Unit 14

THEOREMS RELATED WITH AREA

Student Learning Outcomes (SLOs)

After completing this unit, students will be able to:

- Understand the following theorems along with their corollaries and apply them to solve allied problems.
- Parallelograms on the same base and lying between the same parallel lines (or of the same altitude) are equal in area.
- Parallelograms on equal bases and having the same altitude are equal in area.
- The triangles on the same base and of the same altitude are equal in area.
- Triangles on equal bases and of the same altitude are equal in area.

Introduction

We will study the Theorem related with Area.

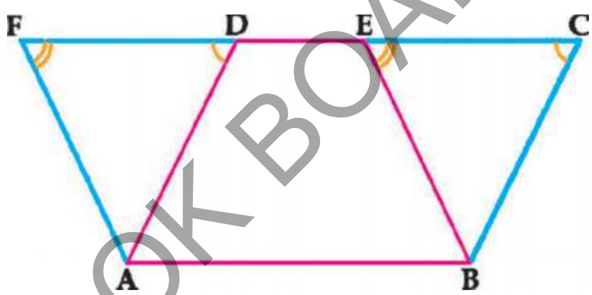
14.1 Theorems Related with Area

Theorem 14.1.1

Parallelograms on the same base and lying between the same parallel lines (or of the same altitude) are equal in area.

Given:

Two parallelograms ABCD and ABEF with the same base $\overline{AB}$ and between the same parallels segments $\overline{AB}$ and $\overline{DE}$.



To prove:

Parallelograms ABCD and ABEF are equal in areas,

i.e. ▰ABCD = ▰ABEF.

Proof:

Statements	Reasons
In ΔADF ↔ ΔBCE	
$m\overline{BC} = m\overline{AD}$ … (i)	Opposite sides of $\|^{m}$ ABCD are equal.
$m\angle BCE = m\angle ADF$ … (ii)	Corresponding angles of $\|^{m}$ ABCD.
$\angle E \cong \angle F$ … (iii)	Corresponding angles of $\|^{m}$ ABEF.
ΔBCE ≅ ΔADE	S.A.A ≅ S.A.A
▲BCE ≅ ▲ADF	Congruent figures are equal in area.
▰ABED + ▲BCE = ▰ABED + ▲ADF	Adding same area on both sides
Thus, ▰ABCD = ▰ABEF.	▰ABCD = ▰ABED + ▲BCE ▰ABEF = ▰ABED + ▲ADF

Q.E.D

Corollary

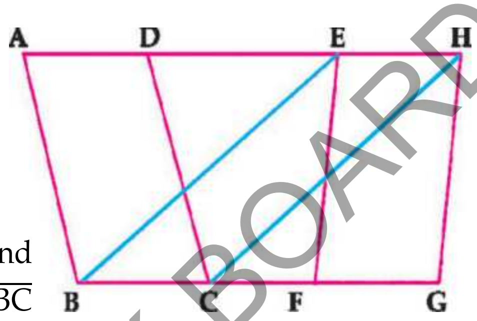
(i) The area of parallelogram is equal to that of a rectangle on the same base and having the same altitude.

Theorem 14.1.2

Parallelograms on equal bases and having the same altitude are equal in area.

Given:

Parallelograms ABCD and EFGH are on the equal bases $\overline{BC}$ and $\overline{FG}$, having equal altitudes.



To Prove: ▰ABCD = ▰EFGH.

Construction:

Place the parallelograms ABCD and EFGH so that their equal bases $\overline{BC}$ and $\overline{FG}$ are on the same straight line. Join B to E and C to H.

Proof:

Statements	Reasons
$\|^m$ ABCD and $\|^m$ EFGH are between the same parallel segments $\overline{AH}$ and $\overline{BG}$. Hence, A, D, E and H are points lying on a straight line parallel to $\overline{BC}$.	Their altitudes are equal (given)
$m\overline{BC} = m\overline{FG}$	Given
$m\overline{BC} = m\overline{EH}$	EFGH is a parallelogram
$m\overline{BC} = m\overline{EH}$ also these are parallel	Segment of parallel lines are also parallel segments.
Hence, EBCH is a parallelogram	A quadrilateral with two parallel opposite sides is a parallelogram
Now ▰ABCD = ▰EBCH ... (i)	Theorem 14.1.1
But ▰EBCH = ▰EFGH ... (ii)	Theorem 14.1.1
Thus, ▰ABCD = ▰EFGH	From (i) and (ii)

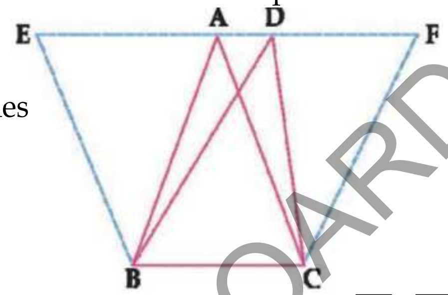
Q.E.D

Theorem 14.1.3

Triangles on the same base and of the same altitude are equal in area.

Given:

ΔABC and ΔDBC are on the same base $\overline{BC}$ and between the same parallel lines $\overline{BC}$ and $\overline{AD}$.



To prove:

▲ABC = ▲DBC

Construction:

Draw $\overline{BE}$ $\overline{CA}$, meeting at $\overline{AD}$ produced, at E and also draw $\overline{CF}$ $\overline{BD}$ meeting at $\overline{AD}$ produced at F.

Proof :

Statements	Reasons
BCAE is a parallelogram.	By construction
▲ABC = $\frac{1}{2}$ (▰BCAE) … (i)	Diagonal $\overline{AD}$ divides parallelogram BCAE into two Δs of equal areas.
Similarly BCFD is a parallelogram	By construction
▲DBC = $\frac{1}{2}$ (▰BCFD) … (ii)	Diagonal CD divides parallelogram BCFD into two triangles of equal areas.
▰BCAE = ▰BCFD … (iii)	Theorem 14.1.1

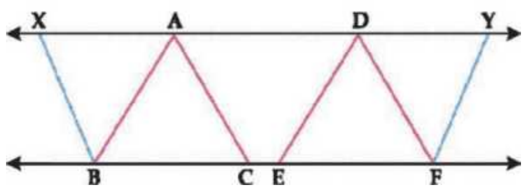
Q.E.D

Theorem 14.1.4

Triangles on equal bases and of equal altitudes are equal in area.

Given:

ΔABC and ΔDEF are on equal bases $\overline{BC}$ and $\overline{EF}$ respectively and having equal altitudes.



To prove:

▲ABC = ▲DEF

Construction:

Draw $\overleftrightarrow{AD}$, $\overleftrightarrow{BF}$ containing points B, C, E, F.

Place the ΔABC and ΔDEF so that their equal bases $\overline{BC}$ and $\overline{EF}$ are on the straight line. Draw $\overline{BX}$ $\overline{CA}$ and $\overline{FY}$ $\overline{ED}$. Such that point X and Y lie on $\overleftrightarrow{AD}$.

Proof :

Statements	Reasons
ΔABC and ΔDEF are between the same parallel lines. $\overleftrightarrow{BF} \parallel \overleftrightarrow{XY}$	Altitudes are equal (given)
$\therefore$ ▰BCAX = ▰EFYD … (i)	Theorem 14.1.2
But, ▲ABC $= \frac{1}{2}$ (▰BCAX) … (ii)	Diagonal of a parallelogram divides $\|^{m}$ into two equal triangles
and ▲DEF $= \frac{1}{2}$ (▰EFYD) … (iii)	By same reason
$\therefore$ ▲ABC = ▲DEF.	From eqs.(i), (ii) and (iii)

Q.E.D

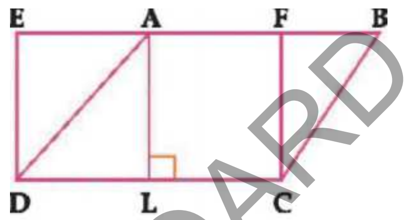
Corollaries: Triangles having a common vertex and equal bases in the same straight line are equal in area.

Exercise 14.1

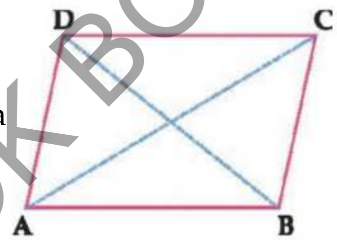
1. In the given figure, ABCD is a parallelogram and EFCD is a rectangle, also $\overline{AL} \perp \overline{DC}$. Prove that

   (i) ▰ABCD = ▰EFCD

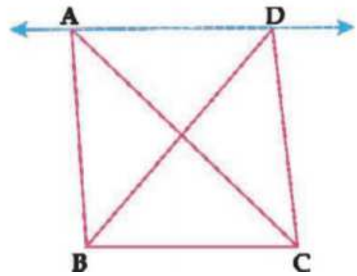
   (ii) ▰ABCD = $m\overline{DC} \times m\overline{AL}$.



2. In the given figure, if the diagonals of a quadrilateral separate it into four triangles of equal area, show that it is a parallelogram.



3. In the given figure $\overline{BC}$ $\overline{AD}$. ABC is a right-angled at vertex B with $m\overline{BC} = 7$ cm and $m\overline{AC} = 11$ cm, also ΔABC and ΔBCD are on the same base $\overline{BC}$. Find the area of ΔBCD.



4. Show that a median of a triangle divides it into two triangles of equal area. Give conditions and use name in questions.
5. Show that the line segment joining the mid-points of the opposite sides of a rectangle, divides it into two equal rectangles.
6. If two parallelograms of equal areas have the same or equal bases, their altitudes are equal.
7. Show that an angle bisector of an equilateral triangle divides it into two triangles of equal areas.
8. Prove that a rhombus is divided by its diagonals into four triangles of equal areas.

Review Exercise 14

1. Mark 'T' for True and 'F' for False in front of each given below:

(i) Area of a closed figure means region enclosed by bounding lines of the figure. **T/F**

(ii) A diagonal of rectangle divides it into two congruent triangles. **T/F**

(iii) Congruent figures have different areas. **T/F**

(iv) The area of parallelogram is equal to the product of base and height. **T/F**

(v) Median of a triangle means perpendicular from a vertex to the opposite side (base). **T/F**

(vi) Perpendicular distance between two parallel lines can sometimes be different. **T/F**

(vii) An altitude drawn from a vertex always bisects the opposite side. **T/F**

(viii) Two triangles are equal in areas, if they have the same base and equal altitude. **T/F**

2. Tick (✓) the correct answer.

(i) If perpendicular distance between two lines is the same. The lines are ______________

(a) Perpendicular to each other (b) Parallel to each other
(c) Intersecting to each other (d) None of these.

(ii) If two triangles have equal areas then they will _____________be congruent as well.

(a) Not necessarily (b) Necessarily
(c) Definitely (d) None of these.

(iii) Perpendicular from a vertex of a triangle to its opposite side is called ______________.

(a) Median (b) Perpendicular bisector
(c) Altitude (d) Angle bisector

(iv) Parallelograms having same base and same altitude are _____.
(a) Congruent (b) Equal in areas
(c) Similar (d) All of these.

(v) Two parallelograms have equal bases. They will be having the same area, if _____________.
(a) Their altitudes are equal
(b) Their altitude is the same
(c) They lies between the same parallel lines
(d) All of these.

(vi) ΔABC and ΔDEF have equal bases and equal altitudes, then triangles are_____________.
(a) Equal in area (b) Congruent
(c) Similar (d) None of these.

Summary

- In this unit we have mentioned some necessary preliminaries, stated and proved the following theorems along with corollaries, if any.
- Parallelograms on the same base and between the same parallel lines (or of the same altitude) are equal in areas.
- Parallelograms on the equal bases and having the same (or equal) altitude are equal in areas.
- Triangles on the same base and of the same (i.e. equal) altitudes are equal in areas.
- Triangles on equal bases and of equal altitudes are equal in areas.
- Area of a figure means region enclosed by the boundary lines of a closed figure.
- A triangular region means the union of triangle and its interior.
- By area of triangle means the area of its triangular region.
- Altitude or height of a triangle means perpendicular distance to base from its opposite vertex.