Bhysics and Measurements

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Pride of Pakistan, World's second highest peak, K-2, summit of 8611 meters.

In this unit student should be able to:

Describe Physics.

Unit

- Describe the scope of Physics in science, technology and society.
- > State SI base units, derive units and supplementary units for various measurements.
- Express derived units as products or quotients of the base units.
- State the conventions for indicating units as set in the SI units.
- Measure, using appropriate techniques, the length, mass, time temperature and electrical quantities by making use of both analogue scales and digital > displays particularly, short time interval by ticker timer and by C.R.O.
- Check the homogeneity of physical equations by using dimensionality and base units.
- Derive formulae in simple cases using dimensions.
- Why all measurements contain some uncertainty.
- Distinguish between systematic errors (including zero errors) and random errors.
- Measure the diameters of few ball bearings of different sizes and estimate their volumes mention the uncertainty in each result.

- Analyze and evaluate the above experiment and suggest improvements.
- Assess the uncertainty in a derived quantity by simple addition of actual, fractional or percentage uncertainties.
- Identify dependent and independent variables.
- Draw line of best fit and error bar
- Draw extrapolation.
- Write answers with correct scientific notation, number of significant figures and units in all numerical and practical work.
- Identify that least count or resolution of a measuring instrument is the smallest increment measurable by it.
- Differentiate between precision and accuracy.
- Explain why it is important to use and instrument of smallest resolution.
- Explain the importance of increasing the number of readings in an experiment.
- Interpret the information from linear or nonlinear graphs / curves by measuring slopes and intercepts.

Measurements in physics play a crucial role in understanding and quantifying physical phenomena. They involve the determination of various properties such as length, mass, time, temperature, and energy. Precise and accurate measurements provide the foundation for scientific investigations and the development of theories and models.

1.1 Scope of Physics:

What do fly birds, moving automobiles, looking blue skies and vibrating cellular phones have in common? They all follow physics or physical laws.

Birds use the difference in air pressures above and below their wings to keep them aloft. Automobiles follow the



Migrating birds use celestial cues and magnetic field generated by Earth's molten core to reach their destination



principles of mechanics and thermodynamics to transfer stored gasoline for moving tires. The sky seems blue when sunlight strikes and scatters off nitrogen and oxygen molecules in the atmosphere. Lastly, cellular phones use electronic components and the principles of electromagnetic waves to transfer energy and information from one to another phone.

1.1.1 Physics:

Physics is a branch of science that seeks to understand and describe the fundamental principles and laws governing the natural world. It encompasses the study of matter, energy, space, and time, and how they interact with each other.

The goal of physics is to develop a set of mathematical models and theories that can explain and predict the behavior of physical systems. These models are based on observations, experiments, and measurements, and are refined through a process of hypothesis testing and verification.

Physics is often divided into several sub-disciplines, each focusing on different aspects of the physical world:

Classical Mechanics: This branch deals with the motion of objects and the forces that act upon them. It includes concepts such as Newton's laws of motion, kinematics, momentum, and energy.

Electromagnetism: Electromagnetic theory explains the behavior of electric and magnetic fields, as well as their interaction with charged particles and currents. It encompasses topics like electric and magnetic forces, electromagnetic waves, and the principles underlying electricity and magnetism.

Thermodynamics: Thermodynamics deals with the study of heat, energy, and their transformations. It explores concepts like temperature, entropy, energy conservation, and the behavior of gases and fluids.

Optics: Optics focuses on the properties and behavior of light. It covers topics such as reflection, refraction, diffraction, and the interaction of light with various materials and optical systems.

Quantum Mechanics: Quantum mechanics is the branch of physics that deals with the behavior of particles at the atomic and subatomic levels. It describes phenomena that classical mechanics cannot explain, such as wave-particle duality, quantum superposition, and quantum entanglement.

Relativity: Relativity theory, including both special relativity and general relativity, deals with the behavior of objects in extreme conditions, such as those involving high speeds or strong gravitational fields. It explores concepts like time dilation, length contraction, and the curvature of space-time.

These are just a few examples of the many subfields within physics. Physics plays a crucial role in understanding the fundamental nature of the universe, from the microscopic world of particles to the vast scales of galaxies and the cosmos. It also underlies many technological advancements and has practical applications in various industries, including engineering, medicine, and telecommunications.

1.1.2 Physics: Scope in Science, Technology and Society:

Physics plays a significant role in science, technology, and society across various domains. Here are some key aspects highlighting the scope of physics in these areas:

Science:

Fundamental Laws: Physics provides fundamental laws and principles that form the basis of understanding the natural world. It explores the behavior of matter, energy, forces, and their interactions, enabling scientists to develop theories and models to explain phenomena.

Advancing Knowledge: Physics drives scientific progress by pushing the boundaries of our understanding. It seeks to uncover new insights into the nature of the universe, from the microscopic realm of particles to the vast expanse of the cosmos.

Interdisciplinary Connections: Physics often intersects with other scientific disciplines, such as chemistry, biology, astronomy, and geology. It provides a framework for understanding complex systems and phenomena, facilitating interdisciplinary research and collaboration.

Technology:

Engineering Applications: Physics principles are employed in various engineering fields. For example, electrical engineers rely on electromagnetism, materials engineers utilize quantum mechanics, and mechanical engineers apply laws of motion to design and optimize technologies.

Energy and Power: Physics plays a crucial role in the generation, transmission, and utilization of energy. It underpins technologies such as renewable energy systems, nuclear power, electrical grids, and energy storage.

Electronics and Communications: Physics principles are fundamental to the development of electronic devices, telecommunications, and information technology. The study of semiconductor physics, quantum mechanics, and electromagnetism is essential for advancing these fields.

Society:

Medical Applications: Physics contributes to medical imaging technologies like X-rays, CT scans, MRI, and ultrasound. It also facilitates advancements in radiation therapy, laser surgery, and medical diagnostics.

Materials Science: Physics research aids in understanding the properties of materials and developing new materials for various applications, including electronics, transportation, construction, and energy technologies.

Environmental Studies: Physics plays a role in studying climate change, atmospheric physics, and environmental monitoring. It helps in developing sustainable technologies and understanding the impact of human activities on the planet.

Education and Scientific Literacy: Physics education fosters critical thinking, problemsolving skills, and a scientific mindset. It promotes scientific literacy, enabling individuals to make informed decisions and engage with science-related topics and issues.

Overall, physics provides the foundation for scientific advancements, technological innovations, and a deeper understanding of the world we live in. Its scope extends across diverse fields, contributing to the progress of science, technology, and society as a whole.

1.2 SI Base, Derived Units and Supplementary Units

Units are standards of measurement that are used to express physical quantities such as length, mass, time, temperature, electric current, and many others. Units provide a way to quantify and compare the magnitude of physical quantities, and to communicate these measurements in a clear and concise manner.

There are various systems of units, including the International System of Units (SI), which is the most widely used system in the world. Other systems of units, such as the British system or the U.S. customary system, are also used in certain regions or industries.

1.2.1 SI Base Units:

SI units, also known as the International System of Units, are a system of units used for measurement that has been officially adopted by the International System of Units (SI). This system is used for scientific, engineering, and technological applications, and it provides a consistent and standardized way of measuring physical quantities.

Table1.1 SI Base Quantities and Units					
Quantity	Unit	Unit Abbreviation			
Length	meter	m			
Time	second	S			
Mass	kilogram	kg			
Electric Current	ampere	А			
Thermodynamic Temperature	kelvin	Κ			
Amount of Substance	mole	mol			
Luminous Intensity	candela	cd			

SI Derived Units:

Derived units, in the context of measurement, are units of measure that are defined **based on** combinations of base units. In the International System of Units (SI), there are many derived units that are used to express different physical quantities.

Derived units are those which are obtained by the multiplication or division of base units. For example, the SI unit of force is the derived unit newton (N): One newton is equal to 1kg m/s^2 .

Table	1.2:	Units	derived	through	base	quantities	and	formula	are	derived	units.
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Physical Quantity	Derived Unit	Symbol
Volume	liter/cubic meter	l/m^3
Area	square meter	m ²
Force	newton	Ν
Speed / Velocity	meter per second	m/s

Supplementary Units:

Supplementary units, in the context of measurement, are units of measure that are not part of the base units in the International System of Units (SI) but are used to express certain physical quantities that are not directly covered by the base units.

Supplementary units are the dimensionless units that are used along with the base units in the SI units. Supplementary quantities are geometrical quantities of circle and sphere.

Physical Quantities	Supplementary unit	Symbol	Definition	
Plane Angle	Radian	rad	A unit of measurement of angles equal to 57.3° , equivalent to the angle subtended at the center of a circle by an arc equal in length to the radius as shown in figure 1.1 (a).	
Solid Angle	Steradian	Sr	The solid angle subtended at the center of a sphere by an area of its surface equal to the square of the radius of that sphere as shown in figure 1.1 (b).	
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1.2.2 Derived Units as Products or Derived units as quotients of the base units.

A derived quantity is defined based on a combination of base quantities and has a derived unit that is the exponent, product or quotient of these base units. Some examples are given in table 1.4

Derived Quantity	Unit	Symbol	Product / Quotient	SI Base Units
Pressure	Pascal	Ра	N / m ²	kg m ⁻¹ s ⁻²
Energy / Work	Joule	J	N. m	kg m ² s ⁻²
Power	Watt	W	J/s	kg m ² s ⁻³
Electric Resistance	Ohm	Ω	V / A	$kg m^2 s^{-3} A^{-2}$
Capacitance	Farad	F	C / V	$kg^{-1} m^{-2} s^4 A^2$

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Table 1.4: Derived units

1.2.3 Conventions for Units:

Conventions for units in physics follow established standards to ensure consistent and standardized communication of measurements. Here are some key conventions for units in physics:

International System of Units (SI): The SI system is the globally accepted standard for units of measurement in physics. It provides a coherent set of base units and derived units that cover various physical quantities. The SI units are used to express measurements consistently and facilitate scientific communication.

Base Units: The SI system defines seven base units, which are used to express fundamental physical quantities.

Prefixes: The SI system incorporates a range of prefixes that can be used with base units to represent multiples or fractions of those units. These prefixes make it easier to express measurements across a wide range of magnitudes. Some common SI prefixes include kilo (k), mega (M), milli (m), micro (μ), and nano (n).

Consistent Formatting: When writing units, it is important to adhere to consistent formatting conventions. Units should be written in lowercase letters, except for symbols derived from names of scientists, such as the Kelvin unit (K). It is also standard practice to write units in singular form, except when quantities are greater than one (e.g., 10 meters).

Appropriate choice of units: It is important to use units that are appropriate for the quantity being measured. Using the correct units helps to avoid confusion and ensures accurate representation of physical quantities. For example, using meters per second (m/s) for speed and meters per second squared (m/s²) for acceleration.

1.2.4 Measurement Techniques:

To measure different physical quantities, various techniques and instruments are used. Here are some common measurement techniques for length, mass, time, temperature, and electrical quantities:

Length:

The length of an object can be measured using a ruler, caliper, or tape measure.

Moreover, there are certain digital methods to measure the length such as laser rangefinders, digital sliding cliper, odometer, etc

Mass:

Physical Balance: A balance or weighing scale can be used to directly measure the mass of an object. In some cases, mass can be inferred from measurements such as weight (force due to gravity) using the appropriate conversion factors.

Time:

Mechanical clocks: This is a traditional method that uses a mechanism, such as a swinging pendulum or a rotating escapement, to keep time. Examples include grandfather clocks, cuckoo clocks, and wristwatches.

Atomic Clock: For highly accurate and precise time measurements, atomic clocks, such as cesium or rubidium atomic clocks, are used.

Measurement of Speed by the ticker timer:

The ticker timer is simply a piece of apparatus that we use to measure time. When you work out the speed of an object you need to know how far it goes in a certain time.





Solar daytime, or simply "daytime," refers to the period of the day when the Sun is visible in the sky. It is the time between sunrise and sunset.



Foot, pace, and yard are some other units of lengths based on body parts. However, these units are not reliable as the length of body parts varies from person to person. Therefore, people realized the need for Standard Units of Measurement



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Ancient Time Measurement Devices

In ancient times sundials, sandglass and pendulum were used to measure time each of them is shown in figure below.



Activity: Make a water clock by using two water bottles with some color liquid. You are going to create ancient time piece.

Temperature:

Thermometer: A thermometer can be used to measure temperature. Various types of thermometers, such as liquid-in-glass thermometers, digital thermometers, or infrared thermometers, are available for different temperature ranges and applications (Fig: A).

Thermocouple: Thermocouples are temperature sensors that utilize the temperature-dependent voltage across the junction of two dissimilar metals. (Fig: B).

Measuring Instruments for electrical quantities:

Voltmeter: Voltage can be measured using a voltmeter, which is connected in parallel to the circuit or component being measured.

Ammeter: Current can be measured using an ammeter, which is connected in series with the circuit or component being measured.

Ohm meter: Resistance can be measured using an ohmmeter or a multimeter, which measures the electrical resistance of a component or circuit (Fig: C).

Cathode Ray Oscilloscope (C.R.O):

A cathode ray oscilloscope (CRO) is a type of electronic instrument that can be used for a variety of measurement techniques as shown in figure 1.4. Some common techniques that can be performed with a CRO include:

Voltage measurement: A CRO can be used to measure the voltage of a signal by displaying its waveform on the screen. The vertical axis of the display represents the voltage, and the horizontal axis represents time. The peak-to-peak voltage of the signal can be measured by using the CRO's vertical and horizontal cursors.

1.3 Dimensionality:

Dimension: the nature of a physical quantity is known as dimension.

The dimensions of length, mass and time is dented by [L], [M] and [T] respectively. Also, for electric current [A] and thermodynamic temperature is [K]. As derived quantities are products or quotients which means formula may differ but their dimensionality is equalized.

The dimensions of a physical quantity are the power to which the units of the base quantity are raised to represent a derived unit of that quantity.

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$$Velocity = \frac{Displacment}{time} = \frac{[L]}{[T]} = [M^0 L T^{-1}]$$



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(A)

Here the dimension of velocity is zero in mass, one in length and negative one in time. Some dimensions of physical quantities are given as under in table 1.5.

Physical Quantities	Expression	Dimensional Formula	
Area	Length ×Breadth	[L ²]	
Density	Mass/ volume	[ML ⁻³]	
Momentum	Mass × velocity	[MLT ⁻¹]	
Work / Energy	Force \times displacement	$[ML^2T^{-2}]$	
Electric Charge	Current × time	[AT]	
Gravitational Constant	[Force \times (distance) ²] / mass ²	$[M^{-1}L^{3}T^{-2}]$	
Moment of Inertia	Mass \times (distance) ²	$[ML^2]$	
Moment of force	Force × distance	$[ML^2T^{-2}]$	
Angular Momentum Linear Momentum × distance		$[ML^2T^{-1}]$	

Table 1.5: Dimensional formula for various physical quantities



Applications of Dimensionality:

Dimensionality is a fundamental aspect of measurement and is applied in real-life physics. We make use of dimensional analysis for three prominent reasons:

- To check the consistency of a dimensional equation
- To derive the relation between physical quantities in physical phenomena
- To change units from one system to another

Limitations of Dimensionality:

Some limitations of dimensionality are:

- It doesn't give information about the dimensional constant.
- The formula containing trigonometric function, exponential functions, logarithmic function, etc. cannot be derived.
- It gives no information about whether a physical quantity is a scalar or vector.

1.3.1 Homogeneity of Physical Equation by Dimensionality:

Dimensionality or dimensional analysis is a technique used in physics to check the consistency of equations and measurements. It involves analyzing the dimensions of each term in an equation to ensure that the units on both sides of the equation are equivalent. If the units on both sides do not match, then the equation is either incorrect or incomplete. Dimensional analysis can also be used to derive equations for physical quantities by analyzing the dimensions of the various terms involved.

For example, $S = V_i t + \frac{1}{2} at^2$ expresses a relation between distance, speed, acceleration and time. The dimensions of each quantity are added on both sides and then found equal i.e., [L]. So, equation is homogenous equation or dimensionally correct.

Worked Example 1.1

Show that the equation for impulse $Ft = m V_f - m V_i = m (V_f - V_i) = m \Delta V$ is dimensionally correct.

Solution:

Step 1: write above equation in dimensional form we have. $[M][L][T^{-2}][T] = [M][L][T]^{-1} + [M][L][T]^{-1}$ Step 2:

Therefore $[M][L][T]^{-1} = [M][L][T]^{-1}$ and the equation is correct, both sides having the dimensions of momentum.

1.3.2 Using dimension to derive equation:

Consider the oscillations of a simple pendulum. We assume that the period of the pendulum [T] depends on following quantities:

- (i) the mass of the pendulum bob [M]
- (ii) the length of string of the pendulum [L], and
- (iii) the gravitational acceleration (g) $[LT^{-2}]$

Therefore, the equation can be written as:

 $T = k m^{x} l^{y} g^{z}$

Where x, y and z are unknown powers and k is a dimensionless constant.

The dimensional form is

 $[T] = [M]^{x} [L]^{y} [L]^{z} [T]^{-2z}$

Equating the indices for M, L and T on both sides of the equation, we get:

M: 0 = x

L: 0 = y + z

T: 1 = -2z

Therefore

x =0, y = $\frac{1}{2}$ and z = - $\frac{1}{2}$

The original equation therefore becomes

 $T = k (1 / g)^{1/2}$

Which is what we would expect for a simple pendulum. Dimensionality does not give us the value of the dimensionless constant k which can be shown by other methods to be 2π , so it becomes

$$T = 2\pi \sqrt{\frac{L}{g}} \qquad \dots \dots (1.1)$$

1.3 Errors and Uncertainty:

Error and uncertainty are two related but distinct concepts in various fields, including statistics, science, and engineering. Here's a distinction between the two:

Error: Error refers to the discrepancy between a measured or observed value and the true or expected value.

Uncertainty: Uncertainty, on the other hand, relates to the lack of precise knowledge or the degree of doubt associated with a measurement, prediction, or estimation. It arises due to limitations in available information or inherent variability in the system being studied.

The main difference between errors and uncertainties is that an *error is the difference between the actual value and the measured value*, while an **uncertainty is an estimate of the range between them, representing the reliability of the measurement.**

1.4.1 Uncertainty in measurements:

Any experiment will have a number of measurements, and which will be made to a certain degree of accuracy. There is always a degree of uncertainty when measurements are taken; the uncertainty can be thought of as the difference between the *actual* reading taken (caused by the equipment or techniques used) and the *standard value*. Uncertainties are not the same as errors

- > Errors can be because of issues with equipment or methodology that cause a reading to be different from the standard value.
- > The uncertainty is a range of values around a measurement within which the true value is expected to lie, and is an **estimate**.

For example: The calculations of velocity require the movement of a time and distance. Using

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a stop watch to measure time nearest tenth of a second. and using a meter scale to find distance to the nearest of millimeter (for small distances in a laboratory). It is very useful to have a rough idea of the kind of result that you might expect before starting an experiment.

1.4.2 Systematic error and Random Error:

Errors are common occurrences in Physics and there are two specific types of errors that may occur during experiments.

Systematic Errors:

Systematic errors are errors that have a clear cause and can be eliminated for future experiments as shown in fig: 1.5.





There are four different types of systematic errors:

Instrumental: When the instrument being used does not Function properly causing error in the experiment (such as a scale that reads 2g more than the actual weight of the object, causing the measured values to read too high **consistently**)

Environmental: When the surrounding environment (such as a lab) causes errors in the experiment (the scientist cell phone's RF waves cause the geiger counters to incorrectly display the radiation)

Observational: When the scientist inaccurately reads a measurement wrong (such as when not standing straight-on when reading the volume of a flask causing the volume to be incorrectly measured)

Theoretical: When the model system being used causes the results to be inaccurate (such as being told that humidity does not affect the results of an experiment when it actually does)

Random Errors:

Random errors occur randomly, and sometimes have no source/cause as shown in fig:1.6. There are two types of random errors:

Observational: When the observer makes consistent observational mistakes (such not reading the scale correctly and writing down values that are constantly too low or too high)

Environmental: When unpredictable changes occur in the environment of the experiment (such as students repeatedly opening and closing the door when the pressure is being measured, causing fluctuations in the reading)



Systematic vs. Random Errors:

Figure 1.6

Systematic errors and random errors are sometimes similar, so here is a way to distinguish between them:

Systematic Errors are errors that occur in the same direction consistently, meaning that if the scale was off by and extra 3lbs, then every measurement for that experiment would contain an extra 3 lbs. This error is identifiable and, once identified, they can be eliminated for future experiments

Random Errors are errors that can occur in any direction and are not consistent, thus they are hard to identify and thus the error is harder to fix for future experiments. An observer might make a

mistake when measuring and record a value that's too low, but because no one else was there when it was measured, the mistake went on unnoticed.

1.4.3 Measure the diameters of a few ball bearings of different sizes and estimate their volumes. Mention uncertainty in each result. analyze and evaluate the above experiment and suggest improvements.

To measure the diameters of different-sized ball bearings and estimate their volumes, assume we have a caliper with a measurement uncertainty of ± 0.01 mm. Here's an example of measuring three ball bearings and estimating their volumes:

Ball Bearing 1:

Diameter measurement: 5.12 mm
$$\pm$$
 0.01 mm (using the caliper)
Radius = $\frac{diameter}{2}$ = 2.56 mm \pm 0.005 mm = $r + \Delta r$
Volume = $\frac{4}{3}\pi r^3 = \frac{4}{3}\pi (r + \Delta r)^3$

 \therefore Δr is very small as compared to r so therefore square and higher power will be neglected

$$Volume = \frac{4}{3}\pi r^3 \pm 4\pi r^2 \Delta r$$
$$V \pm \Delta V = \frac{4}{3}(3.14)(2.56)^3 \pm 4(3.14)(2.56)^2(0.005))$$
$$= (70.24 \pm 0.41)mm^3$$

Volume of ball bearing 1 is found to be 70.24mm³ with uncertainty of 0.41 mm³

Ball Bearing 2:

Diameter measurement: 3.78 mm ± 0.01 mm Radius = diameter/2 = 1.89 mm ± 0.005 mm $V \pm \Delta V = \frac{4}{3}\pi r^3 \pm 4\pi r^2 \Delta r$ $V \pm \Delta V = (28.26 \pm 0.22)mm^3$

Volume of ball bearing 2 is found to be 28.26mm³ with uncertainty of 0.22 mm³

Ball Bearing 3:

Diameter measurement: 7.25 mm \pm 0.01 mm Radius = diameter/2 = 3.62 mm \pm 0.005 mm $V \pm \Delta V = \frac{4}{3}\pi r^3 \pm 4\pi r^2 \Delta r$ $V \pm \Delta V = (198.60 \pm 0.82)mm^3$

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Volume of ball bearing 3 is found to be 198.60mm³ with uncertainty of 0.82 mm³

Analyze and evaluate the above experiment and suggest improvements. Now let's evaluate the experiment and discuss potential improvements:

Measurement Technique: The use of a caliper is a common and practical approach for measuring ball bearing diameters. However, to improve accuracy, consider using a digital caliper with higher precision and readability. This can help reduce measurement errors and enhance the quality of the results.

Replicability: To ensure the reliability of the measurements, it is advisable to take multiple readings for each ball bearing and calculate the average. This helps minimize random errors and provides a more accurate representation of the true diameter.

Calibration: Regular calibration of the measuring instrument, such as the caliper, is crucial to ensure accurate measurements. Ensure the caliper is properly calibrated before each measurement session to minimize systematic errors.

Uncertainty Analysis: While we estimated the uncertainty in the volume calculations based on the uncertainty in the radius measurement, it is important to consider other potential sources of uncertainty, such as the accuracy of the volume formula and the assumption of perfect spherical shape. A more comprehensive analysis of uncertainty should involve considering all potential sources of error.

Quality Control: Implement quality control measures to ensure the ball bearings used in the experiment have consistent shape, surface quality, and are free from defects that could affect the measurements.

By addressing these points and incorporating the improvements mentioned, the experiment can yield more accurate and reliable results.

Worked Example 1.2

If the radius of sphere is measured a 9 cm with an error of 0.02 cm. Find the approximate error in calculating its volume.

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Solution: Step 1: $R = 9 \text{ cm and } \Delta R = 0.02 \text{ cm}$ Volume of sphere $= \frac{4}{3}\pi r^3$ By differentiating both sides, we get $\therefore \frac{\Delta V}{V} = 3 \frac{\Delta R}{R}$

 $\frac{\Delta V}{V} = 3 \times \frac{\Delta R}{R} = 3 \times \left(\frac{0.02}{9}\right) = 0.0067$

So, the error in volume calculation is approximately 0.67%. **Step 2:**

$$V = \frac{4}{3}\pi R^3$$
$$V = \frac{4}{3} \times \pi \times (9^3) = 904 \text{ cm}^3$$

The absolute error in volume is approximately $904 \times 0.0067 = 6.1 \text{ cm}^3$ Therefore, the approximate error in calculating the volume of the sphere is 6.1 cm³.

Worked Example 1.3

If radius of a sphere is measured as 7.5 cm with error of 0.03 cm, find the approximate error in calculating its volume.

Solution: Step 1:

Let R be the radius and V be the volume of the sphere, then

 $V = \frac{4}{3} \pi R^3$ Differentiating both sides, we get

Let ΔR be the error in R and the corresponding error in V is ΔV , then

$$\frac{\Delta V}{V} = 3 \times \frac{\Delta R}{R} = 3 \times \left(\frac{0.03}{7.5}\right) = 0.004$$

Step 2:

If R is given 7.5 cm and ΔR is 0.03 cm

 $V = \frac{4}{3} \pi 7.5^3 = 1767.15 \text{ cm}^3$

The absolute error in volume is approximately $1767.15 \times 0.004 = 7.1 \text{ cm}^3$ Therefore, the approximate error in calculating the volume of a sphere is 7.1 cm³.

1.4.5 Uncertainty Calculation of Derived Quantities:

Uncertainty in a single measurement:

A man weighs himself on his bathroom scale. The smallest divisions on the scale are 1Newton marks, so the least count of the instrument is 1 Newton.

He reads his weight as closest to the 142Newton mark. He knows his weight must be larger than 141.5 Newton (or else it would be closer to the 141-newton mark), but smaller than 142.5 Newtons (or else it would be closer to the 143-newton mark). So, his weight must be Weight = 142 ± 0.5 newtons.

In general, the uncertainty in a single measurement from a single instrument is half the least count of the instrument.

Fractional and Percentage Uncertainty

What is the fractional uncertainty in man's weight?

Fractional Uncertainity = $\frac{\text{Uncertainity in weight}}{\text{Value for weight}}$

$=\frac{0.5 \text{ newtons}}{142 \text{ newtons}}=0.0035$

What is the uncertainty in man's weight, expressed as percentage of his weight?

Percentage Uncertainity =
$$\frac{0.5}{142} \times 100\% = 0.35\%$$

Combining Uncertainties: Adding or Subtracting:

The length of a copper wire at 30°C is 18.2±0.04 cm and at 60°C is 19.7±0.02 cm. Find the absolute uncertainty and the extension of the wire.

Absolute uncertainty = 0.04 + 0.02 = 0.06

Extension of the wire = $(19.7-18.2) \pm 0.06$

Extension of the wire = $1.5 \text{ mm} \pm 0.06$

Multiplication or Division in an equation, percentage uncertainty of each value is added together.

The weight of an iron block is 8.0 ± 0.3 N and is placed on a wooden base of area, 3.5 ± 0.2 m² Find the percentage uncertainties of the values and then calculate the pressure exerted by the block.

Percentage uncertainty in the weight= $\left(\frac{0.3}{8}\right) \times 100 = 3.75\%$

Percentage uncertainty in the area = $\left(\frac{0.2}{3.5}\right) \times 100 = 5.71\%$

% uncertainty = 3.75 % + 5.71 % = 9.46 %

Pressure $=\frac{8}{35}=2.3 Pa$

Absolute Uncertainty in the pressure $=\left(\frac{9.46}{100}\right) \times 2.3 = 0.22 - abolute uncertainity =$ *Percentage uncertainity* \times *mean measurment.*

Since both the weight and the area have been approximated to two significant figures, so the same final answer becomes same form.

Pressure = 2.3 ± 0.22 Pa

Worked Example 1.4

Consider the length of cube is given as 5.75 ± 0.3 cm and you want to find absolute uncertainty in volume.

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Solution:

Step 1: VOLUME = $L^3 = (5.75)^3 = 190 \text{ cm}^3$ Step 2: Percentage uncertainty = $3 \times \left(\frac{0.3}{5.75}\right) \times 100 = 15.65\%$ Absolute uncertainty in volume = 190 ± 15.65 cm³

Self-Assessment Questions:

A girl needs to calculate the volume of her pool, so that she knows how much water she will need to fill it. She measures the length, width and height as under: Length = 5.56 ± 0.14 m Width = 3.12 ± 0.08 m height = 2.94 ± 0.11 m What will be the pool's volume with uncertainty? ($51.0\pm8.8\%$)

1.5 Graph:

Graphs are visuals that show relationships between; intended to display the data in a way that is easy to understand and remember. Graphs are used to demonstrate trends, patterns and relationships between sets of data. Graphs may be preferable to display certain types of data. The graph you choose will often depend on the key points you want others to learn from the data you've collected.

1.5.1 Dependent and Independent Variables:

In statistics and mathematical modeling, dependent and independent variables are terms used to describe the relationship between two variables.

An independent variable is a variable that is manipulated in an experiment or study to observe the effect it has on a dependent variable. The independent variable is also sometimes called the predictor variable, explanatory variable, or input variable.

A dependent variable is a variable that is being measured or observed in an experiment or study, and is expected to change as a result of the manipulation of the independent variable. The dependent variable is also sometimes called the response variable, outcome variable, or output variable.

There are different types of graphs that can be used to represent the relationship between dependent and independent variables, including scatter plots, line graphs, and bar graphs. The choice of graph depends on the type of data and the nature of the relationship between the variables.

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1.5.2 Best fit line graph:

A best fit line graph is a type of graph used to ³² visualize the relationship between two variables ²⁸ and is used to show the general trend in the data. ²⁴ The line of best fit is a straight line that is drawn in ²⁰ in the data. ¹⁶

The line of best fit is shown in figure 1.7 and it is drawn in a way that best represents the underlying pattern in the data.



Error Bar:

Error bars are graphical representations of the uncertainty or variability in a set of data points. They are often used in scientific plots to indicate the precision of the data being plotted as shown in figure 1.8.



Figure 1.8

Error bars are usually plotted as vertical or horizontal lines extending from the data points on a graph. The length of the error bars indicates the degree of uncertainty or variability in the data. Shorter error bars indicate greater precision, while longer error bars indicate less precision.

Error bars are useful in indicating the reliability and accuracy of the data, and they allow the reader to assess the significance of the results. In scientific research, error bars are often used to determine the statistical significance of differences between groups or to compare the results of different experiments.

A simpler display is a plot of the mean for each time point and error bars that indicate the variation in the data.

1.5.3 Extrapolation:

Extrapolation is a statistical technique that involves using observed data to estimate values

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beyond the range of the data that was collected. In other words, it is the process of making predictions or estimates about future or unseen data based on the trends or patterns in the existing data.

For example, in case volume temperature graph as shown in figure 1.9, If we extrapolate the line till it intercepts the temperature axis, in result we reach

zero kelvin temperature.

In summary, extrapolation is a technique used to make predictions or estimates about future or unseen data based on the trends or patterns in the existing data.



1.6 Significant Figures:

Significant figures are the digits in a number that are meaningful in terms of the precision of the measurement. They provide information about the degree of accuracy of a measurement and are used to report the results of experiments and calculations.

1.6.1: Rules of Significant Figures:

The rules for determining significant figures are as follows:

- **1.** All non-zero digits are significant. For example, the number 12.3 has three significant figures.
- 2. Zeros between two non-zero digits are significant. For example, the number 102 has three significant figures.
- **3.** Zeros to the right of the decimal point and to the right of a non-zero digit are significant. For example, the number 0.0056 has three significant figures.
- 4. Zeros to the left of the first non-zero digit in a number are not significant. For example, the number 0.0056 has two significant figures.
- 5. Zeros at the end of a number after the decimal point, but before the last non-zero digit, are significant. For example, the number 12.300 has four significant figures.

It is important to be consistent in reporting the significant figures in a calculation, as this provides information about the accuracy of the results. When performing calculations with numbers of different precision, it is necessary to round the result to the appropriate number of significant figures based on the rules above.

When performing calculations, we must consider the significant figures. When adding, subtracting, multiplying or dividing numbers, the answer should contain only as many significant figures as the number involved in the operation that has the least number of significant figures.

Example:

1. A student measures the length of a metallic rod and it was found to be 264.68 cm. While in next trial the length was measured as 247.1 cm. The error in measurement will be

264.68 cm - 247.1 cm = 17.68 cm

In this operation, the least number of significant figures in the operation is four so the final answer must have five significant figures.

2. An engineer measure the length of wall as 2.345m and width 8.3482m the area of the wall will be

 $2.345 \text{ m x} 3.56 \text{ m} = 8.3482 \text{ m}^2 = 8.35 \text{ m}^2$.

The final answer has three significant figures because the least number of significant figures in the operation is three that is 3.56.

3. The following values are part of a set of experimental data: 618.5 cm and 1450.6mm. Write the sum of these values correct to the right number of significant figures.

Or 1450.6/10 = 145.06cm.

or

618.5 cm + 145.06 cm = 763.56 cm

The least number of significant figures in the original values is 4, so write the answer to this significance. The sum is written as 763.6cm.

Scientific Notations:

In scientific notation, a number is expressed as a power of 10 multiplied by a coefficient. For example, the speed of light in vaccum is 299 792 458 m/s. In scientific notation, this number can be expressed as 2.99792458×10^8 m/s conveniently it can be written as 3×10^8 . It is important to note that scientific notation does not change the value of the number, but it provides a convenient way of expressing and manipulating very large or very small numbers.

1.7.1 Least Count or Resolution:

The least count or resolution of a measuring instrument is the smallest increment that can be measured by that instrument. It is the smallest division on the scale of the instrument and represents the smallest change that can be detected in a measurement. For example, if the least count of a ruler is 0.1 cm, then it can only measure lengths to the nearest 0.1 cm. The least count or resolution of a measuring instrument determines the precision of the instrument and thus the level of detail that can be obtained from a measurement.



Figure 1.10 ruler

Least count is inversely proportional to the precision of measurement equipment. The smaller the minimum value of an instrument can measure, the lower will be L.C., and the higher will be the precision.

For better understanding, let's consider an example of digital and mechanical Vernier calipers. Since digital Vernier L.C. (0.01 mm) is smaller than mechanical Vernier L.C. ((0.02 mm).

Mechanical Vernier will measure a 6 ± 0.21 mm dimension as 6 ± 0.20 mm or 6 ± 0.22 mm. Whereas digital Vernier will measure it 6 ± 0.21 . Therefore, Digital Vernier is more precise compared to mechanical Vernier because it has a smaller Least-Count.

Accuracy is also inversely proportional to the Least-Count. Accuracy of an instrument is always less than its L.C. because it cannot measure better than the minimum value it can measure.

1.7.2 Difference Between Accuracy and Precision.

Accuracy	Precision		
Accuracy is referred to the level of	Precision suggests the level of variation		
agreement between the actual measurement	that happens in the values of several		
and the absolute measurement.	measurements of the same factor.		
It represents how closely the results agree	Represents how closely results agree with		
with the standard value.	one another.		
Single-factor or measurement is required.	Multiple measurements are needed to comment about precision.		
Occasionally, a measurement may happen to be accurate by chance, while consistent accuracy and precision are required for a measurement to be reliable.	Results can be precise without being accurate.		

1.7.3 Significance of Resolution:

Using an instrument of smallest resolution is important for several reasons:

- 1. Precision: The smallest resolution of a measuring instrument determines its precision, which is the degree of reproducibility of a measurement. Using an instrument of smallest resolution ensures that the measurements are made with maximum precision and consistency.
- 2. Accuracy: The smallest resolution of a measuring instrument also affects its accuracy, which is the degree of closeness of a measurement to its true value. Using an instrument of smallest resolution helps to ensure that the measurement is as close as possible to the actual value of the quantity being measured.
- **3.** Detail: The smallest resolution of a measuring instrument determines the level of detail that can be obtained from a measurement. Using an instrument of smallest resolution allows for the measurement of finer details and features, which can be important in various applications, such as scientific experiments or engineering design.
- 4. Reduced uncertainty: The smallest resolution of a measuring instrument is directly proportional to the reduced uncertainty in a measurement. The smaller the resolution, the smaller the uncertainty, and the more accurate the measurement.

1.7.4 Importance of Repeating Experiment:

Increasing the number of readings in an experiment is important for several reasons:

- 1. Improved precision: The more readings taken in an experiment, the more accurate and precise the data becomes. This is because additional readings can help to account for any random errors that may have occurred in the first few readings.
- 2. Reduced uncertainty: Taking more readings in an experiment reduces the uncertainty in the data, which is the degree of random error associated with a measurement. The

larger the number of readings, the smaller the uncertainty, and the more accurate the data becomes.

- **3.** Better representation of the underlying trend: Taking multiple readings in an experiment can help to reveal underlying trends in the data. This can be especially useful in cases where the data may be affected by external known factors, or where the data may be influenced by unknown factors.
- 4. Increased confidence: Taking multiple readings in an experiment increases the confidence in the results of the experiment. The larger the number of readings, the more robust and reliable the data becomes, and the more likely it is to represent the true underlying relationship between variables.

In summary, increasing the number of readings in an experiment is important to improve the precision, reduce the uncertainty, better represent the underlying trend, and increase the confidence in the results of the experiment.

1.7.5 Interpreting Data from Graphs:

Linear and nonlinear graphs are graphical representations of mathematical relationships between variables. By measuring slopes and intercepts, you can interpret important information about the nature of the relationship between the variables represented on the graph.

- 1. *Slope*: The slope of a line or curve represents the rate of change of one variable with respect to another. It is calculated as the change in the vertical (y) coordinate divided by the change in the horizontal (x) coordinate between two points on the line or curve. The slope has units of y/x and represents the steepness of the line or curve. If the slope is positive, the line or curve rises from left to right. If the slope is negative, the line or curve falls from left to right.
- 2. *Intercept*: The intercept of a line or curve is the point at which the line or curve crosses the vertical (y) axis. It is the value of the dependent variable (y) when the independent variable (x) is equal to zero. The intercept represents the initial value of the dependent variable.

Linear graph:

Observe on the graph as shown in figure 1.11, x - axis is showing time and Y axis is showing position. It is observed that position is linearly increasing in positive direction with the time. So, the graph is

linear.

To determine the slope and intercept of the graph in figure 1.11,



 $y = mx + c \dots (1.3)$

where m is the slope, c is the point of intercept. Now slope can be determined as

$$Slope = m = \frac{\Delta y}{\Delta x} \dots \dots (1.4)$$



$$=\frac{30}{15}=2$$
ms⁻¹

As the intercept of the line is at origin so the intercept will be zero (c = 0). Note that slope of a straight line always constant

Non-linear graph

In contrast to the previous example, let's graph the position of an object with a constant, non-zero acceleration starting from rest at the origin as shown in figure 1.12. The primary difference between this curve and those on the previous graph is that this line actually curves. The relation between position and time is quadratic when the acceleration is constant and therefore this curve exhibits a non-linear relationship.



Slope is a characteristic exclusive to straight lines. This emphasizes that there is no single, constant velocity in

Figure 1.12 Non linear graph

such cases. The velocity of an object under these circumstances must be undergoing change, indicating acceleration. In the case of a curved position-time graph, since the slope varies at each point along the curve, it is not possible to determine a uniform velocity by calculating the slope alone.





- Physics: is a branch of science which studies the nature and behavior of matter, energy, and the interaction between them.
- Derived Unit: A derived quantity is defined based on a combination of base quantities and has a derived unit that is the exponent, product or quotient of these base units.
- Ticker Timer: The ticker timer is simply a piece of apparatus that we use to measure time.
- •> Electrical Quantities: The standard units of electrical measurement used for the expression of voltage, current and resistance are the Volt [V], Ampere [A] and Ohm $[\Omega]$ respectively.
- Dimensionality: The way in which the derived quantity is related to the basic quantity can be shown by the dimensions of the quantity.
- Uncertainty in measurements: Any experiment will have a number of measurements, and which will be made to a certain degree of accuracy.
- Systematic Error: These errors happen because of faulty apparatus like an incorrectly labelled scale, an incorrect zero mark on a meter or a stop watch running slowly.
- Random Error: The size of these errors depends upon how well the experimenter can use the apparatus.
- •> Graphs: are visuals that show relationships between; intended to display the data in a way that is easy to understand and remember. Graphs are used to demonstrate trends, patterns and relationships between sets of data.
- Independent Variable is the cause. Its value is independent of other variables in your study.
- Dependent Variable is the effect. Its value depends on changes in the independent variable.
- Significant Figures: The significant figures refer to the number of important single digit (0 through 9 inclusive) in the coefficient of an expression in scientific notation.
- Accuracy: The degree to which the result of a measurement conforms to the correct value or a standard' and essentially refers to how close a measurement is to its agreed value.
- Precision: The quality of being exact' and refers to how close two or more measurements are to each other, regardless of whether those measurements are accurate or not.
- Resolution is the ability of the measurement system to detect and faithfully indicate small changes in the characteristic of the measurement result.



Section (A): Multiple Choice Questions (MCQs)

- 1. The respective number of significant figures for the numbers 23.023, 0.0003 and 2.1×10^{-3} are:
- a. 5, 1, 2 b. 5, 1, 5 d. 4, 4, 2 c. 5, 5, 2 2. Which among the following is the supplementary unit: a. Mass b. Time Solid angle c. d. Luminosity 3. The unit of solid angle is b. Steradian a. Second c. Kilogram d. Candela 4. The quantity having the same unit in all system of unit is: b. time a. mass d. temperature c. length 5. Random errors can be eliminated by: a. taking number of observations and their mean. b. measuring the quantity with more than one instrument c. eliminating the cause d. careful observations 6. Systemic error can be: a. either positive or negative c. positive only b. negative only d. zero error 7. [MLT⁻²] is dimensional formula of: a. strain c. displacement b. force d. pressure 8. Which of the following pair has the same dimension? a. moment of inertia and torque c. impulse and momentum b. surface tension and force d. specific heat and latent heat 9. Dependent variable is: a. cause c. cause and effect b. effect d. reason 10. Dimensions of kinetic energy is the same as that of : a. Acceleration c. Work
 - b. Velocity d. Force

Section (B): Structured Questions

CRQ's:

- 1. Give an example of (I) a physical quantity which has a unit but no dimensions. (II) a physical quantity which has neither unit not dimensions. (III) a constant which has a unit. (IV) a constant which has no unit.
- 2. When rounding the product or quotient of two measurements, is it necessary to consider significant digit?
- 3. Drive the equation for period of oscillations of a mass suspended on a vertical spring by

dimensional analysis. i.e., $T = 2\pi \sqrt{\frac{m}{k}}$

- 4. Find the dimensions of the following.
 (a) work
 (b) energy
 (c) power
 (d) momentum
- 5. You measure the radius of a wheel to be 4.16 cm. If you multiply by 2 to get diameter, should you write the result as 8 cm or as 8.32 cm? Justify your answer.
- 6. If $y = a+bt+ct^2$ where y is in meters and t in seconds, what is the unit of c?
- 7. Differentiate between accuracy and precision.
- 8. Define dependent and independent variables.
- 9. Differentiate systematic error and random error.
- 10. Enlist the limitations of dimensional analysis?
- 11. Describe least count of Vernier and screw gauge micrometer.
- 12. Describe extrapolation methods.

ERQ's:

- 1. Discuss graphs and its various types by supported an example?
- 2. Elaborate rules of significant figures. State the rules for determining the number of significant figures in a given measurement.
- 3. What are the uses of dimensional analysis? Explain each of them.
- 4. A vernier and micrometer are shown as under. Observe their readings and write correctly.



Micrometer



Numerical:

- 1. What is the percent uncertainty in the measurement 3.67 ± 0.25 m? (6.8%)
- 2. What is the area, and its approximate uncertainty, of a circle with radius 3.7×10^4 cm $(4.3 \pm 0.11) \times 10^9$ cm²

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3. An aero plane travels at 850 km/h. How long does it take to travel 1.00 km? (4.23 s)

- A rectangular holding tank 25.0 m in length and 15.0 m in width is used to store water for short period of time in an industrial plant. If 2980 m³ water is pumped into the tank. What is the depth of the water? (7.95 m)
- 5. Find the volume of rectangular underground water tank has storage facility of 1.9 m by 1.2 m by 0.8 m. (1.824 m³)
- 6. Two students derive following equations in which x refers to distance traveled, v the speed, the acceleration, and t the time and the subscript (o) means a quantity at time t=0:
 (a) x = vt² + 2at and

(b) $x = v_0 t + 2at^2$, which of these could possibly be correct according to dimensional check? (a) Incorrect (b) Correct

- 7. One hectare is defined as 10^4 m². One acre is 4×10^4 ft². How many acres are in one hectare? (Hint: 1 m = 3.28 ft.) (2.69 acres)
- 8. A watch factory claims that its watches gain or lose not more than 10 seconds in a year. How accurate is this watch, express as percentage? $(3.16 \times 10^{-5} \%)$
- 9. The diameter of Moon is 3480 km. What is the volume of the Moon? How many Moons would be needed to create a volume equal to that of Earth?

(Hint: Radius of Earth = 6380 km) (2.2×10¹⁹ m³, 49.3)