Unit 3

ALGEBRAIC EXPRESSION AND FORMULAS

Student Learning Outcomes (SLOs)

After completing this unit, students will be able to:

- Know that a rational expressions behaves like a rational numbers.
- Define a rational expression as a quotient $\frac{p(x)}{q(x)}$ of two polynomials $p(x)$ and $q(x)$, where $q(x)$, is not the zero polynomial.
- Examine whether a given algebraic expression is a
  - Polynomial or not,
  - Rational expression or not.
- Define $\frac{p(x)}{q(x)}$ as a rational expression in its lowest term, if $p(x)$ and $q(x)$ are polynomials with integral coefficients and having no common factor.
- Examine whether a given rational algebraic expression is in its lowest form or not.
- Reduce a given rational expression to its lowest form.
- Find the sum, difference and the product of rational expressions.
- Divide a rational expression by another rational expression and express the result in its lowest form.
- Find the values of the algebraic expressions at some particular real numbers.
- Know the formulas
  - $(a + b)^2 + (a - b)^2 = 2(a^2 + b^2)$ and $(a + b)^2 - (a - b)^2 = 4ab$.
  - Find the values of '$a^2 + b^2$' and of 'ab' when the values of '$a + b$' and '$a - b$' are known.
- Know the formula
  $(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$.
  - Find the values of $a^2 + b^2 + c^2$, when the values of $a + b + c$ and $ab + bc + ca$ are given.
  - Find the value of $a + b + c$ when the values of $a^2 + b^2 + c^2$ and $ab + bc + ca$ are given.
  - Find the value of $ab + bc + ca$ when the values of $a^2 + b^2 + c^2$ and $a + b + c$ are given.

- Know the formulas

  $(a + b)^3 = a^3 + 3ab(a + b) + b^3$,

  $(a - b)^3 = a^3 - 3ab(a - b) - b^3$.

  - Find the values of $a^3 \pm b^3$, when the values of '$a \pm b$' and 'ab' are given.
  - Find the values of $x^3 \pm \frac{1}{x^3}$, when the value of $x \pm \frac{1}{x}$ is given.
- Know the formula

  $a^3 \pm b^3 = (a \pm b)(a^2 \mp ab + b^2)$.

  - Find the product of $x + \frac{1}{x}$ and $x^2 + \frac{1}{x^2} - 1$.
  - Find the product of $x - \frac{1}{x}$ and $x^2 + \frac{1}{x^2} + 1$.
  - Find the continued product of $(x + y)(x - y)(x^2 + xy + y^2))(x^2 - xy + y^2)$.
- Recognize the surds and their applications.
- Explain the surds of the second order.
- Use basic operations on surds of second order to rationalize the denominators and to evaluate them.
- Explain rationalization (with precise meaning) of real numbers of the types $\frac{1}{a + b\sqrt{x}}$, $\frac{1}{\sqrt{x} + \sqrt{y}}$ and their combinations, where x and y are natural numbers and a and b are integers.

3.1 Algebraic Expressions

We have already studied about Algebraic expression in previous classes. Let's discuss its types.

Following are the three types of algebraic expressions.

(*a*) Polynomial Expression or polynomial,

(*b*) Rational Expression,

(*c*) Irrational Expression.

(*a*) Polynomial Expression or polynomial.

A polynomial expression (simply say polynomial) in one variable x can be written as:

$$a_0x^n + a_1x^{n-1} + a_2x^{n-2} + a_3x^{n-3} \ldots + a_{n-1}x^1 + a_n$$

Where 'n' is a non–negative integer and the coefficients; $a_0, a_1, a_2, \ldots, a_{n-1}, a_n$ are real numbers. Usually, a polynomial is denoted by $p(x)$, so the above polynomial can be expressed as:

$$P(x)=a_0x^n + a_1x^{n-1} + a_2x^{n-2} + a_3x^{n-3} \ldots + a_{n-1}x^1 + a_n$$

If $a_0 \neq 0$, then the polynomial is said to be a polynomial of degree n, and a_0 is called leading coefficient of the polynomial.

Some examples of polynomials and their degrees are given below.

(i) $8x-5$, degree 1 (ii) $x^4-2x^3+5x^2+1$, degree 4

(iii) $6x^{31}+3$, degree 31 (iv) $12x^4-x^3+\frac{2}{3}x^2-3x+1$, degree 4

(v) 4, degree zero. (vi) $\sqrt{10}x^{12}+2x^6-x^5-18x+1$ degree 12

The algebraic expression $x^3-x^3y^2+x^2y^2-10$ is a polynomial with two variables x and y and its degree is 5.

Similarly, the algebraic expression $x^3y^5x^2-x^3y^2z^3+x^2yz-34$ is a polynomial with three variables x, y and z and having degree 10(highest sum of powers) and so on.

Remember

- A polynomial consisting of only single term is called monomial. $3x$, $7xy$, $6xy^2z^5$ etc. are some examples of monomials.
- A polynomial consisting of two terms is called binomial e.g. $x+4$, $5x+y$, $7x-3$ etc. are some examples of binomials.
- A polynomial consisting of three terms is called trinomial e.g. x^2-2x+1

$\frac{1}{\sqrt{3}}x^2y^2 - 5xy + 3$, etc. are some examples of trinomials.

- Other polynomial which consisting of four or more terms, called multinomial.

(b) Rational Expression.

An algebraic expression which can be written in the form $\frac{p(x)}{q(x)}$ where $q(x) \neq 0$, and $p(x)$ and $q(x)$ are polynomials, called a **rational expression** in x.

For example, $\frac{x+1}{x}, \frac{x^2-x+1}{x-5}, \frac{\sqrt{3}x^2-5x+4}{x^2+6x-5}$ etc. are some examples of rational expressions.

Note: Every polynomial is a rational expression but its converse is not true.

(c) Irrational Expression.

An algebraic expression which cannot be written in the form of $\frac{p(x)}{q(x)}$, where $q(x) \neq 0$, when $p(x)$ and $q(x)$ are polynomials is called **irrational expression** in x.

For example, $\frac{1}{\sqrt{x}}, \frac{\sqrt{x}+1}{x}, \frac{\sqrt{x^3}+2x+3}{\sqrt{x}-9}, \sqrt{x}+\frac{5}{\sqrt{x}}$ etc. are some examples of irrational expressions.

3.1.1 Know that a rational expressions behaves like a rational numbers

Let p and q be two integers, then $\frac{p}{q}$ may be an integer or not. Therefore, the number system is extended and $\frac{p}{q}$ is defined as a rational number, where $p, q \in Z$ provided that $q \neq 0$.

Similarly, if $p(x)$ and $q(x)$ are two polynomials, then $\frac{p(x)}{q(x)}$ is not necessarily a polynomial, where $q(x) \neq 0$. Therefore, it is similar to the idea of rational numbers; the concept of rational expressions is developed.

3.1.2 Define a Rational expression as a quotient $\frac{p(x)}{q(x)}$ of two polynomials $p(x)$ and $q(x)$, where $q(x)$ is not the zero polynomial.

As we know that the expression in the form $\frac{p(x)}{q(x)}$, where $p(x)$ and $q(x)$ are two polynomials provided $q(x)$ is non–zero polynomial; called a rational expression.

For examples $\frac{x^2-5}{3x^2+4}, 3x^2+4 \neq 0$ are rational expressions.

3.1.3 Examine whether a given algebraic expression is a,

(i) Polynomial or not (ii) Rational expression or not

The following examples will help to identify polynomial and rational expressions.

Example 01 Examine whether the following are the polynomials or not?

(i) $2x^2 - \frac{1}{\sqrt{x}}$ (ii) $6x^3 - 4x^2 - 5x$

Solution(i): $2x^2 - \frac{1}{\sqrt{x}}$

It's not a polynomial because the second term does not have positive integral exponent.

Solution(ii): $6x^3 - 4x^2 - 5x$,

It is a polynomial, because each term has positive integral exponent.

Example 02 Examine whether the following are rational expressions or not?

(i) $\frac{x-2}{3x^2+1}$ (ii) $6x^3 - \frac{1}{\sqrt{x+4}}$

Solution (i): $\dfrac{x-2}{3x^2+1}$

The numerator and denominator both are polynomials, so it is a rational expression.

Solution (ii): $6x^3 - \dfrac{1}{\sqrt{x+4}}$

It is not a rational expression, because the denominator of the second term is not a polynomial.

3.1.4 Define $\frac{p(x)}{q(x)}$ as a rational expression in its lowest form, if $p(x)$ and $q(x)$ are polynomials with integral coefficients and having no common factor

The rational expression $\dfrac{p(x)}{q(x)}$ is said to be in its lowest form, if $p(x)$ and $q(x)$ are polynomials with integral coefficients and have no common factor.

For example $\dfrac{x+1}{x-1}$ is the lowest form of $\dfrac{x^2-1}{(x-1)^2}$

3.1.5 Examine whether a given rational algebraic expression is in its lowest form or not

To examine the rational expression $\dfrac{p(x)}{q(x)}$, find common factor(s) of $p(x)$ and $q(x)$. If common factor is 1, then the rational expression is in the lowest form.

For example $\dfrac{x+1}{x-1}$ is in its lowest form because, the common factor of $(x+1)$ and $(x-1)$ is 1.

3.1.6 Reduce a rational expression to its lowest form

Let $\dfrac{p(x)}{q(x)}$ be the rational expression, Where $q(x) \neq 0$

Step–1: Find the factors polynomials $p(x)$ and $q(x)$, If possible

Step–2: Find the common factors of $p(x)$ and $q(x)$

Step–3: Cancel the common factors of $p(x)$ and $q(x)$

Example 01 Reduce the following rational expression to their lowest form.

(i) $\dfrac{(x^2-x)(x^2-5x+6)}{2x(x^2-3x+2)}$ (ii) $\dfrac{5(x^2-4)}{(3x+6)\,(x-3)}$

Solution (i): $\dfrac{(x^2-x)(x^2-5x+6)}{2x(x^2-3x+2)}$

$$=\frac{x(x-1)}{2x}\cdot\frac{x^2-3x-2x+6}{x^2-2x-x+2} \quad \text{(Provided } x\neq 0\text{)}$$

$$=\left(\frac{x-1}{2}\right)\cdot\frac{\{x(x-3)-2(x-3)\}}{\{x(x-2)-1(x-2)\}}$$

$$=\frac{(x-1)(x-3)(x-2)}{2(x-2)(x-1)} \quad \text{Provided } x\neq 1 \text{ and } 2.$$

$$=\frac{(x-3)}{2}$$

$$=\frac{1}{2}(x-3) \text{ which is the required lowest form}$$

Solution (ii): $\dfrac{5(x^2-4)}{(3x+6)\,(x-3)}$

$$=\frac{x^2-4}{x-3}\cdot\frac{5}{3x+6}$$

$$=\frac{x^2-2^2}{x-3}\cdot\frac{5}{3(x+2)}$$

$$= \frac{(x+2)(x-2)}{x-3} \cdot \frac{5}{3(x+2)} \qquad \text{(Factorization)}$$

$$= \frac{(x+2)(x-2)}{x-3} \cdot \frac{5}{3(x+2)} \qquad x \neq -2$$

$$= \frac{5(x-2)}{3(x-3)}$$ which is the required lowest form.

3.1.7 Find the sum, difference and product of rational expressions.

The sum, difference and product of rational expression is explained with the help of the following examples.

Example 01 Simplify $\frac{3}{x+1} + \frac{4x}{x^2-1}$

Solution:

$$\frac{3}{x+1} + \frac{4x}{x^2-1}$$

$$= \frac{3}{x+1} + \frac{4x}{(x-1)(x+1)} \qquad \text{(Factorization)}$$

$$= \frac{3(x-1)+4x}{(x-1)(x+1)}$$

$$= \frac{3x-3+4x}{(x-1)(x+1)}$$

$$= \frac{7x-3}{(x-1)(x+1)}$$

$$= \frac{7x-3}{x^2-1}$$ Hence simplified in the lowest form.

Example 02 Simplify $\frac{1}{x^2-1}-\frac{1}{x^3-1}$

Solution:

$$\frac{1}{x^2-1}-\frac{1}{x^3-1}$$

$$=\frac{1}{x^2-1}-\frac{1}{x^3-1}$$

$$=\frac{1}{(x-1)(x+1)}-\frac{1}{(x-1)(x^2+x+1)}$$

$$=\frac{(x^2+x+1)-(x+1)}{(x+1)(x-1)(x^2+x+1)}$$

$$=\frac{x^2+x+1-x-1}{(x+1)(x-1)(x^2+x+1)}$$

$$=\frac{x^2}{(x+1)(x^3-1)}$$

Hence simplified in the lowest form.

Example 03 Simplify $\frac{x^2}{x^2+x-12}\cdot\frac{x^2-9}{2x^2}$

Solution: Simplification

$$\frac{x^2}{x^2+x-12}\cdot\frac{x^2-9}{2x^2}$$

$$=\frac{x^2}{x^2+4x-3x-12}\cdot\frac{(x-3)(x+3)}{2x^2}$$

$$=\frac{1}{x(x+4)-3(x-4)}\cdot\frac{(x-3)(x+3)}{2} \quad \text{(factorization)}$$

$$=\frac{1}{(x+4)(x-3)}\cdot\frac{(x-3)(x+3)}{2} \quad \text{provided } x\neq 3$$

$$=\frac{(x+3)}{2(x+4)}$$

Hence simplified in the lowest form.

3.1.8 Divide a rational expression by another rational expression and express the result in its lowest form.

In order to divide one rational expression with another, we first invert for changing division to multiplication and simplify the resulting product to lowest form.

Example 01 Simplify $\frac{3x-9y}{2x+10y} \div \frac{x^2-3xy}{4x+20y}$

Solution: Simplification

$$\frac{3x-9y}{2x+10y} \div \frac{x^2-3xy}{4x+20y}$$

$$= \frac{3x-9y}{2x+10y} \times \frac{4x+20y}{x^2-3xy} \quad \text{(Taking Reciprocal)}$$

$$= \frac{3(x-3y)}{2(x+5y)} \times \frac{4(x+5y)}{x(x-3y)}$$

$$= \frac{6}{x}$$

3.1.9 Find the values of algebraic expression at some particular real numbers.

Finding the values of algebraic expressions at some particular real numbers is explained in the following example.

Example 01 Find the value of $\frac{x^2+yz}{x^3+y^2-7yz^4}$ when $x = 3$, $y = 2$ and $z = -1$.

Given $x = 3, y = 2, z = -1$

$$= \frac{x^2+yz}{x^3+y^2-7yz^4}$$

$$= \frac{(3)^2+(2)(-1)}{(3)^3+(2)^2-7(2)(-1)^4}$$

$$= \frac{9-2}{27+4-14}$$

$$= \frac{7}{17}$$

Exercise 3.1

1. Examine whether the following algebraic expressions are polynomials or not.

(i) $2xy^2 - 3x^2 + 5y^3 - 6$ (ii) $3xy^{-2}$

(iii) $6x^2 - 10x + x - \sqrt{45}$ (iv) $5\sqrt{x} - x + 5x^2$

(v) $\dfrac{2}{x+2}$ (vi) $\dfrac{2}{x} + x^3 - 2$

2. Examine whether the following expressions are rational or not.

(i) $\dfrac{x^2+2x+3}{x-4}$ (ii) $\dfrac{x^2+5\sqrt{x}-2x}{3x^2+5x+4}$ (iii) $\dfrac{13x^2-9x+4}{x^2+5x+\sqrt{7}}$

(iv) $\dfrac{\sqrt{x}-\sqrt{y}}{\sqrt{x}+\sqrt{y}}$ (v) $\dfrac{7}{x+7}$ (vi) $5\sqrt{x} - x + 5x^2$

3. Reduce the following into their lowest form.

(i) $\dfrac{p^2-100}{p^2+10}$ (ii) $\dfrac{3ab-3a^2}{3a^2+6ab+3ab^2}$ (iii) $\dfrac{(a-b)}{(a+b)} \times \dfrac{(a^2+ab)}{(2a^2-2b^2)}$

(iv) $\dfrac{(x+y)^2-z^2}{x+y+z}$ (v) $\dfrac{(m^2-6m)(3m+15)}{2m-12}$ (vi) $\dfrac{4t^2-36t+80}{(4t-t^2)(5-t)}$

(vii) $\dfrac{x^2-2x-3}{x^2-x-2}$ (viii) $\dfrac{4x^2+24x+36}{3x^2-27}$ (ix) $\dfrac{(y+z)(y^2-2yz+z^2)}{y^2+yz}$

4. Simplify:

(i) $\dfrac{4x-1}{2x-2} + \dfrac{4x+1}{2x+2}$ (ii) $\dfrac{1}{x+2} + \dfrac{2}{x+3}$ (iii) $\dfrac{y^2-25}{y^2-5y} + \dfrac{y}{5y^2-5y}$

(iv) $\dfrac{xy}{xy+1} + \dfrac{xy+1}{xy-1}$ (v) $\dfrac{4}{z^2-4z-5} + \dfrac{2}{4z^2-4}$ (vi) $\dfrac{1}{a+b} - \dfrac{1}{a-b} + \dfrac{2a}{a^2-b^2}$

(vii) $\dfrac{4y}{y^2-1} + \dfrac{y+1}{y-1} + \dfrac{y-1}{y+1}$ (viii) $\dfrac{1}{2x+y} + \dfrac{1}{2x-y} + \dfrac{1}{4x^2-y^2}$

(ix) $\dfrac{x-2}{x^2+6x+9} - \dfrac{x-2}{2x^2-18}$ (x) $\dfrac{1}{a-1} + \dfrac{1}{a-3} + \dfrac{1}{a+1} + \dfrac{1}{a+3}$.

5. Perform the indicated operations and simplify.

(i) $\left(\dfrac{x^2}{4y^2-x^2}+1\right) \div \left(1-\dfrac{x}{2y}\right)$ (ii) $\dfrac{x+3}{3y-2x} \cdot \dfrac{4x^2-9y^2}{xy+3y}$

(iii) $\left(\dfrac{x^2-1}{x^2+2x+1}\times\dfrac{x+1}{x-1}\right)$ (iv) $\dfrac{8(y+3)}{9}\times\dfrac{12(y+1)}{4(y+3)}\div\dfrac{8(y+1)}{5}$

(v) $\dfrac{q^2-25}{q^2-3q}\div\dfrac{q^2+5q}{q^2-9}$ (vi) $\dfrac{4}{z^2-4z-5}\div\dfrac{2}{4z^2-4}$

6. Find the value of $t+\dfrac{1}{t}$, when $t=\dfrac{x-y}{x+y}$

7. Find the values of

(i) $\dfrac{5(x+y)}{3x^2\sqrt{y}+6}$, if $x=-4$, $y=9$

(ii) $\dfrac{42ab^2c^3}{3a^2b+1}$, if $a=3$, $b=2$ and $c=1$

(iii) $\dfrac{(x+y)^3-z^2}{x^2y^2+z^2}$, if $x=2$, $y=-4$ and $z=3$,

(iv) $\dfrac{3x^2y}{z}-\dfrac{bc}{x+1}$, if $x=2$, $y=-1$, $z=3$, b= 4, $c=\dfrac{1}{3}$

(v) $\dfrac{(ab^2-c)}{(a+cd^2)}\times\dfrac{(c+d)}{(a^2b-d)}$, if $a=1$, $b=3$, $c=-3$ and $d=2$.

3.2 Algebraic Formulas

We have already studied and used some algebraic formulas in previous classes. In this section we will learn some more formulas and their applications.

3.2.1 Know the Formulas

(i) $(a+b)^2+(a-b)^2=2(a^2+b^2)$

Verification:

$$\begin{aligned}\text{L.H.S} &= (a+b)^2+(a-b)^2\\ &= a^2+2ab+b^2+a^2-2ab+b^2\\ &= 2a^2+2b^2\\ &= 2(a^2+b^2)=R.H.S\end{aligned}$$

$(a+b)^2=a^2+2ab+b^2$

and $(a-b)^2=a^2-2ab+b^2$

(ii) $(a+b)^2 - (a-b)^2 = 4ab$

Verification:

$$\begin{aligned} \text{L.H.S} &= (a+b)^2 - (a-b)^2 \\ &= a^2 + 2ab + b^2 - (a^2 - 2ab + b^2) \qquad \therefore (a+b)^2 = a^2 + 2ab + b^2 \\ &= a^2 + 2ab + b^2 - a^2 + 2ab - b^2 \qquad \text{and } (a-b)^2 = a^2 - 2ab + b^2 \\ &= 4ab = R.H.S \end{aligned}$$

The method is explained in the following examples.

Example 01 Find the values of (i) $a^2 + b^2$ and (ii) ab, when $a + b = 6$ and $a - b = 4$.

Solution: Given that,

$a + b = 6$, $a - b = 4$.

(i) $a^2 + b^2 = ?$

We know that, $(a + b)^2 + (a - b)^2 = 2(a^2 + b^2)$

By substituting the values of $a+b = 6$ and $a - b = 4$, we get,

$(6)^2 + (4)^2 = 2(a^2 + b^2)$,

$\Rightarrow$ $36 + 16 = 2(a^2 + b^2)$

$\Rightarrow$ $52 = 2(a^2 + b^2)$

$\Rightarrow$ $26 = a^2 + b^2$

$\Rightarrow$ $\boxed{a^2 + b^2 = 26}$

(ii) $ab = ?$

We also know that $(a + b)^2 - (a - b)^2 = 4ab$.

By substituting the values of $a + b = 6$ and $a - b = 4$, we get,

$\therefore$ $(6)^2 - (4)^2 = 4ab$,

$\Rightarrow$ $36 - 16 = 4ab$,

$\Rightarrow$ $20 = 4ab$,

$\Rightarrow$ $5 = ab$,

or $\boxed{ab = 5}$

Thus, $a^2 + b^2 = 26$ and $ab = 5$.

3.3.2 Know the formula $(a+b+c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$

Verification: $(a+b+c)^2 = (a+b+c)(a+b+c)$

$$= a(a+b+c) + b(a+b+c) + c(a+b+c)$$

$$= a^2 + ab + ac + ab + b^2 + bc + ac + bc + c^2$$

$$= a^2 + b^2 + c^2 + 2ab + 2bc + 2ca = \text{R.H.S}$$

This method is explained in the following example.

Example 01 Find the value of $a^2 + b^2 + c^2$, when $a+b+c = 7$ and $ab+bc+ca = 15$

Solution: Given that,

$a+b+c = 7$ and $ab+bc+ca = 15$

$a^2 + b^2 + c^2 = ?$

We know that $(a+b+c)^2 = a^2 + b^2 + c^2 + 2(ab+bc+ca)$

Now, substituting the values of $a+b+c = 7$ and $ab+bc+ca = 15$, in the above formula we get,

$\therefore \quad (7)^2 = a^2 + b^2 + c^2 + 2(15)$

$\Rightarrow \quad 49 = a^2 + b^2 + c^2 + 30$

$\Rightarrow \quad 49 - 30 = a^2 + b^2 + c^2$

$\Rightarrow \quad 19 = a^2 + b^2 + c^2$

$\Rightarrow \quad \boxed{a^2 + b^2 + c^2 = 19}$

Hence, the value of $(a^2 + b^2 + c^2)$ is 19.

Example 02 Find the value of $(a+b+c)$, when $a^2 + b^2 + c^2 = 38$ and $ab+bc+ac = 31$

Solution: Given that,

$a^2 + b^2 + c^2 = 38$ and $ab + bc + ac = 31$,

We know that $(a+b+c)^2 = a^2 + b^2 + c^2 + 2(ab+bc+ca)$

Now, substituting the values of $a^2 + b^2 + c^2 = 38$ and $ab+bc+ac = 31$, in the above formula, we get,

$(a+b+c)^2 = 38 + 2(31)$

$\Rightarrow \quad (a+b+c)^2 = 38 + 62$

$\Rightarrow \quad (a+b+c)^2 = 100$

$\Rightarrow \quad (a+b+c) = \left(\pm\sqrt{100}\right)$

$\Rightarrow \quad \boxed{(a+b+c) = \pm 10}$

Hence, the value of $(a+b+c)$ is ± 10.

Example 03 Find the value of $(ab+bc+ac)$, when $a+b+c = 8$ and $a^2 + b^2 + c^2 = 20$.

Solution: Given that,

$a+b+c = 8$ and $a^2 + b^2 + c^2 = 20$,

We know that $(a+b+c)^2 = a^2+b^2+c^2+2(ab+bc+ca)$

By substituting the values of $a + b + c = 8$ and $a^2 + b^2 + c^2 = 20$, in the above formula we get,

$(8)^2 = 20 + 2(ab + bc + ac)$

$\Rightarrow \quad 64 = 20 + 2(ab + bc + ac)$

$\Rightarrow \quad 64 - 20 = 2(ab + bc + ac)$

$\Rightarrow \quad 44 = 2(ab + bc + ac)$

$\Rightarrow \quad 22 = ab + bc + ac$

$\Rightarrow \quad ab + bc + ac = 22$

Hence, the value of $(ab + bc + ac)$ is 22.

3.2.3 Know the cubic formulas

(i) $(a + b)^3 = a^3 + b^3 + 3ab(a + b)$

Verification: Here, $(a+b)^3 = (a+b)(a+b)^2$

$$= (a+b)(a^2+2ab+b^2)$$
$$= a^3+2a^2b+ab^2+a^2b+2ab^2+b^3$$
$$= a^3+b^3+3a^2b+3ab^2$$
$$= a^3+b^3+3ab(a+b)$$

(ii) $(a - b)^3 = a^3 - b^3 - 3ab(a - b)$

Verification: Here, $(a-b)^3 = (a-b)(a-b)^2$

$$= (a-b)(a^2-2ab+b^2)$$
$$= a^3-2a^2b+ab^2-a^2b+2ab^2-b^3$$
$$= a^3-b^3-3a^2b+3ab^2$$
$$= a^3-b^3-3ab(a-b)$$

The following examples are helpful for understanding the application for Cubic formulas.

Example 01 Find the value of $a^3 + b^3$, when $a + b = 4$ and $ab = 5$.

Solution: Given that,

$a + b = 4$ and $ab = 5$

We have to find

$a^3 + b^3$

Since, $(a + b)^3 = a^3 + b^3 + 3ab(a + b)$.

By substituting the values of $a + b = 4$ and $ab = 5$, in the above formula we get,

$(4)^3 = a^3 + b^3 + 3(5)(4)$

$\Rightarrow \quad 64 = a^3 + b^3 + 60$

$\Rightarrow \quad 64 - 60 = a^3 + b^3$

$\Rightarrow \quad 4 = a^3 + b^3$

$\Rightarrow \quad \boxed{a^3 + b^3 = 4}$

Hence the value of $(a^3 + b^3)$ is 4.

Example 02 Find the value of ab, when $a^3 - b^3 = 5$ and $a - b = 5$.

Solution: Given that,

$a^3 - b^3 = 5$ and $a - b = 5$

We have to find ab

Since, $(a - b)^3 = a^3 - b^3 - 3ab(a - b)$.

Now, substituting the values of $a^3 - b^3 = 5$ and $a - b = 5$, in the above formula, we get,

$(5)^3 = 5 - 3ab(5)$

$\Rightarrow \quad 125 = 5 - 15ab$

$\Rightarrow \quad 125 - 5 = -15ab$

$\Rightarrow \quad 120 = -15ab$

$\Rightarrow \quad -8 = ab$

$\Rightarrow \quad \boxed{ab = -8}$

Example 03 Find the value of $x^3+\frac{1}{x^3}$ when $x+\frac{1}{x}=3$

Solution: Given that

$$x+\frac{1}{x}=3$$

Taking cube on both sides, we have,

$$\left(x+\frac{1}{x}\right)^3=3^3$$

$$\Rightarrow \quad x^3+\frac{1}{x^3}+3x\cdot\frac{1}{x}\left(x+\frac{1}{x}\right)=27 \qquad \left[\ (a+b)^3=a^3+b^3+3ab(a+b)\ \right]$$

$$\Rightarrow \quad x^3+\frac{1}{x^3}+3(3)=27$$

$$\Rightarrow \quad x^3+\frac{1}{x^3}+9=27$$

$$\Rightarrow \quad x^3+\frac{1}{x^3}=27-9$$

$$\Rightarrow \quad \boxed{x^3+\frac{1}{x^3}=18}$$

Example 04 Find $8x^3-\frac{1}{x^3}$, when $2x-\frac{1}{x}=4$

Solution: Given that

$$=\ 2x-\frac{1}{x}=4$$

Cubing on both Sides

$$(2x-\frac{1}{x})^3=(4)^3$$

$$(2x)^3-(\frac{1}{x})^3-3(2x)(\frac{1}{x})(2x-\frac{1}{x})=64$$

$$8x^3-\frac{1}{x^3}-6(4)=64$$

$$8x^3-\frac{1}{x^3}-24=64 \qquad \Rightarrow 8x^3-\frac{1}{x^3}=88$$

3.2.4 Know the formula $a^3 \pm b^3 = (a \pm b)(a^2 \mp ab + b^2)$.

(i) $a^3 + b^3 = (a+b)(a^2 - ab + b^2)$

Verification: Here, $a^3 + b^3 = (a+b)(a^2 - ab + b^2)$

$$= a^3 - a^2b + ab^2 + a^2b - ab^2 + b^3$$

$$= a^3 + b^3$$

(ii) $a^3 - b^3 = (a-b)(a^2 + ab + b^2)$

Verification: Here, $a^3 - b^3 = (a-b)(a^2 + ab + b^2)$

$$= a^3 + a^2b + ab^2 - a^2b - ab^2 - b^3$$

$$= a^3 - b^3$$

Example 01 Find the product of $\left(x+\frac{1}{x}\right)$ and $\left(x^2+\frac{1}{x^2}-1\right)$

Solution: $\left(x+\frac{1}{x}\right)\left(x^2-1+\frac{1}{x^2}\right)$

$\therefore\ (a+b)(a^2-ab+b^2) = a^3+b^3$

Thus,

$$\left(x+\frac{1}{x}\right)\left(x^2-1+\frac{1}{x^2}\right) = x^3 + \frac{1}{x^3}$$

Example 02 Find the product of $\left(x-\frac{1}{x}\right)\left(x^2+1+\frac{1}{x^2}\right) = x^3 - \frac{1}{x^3}$

Solution: $\left(x-\frac{1}{x}\right)\left(x^2+1+\frac{1}{x^2}\right)$

$\therefore\ (a-b)(a^2+ab+b^2) = a^3-b^3$

Thus,

$$\left(x-\frac{1}{x}\right)\left(x^2+1+\frac{1}{x^2}\right) = x^3 - \frac{1}{x^3}$$

Example 03 Find the continued product.

(i) $(x+y)(x^2-xy+y^2)(x-y)(x^2+xy+y^2)$

Solutions (i): $(x+y)(x^2-xy+y^2)(x-y)(x^2+xy+y^2)$

$= (x^3+y^3)(x^3-y^3)$ $\quad [\because (a \pm b)(a^2 \mp ab + b^2) = a^3 \pm b^3]$

$= x^6 - y^6.$

Exercise 3.2

1. Find the value of $a - b$, when $a + b = 9$ and $ab = 20$.
2. Find the value of $a - b$, when $a + b = 5$ and $ab = -6$.
3. Find the value of $a^2 + b^2$ and ab, when $a + b = 8$ and $a - b = 6$.
4. Find the value of $a^2 + b^2$ and ab, when $a + b = 5$ and $a - b = 3$.
5. Find the value of $a^2 + b^2 + c^2$, when $a + b + c = 9$ and $ab + bc + ac = 13$.
6. Find the value of $a^2 + b^2 + c^2$, when $a + b + c = \frac{1}{3}$ and $ab + bc + ac = \frac{-2}{9}$.
7. Find the value of $a + b + c$, when $a^2 + b^2 + c^2 = 29$ and $ab + bc + ac = 10$.
8. Find the value of $a + b + c$, when $a^2 + b^2 + c^2 = 0.9$ and $ab + bc + ac = 0.8$.
9. Find the value of $ab + bc + ac$, when $a + b + c = 10$ and $a^2 + b^2 + c^2 = 20$.
10. Find the value of $a^3 + b^3$, when $a + b = 4$ and $ab = 3$.
11. Find the value of ab, when $a^3 - b^3 = 5$ and $a - b = 5$.
12. Find the value of ab, when $a^3 - b^3 = 16$ and $a - b = 4$.
13. Find the value of $a^3 - b^3$, when $a - b = 5$ and $ab = 7$.
14. Find the value of $125x^3 + y^3$ when $5x + y = 13$ and $xy = 10$.
15. Find the value of $216a^3 - 343b^3$, when $6a - 7b = 11$ and $ab = 8$.
16. Find the value of $x^3 + \frac{1}{x^3}$, when $x + \frac{1}{x} = 7$.
17. Find the value of $x^3 - \frac{1}{x^3}$, when $x - \frac{1}{x} = 11$
18. Find the continued product by using the relevant formulas.
    (i) $(2x^2 + 3y^2)(4x^4 - 6x^2y^2 + 9y^4)$
    (ii) $(x - y)(x + y)(x^2 + y^2)(x^4 + y^4)$
    (iii) $(x - y)(x + y)(x^2 + y^2)(x^2 + xy + y^2)(x^2 - xy + y^2)(x^4 + x^2y^2 + y^4)$.
    (iv) $(2x + 3y)(2x - 3y)(4x^2 + 9y^2)(16x^4 + 81y^4)$
19. Find product of
    (i) $\left(\frac{3}{2}b + \frac{2}{3b}\right)\left(\frac{9b^2}{4} + \frac{4}{9b^2} - 1\right)$ (ii) $\left(\frac{7y^2}{9} + \frac{9}{7y^2}\right)\left(\frac{49y^4}{81} + \frac{81}{49y^4} - 1\right)$

(iii) $\left(\frac{x^4}{12}+\frac{12}{x^4}\right)\left(\frac{x^8}{144}+\frac{144}{x^8}+1\right)$ (iv) $\left(c^2-\frac{1}{c^2}\right)\left(c^4+\frac{1}{c^4}+1\right)$

3.3 Surds and their Applications

3.3.1 Recognize the surds and their applications.

Surd: An expression is called a surd which has at least one term involving a radical term in its simplified form.

For examples, $\sqrt{2}, \sqrt{a-4}, \sqrt[3]{\frac{5}{10}}, \left(\frac{1}{3}+\sqrt{3}\right), \left(\sqrt[5]{2}-\frac{1}{2}\right)$ are surds.

All surds are irrational numbers.

If $\sqrt[n]{a}$ is an irrational number and 'a' is not a perfect n^{th} power then it is called a surd of n^{th} order. The result of $\sqrt[n]{a}$ is an irrational number. It is also called an irrational radical with rational radicand.

For examples: $\sqrt{\frac{5}{7}}, \sqrt[3]{5}, \sqrt[4]{6}, \sqrt[5]{2}, \sqrt[7]{10}$ are surds of order 2^{nd}, 3^{rd}, 4^{th}, 5^{th} and 7^{th} respectively. But, $\sqrt[3]{27}, \sqrt{\frac{1}{4}}$ are not surds because they represent the number 3 and $\frac{1}{2}$ respectively.

3.3.2 Explain the surds of the second order use basic operations on surds of second order to rationalize the denominators and to evaluate them.

(a) Surds of the second order:

(i) A surd which contains a single term is called a monomial surds.

For examples, $\sqrt{53}, \sqrt{a-9}, \sqrt{\frac{4}{5}}$ etc. are monomials and of 2^{nd} orders.

(ii) A surd which contains sum or difference of two monomial surds or sum of a monomial surd and a rational number is called a binomial surd.

For examples, $\sqrt{17}+\sqrt{11}, \sqrt{2}-13, \sqrt{3}-35$ etc. are binomial surds and of 2^{nd} order.

(iii) Conjugate of Binomial Surds

Expressions of the type

(a) $\left(\sqrt{a}+c\sqrt{b}\right)$ and $\left(\sqrt{a}-c\sqrt{b}\right)$ are conjugate surds of each other.

(b) $a+\sqrt{b}$ and $a-\sqrt{b}$ are conjugate surds of each other.

(b) Basic operations on surds of second order to rationalize the denominators and to evaluate them.

(i) Addition and subtraction of Surds.

The addition and subtraction of surds can be done by using following law.

For example, $a\sqrt{c}+b\sqrt{c}=(a+b)\sqrt{c}$ and $a\sqrt{c}-b\sqrt{c}=(a-b)\sqrt{c}$

Example 01 Simplify: $\sqrt{343}-3\sqrt{7}-2\sqrt{7}$

Solution:

$$\sqrt{343}-3\sqrt{7}-2\sqrt{7}$$
$$=\sqrt{7\times7\times7}-3\sqrt{7}-2\sqrt{7}$$
$$=7\sqrt{7}-3\sqrt{7}-2\sqrt{7}$$
$$=(7-3-2)\sqrt{7}$$
$$=(7-5)\sqrt{7}$$
$$=2\sqrt{7}$$

Example 02 Simplify: $\sqrt{32}+5\sqrt{2}+\sqrt{128}+7\sqrt{2}$

Solution:

$$\sqrt{32}+5\sqrt{2}+\sqrt{128}+7\sqrt{2}$$
$$=\sqrt{16\times 2}+5\sqrt{2}+\sqrt{64\times 2}+7\sqrt{2}$$
$$=\sqrt{(4)^2\times 2}+5\sqrt{2}+\sqrt{(8)^2\times 2}+7\sqrt{2}$$
$$=4\sqrt{2}+5\sqrt{2}+8\sqrt{2}+7\sqrt{2}$$
$$=(4+5+8+7)\sqrt{2}$$
$$=24\sqrt{2}$$

(ii) Multiplication and Division of Surds.

The Multiplication and division of the surds can be simplified by using the following laws:

(a) $\sqrt{a}\times\sqrt{b}=\sqrt{ab}$ (b) $\dfrac{\sqrt{a}}{\sqrt{b}}=\sqrt{\dfrac{a}{b}}$, provided $a>0$ and $b>0$.

Example 01 Simplify: $\sqrt{125}\times\sqrt{48}$

Solution: Simplification

$$\sqrt{125}\times\sqrt{48}$$
$$=\sqrt{(5)^2\times 5}\times\sqrt{(4)^2\times 3}$$
$$=5\sqrt{5}\times 4\times\sqrt{3}$$
$$=(5\times 4)\left(\sqrt{5}\times\sqrt{3}\right)$$
$$=20\sqrt{15}$$

Example 02 Simplify: $\dfrac{\sqrt{162}}{\sqrt{144}}$

Solution:

$$\frac{\sqrt{162}}{\sqrt{144}}$$
$$=\frac{\sqrt{2\times 81}}{\sqrt{12\times 12}}$$
$$=\frac{\sqrt{2\times(9)^2}}{\sqrt{(12)^2}}=\frac{9\sqrt{2}}{12}=\frac{3\sqrt{2}}{4}$$

Exercise 3.3

1. Simplify

(i) $\sqrt[4]{81x^{-8}z^{4}}$ (ii) $\sqrt[3]{256a^{6}b^{12}c^{9}}$ (iii) $\sqrt[7]{128}$ (iv) $\sqrt{7776}$

(v) $\dfrac{\sqrt[3]{(125)^{2}\times 8}}{\sqrt{(2\times 32)^{2}}}$ (vi) $\dfrac{\sqrt{21}\times\sqrt{28}}{\sqrt{121}}$ (vii) $\sqrt{\dfrac{(216)^{\frac{2}{3}}\times(125)^{\frac{1}{2}}}{(0.04)^{\frac{-3}{2}}}}$

(viii) $\dfrac{\sqrt[6]{4}\times\sqrt[3]{27}\times\sqrt{60}}{\sqrt{180}\times\sqrt[3]{0.25}\times\sqrt[4]{9}}$

2. Find the conjugate of

(i) $\left(8-4\sqrt{3}\right)$ (ii) $\left(6\sqrt{6}+2\sqrt{3}\right)$ (iii) $\left(8\sqrt{12}+\sqrt{8}\right)$

(iv) $\left(2-\sqrt{3}\right)$

3. Simplify

(i) $\left(6\sqrt{3}+4\sqrt{2}+7\sqrt{128}\right)$ (ii) $\dfrac{\sqrt{2}}{10}+\sqrt{8}$

(iii) $\left(13+15\sqrt{3}\right)+\left(7-6\sqrt{3}\right)$ (iv) $\sqrt{250}+\sqrt{490}+3\sqrt{10}$

(v) $\sqrt{245}+\sqrt{625}-\sqrt{45}$ (vi) $10\sqrt{11}-\sqrt{396}-3\sqrt{11}$

(vii) $\sqrt{17}\left(10\sqrt{17}-2\sqrt{17}\right)$ (viii) $\dfrac{3}{2}\left(\sqrt{18}+\sqrt{32}-\sqrt{50}\right)$

(ix) $\left(\dfrac{\sqrt{2}}{\sqrt{3}}+\dfrac{1}{\sqrt{3}}\right)\left(\dfrac{\sqrt{2}}{\sqrt{3}}-\dfrac{1}{\sqrt{3}}\right)$ (x) $\left(\sqrt{13}+\sqrt{11}\right)\left(\sqrt{13}-\sqrt{11}\right)$

(xi) $\left(3\sqrt{6}-4\sqrt{5}\right)^{2}$ (xii) $\left(2\sqrt{3}+3\sqrt{2}\right)^{2}$

4. Simplify

(i) $\sqrt{32}-\sqrt{8}$ (ii) $8\sqrt{243}-2\sqrt{27}$ (iii) $\dfrac{3}{2}\sqrt{18}-\dfrac{3}{8}\sqrt{32}-\dfrac{6}{4}\sqrt{50}$

(iv) $10\sqrt{289}$ (v) $3\sqrt{2}+\sqrt{8}$ (vi) $(3\sqrt{27}-5\sqrt{3})+(\sqrt{3}+\sqrt{27})$

3.4 Rationalization

3.4.1 Explain rationalization (with precise meaning) of real numbers on surds of the types $\frac{1}{a+b\sqrt{x}}$, $\frac{1}{\sqrt{x}+\sqrt{y}}$ and their combinations, where *x,y* are natutal number and *a* and *b* are integer.

- If the product of two surds is a rational number, then each surd is called the rationalizing factor of the other.
  For example, $\left(35+\sqrt{31}\right)$ and $\left(35-\sqrt{31}\right)$
- The process of multiplying a given surd by its rationalizing factor to get a rational number as product is called rationalization of the given surd. The product of the conjugate surds is a rational number.

Example 01 Find the product of $\left(\sqrt{3}+\sqrt{2}\right)\left(\sqrt{3}-\sqrt{2}\right)$

Solution: $\left(\sqrt{3}+\sqrt{2}\right)\left(\sqrt{3}-\sqrt{2}\right)$

$=\left(\sqrt{3}\right)^2-\left(\sqrt{2}\right)^2$

$=3-2=1$ which is a rational number.

Rationalization of denominator

Keeping the above discussion in mind, we observe that, in order to rationalize a denominator of the form $(a+b\sqrt{x})$ or $(a-b\sqrt{x})$, we multiply both numerator and denominator by the conjugate factor $(a-b\sqrt{x})$ or $(a+b\sqrt{x})$, by doing this we eliminate the radical and thus obtain a denominator free of any surd.

Rationalization of real numbers of the Types.

(i) $\frac{1}{a+b\sqrt{x}}$ (ii) $\frac{1}{\sqrt{x}+\sqrt{y}}$

For the expressions $\frac{1}{a+b\sqrt{x}}$ and $\frac{1}{\sqrt{x}+\sqrt{y}}$ also their rationalization, where $x,y\in\mathbb{N}$ and $a,b\in \mathbf{Z}$. The following examples will help to understand the concept of rationalization.

Example 01 Rationalize: $\dfrac{1}{5+2\sqrt{3}}$

Solution:

(Multiply and divide the denominator by $5-2\sqrt{3}$ the conjugate of $5+2\sqrt{3}$)

$$\frac{1}{5+2\sqrt{3}}$$

$$=\frac{1}{5+2\sqrt{3}}\times\frac{5-2\sqrt{3}}{5-2\sqrt{3}}$$

$$=\frac{5-2\sqrt{3}}{(5)^2-(2\sqrt{3})^2}$$

$$=\frac{5-2\sqrt{3}}{25-12}$$

$$=\frac{5-2\sqrt{3}}{13}$$

$$=\frac{5}{13}-\frac{2}{13}\sqrt{3}$$

Example 02 Rationalize: $\dfrac{5}{\sqrt{3}+\sqrt{2}}$

Solution:

$$\frac{5}{\sqrt{3}+\sqrt{2}}=\frac{5}{\sqrt{3}+\sqrt{2}}\times\frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}-\sqrt{2}}$$

$$=\frac{5\left(\sqrt{3}-\sqrt{2}\right)}{\left(\sqrt{3}\right)^2-\left(\sqrt{2}\right)^2}$$

$$=\frac{5\left(\sqrt{3}-\sqrt{2}\right)}{3-2}$$

$$=\frac{5\left(\sqrt{3}-\sqrt{2}\right)}{1}$$

$$=5\left(\sqrt{3}-\sqrt{2}\right)$$

Exercise 3.4

1. Find the conjugate of the following.

(i) $\sqrt{6}-2$ (ii) $10+\sqrt{10}$ (iii) $3-2\sqrt{2}$

(iv) $3\sqrt{2}+2\sqrt{3}$ (v) $5\sqrt{10}+\sqrt{3}$ (vi) $4\sqrt{5}-\sqrt{11}$

(vii) $\sqrt{11}+\sqrt{7}$ (viii) $9-3\sqrt{7}$

2. Rationalize the denominator of the following.

(i) $\dfrac{1}{\sqrt{2}+10}$ (ii) $\dfrac{1}{3+2\sqrt{5}}$ (iii) $\dfrac{1}{4\sqrt{3}-3\sqrt{6}}$

(iv) $\dfrac{16}{\sqrt{12}+\sqrt{11}}$ (v) $\dfrac{9-\sqrt{2}}{9+\sqrt{2}}$ (vii) $\dfrac{\sqrt{13}+3}{\sqrt{13}-3}$

(viii) $\dfrac{15}{7-2\sqrt{5}}$ (viii) $\dfrac{11-\sqrt{2}}{\sqrt{2}+11}$ (ix) $\dfrac{1}{\sqrt{17}+4}$

(x) $\dfrac{1}{\sqrt{17}-4}$

3. (i) If $x=8-3\sqrt{7}$, find $\left(x+\dfrac{1}{x}\right)^2$ (ii) If $\dfrac{1}{x}=2\sqrt{28}-11$, find x.

(iii) If $x=3-2\sqrt{2}$

find: $x+\dfrac{1}{x}, x-\dfrac{1}{x}, x^2+\dfrac{1}{x^2}, x^2-\dfrac{1}{x^2}$ and $x^4+\dfrac{1}{x^4}$

4. Simplify:

(i) $\dfrac{\sqrt{x+3}+\sqrt{x-3}}{\sqrt{x+3}-\sqrt{x-3}}$ (ii) $\dfrac{\sqrt{y^2+5}-y}{\sqrt{y^2+5}+y}$ (iii) $\dfrac{10}{6-\sqrt{y^2+36}}$

(iv) $\dfrac{5}{6+\sqrt{y^2+36}}$ (v) $\dfrac{15}{3-2\sqrt{2}}-\dfrac{5}{3+2\sqrt{2}}$

(vi) $\dfrac{6}{12+\sqrt{6}}-\dfrac{3}{12-\sqrt{6}}$

Review Exercise 3

1. Encircle the correct answer.

(i) Every Polynomial is:
(a) an irrational expression (b) a rational expression
(c) a sentence (d) none of these

(ii) A surd which contains sum of two monomial surds is called
(a) Trinomial surd (b) Binomial surd
(c) Conjugate surd (d) Monomial surd

(iii) $3x + 2y - 3$ is an algebraic
(a) Expression (b) Equation
(c) Sentence (d) In equation

(iv) The degree of the $3x^2y + 5y^4 - 10$ is
(a) 4 (b) 5 (c) 6 (d) 10

(v) $\sqrt{7}$ is an example of
(a) Monomial surd (b) Trinomial surd
(c) Binomial surd (d) Conjugate surd

(vi) Quotient $\frac{p(x)}{q(x)}$ of two polynomials $p(x)$ and $q(x)$, where $q(x) \neq 0$ is called
(a) Rational expression (b) Irrational expression
(c) Polynomial (d) Conjugate

(vii) $\frac{1}{x-y} - \frac{1}{x+y}$ is equal to
(a) $\frac{2x}{x^2-y^2}$ (b) $\frac{2y}{x^2-y^2}$ (c) $\frac{-2x}{x^2-y^2}$ (d) $\frac{-2y}{x^2-y^2}$

(viii) Conjugate of $2 - \sqrt{3}$ is
(a) $2+\sqrt{3}$ (b) $-2-\sqrt{3}$ (c) $-2+\sqrt{3}$ (d) $\sqrt{3}-4$

(ix) $a^3 - 3ab(a-b) - b^3$ is equal to
(a) $(a-b)^3$ (b) $(a+b)^3$ (c) $a^3 + b^3$ (d) $a^3 - b^3$

(x) If $a+b=5$ and $a-b=3$, then the value of ab is
(a) 4 (b) 5 (c) 3 (d) 6

(xi) $\left(5+\sqrt{15}\right)\left(5-\sqrt{15}\right)$ is equal to

(a) 10 (b) 15 (c) 25 (d) 30

(xii) $a^2+b^2+c^2+2ab+2bc+2ca$ is equal to

(a) $(a+b-c)^2$ (b) $(a+b+c)^2$

(c) $(a-b+c)^2$ (d) $(a+b+c)^3$

2. Fill in the blanks.

(i) Degree of any polynomial is __________ .

(ii) Conjugate of surd $2-\sqrt{3}$ is __________ .

(iii) Degree of polynomial $2x^3+x^2-4x^4+7x-9$ is __________ .

(iv) $\dfrac{\sqrt{x}}{3x+5}$ is a __________ Expression $x \neq \dfrac{-5}{3}$.

(v) $(x-y)(x+y)(x^2+y^2)=$__________ .

Summary

- A polynomial expression (simply say polynomial) in one variable x can be written as: $a_0x^n + a_1x^{n-1} + a_2x^{n-2} + a_3x^{n-3} + \cdots + a_{n-1}x^1 + a_n$
  A polynomial is usually denoted by $p(x)$.
- An algebraic expression which can be written in the form $\dfrac{p(x)}{q(x)}$, where $q(x) \neq 0$, and $p(x)$, $q(x)$ are both polynomials, called **rational expression** in x.
- An algebraic expression which cannot be written in form $\dfrac{p(x)}{q(x)}$, where $q(x) \neq 0$, and $p(x)$, $q(x)$ are both polynomials, called irrational expression in x.
- A polynomial expression consisting of only single term is called monomial.
- A polynomial expression consisting of two terms is called binomial.
- A polynomial expression consisting of three terms is called trinomial.
- Polynomial expression consisting two or more than two terms is called multinomial.

- The rational expression $\frac{p(x)}{q(x)}$, is said to be in its lowest form, if $p(x)$ and $q(x)$ are polynomials with integral coefficients and have no common factor.
- $(a+b)^2 + (a-b)^2 = 2(a^2+b^2)$ and $(a+b)^2 - (a-b)^2 = 4ab$.
- $(a+b+c)^2 = a^2+b^2+c^2+2ab+2bc+2ac$.
- $(a+b)^3 = a^3+b^3+3ab(a+b)$ and $(a-b)^3 = a^3-b^3-3ab(a-b)$
- $a^3+b^3 = (a+b)(a^2-ab+b^2)$.
- $a^3-b^3 = (a-b)(a^2+ab+b^2)$.
- An expression is called a surd which has at least one term involving a radical sign. For example, $\sqrt{2}, \sqrt{a-4}, \sqrt{\frac{3}{10}}$ are surds.
- If $\sqrt[n]{a}$ is an irrational number and '*a*' is not a perfect n^{th} power then it is called a surd of n^{th} order.
- A surd which contains a single term is called a monomial.
- A surd which contains sum or difference of two surds or sum of monomial surd and a rational number is called binomial surd.
- Expressions of the type $\left(\sqrt{a}+c\sqrt{b}\right)$ and $\left(\sqrt{a}-c\sqrt{b}\right)$ are conjugate surds of each other.
- $\sqrt{a}\times\sqrt{b}=\sqrt{ab}$ and $\frac{\sqrt{a}}{\sqrt{b}}=\sqrt{\frac{a}{b}}$, provided $a>0$ and $b>0$.
- If the product of two surds is a rational number, then each surd is called the rationalizing factor of the other.
- The process of multiplying a given surd by its rationalizing factor to get a rational number as product is called rationalization of the given surd.