Unit 13

PRACTICAL GEOMETRY – TRIANGLES

Student Learning Outcomes (SLOs)

After completing this unit, students will be able to:

- Construct a triangle having given:
  - Two sides and the included angle,
  - One side and two of the angles,
  - Two of its sides and the angle opposite to one of them, (with all the three possibilities)
- Draw :
  - Angle bisectors,
  - Altitudes,
  - Perpendicular bisectors,
  - Medians of a given triangle and verify their given concurrency
- Construct a triangle equal in area to a given quadrilateral.
- Construct a rectangle equal in area to a given triangle.
- Construct a square equal in area to a given rectangle.
- Construct a triangle of equivalent area on a base of given length.

13.1 Construction of a Triangle

13.1.1 Construct a triangle having given:

- **Two sides and included angles.**
- **One side and two angles.**
- **Two of its sides and the angle apposite to one of them. With all three possibilities.**

When two sides and the included angle are given.

Example Construct a triangle ABC in which
$m\overline{AB} = 4.8\,\text{cm},\ m\overline{AC} = 3.2\,\text{cm}$ and $m\angle B = 75°$

Construction:

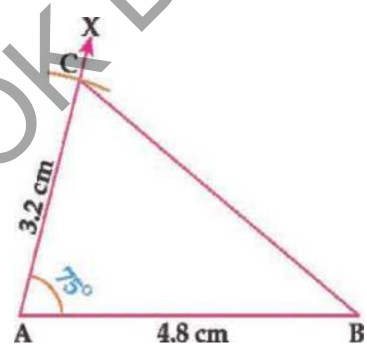
i) Draw the line segment $\overline{AB}$ of measure 4.8 cm.

ii) At point A, draw on angle XAB of measure 75°.

iii) Cut $\overline{AC}$ of measure 3.2 cm from AX

iv) Draw $\overline{BC}$

Thus ΔABC is the required triangle.



When one side and two angles are given.

Example 01 Construct a triangle ΔXYZ in which
$m\overline{AB} = 5.5\,\text{cm},\ m\angle A = 75°$ and $m\angle B = 40°$

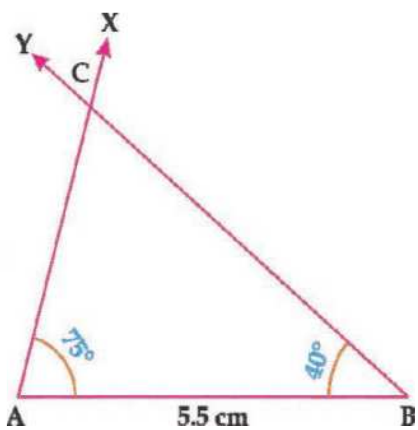
Construction:

i) Draw the line segment $\overline{AB}$ of measure 5.5 cm.

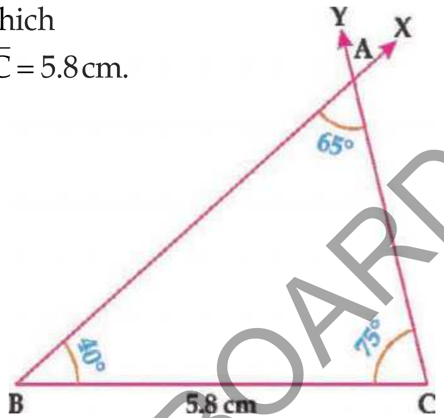
ii) At point A, draw on angle XAB of measure 75°.

iii) At point B draw an angle YBA of measure 40°, such that BY cuts AX at point C.

Thus ΔABC is the required triangle.



Example 02 Construct a triangle ΔXYZ in which
$m\angle A = 65^\circ$, $m\angle B = 40^\circ$ and $m\overline{BC} = 5.8\,cm$.



Construction:

We know that in ΔABC
$m\angle A + m\angle B + m\angle B = 180^\circ$
Here $m\angle A = 65^\circ$ and $m\angle B = 40^\circ$
So, $m\angle C = 180^\circ - (m\angle A + m\angle B)$
$= 180^\circ - (65^\circ + 40^\circ)$
$= 180^\circ - 105^\circ$
$= 75^\circ$

We now construct the triangle with
$m\overline{BC} = 5.8\,cm$, $m\angle B = 40^\circ$ and $m\angle C = 75^\circ$

i) Draw $\overline{BC}$ of measure 5.8 cm.
ii) Draw an angle $XAB = 40^\circ$, at point B.
iii) Draw $m\angle YCB = 75^\circ$, at point C.
iv) Rays BX and CY intersect each other at point A,
Thus ΔABC is the required triangle.

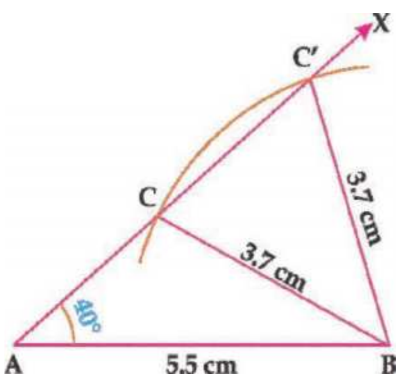
Two of its sides and the angle apposite to one of them. With all three possibilities.

Case I

Example 01 Construct a triangle ABC in which
$m\angle A = 40^\circ$, $m\overline{BC} = 3.7$ cm and $m\overline{AB} = 5.3\,cm$

Construction:

i) Draw $\overline{AB}$ of measure 5.3 cm.
ii) At point A, draw $\angle BAX$ of measure 40°.
iii) With center B and radius 3.7 cm, draw an arc which cuts AX at point C and C′.
iv) Draw $\overline{BC}$ and $\overline{BC'}$
ΔABC and ΔABC' are the required triangle.



Case II

Example 02 Construct a triangle $\overline{DEF}$ when

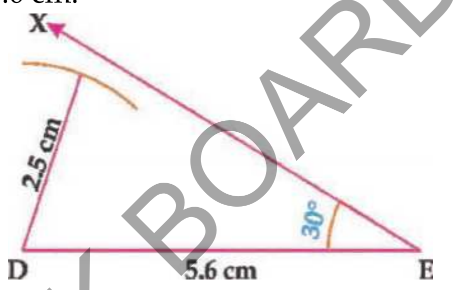
$m\overline{DE} = 5.6\,\text{cm}$, $m\overline{DF} = 2.5\,\text{cm}$ and $m\angle E = 30^\circ$

Construction:

i) Draw a line segment $\overline{DE}$ of measure 5.6 cm.

ii) Draw and angle DEX of measure 40° at point E.

iii) With D as a center draw an arc of radius 2.5 cm, which does not cut EX at any point. In this case no triangle can be constructed satisfying the given data.



Case III

Example 03 Construct a triangle PQR when

$m\overline{PR} = m\overline{QR} = 4.7\,\text{cm}$ and $m\angle P = 55^\circ$

Construction:

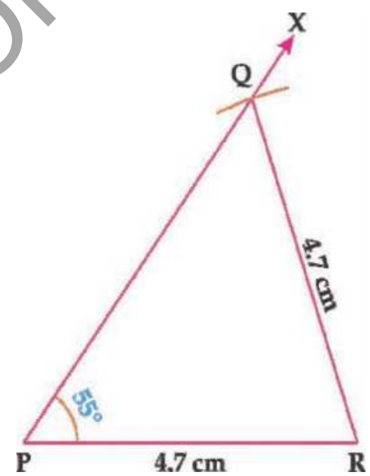
i) Draw a line segment $\overline{PR}$ of measure 4.7 cm.

ii) Draw and angle $\angle XPR$ of measure 55° at point P.

iii) With point R as a center draw an arc of radius 4.7 cm, which cuts EX at point Q.

iv) Join point Q and R.

ΔPQR is the required triangle.



Note: The above case I, case II and case III are called ambiguous cases.

Exercise 13.1

1. Construct ΔPQR such that, $m\overline{PQ} = m\overline{QR} = 4.6$ cm and $m\angle Q = 35^\circ$
2. Construct ΔABC such that, $m\overline{AB} = m\overline{AC} = 5.1$ cm and $m\angle A = 65^\circ$
3. Construct ΔLMN such that, $m\overline{LM} = 3.7$ cm, $m\overline{MN} = 2.5$ cm and $\angle M = 50^\circ$
4. Construct ΔABC such that, $m\overline{AB} = 3.5$ cm, $m\overline{BC} = 2.7$ cm and $\angle B = 110^\circ$
5. Construct ΔXYZ such that, $m\overline{XY} = 4.1$ cm, $m\overline{YZ} = 5$ cm and $\angle Z = 80^\circ$

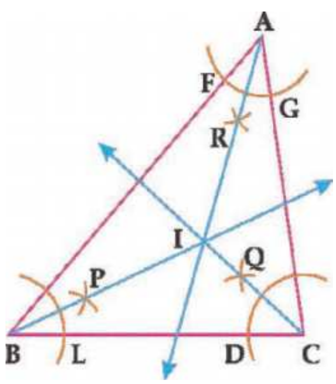
6. Construct the ΔDEF, ΔLMN and ΔABC in the following.
   (i) $m\overline{DE} = 5cm$, $m\angle D = 45°$ and , $m\angle E = 60°$
   (ii) $m\overline{LM} = 6cm$, $m\angle L = 75°$ and , $m\angle M = 45°$
   (iii) $m\overline{BC} = 5.8cm$, $m\angle A = 30°$ and , $m\angle B = 45°$
7. Construct a ΔABC, when lengths of two of its sides and measure of an angle opposite one of the side is given as under:
   (i) $m\overline{AC} = 4.5cm$, $m\overline{BC} = 4.1cm$ and $m\angle B = 75°$
   (ii) $m\overline{BC} = 5cm$, $m\overline{AB} = 5.5$ cm and $m\angle C = 70°$
   (iii) $mAB = 5cm$, $m\overline{BC} = 5.5cm$ and $m\angle A = 45°$

13.1.2 Draw

- **Angle bisectors**
- **Altitudes**
- **Perpendicular bisectors**
- **Medians**

(i) Draw the angle bisector of a given triangle

Example Draw bisectors of angle of ΔABC.



Given:

ABC is a triangle $\angle A$, $\angle B$ and $\angle C$ are its angles.

Required:

To draw bisectors of $\angle A$, $\angle B$ and $\angle C$.

Construction:

i) Draw the triangle ABC.
ii) With point B as a center draw an arc of any radius, intersecting the sides $\overline{BC}$ and $\overline{BA}$ at points L and M.
iii) Take point L as a center and draw an arc of any radius.
iv) Now take point M as a center and with the same radius draw another arc, which cuts the pervious arc at point P.
v) Join point P to B and produce it.
   BP is the bisector of $\angle B$.
vi) Repeat steps (ii) to (v) to draw $\overleftrightarrow{CQ}$ and $\overleftrightarrow{AR}$ the bisectors of $\angle C$ and $\angle A$ respectively.

(ii) Draw the altitudes of a given triangle

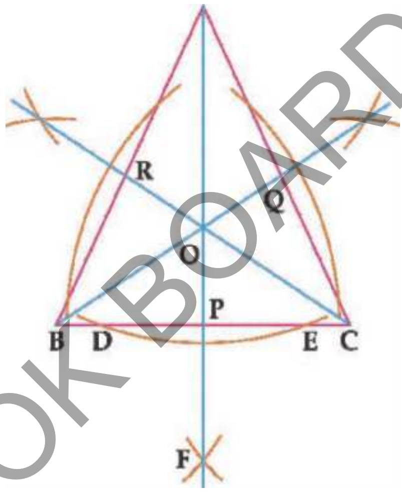
Example Take any triangle ABC and draw its altitudes.

Given:

A ΔABC

Required:

To draw altitudes of the ΔABC.



Construction:

i) Draw the triangle ABC.

ii) Take point A as center and draw an arc of suitable radius, which cuts $\overline{BC}$ at points D and E.

iii) From D as center, draw an arc of radius more than $\frac{1}{2}m\overline{DE}$.

iv) Again from point E draw another arc of same radius, cutting first arc at point F.

v) Join the points A and F. Such that $\overline{AF}$ intersects $\overline{BC}$ at point P. Then $\overline{AP}$ is the altitude of the ΔABC from the vertex A.

vi) Repeat the steps (ii) to (v) and draw $\overline{BQ}$ and $\overline{CR}$, the altitudes of ΔABC from the vertices B and C, respectively.

Hence $\overline{AP}$, $\overline{BQ}$ and $\overline{CR}$ are the feet of these altitudes.

(iii) Draw the perpendicular bisector of a given triangle

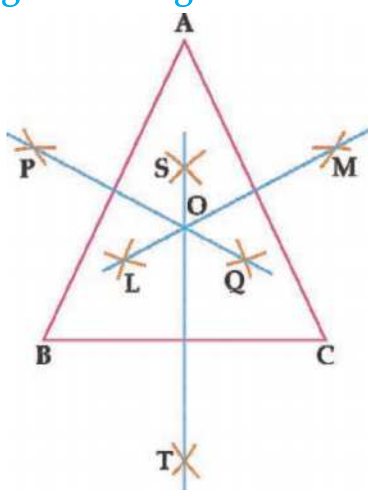
Example Draw the perpendicular bisector of sides of a triangle ABC.

Given:

A triangle ABC.

Required:

To draw perpendicular bisectors of the sides $\overline{AB}$, $\overline{BC}$ and $\overline{CA}$.



Construction:

i) Draw the triangle ABC.

ii) To draw perpendicular bisector of the side $\overline{AB}$, with B as a center and radius more than half of $\overline{AB}$, draw arcs on either sides of $\overline{AB}$.

iii) Now with A as a center and with the same radius, draw arcs on either sides of $\overline{AB}$, cutting previous arcs at P and Q.

iv) Join P and Q.

$\overline{PQ}$ is the perpendicular bisector of the $\overline{AB}$.

v) Repeat the steps (ii) to (iv) and draw $\overline{ST}$ and $\overline{LM}$, the perpendicular bisectors of $\overline{BC}$ and $\overline{AC}$, respectively.

Hence $\overline{PQ}$, $\overline{ST}$ and $\overline{LM}$ are the required perpendicular bisector of the sides $\overline{AB}$, $\overline{BC}$ and $\overline{AC}$, respectively, of the ΔABC.

(iv) Draw the median of a given triangle

Example Take any triangle ABC and draw medians of this triangle.

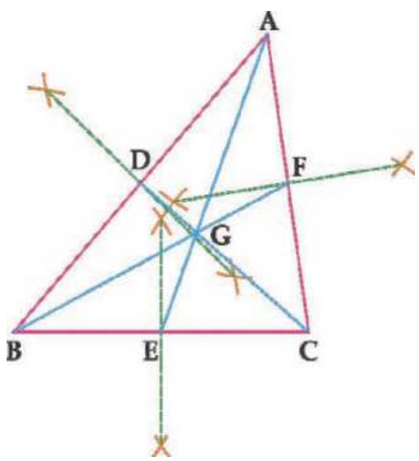
Given:

A ΔABC

Required:

To draw medians of the ΔABC.

Construction:



i) Draw the triangle ABC.

ii) Bisect the sides $\overline{AB}$, $\overline{BC}$ and $\overline{AC}$ at points D, E and F, respectively.

iii) Join A to E; B to F and C to D.

Thus $\overline{AE}$, $\overline{BF}$ and $\overline{CD}$ are the required medians of the ΔABC, which meet in a point G.

It may be noted that medians of every triangle are concurrent (i.e., meet in one point) and their point of concurrency, called centroid, divides each of them in 2 : 1.

By actual measurement it can be proved that

$$\frac{m\overline{AG}}{m\overline{GE}} = \frac{m\overline{BG}}{m\overline{GF}} = \frac{m\overline{CG}}{m\overline{GD}} = \frac{2}{1}$$

Exercise 13.2

1. Take a Δ and draw the medians and prove that they are concurrent.
2. Take a Δ and draw the altitudes and prove that they are concurrent.
3. Take a Δ and draw the internal bisectors of angles and prove that they are concurrent.
4. Construct a triangle ABC in which $m\overline{BC} = 6$ cm, $m\overline{CA} = 4$ cm and $m\overline{AB} = 5$ cm, draw the bisectors of angle A and B.
5. Construct a triangle PQR in which $m\overline{PQ} = 5.7$cm, $m\overline{QR} = 6.4$cm and $m\overline{PR} = 4.4$ cm, draw the altitudes from vertex R and vertex Q.
6. Construct a triangle STU in which $\angle T = 60°$, $\angle U = 30°$ and $m\overline{TU} = 7$ cm. Find the perpendicular bisectors of the sides of triangle and prove that they are concurrent.
7. Construct a right triangle ABC in which $\angle C = 90°$, $\angle B = 45°$ and $m\overline{CB} = 5$ cm. Draw the medians of the triangle.
8. Construct the following ΔXYZ. Draw their three medians and show that they are concurrent.
   (i) $m\overline{YZ}=4.4$cm, $m\angle Y = 45°$ and $m\angle Z = 75°$
   (ii) $m\overline{XY}=4.6$cm, m $m\overline{XZ} = 4.6$cm and $m\angle Y = 60°$
9. Construct the ΔKLM, in which $m\,\overline{KL} =4.8$cm, $m\,\overline{LM} = 5.2$cm and $m\overline{MK}=4.5$cm, draw their altitudes and verify their concurrency.
10. Construct the ΔPQR, in which $m\,\overline{PQ} =7$cm, $m\,\overline{QR} = 6.5$cm and $m\,\overline{PR} =5.8$cm, find their perpendicular bisectors and verify their concurrency.

13.2 Figures with equal Areas

13.2.1 Construct a triangle equal in area to a given quadrilateral.

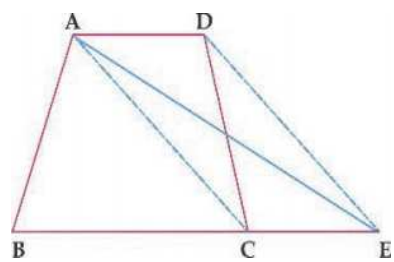
E.g. draw an angle equal in area to given quadrilateral ABCD. We know that, Area of all triangles with same base equal of vertices are on the line perpendicular to base.

1. ABCD is a given quadrilateral.

2. Join A to C.

4. Through D, draw $\overline{DE}$ parallel to $\overline{AC}$ meeting $\overline{BC}$ produced at point E.

5. Join A to E, then ABE is the required triangle.



13.2.2 Construct a rectangle equal in area to a given triangle.

E.g. Construct a rectangle equal in area to given ΔABC

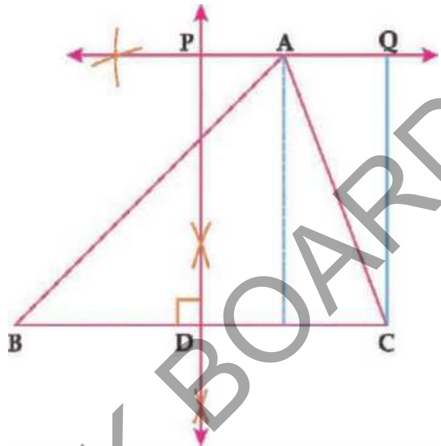
1. Draw a triangle ABC.

2. Draw a perpendicular bisector $\overleftrightarrow{PD}$ of $\overline{BC}$.

3. Through A, draw a line PQ parallel to $\overline{BC}$.

4. Take $m\overline{PQ} = m\overline{DC}$.

5. Then CDPQ is the required rectangle.

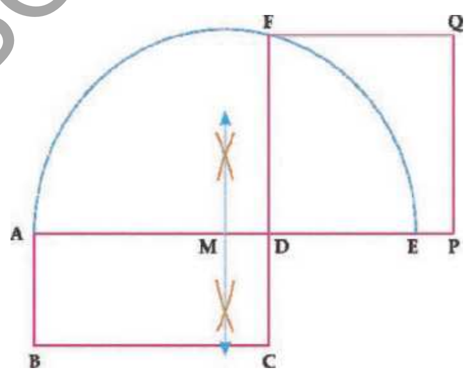


13.2.3 Construct a Square equal in area to a given rectangle.

E.g. Construct a square equal in Area to given rectangle ABCD.

Following are the steps of construction

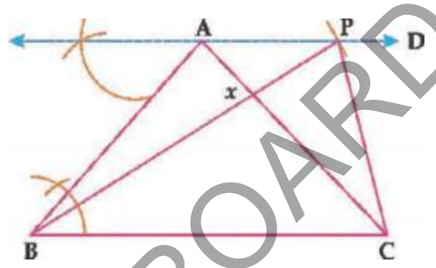
1. ABCD is a given rectangle.
2. Produce side $\overline{AD}$ to E making $m\overline{DE} = m\overline{CD}$.
3. Bisect $\overline{AE}$ at M.
4. With centre M and radius $m\overline{AM}$ construct a semi circle.
5. Produce $\overline{CD}$ to meet the semi circle at F.
6. On $\overline{DF}$ as a side construct a square DFQP. This shall be required square.



13.2.4 Construct a Triangle of equivalent area on a base of given length:

Following are the steps of construction

1. ABC is given triangle.
2. Draw $\overleftrightarrow{AD} \parallel \overline{BC}$.
3. With B as centre, and radius = x, such that $m\overline{BC} = x$ draw an arc cutting $\overleftrightarrow{AD}$ at P.
4. Join $\overline{BP}$ and $\overline{CP}$.
5. Then ΔBCP is the required triangle with equal base $\overline{BP} = x$ and area equivalent to ΔABC.



Exercise 13.3

1. Construct a rectangle whose adjacent sides are 2.5 cm and 5 cm respectively. Construct a square having area equal to the given rectangle.
2. Construct a square equal in area to a rectangle whose adjacent sides are 4.5 cm and 2.2 cm respectively. Measure the sides of the square and find its area and compare with the area of the rectangle. Also verify by measurement that the perimeter of the square is less than that of the rectangle.
3. Construct a triangle having base 4 cm and other two sides equal to 3.6cm and 3.8 cm each. Transform it into a rectangle with equal Area.
4. Construct a triangle having base 6cm and other sides equal to 5cm and 6cm each. Construct a rectangle equal in area to given Δ.

Review Exercise 13

1. **Fill in the blanks.**

   i) The side of a triangle opposite to the greatest angle is__________

   ii) The line segment joining a vertex of a triangle and perpendicular to its opposite side is called an__________

   iii) A line segment drawn from a vertex of a triangle and meeting the mid-point of its opposite side is called a __________

iv) The perpendicular bisectors of the sides of a triangle are _________

v) Two or more than two triangles are said to be congruent if they are equiangular and measures of their corresponding sides are ______

2. Tick (✓) the correct answer.

i) A triangle having all the three sides congruent is called _________ triangle.

(a) scalene (b) right angled
(c) equilateral (d) isosceles

ii) A quadrilateral having each angle equal to 90° and all the sides are congruent is called _________

(a) parallelogram (b) rectangle
(c) trapezium (d) square

iii) The medians of a triangle are _________

(a) collinear (b) congruent
(c) concurrent (d) parallel

iv) The _________ altitudes of an equilateral triangle are congruent.

(a) two (b) three
(c) four (d) none

v) The diagonals of a rectangle _________ each other.

(a) bisect (b) trisect
(c) bisect at right angle (d) none of these

vi) The _________ of a triangle cut each other in the ratio 2:1.

(a) Altitudes (b) Angle bisectors
(c) Right bisectors (d) Medians

vii) If each angle on the base of an isosceles triangle is 45°, then the measure of the third angle is _________

(a) 30° (b) 60°
(c) 90° (d) 120°

viii) If the three medians of a triangle are congruent then the triangle is ___________

(a) Right angled (b) equilateral
(c) Isosceles (d) acute angled

ix) If two _________of a triangle are congruent, then the triangle will be isosceles.

(a) Altitudes (b) Medians
(c) Right bisectors (d) sides

Summary

- In this unit we have learnt the construction of the following figures and relevant concepts.
- To construct a triangle, having given two sides and the included angle.
- To construct a triangle, having given one side and two of the angles.
- To construct a triangle, having given two of its sides and the angle opposite to one of them.
- To draw angle bisectors of a given triangle and to verify their concurrency.
- To draw altitudes of a given triangle and verify their concurrency.
- To draw perpendicular bisectors of the sides of a given triangle and to verify their concurrency.
- To draw medians of a given triangle and verify their concurrency.
- To construct a triangle equal in area to a given quadrilateral.
- To construct a rectangle equal in area to given triangle.
- To construct a square equal in area to given rectangle.
- To construct a triangle of equivalent area on the base of given length.
- Two or more than three lines are said to be concurrent if these passes through a common point and that point is called the point of concurrency.
- The point where the internal bisectors of the angles of a triangle intersect is called the in-centre of a triangle.
- The point of concurrency of the perpendicular bisectors of the sides of a triangle is called its circum-centre.
- Median of a triangle means a line segment joining a vertex of a triangle to the mid-point of the opposite side.
- Ortho-centre of a triangle means the point of concurrency of three altitudes of a triangle.