the force on the conductor will be directed towards right in a direction at right angles to both the

For Your Information



Convention to represent direction

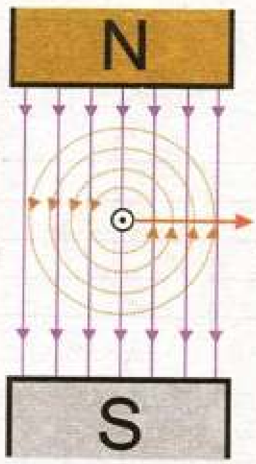


Fig. 14.4 The magnetic force on the current carrying conductor placed at right angle to a magnetic field.

conductor and the magnetic field. This rule is often referred as extension of right hand rule. It can be seen that the direction of the force is the same as given by the direction of the vector **L** x **B**.

Example 14.1: A 20.0 cm wire carrying a current of 10.0 A is placed in a uniform magnetic field of 0.30 T. If the wire makes an angle of 40° with the direction of magnetic field, find the magnitude of the force acting on the wire.

Solution:

Length of the wire = L = 20.0 cm = 0.20 m

Current = I = 10.0 A

Strength of magnetic field = B = 0.30 T

Angle = α = 40°

Substituting these values in the equation

$$F = I B L \sin\alpha$$

$$F = 10.0\,A \times 0.30\,T \times 0.20\,m \times \sin 40^\circ = 0.39\,N$$

14.3 MAGNETIC FLUX AND FLUX DENSITY

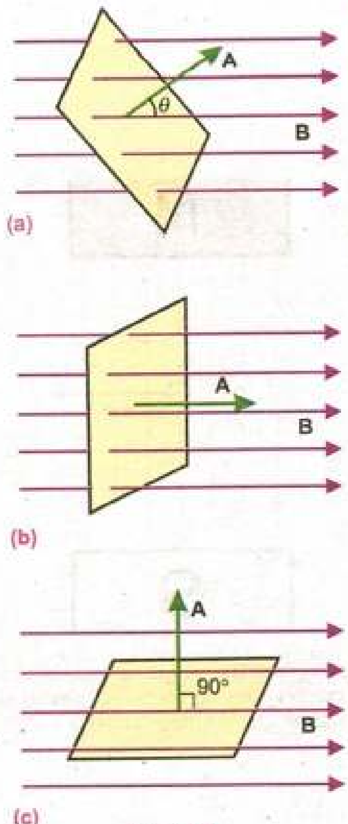


Fig. 14.5

Like electric flux, the magnetic flux Φ_B through a plane element of area **A** in a uniform magnetic field **B** is given by dot product of **B** and **A** (Fig. 14.5).

$$\Phi_B = \mathbf{B.A}$$

$$\Phi_B = BA\cos\theta \quad .... \quad (14.3)$$

Note that **A** is a vector whose magnitude is the area of the element and whose direction is along the normal to the surface of the element, θ is the angle between the directions of the vectors **B** and **A**.

In Fig. 14.5 (b) the field is directed along the normal to the area, so θ is zero and the flux is maximum, equal to BA. When the field is parallel to the plane of the area (Fig.14.5 c), the angle between the field and normal to area is 90° i.e., θ = 90°, so the flux through the area in this position is zero.

In case of a curved surface placed in a non uniform magnetic field, the curved surface is divided into a number of small surface elements, each element being assumed plane and the flux through the whole curved surface is calculated by sum of the contributions from all the elements of the surface. From the definition of tesla, the unit of magnetic flux is NmA^{-1} which is called weber (Wb).

According to Eq.14.3, the magnetic induction **B** is the flux per unit area of a surface perpendicular to **B**, hence it is also called as flux density. Its unit is then, Wbm^{-2}. Therefore, magnetic induction, i.e., the magnetic field strength is measured in terms of Wbm^{-2} or $NA^{-1}m^{-1}$ (tesla).

Example 14.2: The magnetic field in a certain region is given by $\mathbf{B} = (40\hat{i} - 18\hat{k})\ Wbm^{-2}$. How much flux passes through a 5.0 cm^2 area loop in this region if the loop lies flat in the xy- plane?

Solution:

$$\text{Magnetic induction} = \mathbf{B} = 40\hat{i} - 18\hat{k}$$

$$\text{Area of the loop} = \Delta\mathbf{A} = 5.0 \times 10^{-4}\ m^2\ \hat{k}$$

$$Flux = \Phi_B = \mathbf{B}.\Delta\mathbf{A}$$

$$= (40\hat{i} - 18\hat{k}).(5.0 \times 10^{-4} m^2 \hat{k})$$

$$\Phi_B = 90 \times 10^{-4}\ Wb$$

14.4 AMPERE'S LAW AND DETERMINATION OF FLUX DENSITY B

We know that an electric current produces a magnetic field. Ampere, after carrying out a series of experiments, generalized his results into a law known as Ampere circuital law by which the magnetic flux density **B** at any point due to a current carrying conductor can be easily computed as explained below:

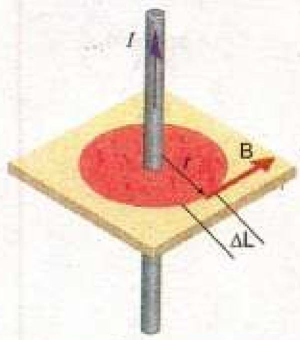


Fig. 14.6 Ampere's law to find the magnetic field in the vicinity of this long, straight, current-carrying wire.

Consider a closed path in the form a circle of radius r enclosing the current carrying wire (Fig.14.6). This closed path is referred as Amperean path. Divide this path into small elements of length like $\Delta\mathbf{L}$. Let **B** be the value of flux density at the site of $\Delta\mathbf{L}$. Determine the value of $\mathbf{B}.\Delta\mathbf{L}$. If θ is the angle between **B** and $\Delta\mathbf{L}$, then

$$\mathbf{B}.\Delta\mathbf{L} = B\Delta L\cos\theta$$

$B\cos\theta$ represents the component of **B** along the element of length $\Delta\mathbf{L}$ i.e., Component of **B** parallel to $\Delta\mathbf{L}$. Thus $\mathbf{B}.\Delta\mathbf{L}$ represents the product of the length of the element $\Delta\mathbf{L}$ and the component of **B** parallel to $\Delta\mathbf{L}$. Ampere stated that the sum of the quantities $\mathbf{B}.\Delta\mathbf{L}$ for all path elements into which the complete loop has been divided equals μ_o times the total current enclosed by the loop, where μ_o is a constant, known as permeability of free space. In SI units its value is $4\pi \times 10^{-7} WbA^{-1}m^{-1}$. This can be mathematically expressed as

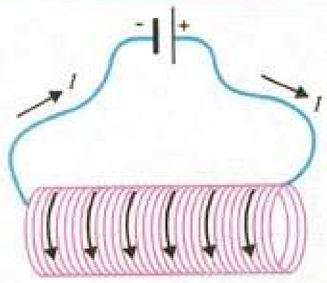
$$(\mathbf{B}.\Delta \mathbf{L})_1 + (\mathbf{B}.\Delta \mathbf{L})_2 + ... + (\mathbf{B}.\Delta \mathbf{L})_r + ... + (\mathbf{B}.\Delta \mathbf{L})_N = \mu_o I$$

or $$\sum_{r=1}^{N} (\mathbf{B}.\Delta \mathbf{L})_r = \mu_o I \quad .... \quad (14.4)$$

where $(\mathbf{B}.\Delta \mathbf{L})_r$ is the value of $\mathbf{B}.\Delta \mathbf{L}$ along the r th element and N is the total number of elements into which the loop has been divided. This is known as Ampere's circuital law.

Field Due to a Current Carrying Solenoid

For Your Information



A solenoid

A solenoid is a long, tightly wound, cylindrical coil of wire. When current passes through such a coil, it behaves like a bar magnet. The magnetic field produced by it is shown in Fig. 14.7(a). The field inside a long solenoid is uniform and much strong whereas out side the solenoid, it is so weak that it can be neglected as compared to the field inside.

The value of magnetic filed **B** can be easily determined by applying Ampere's circuital law. Consider a rectangular loop abcd as shown in Fig. 14.7 (b). Divide it into four elements of length ab = ℓ_1, bc = ℓ_2, cd = ℓ_3 and da = ℓ_4.

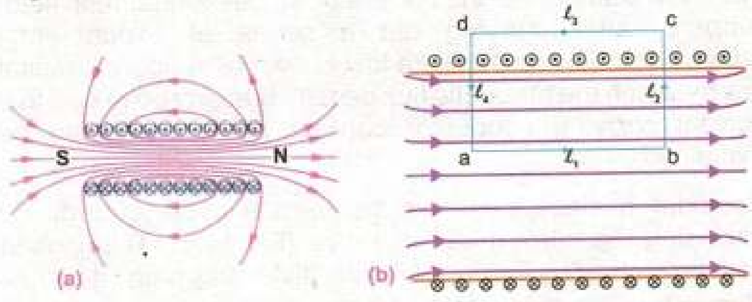


Fig. 14.7

Applying Ampere's law, we have

$$\sum_{r=1}^{4} (\mathbf{B}.\Delta \mathbf{L})_r = \mu_o \times \text{current enclosed}$$

$$(\mathbf{B}.\Delta \mathbf{L})_1 + (\mathbf{B}.\Delta \mathbf{L})_2 + (\mathbf{B}.\Delta \mathbf{L})_3 + (\mathbf{B}.\Delta \mathbf{L})_4 = \mu_o \times \text{ current enclosed}$$

Now we will calculate the value of $\mathbf{B}.\Delta \mathbf{L}$ for each of the elements. First we will consider the element ab = ℓ_1 that lies inside the solenoid. Field inside the solenoid is uniform and is parallel to (Fig. 14.7b), so

$$(\mathbf{B}.\Delta\mathbf{L})_1 = \ell_1 B\cos 0^\circ$$
$$= \ell_1 B$$

For the element cd = ℓ_3, that lies outside the solenoid , the field **B** is zero, so

$$(\mathbf{B}.\Delta\mathbf{L})_3 = 0$$

Again **B** is perpendicular to ℓ_2 and ℓ_4 inside the solenoid and is zero outside, so

$$(\mathbf{B}.\Delta\mathbf{L})_2 = (\mathbf{B}.\Delta\mathbf{L})_4 = 0$$

$$\therefore \quad \sum_{r=1}^{4}(\mathbf{B}.\Delta\mathbf{L})_r = B\,\ell_1 = \mu_o \times \text{current enclosed}$$

To find the current enclosed, consider the rectangular surface bounded by the loop abcda.

If n is the number of turns per unit length of the solenoid, the rectangular surface will intercept $n\ell_1$ turns, each carrying a current I. So the current enclosed by the loop is $n\ell_1 I$. Thus Ampere's law gives

$$B\,\ell_1 = \mu_o \times n\,\ell_1 I$$

or $\quad B = \mu_o n I \quad$ .... (14.5)

The field **B** is along the axis of the solenoid and its direction is given by right hand grip rule which states "hold the solenoid in the right hand with fingers curling in the direction of the current, the thumb will point in the direction of the field".

Example 14.3: A solenoid 15.0 cm long has 300 turns of wire. A current of 5.0 A flows through it. What is the magnitude of magnetic field inside the solenoid?

Solution:

Length of the solenoid = L = 15.0 cm = 0.15 m

Total number of turns = N = 300

Current = I = 5.0A

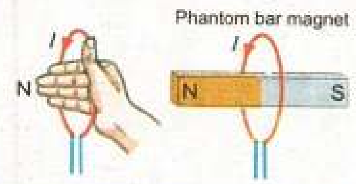
Permeability of free space = $\mu_o = 4\pi \times 10^{-7}\ \text{WbA}^{-1}\text{m}^{-1}$

Number of turns per unit length = $n = \dfrac{N}{\ell} = \dfrac{300}{0.15\ \text{m}}$

$= 2000$ turns/m

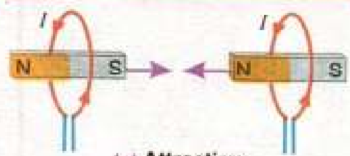
Magnetic field = $B = \mu_o n I$

Do You Know?

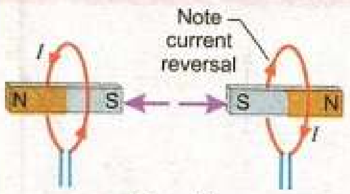


The current loop can be imagined to be a phantom bar magnet with a north pole and a south pole.

For Your Information



(a) **Attraction**



(b) **Repulsion**

The "phantom" magnet included for each loop helps to explain the attraction and repulsion between the loops.

$= 4 \times 10^{-7}\,WbA^{-1}m^{-1} \times 2000\,m^{-1} \times 5.0\,A$

$B = 1.3 \times 10^{-2}\,Wbm^{-2}$

14.5 FORCE ON A MOVING CHARGE IN A MAGNETIC FIELD

We have seen that a current carrying conductor, when placed in a magnetic field, experiences a force. The current through the conductor is because of the motion of charges. Actually the magnetic field exerts force on these moving charges due to which the conductor experiences force. We are interested in calculating the force exerted on the moving charges.

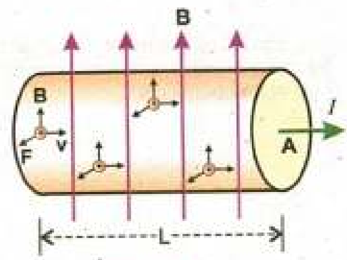


Fig. 14.8

Consider the situation as shown in Fig. 14.8 where we see a portion of the wire that is carrying a current I. Suppose there are n charge carriers per unit volume of the wire, and that each is moving with velocity **v** as shown. We will now find how long it takes for all the charge carriers originally in the wire segment shown to exit through the end area **A**.

The volume of the wire segment is AL. Because there are n charge carriers per unit volume, the number of charge carrier in the segment is $n\,AL$. If the charge on a charge carrier is q, each of it, as it crosses the end area, will transport a charge q through it. Assuming the speed of the carriers to be v, the carrier entering the left face of the segment takes a time $\Delta t = L/v$ to reach the right hand face. During this time, all the charge carriers originally in the segment, namely $n\,AL$, will exit through the right hand face. As each charge carrier has a charge q, the charge ΔQ that exits through the end area in time $\Delta t = L/v$ is

$$\Delta Q = n\,ALq$$

Then, from the definition of the current, the current I through the conductor is

$$I = \frac{\Delta Q}{\Delta t} = \frac{n\,ALq}{L/v}$$

$$= nAqv \qquad .... \qquad (14.6)$$

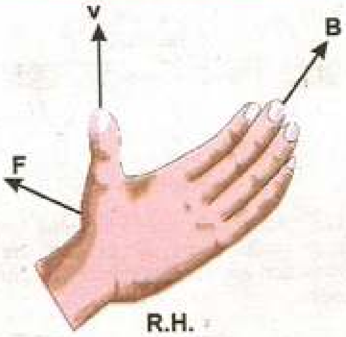
By Eq.14.2, the force on the segment **L** of a conductor, carrying current I is given by

$$\mathbf{F}_L = I\,\mathbf{L} \times \mathbf{B}$$

Substituting the value of the current I,

$$\mathbf{F}_L = nAqv\,\mathbf{L} \times \mathbf{B} \qquad .... \qquad (14.7)$$

For Your Information



The moving charge experiences a magnetic force **F** because of the magnetic field **B**.

In Fig.14.8, it can be seen that the direction of the segment **L** is the same as the direction of the velocity of the charge carriers. If $\hat{L}$ is a unit vector along the direction of the segment **L** and $\hat{v}$, a unit vector along the velocity vector **v**, then $\hat{L} = \hat{v}$

$$v\mathbf{L} = v\hat{L}L$$

$$= v\hat{v}L = \mathbf{v}L$$

Substituting the value of $v\mathbf{L}$ in Eq.14.7, we have

$$\mathbf{F}_L = nAq(\mathbf{v}L) \times B$$

$$= nALq\,\mathbf{v} \times \mathbf{B}$$

nAL is the total number of charge carries in the segment **L**, so the force experienced by a single charge carrier is

$$F = \frac{F_L}{n\,AL} = q\mathbf{v} \times \mathbf{B}$$

Thus the force experienced by a single charge carrier moving with velocity **v** in magnetic field of strength **B** is

$$\mathbf{F} = q(\mathbf{v} \times \mathbf{B}) \quad \text{..........} \quad (14.8)$$

Although the Eq.14.8 has been derived with reference to charge carrier moving in a conductor but it does not involve any parameter of the conductor, so the Eq.14.8 is quite general and it holds for any charge carrier moving in a magnetic field.

If an electron is projected in a magnetic field with a velocity **v**, it will experience a force which is given by putting $q = -e$ in Eq.14.8 where e is the magnitude of the electronic charge.

$$\therefore \quad \mathbf{F} = -e\mathbf{v} \times \mathbf{B} \quad \text{..........} \quad (14.9)$$

In case of proton, F is obtained by putting $q = +e$.

$$\therefore \quad \mathbf{F} = +e\mathbf{v} \times \mathbf{B} \quad \text{..........} \quad (14.10)$$

Note that in case of proton or a positive charge the direction of the force is given by the direction of the vector **v** x **B** i.e., rotate **v** to coincide with **B** through the smaller angle of rotation and curl the fingers of right hand in the direction of rotation. Thumb will point in the direction of the force. This is illustrated in Fig.14.9 in which the proton enters into a magnetic field, as shown in figure, along the direction of dotted line. It experiences a force in the upward direction as given by the vector **v** x ***B***. As a result of this force the proton is deflected upwards as shown in Fig. 14.9. The direction of the force on a moving negative charge will be opposite to that of positive charge. Due to this force, the electron is deflected in the downward direction as it enters into a magnetic field. It

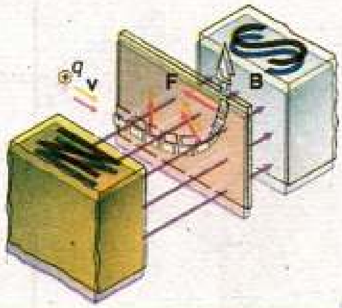


Fig. 14.9 Magnetic force **F** is perpendicular to both the magnetic field **B** and the velocity **v** and causes the particle's trajectory to bend in a vertical plane.

may be noted that the magnitude of the force on a moving charge carrier is $qvB\sin\theta$ where θ is the angle between the velocity of the carrier and the magnetic field. It is maximum when $\theta = 90^\circ$ i.e., when the charged particle is projected at right angles to the field. It is zero when $\theta = 0^\circ$ i.e., a charged particle projected in the direction of the field experiences no force.

14.6 MOTION OF CHARGED PARTICLE IN AN ELECTRIC AND MAGNETIC FIELD

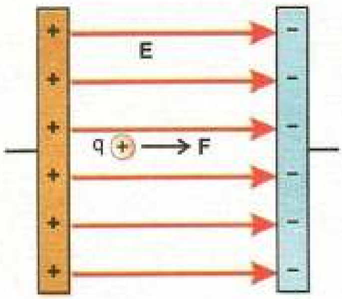


Fig. 14.10

When an electric charge q is placed in an electric field **E**, It experiences a force **F** parallel to electric field (Fig. 14.10). It is given by

$$\mathbf{F} = q\mathbf{E}$$

If the charge is free to move, then it will accelerate according to Newton's second law as

$$\mathbf{a} = \frac{\mathbf{F}}{m} = \frac{q\mathbf{E}}{m} \quad .... \quad (14.11)$$

If electric field is uniform, then acceleration is also uniform and hence, the position of the particle at any instant of time can be found by using equations of uniformly accelerated motion.

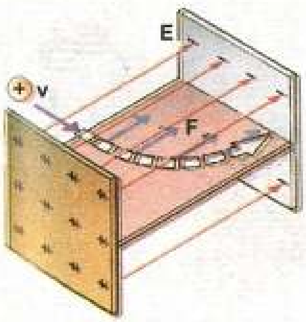
When a charge particle q is moving with velocity **v** in a region where there is an electric field **E** and magnetic field **B**, the total force **F** is the vector sum of the electric force $q\mathbf{E}$ and magnetic force $q(\mathbf{v}\times\mathbf{B})$ that is,

$$\mathbf{F} = \mathbf{F}_e + \mathbf{F}_b$$

$$\mathbf{F} = q\mathbf{E} + q(\mathbf{v}\times\mathbf{B}) \quad .... \quad (14.12)$$

This force **F** is known as the Lorentz force. It is to be pointed out that only the electric force does work, while no work is done by the magnetic force which is simply a deflecting force.

Do You Know?



The electric force **F** that acts on a positive charge is parallel to the electric field **E** and causes the particle's trajectory to bend in a horizontal plane.

14.7 DETERMINATION OF *e/m* OF AN ELECTRON

Let a narrow beam of electrons moving with a constant speed v be projected at right angles to a known uniform magnetic field **B** directed into plane of paper. We have seen that electrons will experience a force

$$\mathbf{F} = -e\,\mathbf{v}\times\mathbf{B}$$

The direction of the force will be perpendicular to both **v** and **B**. As the electron is experiencing a force that acts at right angle to its velocity, so it will change the direction of the velocity. The magnitude of velocity will remain unchanged. The magnitude of the force is $evB\sin\theta$. As θ is $90°$, so $F = evB$. As both v and B do not change, the magnitude of F is constant. Thus the electrons are subjected to a constant force evB at right angle to their direction of motion. Under the action of this force, the electrons will move along a circle as shown in Fig. 14.11.

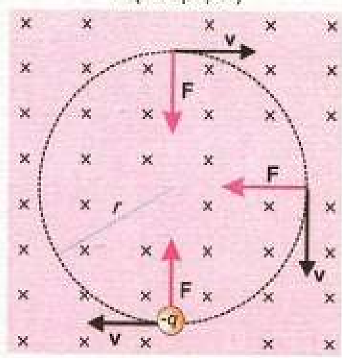


Fig 14.11 An electron is moving perpendicular to a constant magnetic field. The magnetic force **F** causes the particle to move on a circular path.

The magnetic force $F = Bev$ provides the necessary centripetal force $\frac{mv^2}{r}$ to the electron of mass m to move along a circular trajectory of radius r. Thus we have

$$Bev = \frac{mv^2}{r}$$

or $$\frac{e}{m} = \frac{v}{Br} \quad \text{..........} \quad (14.13)$$

If v and r are known, e/m of the electron is determined. The radius r is measured by making the electronic trajectory visible. This is done by filling a glass tube with a gas such as hydrogen at low pressure. This tube is placed in a region occupied by a uniform magnetic field of known value. As electrons are shot into this tube, they begin to move along a circle under the action of magnetic force. As the electrons move, they collide with atoms of the gas. This excites the atoms due to which they emit light and their path becomes visible as a circular ring of light (Fig. 14.12). The diameter of the ring can be easily measured.

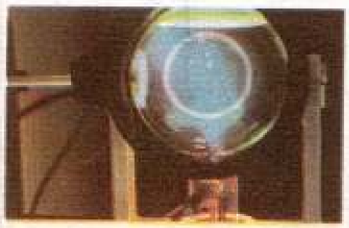


Fig. 14.12

In order to measure the velocity v of the electrons, we should know the potential difference through which the electrons are accelerated before entering into the magnetic field. If V is this potential difference, the energy gained by electrons during their acceleration is Ve. This appears as the kinetic energy of electrons

$$\frac{1}{2}mv^2 = Ve$$

or $$v = \sqrt{\frac{2Ve}{m}}$$

Substituting the value of v in Eq. 14.13, we have

$$\frac{e}{m} = \frac{2V}{B^2r^2} \quad \text{..........} \quad (14.14)$$

Example 14.4: Find the radius of an orbit of an electron moving at a rate of 2.0×10^7 ms^{-1} in a uniform magnetic field of 1.20×10^{-3}T.

Solution:

Speed of the electron $= v = 2.0 \times 10^7 \text{ ms}^{-1}$

Magnetic field strength $= B = 1.20 \times 10^{-3}\text{T}$

Mass of the electron $= m = 9.11 \times 10^{-31}\text{kg}$

Charge on electron $= e = 1.61 \times 10^{-19}\text{C}$

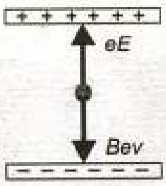
The radius of the orbit is

$$r = \frac{mv}{eB}$$

$$= \frac{9.11 \times 10^{-31}\text{ kg} \times 2.0 \times 10^{7}\text{ ms}^{-1}}{1.61 \times 10^{-19}\text{ C} \times 1.20 \times 10^{-3}\text{ T}}$$

$$r = 9.43 \times 10^{-2}\text{ m}$$

Example 14.5: Alpha particles ranging in speed from 1000 ms^{-1} to 2000 ms^{-1} enter into a velocity selector where the electric intensity is 300 Vm^{-1} and the magnetic induction 0.20 T. Which particle will move undeviated through the field?



Solution:

$E = 300\text{ Vm}^{-1} = 300\text{ NC}^{-1}$, $B = 0.20\text{ T}$

Only those particles will be able to pass through the plate for which the electric force eE acting on the particles balances the magnetic force Bev on the particle as shown in the figure.

Therefore $eE = Bev$

Thus, the selected speed is

$$v = \frac{E}{B} = \frac{300\text{ NC}^{-1}}{0.20\text{ NA}^{-1}\text{m}^{-1}} = 1500\text{ ms}^{-1}$$

The alpha particles having a speed of 1500 ms^{-1} will move undeviated through the field.

14.8 CATHODE RAY OSCILLOSCOPE

Cathode ray oscilloscope (CRO) is a very versatile electronic instrument which is, in fact, a high speed graph plotting device. It works by deflecting beam of electrons as they pass through uniform electric field between the two sets of parallel plates as shown in the Fig. 14.13(a). The deflected beam then falls on a fluorescent screen where it makes a visible spot.

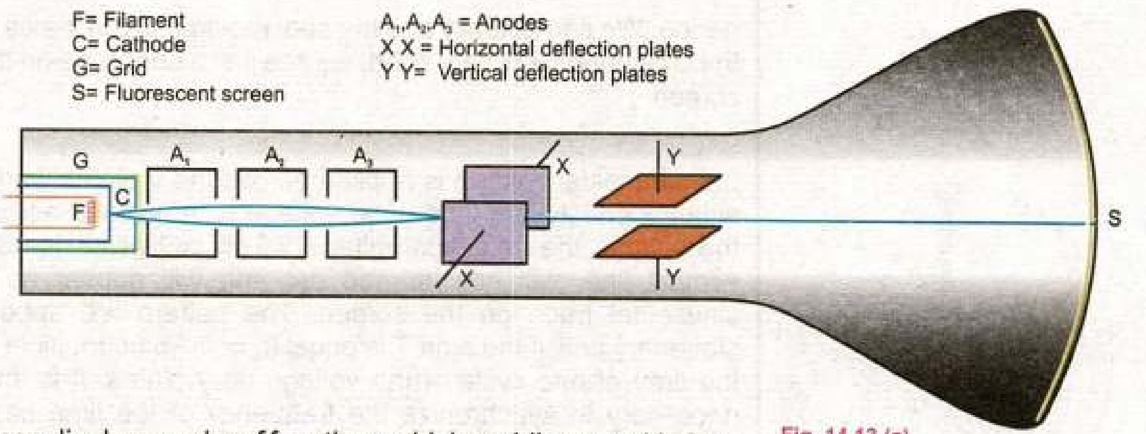
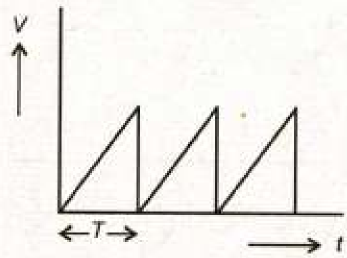


Fig. 14.13 (a)

It can display graphs of functions which rapidly vary with time. It is called cathode ray oscilloscope because it traces the desired waveform with a beam of electrons which are also called cathode rays.

The beam of the electrons is provided by an electron gun which consists of an indirectly heated cathode, a grid and three anodes. The filament F heats the cathode C which emits electrons. The anodes A_1, A_2, A_3 which are at high positive potential with respect to cathode, accelerate as well as focus the electronic beam to fixed spot on the screen S. The grid G is at a negative potential with respect to cathode. It controls the number of electrons which are accelerated by anodes, and thus it controls the brightness of the spot formed on the screen.



Saw tooth voltage waveform

Fig. 14.13 (b)

Now we would explain how the waveform of various voltages is formed in CRO.

The two set of deflecting plates, shown in Fig. 14.13(a) are usually referred as x and y deflection plates because a voltage applied between the x plates deflects the beam horizontally on the screen i.e., parallel to x-axis. A voltage applied across the y plates deflects the beam vertically on the screen i.e., along the y-axis. The voltage that is applied across the x plates is usually provided by a circuit that is built in the CRO. It is known as sweep or time base generator. Its output waveform is a saw tooth voltage of period T (Fig. 14.13-b). The voltage increases linearly with time for a period T and then drops to zero. As this voltage is impressed across the x plates, the spot is deflected linearly with time along the x-axis for a time T. Then the spot returns to its starting point on the screen very quickly because a saw tooth voltage rapidly falls to its initial value at the end of each

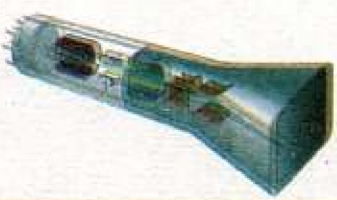


Fig. 14.13 (b)
hree dimensional view of CRO

period. We can actually see the spot moving on the x-axis. If the time period T is very short, we see just a bright line on the screen.

If a sinusoidal voltage is applied across the y plates when, simultaneously, the time base voltage is impressed across the x plates, the sinusoidal voltage, which itself gives rise to a vertical line, will now spread out and will appear as a sinusoidal trace on the screen. The pattern will appear stationary only if the time T is equal to or is some multiple of the time of one cycle of the voltage on y plates. It is thus necessary to synchronize the frequency of the time base generator with the frequency of the voltage at the y plates. This is possible by adjusting the synchronization controls provided on the front panel of the CRO.

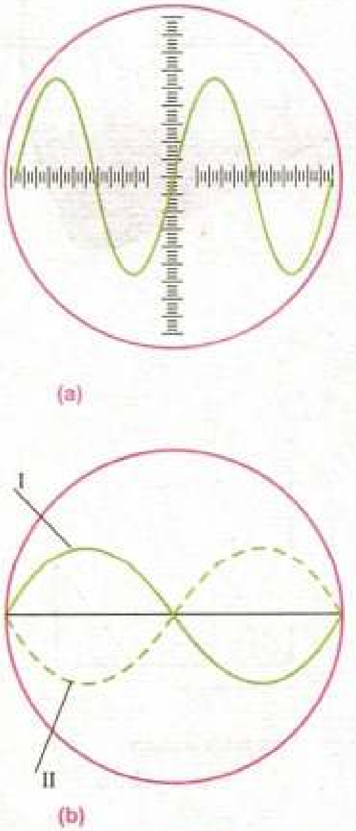


Fig. 14.14

Uses of CRO

The CRO is used for displaying the waveform of a given voltage. Once the waveform is displayed, we can measure the voltage, its frequency and phase. For example, Fig. 14.14(a) shows the waveform of an alternating voltage. As the y-axis is calibrated in volts and the x-axis in time, we can easily find the instantaneous value and peak value of the voltage. The time period can also be determined by using the time calibration of x-axis. Information about the phase difference between two voltages can be obtained by simultaneously displaying their waveforms. For example, the waveforms of two voltages are shown in Fig. 14.14(b). These waveforms show that when the voltage of I is increasing, that of II is decreasing and vice versa. Thus the phase difference between these voltages is 180°.

14.9 TORQUE ON A CURRENT CARRYING COIL

Consider a rectangular coil carrying a current I. The coil is capable of rotation about an axis . Suppose it is placed in uniform magnetic field **B** with its plane along the field (Fig. 14.15). We know that a current carrying conductor of length **L** when placed in a magnetic field experiences a force $F = I\,L\,B \sin\theta$ where θ is the angle between conductor and the field. In case of sides AB and CD of the coil, the angle θ is zero or 180°, so the force on these sides will be zero. In case of sides DA and BC, the angle θ is 90° and the force on these sides will be

$$F_1 = F_2 = ILB$$

where L is the length of these sides, $\mathbf{F}_1$ is the force on the side DA and $\mathbf{F}_2$ on BC. The direction of the force is given by the vector $I\,\mathbf{L} \times \mathbf{B}$. It can be seen that $\mathbf{F}_1$ is directed out of the plane of paper and $\mathbf{F}_2$ into the plane of paper (Fig. 14.15 a). Therefore, the forces $\mathbf{F}_1$ and $\mathbf{F}_2$ being equal and opposite form a couple which tends to rotate it about the axis. The torque of this couple is given by

$$\tau = \text{Force} \times \text{Moment arm}$$

$$= ILB \times a$$

where a is the moment arm of the couple and is equal to the length of the side AB or CD. La is the area A of the coil,

$$\tau = IBA \quad \text{..........} \quad (14.15)$$

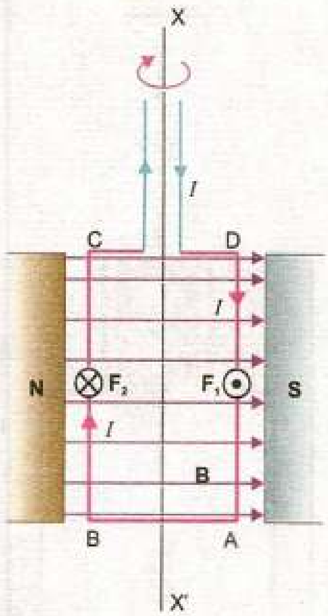


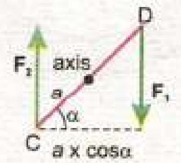
Fig 14.15 (a)

Note that the Eq.14.15 gives the value of torque when the field **B** is in the plane of the coil. However if the field makes an angle α with the plane of the coil, as shown in Fig 14.15(b), the moment arm now becomes $a\cos\alpha$. So

$$\tau = ILB \times a\cos\alpha$$

or

$$\tau = IBA\cos\alpha \quad \text{..........} \quad (14.16)$$



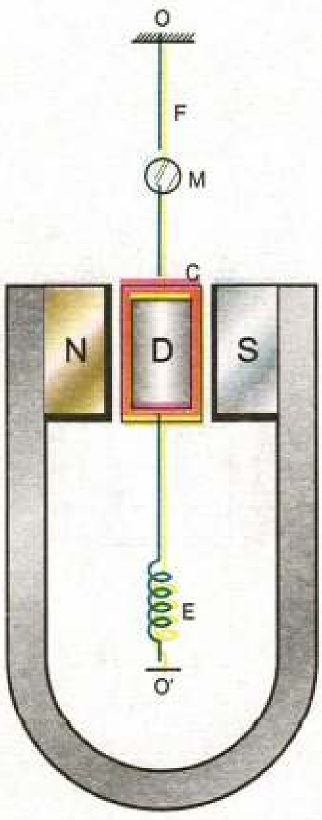
(Top view of coil)
Fig 14.15 (b)

14.10 GALVANOMETER

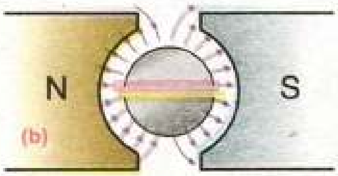
A galvanometer is an electrical instrument used to detect the passage of current. Its working depends upon the fact that when a conductor is placed in a magnetic field, it experience a force as soon as a current passes through it. Due to this force, a torque τ acts upon the conductor if it is in the form of a coil or loop.

$$\tau = NIBA\cos\alpha$$

where N is the number of turns in the coil, A is its area, I is current passing through it, **B** is the magnetic field in which the coil is placed such that its plane makes an angle α with the direction of **B**. Due to action of the torque, the coil rotates and



(a)



(b)

Fig. 14.16 (a) Moving coil galvanometer (b) Concave pole piece and soft iron cylinder makes the field radial and stronger.

thus it detects the current. The construction of a moving coil galvanometer is shown in Fig. 14.16 (a).

A rectangular coil C is suspended between the concave shaped poles N and S of a U-shaped magnet with the help of a fine metallic suspension wire. The rectangular coil is made of enameled copper wire. It is wound on a frame of non-magnetic material. The suspension wire F is also used as one current lead to the coil. The other terminal of the coil is connected to a loosely wound spiral E which serves as the second current lead. A soft iron cylinder D is placed inside the coil to make the field radial and stronger near the coil as shown in Fig. 14.16 (b).

When a current is passed through the coil, it is acted upon by a couple which tends to rotate the coil. This couple is known as deflecting couple and is given by $NIBA\cos\alpha$. As the coil is placed in a radial magnetic field in which the plane of the coil is always parallel to the field (Fig. 14.16 b), so α is always zero. This makes $\cos\alpha = 1$ and thus,

$$\text{Deflecting couple} = NIBA$$

As the coil turns under the action of deflecting couple, the suspension wire Fig. (14.16 a) is twisted which gives rise to a torsional couple. It tends to untwist the suspension and restore the coil to its original position. This couple is known as restoring couple. The restoring couple of the suspension wire is proportional to the angle of deflection θ as long as the suspension wire obeys Hooke's law. Thus

$$\text{Restoring torque} = c\,\theta$$

where the constant c of the suspension wire is known as torsional couple and is defined as couple for unit twist.

Under the effect of these two couples, coil comes to rest when

$$\text{Deflecting torque} = \text{Restoring torque}$$

$$NIBA = c\,\theta$$

or $$I = \frac{c}{BAN}\theta \qquad \text{..........} \qquad (14.17)$$

Thus $I \propto \theta$ since $\frac{c}{BAN} = \text{Constant}$

Thus the current passing through the coil is directly proportional to the angle of deflection.

There are two methods commonly used for observing the angle of deflection of the coil. In sensitive galvanometers the angle of deflection is observed by means of small mirror attached to the coil along with a lamp and scale arrangement (Fig.14.17). A beam of light from the lamp is directed towards the mirror of the galvanometer. After reflection from the mirror it produces a spot on a translucent scale placed at a distance of one metre from the galvanometer. When the coil rotates, the mirror attached to coil also rotates and spot of light moves along the scale. The displacement of the spot of light on the scale is proportional to the angle of deflection (provided the angle of deflection is small).

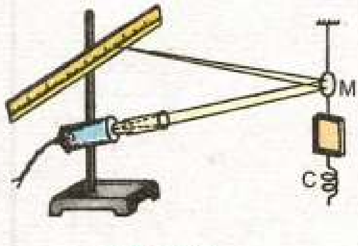


Fig. 14.17

The galvanometer used in school and college laboratories is a pivoted type galvanometer. In this type of galvanometer, the coil is pivoted between two jewelled bearings. The restoring torque is provided by two hair springs which also serve as current leads. A light aluminium pointer is attached to the coil which moves over a scale (Fig.14.18). It gives the angle of deflection of the coil.

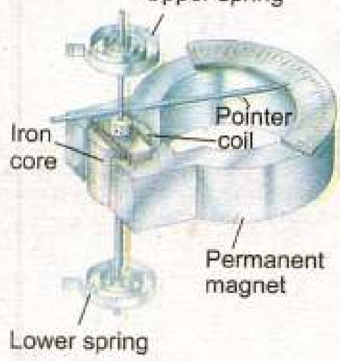


Fig. 14.18

It is obvious from Eq.14.17 that a galvanometer can be made more sensitive (to give large deflection for a given current) if c/BAN is made small. Thus, to increase sensitivity of a galvanometer, c may be decreased or B, A and N may be increased. The couple c for unit twist of the suspension wire can be decreased by increasing its length and by decreasing its diameter. This process, however, cannot be taken too far, as the suspension must be strong enough to support the coil. Another method to increase the sensitivity of galvanometer is to increase N, the number of turns of the coil. In case of suspended coil type galvanometer, the number of turns can not be increased beyond a limit because it will make the coil heavy. To compensate for the loss of sensitivity, in case fewer turns are used in the coil, we increase the value of the magnetic field employed. We define current sensitivity of a galvanometer as the current, in microamperes, required to produce one millimetre deflection on a scale placed one metre away from the mirror of the galvanometer.

When the current passing through the galvanometer is discontinued, the coil will not come to rest as soon as the current flowing through the coil is stopped. It keeps on oscillating about its mean position before coming to rest. In the same way if the current is established suddenly in a galvanometer, the coil will shoot beyond its final equilibrium position and will oscillate several times before coming to rest at its equilibrium position. As it is annoying and time consuming to wait for the coil to come to rest, artificial ways are employed to make the coil come to rest quickly. Such galvanometer in which the coil comes to rest quickly after the current passed through it or the current is stopped from flowing through it, is called stable or a dead beat galvanometer.

Ammeter

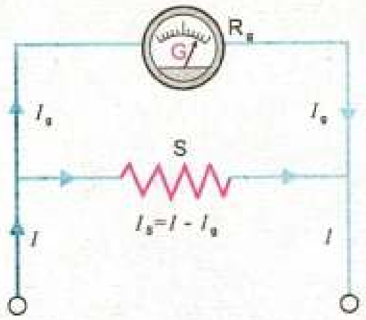


Fig. 14.19 An ammeter is a galvanometer which is shunted by a proper low resistance.

An ammeter is an electrical instrument which is used to measure current in amperes. This is basically a galvanometer. The portion of the galvanometer whose motion causes the needle of the device to move across the scale is usually known as meter - movement. Most meter movements are very sensitive and full scale deflection is obtained with a current of few milliamperes only. So an ordinary galvanometer cannot be used for measuring large currents without proper modification.

Suppose we have a galvanometer whose meter - movement (coil) has a resistance R_g and which gives full scale deflection when current I_g is passed through it. From Ohm's law we know that the potential difference V_g which causes a current I_g to pass through the galvanometer is given by

$$V_g = I_g R_g$$

If we want to convert this galvanometer into an ammeter which can measure a maximum current I, it is necessary to connect a low value bypass resistor called shunt. The shunt resistance is of such a value so that the current I_g for full scale deflection of the galvanometer passes through the galvanometer and the remaining current $(I - I_g)$ passes through the shunt in this situation (Fig. 14.19).

The shunt resistance R_s can be calculated from the fact that as the meter - movement and the shunt are connected in parallel with each other, the potential difference across the meter - movement is equal to the potential difference across the shunt.

$$\therefore \qquad I_g R_g = (I - I_g) R_s$$

or $$R_s = \frac{I_g}{I - I_g} R_g \qquad \text{..........} \qquad (14.18)$$

The resistance of the shunt is usually so small that a piece of copper wire serves the purpose. The resistance of the ammeter is the combined resistance of the galvanometer's meter - movement and the shunt. Usually it is very small. An ammeter must have a very low resistance so that it does not disturb the circuit in which it is connected in series in order to measure the current.

Voltmeter

A voltmeter is an electrical device which measures the potential difference in volts between two points. This, too, is made by modifying a galvanometer. Since a voltmeter is always connected in parallel, it must have a very high resistance so that it will not short the circuit across which the voltage is to be measured. This is achieved by connecting a very high resistance R_h placed in series with the meter - movement (Fig.14.20). Suppose we have a meter - movement whose resistance is R_g and which deflects full scale with a current I_g. In order to make a voltmeter from it which has a range of V volts, the value of the high resistance R_h should be such that full scale deflection will be obtained when it is connected across V volt. Under this condition the current through the meter - movement is I_g. Applying Ohm's law (Fig.14.20) we have

$$V = I_g(R_g + R_h)$$

$$R_h = \frac{V}{I_g} - R_g \qquad \text{..........} \qquad (14.19)$$

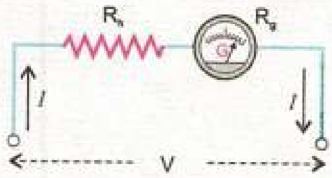


Fig. 14.20 A galvanometer in series with a high resistance acts as a voltmeter.

If the scale of the galvanometer is calibrated from 0 *to* V volts, the combination of galvanometer and the series resistor acts as a voltmeter with range 0 - V volts. By properly arranging the resistance R_h any voltage can be measured. Thus, we see that a voltmeter possesses high resistance.

It may be noted that a voltmeter is always connected across the two points between which potential difference is to be measured. Before connecting a voltmeter, it should be assured that its resistance is very high in comparison with the resistance of the circuit across which it is connected otherwise it will load the circuit and will alter the potential difference which is required to be measured.

Example 14.6: What shunt resistance must be connected across a galvanometer of 50.0 Ω resistance which gives full scale deflection with 2.0 mA current, so as to convert it into an ammeter of range 10.0 A?

Solution:

Resistance of galvanometer $= R_g = 50.0\ \Omega$

Current for full scale deflection $= I_g = 2.0$ mA

Current to be measured $= I = 10.0$ A

The shunt resistance R_s is given by

$$R_s = \frac{I_g}{I - I_g} R_g = \frac{2.0 \times 10^{-3}\ \text{A}}{10.0\ \text{A} - 2.0 \times 10^{-3}\ \text{A}} \times 50.0\ \text{A} = 0.01\ \Omega$$

Ohmmeter

It is a useful device for rapid measurement of resistance. It consists of a galvanometer, and adjustable resistance r_s and a cell connected in series (Fig.14.21-a). The series resistance r_s is so adjusted that when terminals c and d are short circuited, i.e., when $R = 0$, the galvanometer gives full scale deflection. So the extreme graduation of the usual scale of the galvanometer is marked 0 for resistance measurement. When terminals c and d are not joined, no current passes through the galvanometer and its deflection is zero. Thus zero of the scale is marked as infinity (Fig. 14.21-b). Now a known resistance R is connected across the terminals c and d. The galvanometer deflects to some intermediate point. This point is calibrated as R. In this way the whole scale is calibrated into resistance. The resistance to be measured is connected across the terminals c and d. The deflection on the calibrated scale reads the value of the resistance directly.

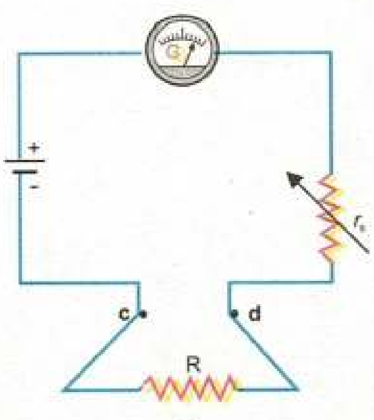


Fig. 14.21 A moving coil galvanometer is converted into an ohmmeter.

14.11 AVO METER - MULTIMETER

It is an instrument which can measure current in amperes, potential difference in volts and resistance in ohms. It basically consists of a sensitive moving coil galvanometer which is converted into a multirange ammeter voltmeter or ohmmeter accordingly as a current measuring circuit or a voltage measuring circuit or a resistance measuring circuit is connected with the galvanometer with the help of a switch known as function switch (Fig.14.22). Here X, Y are the main terminals of the AVO meter which are connected with the circuit in which measurement is required. FS is the function selector switch which connects the galvanometer with relevant measuring circuit.

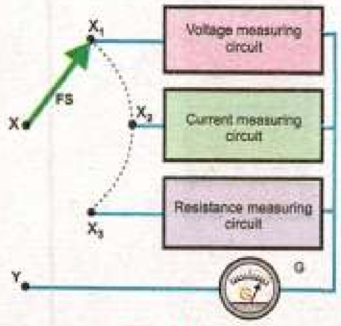


Fig. 14.22

Voltage Measuring Part of AVO Meter

The voltage measuring part of the AVO meter is actually a multirange voltmeter. It consists of a number of resistances each of which can be connected in series with the moving coil galvanometer with the help of a switch called the range switch (Fig. 14.23). The value of each resistance depends upon the range of the voltmeter which it controls.

Alternating voltages are also measured by AVO meter. AC voltage is first converted into DC voltage by using diode as rectifier and then measured as usual.

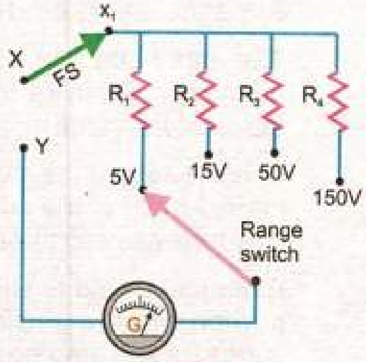


Fig. 14.23

Current Measuring Part of AVO Meter

The current measuring part of the AVO meter is actually a multirange ammeter. It consists of a number of low resistances connected in parallel with the galvanometer. The values of these resistances depend upon the range of the ammeter (Fig. 14.24).

The circuit also has a range selection switch RS which is used to select a particular range of the current.

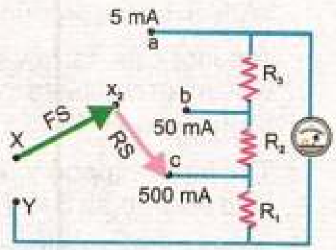


Fig. 14 .24

Resistance Measuring Part of AVO Meter

The resistance measuring part of AVO meter is, in fact, a multirange ohmmeter. Circuit for each range of this meter consists of a battery of emf V_0 and a variable resistance r_s connected in series with galvanometer of resistance R_g. When the function switch is switched to position X_3 (Fig. 14.22), this circuit is connected with the terminals X, Y of the AVO meter (Fig.14.25 a).

Before measuring an unknown resistance by an ohmmeter it is first zeroed which means that we short circuit the terminals X, Y and adjust r_s, to produce full scale deflection.

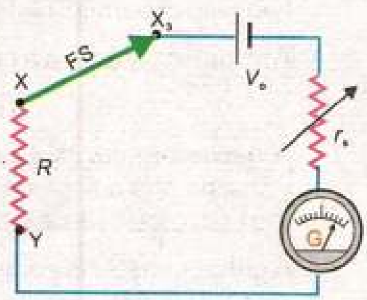


Fig.14.25

Digital Multimeter (DMM)

Another useful device to measure resistance, current and voltage is an electronic instrument called digital multimeter.

Fig.14.26

It is a digital version of an AVO meter. It has become a very popular testing device because the digital values are displayed automatically with decimal point, polarity and the unit for V, A or Ω. These meters are generally easier to use because they eliminate the human error that often occurs in reading the dial of an ordinary AVO meter. A portable DMM is shown in Fig. 14.26.

SUMMARY

- A magnetic field is set up in the region surrounding a current carrying conductor.
- The right hand rule states, "If the wire is grasped in the fist of right hand with the thumb pointing in the direction of current, the fingers of the hand will circle the wire in the direction of the magnetic field".
- The strength of the magnetic field or magnetic induction is the force acting on one metre length of the conductor placed at right angle to the magnetic field when 1 A current is passing through it.
- A magnetic field is said to have a strength of one tesla if it exerts a force of one newton on one metre length of the conductor placed at right angle to the field when a current of one ampere passes through the conductor.
- The magnetic flux Φ_B through plane element of area **A** in a uniform magnetic field **B** is given by dot product of **B** and **A**.
- Ampere circuital law states the sum of the quantities $\mathbf{B}.\Delta\mathbf{L}$ for all path elements into which the complete loop has been divided equals μ_o times the total current enclosed by the loop.
- The force experienced by a single charge carrier moving with velocity **v** in magnetic field of strength **B** is $\mathbf{F} = q(\mathbf{v} \times \mathbf{B})$.
- Cathode ray oscilloscope (CRO) is a high speed graph plotting device. It works by deflecting beam of electrons as they pass through uniform electric field between the two sets of parallel plates.
- A torque may act on a current carrying coil placed in a magnetic field.

$$\tau = I A B \cos\alpha$$

- A galvanometer is an electric device which detects the flow of current. It usually consists of a coil placed in a magnetic field. As the current passes through the coil, the coil rotates, thus indicating the flow of current.
- A galvanometer is converted into an ammeter by properly shunting it.
- A galvanometer is converted into a voltmeter by connecting a high resistance in series.

QUESTIONS

14.1 A plane conducting loop is located in a uniform magnetic field that is directed along the x-axis. For what orientation of the loop is the flux a maximum? For what orientation is the flux a minimum?

14.2 A current in a conductor produces a magnetic field, which can be calculated using Ampere's law. Since current is defined as the rate of flow of charge, what can you conclude about the magnetic field due to stationary charges? What about moving charges?

14.3 Describe the change in the magnetic field inside a solenoid carrying a steady current I, if (a) the length of the solenoid is doubled but the number of turns remains the same and (b) the number of turns is doubled, but the length remains the same.

14.4 At a given instant, a proton moves in the positive x direction in a region where there is magnetic field in the negative z direction. What is the direction of the magnetic force? Will the proton continue to move in the positive x direction? Explain.

14.5 Two charged particles are projected into a region where there is a magnetic field perpendicular to their velocities. If the charges are deflected in opposite directions, what can you say about them?

14.6 Suppose that a charge q is moving in a uniform magnetic field with a velocity v. Why is there no work done by the magnetic force that acts on the charge q?

14.7 If a charged particle moves in a straight line through some region of space, can you say that the magnetic field in the region is zero?

14.8 Why does the picture on a TV screen become distorted when a magnet is brought near the screen?

14.9 Is it possible to orient a current loop in a uniform magnetic field such that the loop will not tend to rotate? Explain.

14.10 How can a current loop be used to determine the presence of a magnetic field in a given region of space?

14.11 How can you use a magnetic field to separate isotopes of chemical element?

14.12 What should be the orientation of a current carrying coil in a magnetic field so that torque acting upon the coil is (a) maximum (b) minimum?

14.13 A loop of wire is suspended between the poles of a magnet with its plane parallel to the pole faces. What happens if a direct current is put through the coil? What happens if an alternating current is used instead?

14.14 Why the resistance of an ammeter should be very low?

14.15 Why the voltmeter should have a very high resistance?

PROBLEMS

14.1 Find the value of the magnetic field that will cause a maximum force of 7.0×10^{-3} N on a 20.0 cm straight wire carrying a current of 10.0 A.
(**Ans:** 3.5×10^{-3} T)

14.2 How fast must a proton move in a magnetic field of 2.50×10^{-3} T such that the magnetic force is equal to its weight? (**Ans:** 4.09×10^{-5} ms^{-1})

14.3 A velocity selector has a magnetic field of 0.30 T. If a perpendicular electric field of 10,000 Vm^{-1} is applied, what will be the speed of the particle that will pass through the selector? (**Ans:** 3.3×10^4 ms^{-1})

14.4 A coil of 0.1 m x 0.1 m and of 200 turns carrying a current of 1.0 mA is placed in a uniform magnetic field of 0.1 T. Calculate the maximum torque that acts on the coil. (**Ans:** 2.0×10^{-4} Nm)

14.5 A power line 10.0 m high carries a current 200 A. Find the magnetic field of the wire at the ground. (**Ans:** 4.0×10^{-6} T)

14.6 You are asked to design a solenoid that will give a magnetic field of 0.10 T, yet the current must not exceed 10.0 A. Find the number of turns per unit length that the solenoid should have. (**Ans:** 7.96×10^3)

14.7 What current should pass through a solenoid that is 0.5 m long with 10,000 turns of copper wire so that it will have a magnetic field of 0.4 T? (**Ans:** 16.0 A)

14.8 A galvanometer having an internal resistance R_g = 15.0 Ω gives full scale deflection with current I_g = 20.0 mA. It is to be converted into an ammeter of range 10.0 A. Find the value of shunt resistance R_s. (**Ans:** 0.030 Ω)

14.9 The resistance of a galvanometer is 50.0 Ω and reads full scale deflection with a current of 2.0 mA. Show by a diagram how to convert this galvanometer into voltmeter reading 200 V full scale. (**Ans:** R_h = 99950 Ω)

14.10 The resistance of a galvanometer coil is 10.0 Ω and reads full scale with a current of 1.0 mA. What should be the values of resistances R_1, R_2 and R_3 to convert this galvanometer into a multirange ammeter of 100, 10.0 and 1.0 A as shown in the Fig.P.14.10? (**Ans:** R_1 = .0001 Ω, R_2 = 0.001 Ω, R_3 = 0.01 Ω)

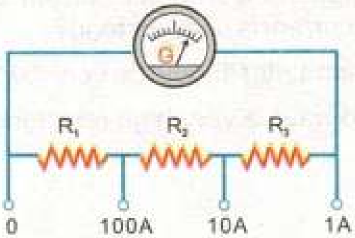


Fig. P.14.10