Unit 5

Gravitation

This unit is built on
Gravitation - Science-V
Earth & Space - Science-VI

This unit leads to:
Gravitational Potential, Escape Velocity and Artificial Satellite

Major Concepts

5.1 Law of Gravitation
5.2 Measurement of mass of Earth
5.3 Variation of g with altitude
5.4 Motion of artificial satellites

STUDENT'S LEARNING OUTCOMES

After studying this unit, the students will be able to:

- state Newton's law of gravitation.
- explain that the gravitational forces are consistent with Newton's third law.
- explain gravitational field as an example of field of force.
- define weight (as the force on an object due to a gravitational field.)
- calculate the mass of Earth by using law of gravitation.
- solve problems using Newton's law of gravitation.
- explain that value of *g* decreases with altitude from the surface of Earth.
- discuss the importance of Newton's law of gravitation in understanding the motion of satellites.

SCIENCE, TECHNOLOGY AND SOCIETY CONNECTION

The students will be able to:

- gather information to predict the value of the acceleration due to gravity g at any planet or moon's surface using Newton's law of gravitation.
- describe how artificial satellites keep on moving around the Earth due to gravitational force.

The first man who came up with the idea of gravity was Isaac Newton. It was an evening of 1665 when he was trying to solve the mystery why planets revolve around the Sun. Suddenly an apple fell from the tree under which he was sitting. The idea of gravity flashed in his mind. He discovered not only the cause of falling apple but also the cause that makes the planets to revolve around the Sun and the moon around the Earth. This unit deals with the concepts related to gravitation.

5.1 THE FORCE OF GRAVITATION

On the basis of his observations, Newton concluded that the force which causes an apple to fall on the Earth and the force which keeps the moon in its orbit are of the same nature. He further concluded that there exists a force due to which everybody of the universe attracts every other body. He named this force the force of gravitation.

LAW OF GRAVITATION

According to Newton's law of universal gravitation:

Everybody in the universe attracts every other body with a force which is directly proportional to the product of their masses and inversely proportional to the square of the distance between their centres.

Consider two bodies of masses m_1 and m_2. The distance between the centres of masses is d as shown in figure 5.1.

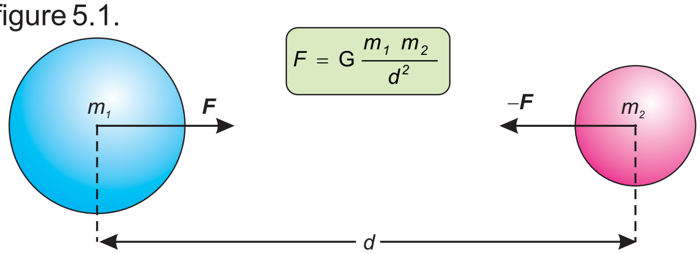

Figure 5.1: Two masses attract each other with a gravitational force of equal magnitude.

According to the law of gravitation, the gravitational force of attraction F with which the two masses m_1 and m_2 separated by a distance d attract each other is given by:

$$F \propto m_1 m_2$$

$$F \propto \frac{1}{d^2}$$

or $$F \propto \frac{m_1 m_2}{d^2}$$

$$F = G\frac{m_1 m_2}{d^2} \quad \dots \dots \dots \dots \quad (5.1)$$

Here G is the proportionality constant. It is called the universal constant of gravitation. Its value is same everywhere. In SI units its value is 6.673×10^{-11} Nm^2kg^{-2}. Due to small value of G, the gravitational force of attraction between objects around us is very small and we do not feel it. Since the mass of Earth is very large, it attracts nearby objects with a significant force. The weight of an object on the Earth is the result of gravitational force of attraction between the Earth and the object.

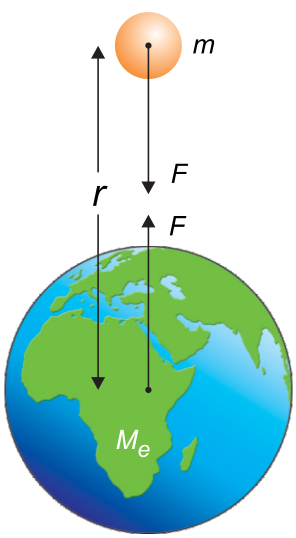

Figure 5.2: Weight of a body is due to the gravitational force between the body and the Earth.

LAW OF GRAVITATION AND NEWTON'S THIRD LAW OF MOTION

It is to be noted that mass m_1 attracts m_2 towards it with a force F while mass m_2 attracts m_1 towards it with a force of the same magnitude F but in opposite direction. If the force acting on m_1 is considered as action then the force acting on m_2 will be the reaction. The action and reaction due to force of gravitation are equal in magnitude but opposite in direction. This is consistent with Newton's third law of motion which states, to every action there is always an equal but opposite reaction.

EXAMPLE 5.1

Two lead spheres each of mass 1000 kg are kept with their centres 1 m apart. Find the gravitational force with which they attract each other.

SOLUTION

Here $m_1 = 1000 \text{ kg}$

$m_2 = 1000 \text{ kg}$

$d = 1 \text{ m}$

Since $F = G\dfrac{m_1 m_2}{d^2}$

Putting the values, we get

$$F = 6.673 \times 10^{-11}\ \text{Nm}^2\text{kg}^{-2} \times \frac{1000 \text{ kg} \times 1000 \text{ kg}}{(1 \text{ m})^2}$$

$$= 6.673 \times 10^{-5}\ \text{N}$$

Thus, gravitational force of attraction between the lead spheres is 6.673×10^{-5} N.

GRAVITATIONAL FIELD

According to the Newton's law of gravitation, the gravitational force between a body of mass m and the Earth is given by

$$F = G\frac{m M_e}{r^2} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \ldots \ldots \quad (5.2)$$

where M_e is the mass of the Earth and r is the distance of the body from the centre of the Earth.

The weight of a body is due to the gravitational force with which the Earth attracts a body. Gravitational force is a non-contact force. For example, the velocity of a body, thrown up, goes on decreasing while on return its velocity goes on increasing. This is due to the gravitational pull of the Earth acting on the body whether the body is in contact with the Earth or not. Such a force is called the field force. It is assumed that a gravitational field exists all around the Earth. This field is directed towards the centre of the Earth as shown by arrows in figure 5.3. The gravitational field becomes weaker and weaker as we go farther and farther away from the Earth. In the gravitational field of the Earth, the gravitational force per unit mass is called

Figure 5.3: Gravitational field around the Earth is towards its centre.

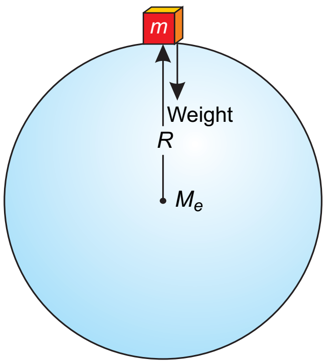

Figure 5.4: Weight of a body is equal to the gravitational force between the body and the Earth.

the **gravitational field strength** of the Earth. At any place its value is equal to the value of g at that point. Near the surface of the Earth, the gravitational field strength is 10 N kg^{-1}.

5.2 MASS OF THE EARTH

Consider a body of mass *m* on the surface of the Earth as shown in figure 5.4. Let the mass of the Earth be M_e and radius of the Earth be *R*. The distance of the body from the centre of the Earth will also be equal to the radius *R* of the Earth. According to the law of gravitation, the gravitational force *F* of the Earth acting on a body is given by

$$F = G\frac{mM_e}{R^2} \quad \ldots \ldots \ldots \ldots \ldots \quad (5.3)$$

But the force with which Earth attracts a body towards its centre is equal to its weight w. Therefore,

$$F = w = mg \quad \ldots \ldots \ldots \ldots \ldots \quad (5.4)$$

$$\text{or} \quad mg = G\frac{mM_e}{R^2} \quad \ldots \ldots \ldots \ldots \quad (5.5)$$

$$\therefore \quad g = G\frac{M_e}{R^2} \quad \ldots \ldots \ldots \ldots \quad (5.6)$$

$$\text{and} \quad M_e = \frac{R^2 g}{G} \quad \ldots \ldots \ldots \quad (5.7)$$

Mass M_e of the Earth can be determined on putting the values in equation (5.7).

$$M_e = \frac{(6.4 \times 10^6 \text{ m})^2 \times 10 \text{ m s}^{-2}}{6.673 \times 10^{-11} \text{ N m}^2 \text{kg}^{-2}}$$

$$= 6.0 \times 10^{24} \text{ kg}$$

Thus, mass of the Earth is 6 x 10 kg.

5.3 VARIATION OF G WITH ALTITUDE

Equation (5.6) shows that the value of acceleration due to gravity g depends on the radius of the Earth at its surface. The value of g is inversely proportional to the square of the radius of the Earth. But it does not remain constant. It decreases with **altitude.** Altitude is the height of an object or place above sea level. The value of g is greater at sea level than at the hills.

Consider a body of mass m at an altitude h as shown in figure 5.5. The distance of the body from the centre of the Earth becomes $R + h$. Therefore, using equation (5.6), we get

$$g_h = G\frac{M_e}{(R+h)^2} \quad \ldots \ldots \ldots \ldots \quad (5.8)$$

According to the above equation, we come to know that at a height equal to one Earth radius above the surface of the Earth, g becomes one fourth of its value on the Earth. Similarly at a distance of two Earths radius above the Earth's surface, the value of g becomes one ninth of its value on the Earth.

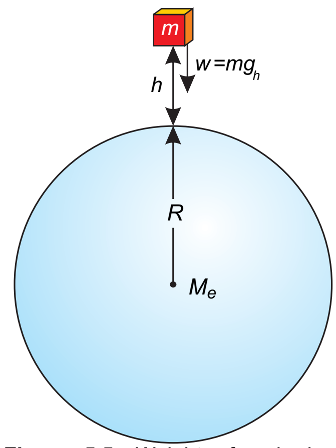

Figure 5.5: Weight of a body decreases as its height increases from the surface of the Earth.

Mini Exercise

1. Does an apple attract the Earth towards it?
2. With what force an apple weighing 1N attracts the Earth?
3. Does the weight of an apple increase, decrease or remain constant when taken to the top of a mountain?

EXAMPLE 5.2

Calculate the value of g, the acceleration due to gravity at an altitude 1000 km. The mass of the Earth is 6.0×10^{24} kg. The radius of the Earth is 6400 km.

SOLUTION

Here R = 6400 km

h = 1000 km

$M_e = 6.0 \times 10^{24}$ kg

g_h = ?

As $R + h$ = 6400 km + 1000 km = 7400 km

$= 7.4 \times 10^{6}$ m

$$g_h = G\frac{M_e}{(R+h)^2}$$

$$\therefore \quad g_h = \frac{6.673\times10^{-11}\,\text{Nm}^2\,\text{kg}^{-2}\times6.0\times10^{24}\,\text{kg}}{(7.4\times10^{6}\,\text{m})^2}$$

DO YOU KNOW?

Value of g on the surface of a celestial object depends on its mass and its radius. The value of g on some of the objects is given below:

Object	g (ms^{-2})
Sun	274..2
Mercury	3.7
Venus	8.87
Moon	1.62
Mars	3.73
Jupiter	25.94

$$= 7.3 \text{ N kg}^{-1} = 7.3 \text{ ms}^{-2}$$

Thus the value of g, the acceleration due to gravity at an altitude of 1000 km will be 7.3 ms^{-2}

5.4 ARTIFICIAL SATELLITES

An object that revolves around a planet is called a satellite. The moon revolves around the Earth so moon is a natural satellite of the Earth. Scientists have sent many objects into space. Some of these objects revolve around the Earth. These are called artificial satellites. Most of the artificial satellites, orbiting around the Earth are used for communication purposes. Artificial satellites carry instruments or passengers to perform experiments in space.

DO YOU KNOW?

The height of a geostationary satellite is about 42,300 km from the surface of the Earth. Its velocity with respect to Earth is zero.

Figure 5.6: A satellite is orbiting around the Earth at a height *h* above the surface of the Earth.

DO YOU KNOW?

Global Positioning System (GPS) is a satellites navigation system. It helps us to find the exact position of an object anywhere on the land, on the sea or in the air. GPS consists of 24 Earth satellites. These satellites revolve around the Earth twice a day with a speed of 3.87 km s^{-1}.

Large number of artificial satellites have been launched in different orbits around the Earth. They take different time to complete their one revolution around the Earth depending upon their distance *h* from the Earth. Communication satellites take 24 hours to complete their one revolution around the Earth. As Earth also completes its one rotation about its axis in 24 hours, hence, these communication satellites appear to be stationary with respect to Earth. It is due to this reason that the orbit of such a satellite is called geostationary orbit. Dish antennas sending and receiving the signals from them have fixed direction depending upon their location on the Earth.

MOTION OF ARTIFICIAL SATELLITES

A satellite requires centripetal force that keeps it to move around the Earth. The gravitational force of attraction between the satellite and the Earth provides the necessary centripetal force.

Consider a satellite of mass m revolving round the Earth at an altitude h in an orbit of radius r_o with orbital velocity v_o. The necessary centripetal force required is given by equation (3.26).

$$F_c = \frac{mv_o^2}{r_o}$$

This force is provided by the gravitational force of attraction between the Earth and the satellite and is equal to the weight of the satellite w' (mg.). Thus

$$F_c = w' = mg_h \quad \dots \quad \dots \quad \dots \quad (5.9)$$

or $$mg_h = \frac{mv_o^2}{r_o}$$

or $$v_o^2 = g_h r_o$$

or $$v_o = \sqrt{g_h r_o} \quad \dots \quad \dots \quad \dots \quad (5.10)$$

as $$r_o = R + h$$

$\therefore$ $$v_o = \sqrt{g_h (R+h)} \quad \dots \quad \dots \quad \dots (5.11)$$

Equation (5.10) gives the velocity, which a satellite must possess when launched in an orbit of radius $r_o = (R + h)$ around the Earth. An approximation can be made for a satellite revolving close to the Earth such that $R >> h$.

$$R+h = R$$

and $$g_h = g$$

$\therefore$ $$v_o = \sqrt{g R} \quad \dots \quad \dots \quad \dots \quad \dots (5.12)$$

A satellite revolving around very close to the Earth, has speed v_0 nearly 8 kms^{-1} or 29000 kmh^{-1}.

DO YOU KNOW?

Moon is nearly 3,80,000 km away from the Earth. It completes its one revolution around the Earth in 27.3 days.

SUMMARY

- Newton's law of universal gravitation states that everybody in the universe attracts every other body with a force which is directly proportional to the product of their masses and inversely proportional to the square of the distance between their centres.
- The Earth attracts a body with a force equal to its weight.
- It is assumed that a gravitational field exists all around the Earth due to the gravitational force of attraction of the Earth.
- In the gravitational field of the Earth, the gravitational force per unit mass is called the gravitational field strength of the Earth. It is 10 N kg^{-1} near the surface of the Earth.
- *Acceleration* $g = G\frac{M_e}{R^2}$
- Mass of Earth $M_e = \frac{R^2 g}{G}$
- g at an altitude $h = G\frac{M}{(R+h)^2}$
- An object that revolves around a planet is called a satellite.
- The moon revolves around the Earth so moon is a natural satellite of the Earth.
- Scientists have sent many objects into space. Some of these objects revolve around the Earth. These are called artificial satellites.
- Orbital velocity $v_o = \sqrt{g_h (R+h)}$

QUESTIONS

5.1 Encircle the correct answer from the given choices:

i. Earth's gravitational force of attraction vanishes at

(a) 6400 km (b) infinity

(c) 42300 km (d) 1000 km

ii. Value of g increases with the

(a) increase in mass of the body

(b) increase in altitude

(c) decrease in altitude

(d) none of the above

iii. The value of g at a heightone Earth's radius above the surface of the Earth is:

(a) 2 g (b) ½ g

(c) ⅓ g (d) ¼ g

iv. The value of g on moon's surface is 1.6 ms^{-2} . What will be the weight of a 100 kg body on the surface of the moon?

(a) 100 N (b) 160 N

(c) 1000 N (d) 1600 N

v. The altitude of geostationary orbits in which communication satellites are launched above the surface of the Earth is:

(a) 850 km (b) 1000 km

(c) 6400 km (d) 42,300 km

vi The orbital speed of a low orbit satellite is:

(a) zero (b) 8 ms^{-1}

(c) 800 ms^{-1} (d) 8000 ms^{-1}

5.2 What is meant by the force of gravitation?

5.3 Do you attract the Earth or the Earth attracts you? Which one is attracting with a larger force? You or the Earth.

5.4 What is a field force?

5.5 Why earlier scientists could not guess about the gravitational force?

5.6 How can you say that gravitational force is a field force?

5.7 Explain, what is meant by gravitational field strength?

5.8 Why law of gravitation is important to us?

5.9 Explain the law of gravitation.

5.10 How the mass of Earth can be determined?

5.11 Can you determine the mass of our moon? If yes, then what you need to know?

5.12 Why does the value of *g* vary from place to place?

5.13 Explain how the value of g varies with altitude.

5.14 What are artificial satellites?

5.15 How Newton's law of gravitation helps in understanding the motion of satellites?

5.16 On what factors the orbital speed of a satellite depends?

5.17 Why communication satellites are stationed at geostationary orbits?

PROBLEMS

5.1 Find the gravitational force of attraction between two spheres each of mass 1000 kg. The distance between the centres of the spheres is 0.5 m.

(2.67×10^{-4} N)

5.2 The gravitational force between two identical lead spheres kept at 1 m apart is 0.006673 N. Find their masses.

(10,000 kg each)

5.3 Find the acceleration due to gravity on the surface of the Mars. The mass of Mars is 6.42×10^{23} kg and its radius is 3370 km. **(3.77 ms^{-2})**

5.4 The acceleration due to gravity on the surface of moon is 1.62 ms^{-2}. The radius of moon is 1740 km. Find the mass of moon.

(7.35×10^{22} kg)

5.5 Calculate the value of g at a height of 3600 km above the surface of the Earth. ($4.0\ ms^{-2}$)

5.6 Find the value of g due to the Earth at geostationary satellite. The radius of the geostationary orbit is 48700 km. ($0.17\ ms^{-2}$)

5.7 The value of g is $4.0\ ms^{-2}$ at a distance of 10000 km from the centre of the Earth. Find the mass of the Earth. (5.99×10^{24} kg)

5.8 At what altitude the value of *g* would become one fourth than on the surface of the Earth? (one Earth's radius)

5.9 A polar satellite is launched at 850 km above Earth. Find its orbital speed. ($7431\ ms^{-1}$)

5.10 A communication satellite is launched at 42000 km above Earth. Find its orbital speed. ($2876\ ms^{-1}$)