Unit 12

Acoustics

Teaching Periods 11 **Weightage %** 8

Sound waves are vibrations occur as a result of interaction of particles either of same nature or different. Humans and other creatures use these sound waves, not only to communicate but also to perform various tasks. To detecting an enemy fighter plane or submarine through RADAR **or** SONAR

In this unit student should be able to:

- Explain the speed of sound depends on the properties of medium. In which it propagates. And describe the Newton's formula for speed of sound.
- Describe the Laplace correction in Newton's formula for speed of sound.
- Identify the factors on which the speed of sound in air depends.
- Solve problems using the formula $v_t = v_o\sqrt{\frac{T}{273}}$
- Describe the principle of superposition of two waves from coherent sources.
- Understand the phenomena of interference of sound waves.
- Describe the phenomenon of formation of BEATS due to interference of non-coherent source
- Explain the tuning of musical instrument by BEATS
- Explain the formation of stationary waves in using graphical method
- Define the term Node and Anti- Node.
- Describe modes vibration of stationary waves in a string.
- Describe the formation of stationary waves in vibrating an air column.
- Explain the observed change the frequency of mechanical waves coming from moving object approaches or moves away.
- Recall the application of Doppler Effect such as RADAR, SONAR, Astronomy, Satellite and radar speed and traps.
- Outline some cardiac problems that can be defected through the use of the Doppler's Effect.

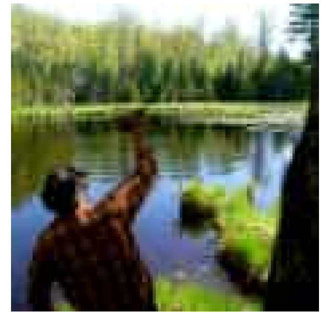


Fig: 12.1

Speed of sound waves:

Physicists use only a few basic models to describe the physical world. One such model is the particle: a point like object with no inner structure and with certain characteristics such as mass and electric charge. Another basic model is the *wave*.

A wave is characterized by some sort of disturbance in an elastic medium that travels away from its source.

In mechanical waves the particles in the medium are disturbed from the equilibrium position as the wave propagates. After the wave has passed the particles return to their equilibrium position. Water waves, seismic waves/earthquake and sound waves are the most common forms of mechanical waves. The longitudinal and transverse (stationary or standing) waves both have required a medium to propagate.

12.1 Sound Waves:

Sound is a *longitudinal/mechanical* wave that is created by a vibrating object, such as a guitar string, the human vocal cords, or the diaphragm of a loudspeaker (Fig.12.2). **Sound can be produced and transmitted only in a medium, such as a gas, liquid, or solid**

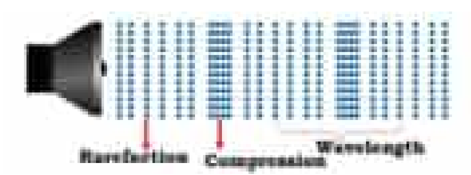


Fig: 12.2
The Physics of a loudspeaker diaphragm.

Sound cannot exist in a vacuum:

When a guitar string is plucked, a transverse wave travels along the string. The vibration of the string is transmitted through the bridge to the body of the guitar, which in turn transmits the vibration to air, called *sound* wave. In the absence of sound wave, air molecules dart around in random directions. On average they are uniformly distributed and the pressure is the same everywhere.

During the propagation of sound wave, the uniform distribution of air molecules (or any medium) is disturbed.

Frequencies of Sound Waves:

The human ear responds to sound waves within a limited

DO YOU KNOW?

When the end of the Slinky is moved up and down continuously, a **Transverse wave** is produced. In such type of wave particles of the medium vibrate perpendicular to the direction of motion of wave.

A back and forth motion of slinky produces **Longitudinal wave.** In such type of wave particles of the medium vibrate along the direction of motion of wave.

range of frequencies. We generally consider the **audible range** to extend from 20 Hz to 20 kHz.
The terms **infrasonic** and **ultrasonic** are used to describe sound waves with frequencies below 20 Hz and above 20 kHz.

12.1.1 Speed of Sound in Air:

The speed of sound in a medium, solid, liquid or gas depends on the elasticity and density of medium.

$$v = \sqrt{\frac{\text{Elastic modulus of medium}}{\text{Density of the medium}}}$$

$$v = \sqrt{\frac{E}{\rho}} \qquad \text{......(12.1)}$$

Newton assumed that the temperature of air (other gases) remains constant when sound waves travel through air. The process is isothermal and Boyle's law can be applied. At compressed regions, pressure increases and volume decreases and in rarefied regions the pressure decreases and volume increases. Under these conditions, the modulus of elasticity is equal to the pressure of the air (gases).

Suppose;

P = initial pressure
V = initial volume
ΔP = increase in pressure
ΔV = decrease in volume
$P + \Delta P$ = final pressure
$V - \Delta V$ = final volume

Using Boyle's law under these conditions,

$$PV = (P + \Delta P)(V - \Delta V) \quad \text{--------(12.2)}$$
$$PV = PV - P\Delta V + \Delta PV - \Delta P\Delta V$$
$$P\Delta V = \Delta PV - \Delta P\Delta V$$

If the change in pressure is small then the corresponding change in volume is also negligible, hence neglecting $\Delta P\Delta V$

Therefore, $P\Delta V = \Delta PV$

$$P = \frac{\Delta PV}{\Delta V}$$

$$P = \frac{\Delta P}{\frac{\Delta V}{V}}$$

$$P = \frac{\textit{Volumetric stress}}{\textit{Volumetric strain}} \qquad E = B \text{ (Bulk Modulus)}$$

Hence, now be written as,

$$\boldsymbol{v} = \sqrt{\frac{B}{\rho}} \Rightarrow \qquad \boldsymbol{v} = \sqrt{\frac{P}{\rho}} \qquad \text{------ (12.3)}$$

DO YOU KNOW?

The audible ranges for animals

Dogs can hear frequencies as high as 50 kHz

Dolphins make use of frequencies as high as 250 kHz (these are **Ultrasound waves**).

Elephants communicate over long distances (up to 4 km) using sounds with fundamental frequencies as low as 14 kHz.

A rhinoceros uses frequencies down to 10 Hz.

Eq. 12.3 is called Newton's formula for speed of sound in air.
Since; P = $\rho_{mercury}$ gh, Eq. 12.3 can now be written as.

$$v = \sqrt{\frac{\rho_{mercury} gh}{\rho}} \quad \text{------ (12.4)}$$

at STP, $\rho_{mercury} = 13.6 gcm^{-3}$, g = 981 cms^{-2} and h = 76 cm and density of air $\rho = 1.293 \times 10^{-3}\ gcm^{-3}$
Using these quantities in Eq. 12.4 the speed of sound in air at S.T.P is found to be

$$v = \sqrt{\frac{13.6 \times 981 \times 76}{1.293 \times 10^{-3}}}$$

$v = 28003.447\ cms^{-1}$

The experimental value determined from various experiments for speed of sound waves in air at S.T.P is found to be 33200 cm s^{-1}.
Therefore the theoretical value is 15.6% less than the experimental value of speed of sound.
The large difference in the theoretical and experimental values cannot be attributed to the experimental errors. Newton was unable to explain the error in his formula and the correction was explained by a French scientist Lalplace.

12.1.2 Newton's Defect and Laplace Correction:

Newton assumed that propagation of sound waves is an isothermal process. But Lapalce thought other way, according to Laplace when sound waves travel through air, there is compression and rarefaction in the particles of the medium. Where there is compression, particles come near to each other and the temperature rises. At rarefaction particles go apart and there is fall of temperature. Therefore, the temperature does not remain constant when sound waves travel through air or any other gas. As sound waves travel through air with a speed of 330 ms^{-1}, the changes in air pressure, volume and temperature is taken place so rapidly. The process is not isothermal but it is adiabatic (Fig.12.3), hence Boyle's law is not applicable. The total quantity of heat of the system as a whole remains constant. It neither gains nor loses any heat through the surroundings.

Table 12.1
Speed of Sound in Various Materials at 0°C and 1 atm (Unless the temperature is mentioned in parenthesis)

Medium	Speed (m/s)
Carbon dioxide	259
Air	331
Nitrogen	334
Air (20°C)	343
Helium	972
Hydrogen	1284
Mercury(25°C)	1450
Fat (37°C)	1450
Water (25°C)	1493
Seawater (25°C)	1533
Blood (37°C)	1570
Muscle (37°C)	1580
Lead	1322
Concrete	3100
Copper	3560
Bone (37°C)	4000
Pyrex glass	5640
Aluminum	5100
Steel	5790
Granite	6500

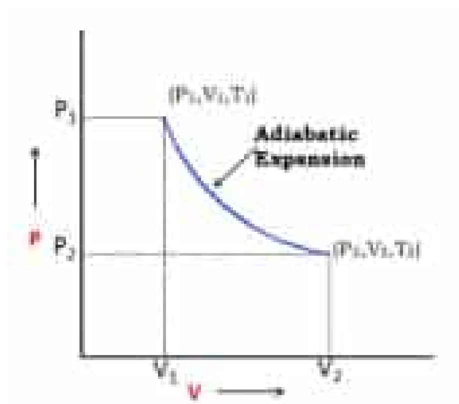


Fig: 12.3
An adiabatic behavior of gas

During an adiabatic process

$$PV^{\gamma} = \text{constant} \qquad (12.5)$$

If pressure of the given mass of the gas changes from P to P+ΔP resulting to a change of V+ ΔV in volume. Eq. 12.5 can now be written as;

$$PV^{\gamma} = (P + \Delta P)(V - \Delta V)^{\gamma}$$

Divide and multiply volume term of (Eq. 12.6) on right hand side with V^{γ}, we get.

$$PV^{\gamma} = (P + \Delta P)V^{\gamma}(1 - \frac{\Delta V}{V})^{\gamma}$$

$$P = (P + \Delta P)(1 - \frac{\Delta V}{V})^{\gamma}$$

Expanding through Binomial theorem and neglecting the square and higher powers of $\frac{\Delta V}{V}$ we get.

$$P = (P + \Delta P)(1 - \frac{\gamma \Delta V}{V})$$

$$P = P - \frac{\gamma P \Delta V}{V} + \Delta P - \frac{\gamma \Delta P \Delta V}{V}$$

Neglecting the term $\frac{\gamma \Delta P \Delta V}{V}$, $\qquad \Delta P = \frac{\gamma P \Delta V}{V}$

Hence, $\qquad \gamma P = \frac{\Delta P}{\frac{\Delta V}{V}} = \frac{\textit{Volumetric stress}}{\textit{Volumetric strain}} = B$

Eq. 12.3 can now be written as,

$$v = \sqrt{\frac{\gamma P}{\rho}} \qquad \ldots\ldots (12.6)$$

Here γ is the ratio between the two molar specific heats, **(C_p, molar specific heat at constant pressure and C_v molar specific heat at constant volume)** of air or gas.

For air, $\gamma = 1.42$.

Hence; according to Laplace correction the speed of sound in air is,

$$v = \sqrt{\frac{1.42 \times 13.6 \times 981 \times 76}{1.293 \times 10^{-3}}}$$

$$v = 33369.9 \text{ cms}^{-1} \approx 333 \text{ ms}^{-1}$$

The value of speed of sound is in good agreement with the experimental value 332 ms^{-1} at 0°C.

12.1.3 Factors Affecting the Speed of Sound in Air:

As mentioned earlier that sound waves are compressible mechanical waves, hence the nature and state of the medium affects the propagation of sound wave through the medium. Density of medium, air moisture, pressure, temperature and speed of wind are some of main factors which affect the speed of sound in air.

1. Density of air:

From the relation, $v = \sqrt{\frac{\text{Elastic modulus of medium}}{\text{Density of the medium}}}$

It is clear that speed of sound varies inversely to the square root of density of air.

2. Moisture:

Moisture is the presence of a liquid, especially water in other media, small amounts of water may be found, for example, in the air (called humidity). The presence of moisture in the air decreases the resultant density of air which increases the speed of sound in humidity. Hence the speed of sound in damp (wet) air is greater than in dry air.

3. Pressure:

According to Ideal Gas equation, for 'n' mole.

$$PV = nRT$$

Where, R is Universal gas constant = 8.314 Joule $^{-1}K^{-1}$ an T is the temperature in Kelvin.

$$P = \frac{RT}{V}; \qquad \ldots\ldots (12.7)$$

where n is the number of moles

Let m is the mass of the air then ρ (density of air) is given as,

$$\rho = \frac{m}{V} \qquad \ldots\ldots (12.8)$$

Hence the speed of sound in Eq. 12.6 can now be written as,

$$v = \sqrt{\frac{\gamma P}{\rho}} \qquad \Rightarrow v = \sqrt{\frac{\gamma n \frac{RT}{V}}{\frac{m}{V}}} \quad \Rightarrow v = \sqrt{\frac{\gamma nRT}{m}}$$

Here; $\frac{m}{n}$ = M; called molar mass of the gas (air)

$$v = \sqrt{\frac{\gamma RT}{M}} \qquad \ldots\ldots (12.9)$$

Eq.(12.9) shows that speed of sound is independent of pressure of gas (air)

4. Temperature:

Temperature changes do not affect the speed of sound in liquid and solid media quite significantly. But for a gas (air) the rise and fall of temperature at constant pressure significantly increases or decreases the volume of gas, and thus the density of gas is changes inversely. There is an increase of 0.6 ms^{-1}in the speed of sound in air for each rise of 1°C in temperature.thus the equation for velocity of sound in air is

$v_t = v_o + 0.61t\,°C$ ------(12.10)

Where, v_o is the speed of sound at (0°C) = 273K = T_o and

v_t is the speed of sound at (t°C+273)K = T

5. Wind:

If v_w is the wind speed then the speed of sound along the direction of wind relative to ground is $(v + v_w)$ and $(v - v_w)$ against the direction of wind.

Worked Example 12.1

If the velocity of sound in air at 27°C and at a pressure of 76 cm of mercury is 345 ms^{-1}. Find the velocity at 127°C and 75 cm of mercury.

Solution:

There is no effect of change of pressure on the velocity of sound.

Step 1:

Write the known quantities and point out quantities to be found.

T_1 = 27°C = 27+273 = 300K

T_2 = 127°C = 127+273 = 400K

v_1 = 345ms^{-1}

Required: Speed of sound v_2 at 127°C

Step 2:

Write down the formula and rearrange if necessary

Using equation

$$\frac{v_t}{v_o} = \sqrt{\frac{T}{T_o}}$$

Step 3:

Put the values in the formula and calculate.

$$\frac{v_2}{v_1} = \sqrt{\frac{273+127}{273+27}} = \sqrt{\frac{4}{3}}$$

Hence;

$v_2 = \frac{2}{\sqrt{3}} \times v_1 = \frac{2}{\sqrt{3}} \times 345 =$ **398.4 ms^{-1}**

Worked Example 12.2

In a game of cricket match a spectator is sitting in the stand at a distance of 60.0 m away from the batsman. How long does it take the **sound** of the bat connecting with the ball hit for a six to travel to the spectator's **ears**? if the temperature of air is 27°C.

Solution:

Step 1: Write the known quantities and point out quantities to be found.

s = 60.0 m

T = 27°C = 27+273 = 300K

Required: t,

time in which spectator will hear the sound.

Step 2: Write down the formulae and rearrange if necessary

Using equation $v_t = v_o + 0.61t$ °C

and $s = vt$

Step 3: Put the values in the formula and calculate.

v_t = 332 + 0.61 x 27 = 348.47 ms^{-1}

Using $s = v \times t$ ∴ $t = s/v$

$t = \frac{60.0}{348.47} =$ **0.172 s**

Self-Assessment Questions:

Q.1 What is meant by the terms *isothermal* and *adiabatic* in terms of propagation of sound?

Q.2 Why is speed of sound in solids generally much faster than speed of sound in air (gas)?

Q.3 Why the change in pressure rather than actual pressure is considered in determining the speed of sound in air?

12.2 Superposition of Sound Waves:

Suppose two sound waves of same type pass through the same region of space. Do the waves affect each other? If the amplitudes of waves are large enough, then particles in the medium are displaced far enough from their equilibrium positions that Hooke's law (restoring force ∝ displacement) no longer holds; in that case the waves do affect each other. However, for small amplitudes, the waves can pass through each other and emerge unchanged. More generally, when the amplitudes are not too large, the principle of superposition applies:

12.2.1 Principle of Superposition:

When two or more waves overlap, the net disturbance at any point is the sum of the individual disturbances due to each wave.

$Y_{total} = Y_1 + Y_2$

Suppose two wave pulses are travelling toward each other on a string (Fig. 12.4). If one of the pulses (acting alone) would produce a displacement y_1 at a certain point and the other would produce a displacement y_2 at the same point, the result when the two overlap is a displacement of $y_1 + y_2$. (Fig..12.4b, c) show in greater detail how y_1 and y_2 add together to produce the net displacement when the pulses overlap.

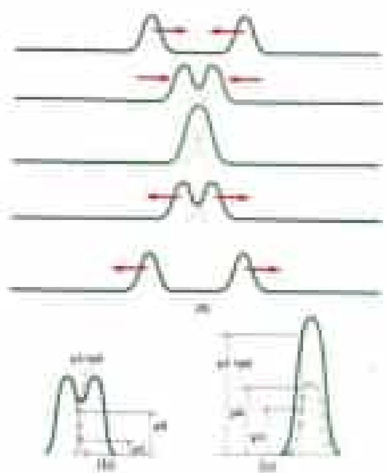


Fig: 12.4
(a) Two identical wave pulses travelling toward and through each other. (b) and (c), Details of the wave pulse summation; dashed lines are the separate wave pulse and solid line is the sum

The dashed curves represent the individual pulses; the solid line represents the superposition of the pulses. In (Fig. 12.4 b) the pulses are starting to overlap and in (Fig. 12.4 c) they are just about to coincide. The wave pulses pass right through each other without affecting each other; once they have separated; their shapes and heights are the same as before the overlap (Fig.12.4 a). The principle of super position enables us to distinguish two voices speaking in the same room at the same time; the sound waves pass through each other unaffected.

12.2.2 Interference of Sound Waves:

The principle of superposition leads to extraordinary effects when applied to coherent sound waves. One way to obtain coherent waves is to get them from same source.

Two waves are coherent if they have the same frequency and they maintain a fixed relationship with each other (Fig: 12.5 a).

Waves are incoherent if the phase relationship between them varies randomly. Whenever two waves come from two different sources they are incoherent (Fig: 12.5 b).

Suppose coherent waves with amplitudes A_1 and A_2 pass through the same point in space. If the waves are in phase at that point, that is, the phase difference is any even integral multiple of π radians, and then the two waves consistently reach their maxima at exactly the same instant of time (Fig.12.5a).

The superposition of the waves that are in phase that is crest of one falls on the crest of other or trough of one wave falls on the trough of other wave is called *constructive interference.*

The resultant amplitude is the algebraic sum of the individual waves $A = A_1 + A_2$.

Two waves that are 180° out of phase at a given point have a phase difference of π radians, 3π radians, 5π radians and so on. The waves are half a cycle apart; when one reaches its maximum, the other reaches its minimum (Fig.12.5 c).

The superposition of waves that are 180° out of phase is called *destructive interference*.

The amplitude of the combined waves is the difference of the amplitudes of the two individual waves.

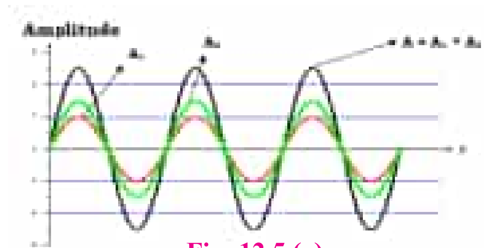


Fig: 12.5 (a)
Two coherent waves
with phase difference φ (Red and Green)
showing constructive interference.
Desstructive Interference

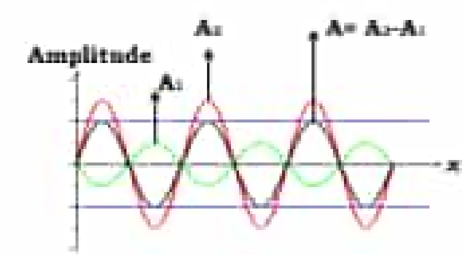


Fig: 12.5 (a) Two non coherent waves red and green are out of phase. The black wave is the resulting wave with smaller amplitude. If both waves have same amplitude then the resulting amplitude shall

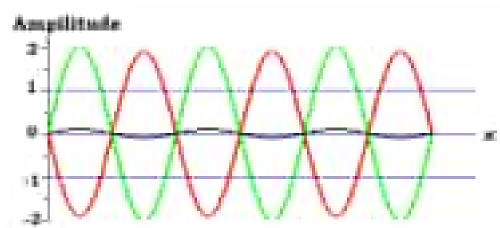


Fig: 12.5 (c) Two coherent waves with 180° phase difference (Red and Green) destructive interference. In order to show the diminishing amplitude of resultant wave (Black)

12.2.3 Formation of Beats due to interference of non-coherent sources:

The principle of superposition shall be applied to two harmonic travelling waves in the same direction in a medium. The two waves are travelling to the right with same frequency, same wavelength and same amplitude, but differ in phase.

Let y_1 and y_2 are representing the individual displacement of two waves.

$y_1 = A_o \sin(kx - \omega t)$ and $y_2 = A_o \sin(kx - \omega t - \varphi)$

Here $\mathbf{k} = \frac{2\pi}{\lambda}$ called wave number

Hence the resultant wave, function displacement is given by,

$$Y = y_1 + y_2 = A_o \sin(kx - \omega t) + A_o \sin(kx - \omega t - \varphi) \quad \ldots\ldots (12.11)$$

Using trigonometric identity

$$\sin a + \sin b = 2\cos\left(\frac{a-b}{2}\right)\sin\left(\frac{a+b}{2}\right)$$

Let $a = kx - \omega t$ and $b = kx - \omega t - \varphi$, simplifying equation 12.11, we get

$$Y = 2A_o\cos\left(\frac{\varphi}{2}\right)\sin\left(kx - \omega t - \frac{\varphi}{2}\right) \quad \ldots\ldots (12.12)$$

The resultant wave function Y is also harmonic having same frequency and wavelength as the individual waves with $2A_o\cos\left(\frac{\varphi}{2}\right)$ is the resultant amplitude with phase $\frac{\varphi}{2}$. If the phase constant $\varphi = 0$ then $\cos\left(\frac{\varphi}{2}\right) = 1$ and the resultant amplitude is $2A_o$. This represents the constructive interference.

In general $\varphi = 0, 2\pi, 4\pi, 6\pi \ldots\ldots 2n\pi$. Where (n = 1, 2, 3, ……).

On the other hand if $\varphi = \pi, 3\pi, 5\pi \ldots\ldots n\pi$. Where (n = 1, 3, 5, ………), then the resultant amplitude shall be zero everywhere. This represents destructive interference.

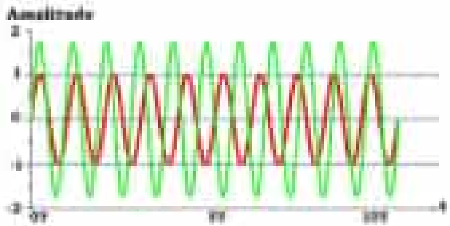


Fig: 12.6
Graph of two sound waves with frequencies f_1(Red) and f_2 (Green)

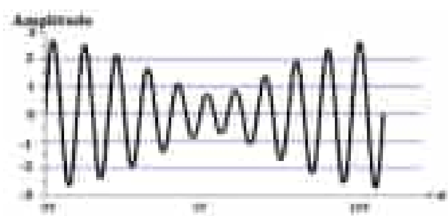


Fig: 12.7
The superposition of two waves has minimum amplitude at t = 5T and maximum amplitude at t = 10T

Interference of Sound Waves in Time (BEATS):

When two sound waves are close in frequency (within about 15 Hz of each other), the superposition of the two produces a pulsation that we call beats.

Beats can be produced by any kind of wave; they are a general result of the principle of superposition when applied to two waves of *slightly different frequency*.

Beats are caused by the slow change in the phase difference between the two waves.

Suppose at one instant (t=0T in Fig.12.6), the two waves are in phase with each other and interfere constructively. According to the principle of superposition principle, the resultant amplitude is maximum. The sum of the amplitudes of the two waves, shown in Fig.12.7

However since the frequencies are different, the waves do not stay in phase. The higher frequency wave has a shorter cycle, so it gets ahead of the other

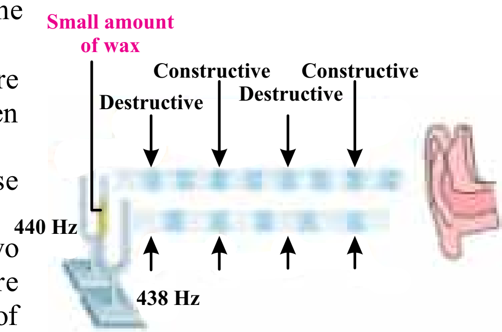


Fig: 12.8
The ear identify the amplitude (intensity) cycling from large to small then small to large and then large to small as pulsation in loudness. Thus, we refer this phenomenon of beats as interference of sound waves in time.

one. The phase difference between the two steadily increases and the resultant amplitude decreases. At a later time (t = 5T) the phase difference reaches 180°; now the waves are half a cycle out of phase and interfere destructively (Fig. 12.8). Now the resultant amplitude is minimum. As the phase difference continues to increase, the amplitude increases until constructive interference occurs.

Beat Frequency:

At what frequency do the beats occur? It depends on how far apart the frequencies of the two waves are. We can measure the time between the beats $T_{beat} = 1/f_{beat}$, as the time to go from one constructive interference to the next constructive interference. During that time, each wave must go through a whole number of cycles, with one of them going through one more cycle than the other.. Since the frequency f is the number of cycles per second. The number of cycles a wave goes through during a time T_{beat} is $f\,T_{beat}$. From (Fig.12.6) T_{beat} is 10T. During that time wave I (red) goes through $f_1\,T_{beat}$ = 10 cycles, while wave 2 (green) goes through $f_2\,T_{beat}$ = 1.1/T x 10T = 11 cycles. If f_2 is greater than f_1 then wave two goes through one more cycle than wave 1. Hence;

$$f_2\,T_{beat} - f_1\,T_{beat} = f\,T_{beat} = 1$$

$$\boldsymbol{f_{beat} = 1/\,T_{beat}}$$

$$\boldsymbol{f_{beat} = f_2 - f_1} \quad \ldots\ldots (12.\ 13)$$

In this way we obtain a very simple result that the beat frequency is the difference between the frequencies of the two waves. The maximum beat frequency a human ear can detect is about 7 beats per second.

Fig: 12.9 (a)
A pianist is tuning his piano with proper tuning, care, and maintenance.

12.2.4 Tuning of Musical Instruments:

All the musical instruments like piano, guitar and clarinet (shehnai) are need to tune after excessive use. The Piano tuners listen the beats as they tune. The tuner sounds two strings and listens for the beats. The beat frequency indicates whether the interval is correct or not. If the two strings are played by the same key, they are tuned to the same fundamental frequency, so the beat frequency should be nearly zero. If the two strings belong to two different notes, the beat frequency is non zero. Actually, in this case the tuner listens to beats between two overtones that are close in frequency, not the fundamentals. The fundamental frequencies are too far apart for the beats to be detected.

Fig: 12.9 (b)
Sindhi Folk musicians are demonstrating their art. With the help of BEATS phenomenon enjoy the beats of dhool and Shahnai

12.2.5 Stationary Waves:

Stationary (**Standing**) (Fig.12.10), waves occur when a string is tightly stretched between two rigid supports. If the string is plucked from the half of its length, the crest extends the whole distance between the supports. This distance is clearly half the wavelength of the transverse wave produce in the stretched string. This wave is reflected at the boundary and the reflected wave interferes with the incident wave so that the wave appears to stand still. Suppose that a harmonic wave on a string, coming from left, hits a boundary where the string is fixed.

The equation of incident harmonic sine wave is; $\mathbf{y_1 = A \sin (kx - \omega t)}$

and the equation of the reflected sine wave travelling to right is; $\mathbf{y_2 = A \sin (kx + \omega t)}$

y_1 and y_2 represents the displacements of incident and reflected waves.

Applying the principle of superposition the motion of the stretched string is described by

$$y = y_1 + y_2 = A [\sin (kx - \omega t) + \sin (kx + \omega t)]$$

Using the trigonometric identity

$$\sin \alpha + \sin \beta = 2[\tfrac{1}{2} \sin (\alpha + \beta)] [\tfrac{1}{2} \cos (\alpha - \beta)]$$

Here, $\alpha = kx - \omega t$ and $\beta = kx + \omega t$

we have;

$$\mathbf{y = 2A \sin kx \cos \omega t} \quad \ldots\ldots (12.14)$$

The resultant expression is the wave function of a standing wave obtained due to the superposition of two waves of the same amplitude and frequency moving with same speed in opposite direction along same axis.

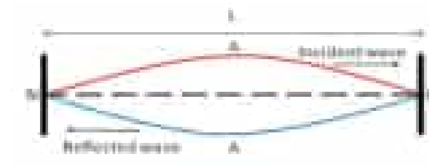


Fig: 12.10
Standing waves on a string fixed at both ends

Reflection of Waves and Phase change

Fig: 12.10
At the interface of two different media the reflection of waves occurs. A reflected wave carrying some of the energy of the incident wave travels backward from the boundary.

Fig: 12.12
Sindhi lok musicians playing Tamboora producing stationary (standing waves).

12.2.6 Nodes and Antinodes:

In standing wave, different points move with different amplitudes, but every point reaches its maximum distance (amplitude) from equilibrium at the same time and same frequency. The amplitude of a particle at any point x is $2A\sin kx$.

Figure.12.13 shows that,

The points which never move labeled as 'N' having minimum (ZERO) amplitude are called NODES.

From equation 12.14 we can determine the position of nodes. The nodes are the points where $\sin kx = 0$. Since $kx = n\pi$ $(n = 0,1, 2, 3,\ldots\ldots)$. By putting $k = \frac{2\pi}{\lambda}$, the wave number, the position of nodes are

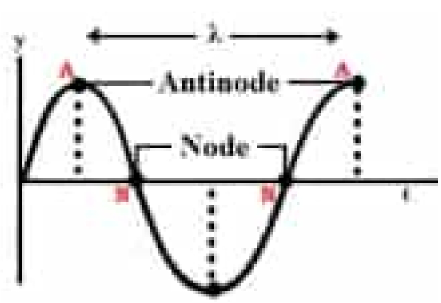


Fig: 12.13 Nodes and Antinodes on a stationary wave.

easily calculated at $x = \frac{n\lambda}{2}$. Thus the distance between two adjacent nodes is $\frac{\lambda}{2}$.

The point where displacement is maximum is called ANTI-NODE labeled as 'A'.

The points of anti-nodes occur where $\sin kx = \pm 1$, which is exactly half way between a pair of nodes. For the points of antinode $x = \frac{n\lambda}{4}$ (n = 1, 3, 5…….) So the nodes and anti-nodes alternate, with one quarter of a wavelength between a node and the neighboring antinode.

12.2.7 Modes of Vibrations in A Stretched String:

Fundamental mode of Vibration or First Harmonic:

Let a string of length L is plucked at its middle point, two transverse waves originate from this point. This superposition forms a transverse stationary wave. The whole string will vibrate in one loop, with nodes at fixed ends and anti-node at the middle as shown in the Fig.12.14. As we know that the distance between successive nodes is equal to half a wavelength λ_1, for first harmonic wave.

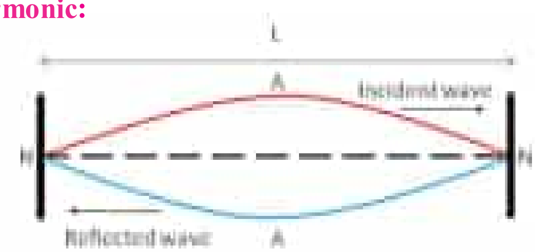


Fig: 12.14
Standing wave in its fundamental mode

$$L = \frac{\lambda_1}{2} \qquad \text{or } \lambda_1 = 2L \qquad \text{...... (12.15)}$$

If v is the speed of either of the component progressive wave, then

$$f_1 = \frac{v}{\lambda_1} = \frac{v}{2L} \qquad \text{...... (12.16)}$$

if M is the total mass of the string then the speed v of the progressive wave along the string is given by

$$\boldsymbol{v} = \sqrt{\frac{T\,X\,L}{M}} \qquad \text{Where T is tension in the string} \qquad \text{...... (12.17)}$$

Replacing $\boldsymbol{v}$ from Eq. 12.17 in Eq. 12.16. So the frequency f_1 is

$$f_1 = \frac{1}{2L}\sqrt{\frac{T\ X\ L}{M}} \qquad \text{...... (12.18)}$$

If μ is the mass per unit length (linear density), $\mu = \frac{M}{L}$ then the above equation becomes

$$f_1 = \frac{1}{2L}\sqrt{\frac{T}{\mu}} \qquad \text{...... (12.19)}$$

This characteristic f_1 of vibration is called the **fundamental frequency** or **first harmonic.**

Second mode of Vibration (Second Harmonic) or First Overtone:

If the string is now plucked from one quarter of its length then the string will vibrate in two loops. The stationary wave set up in the string will have frequency f_2(Fig.12.15).

If λ_2 is the wavelength of second mode of vibrations

$$\boldsymbol{L} = \frac{\lambda_2}{2} + \frac{\lambda_2}{2} \qquad \text{...... (12.20)}$$

$$\lambda_2 = \frac{2L}{2} \quad \text{or } \lambda_2 = L$$

Fig: 12.15 Second harmonic or (First overtone) of vibration

If v is the speed of either of the component of standing wave, then

$$f_2 = \frac{v}{\lambda_2} = \frac{v}{L}$$

$$f_2 = \frac{2}{2L}\sqrt{\frac{T}{\mu}} \quad \ldots\ldots (12.21)$$

Since; $f_1 = \frac{1}{2L}\sqrt{\frac{T}{\mu}}$

Therefore; $\boldsymbol{f_2 = 2f_1}$ …… (12.22)

Thus the frequency of second harmonic is twice the frequency of first harmonic.

In general when the string vibrates in **n** number of loops with (n+1)th nodes and nth anti-nodes.

Third harmonic or (second overtone) of vibration:

$L = \frac{\lambda_3}{2} + \frac{\lambda_3}{2} + \frac{\lambda_3}{2}$ $\lambda_3 = \frac{2L}{3}$

Hence the characteristic frequency for third harmonic shall be (Fig: 12.16)

$$f_3 = \frac{v}{\lambda_3} = \frac{3v}{2L}$$

$$f_3 = \frac{3}{2L}\sqrt{\frac{T}{\mu}}$$

Or $\boldsymbol{f_3 = 3f_1}$ …… (12.23)

Third mode of Vibration or Second Overtone

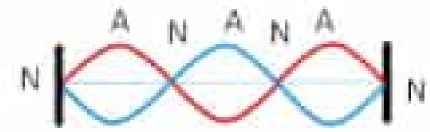


Fig: 12.16

The wavelength for such a mode of vibration is

$$\lambda_n = \frac{2L}{n} \quad \ldots\ldots (12.24)$$

And the characteristic $\boldsymbol{f_n}$ is

$$\boldsymbol{f_n = nf_1} \quad \ldots\ldots (12.25)$$

Following conclusions can be made from the above discussion.

i) A fastened string from both ends shall always vibrate in complete loops as the nodes on the end points.

ii) The length of a loop shall be the distance between two adjacent nodes and shall be equal to half integral multiple of the wavelength.

$$L = \frac{\lambda}{2}, \frac{2\lambda}{2}, \frac{3\lambda}{2}, \ldots\ldots\ldots \frac{n\lambda}{2} \quad (n = 1,2,3\ldots\ldots)$$

iii) The frequency of vibration is directly proportional to the number of loops, i.e. and the corresponding wavelength is decreasing. However for any mode of vibration the product of frequency and wavelength shall remain constant.

iv) Quantization of Frequencies.

The modes of vibration of stationary waves produced in a fixed string are always in a discrete set of frequencies and shall always be the integral multiple of the frequency of fundamental mode or first harmonic of vibration. This phenomenon is called quantization.

Self-Assessment Questions:
1. what is tuning of musical instruments? What is the importance of Beats in this process?
2. Give an expression for the velocity of a transverse wave along a thin flexible string and show that it is dimensionally correct.

12.2.8 Stationary Sound Waves (Organ Pipes):

Stationary sound waves are also caused by reflections at the boundaries. Since sound is a three dimensional wave i.e. it propagates in all direction. The air column inside a pipe open at both or one end only gives rise to standing (stationary) waves comparable to those on a string, as long as the diameter of the pipe is small compared to its length. Organ pipes and flutes are the best models to demonstrate this phenomenon.

Pipe Open at Both Ends:

If the pipe is open at both ends, then the pipe has same boundary condition at each end. At each open end, the column of air inside the pipe communicates with the outside air, so the pressure at the ends can't deviate much from atmospheric pressure. The open ends are therefore pressure nodes (Fig.12.17)

There are also displacements anti-nodes, elements of air vibrate back and forth with maximum amplitude at the ends. Since nodes and anti-nodes alternate with equal spacing (λ/4).

Hence; from equation (12.24).

$$\lambda_n = \frac{2L}{n}$$

n = 1,2,3,…………

$$f_n = \frac{v}{\lambda_n} = \frac{nv}{2L}$$

$$f_n = nf_1 \quad \text{…… (12.26)}$$

Fig: 12.17 Standing waves in an open pipe from both ends, for First, Second and third harmonics.
The wavelength of standing waves in a open pie at both ends are the same as for a string fixed at both ends regardless of your choice of pressure or displacement description on both ends.

Pipe Closed at One End:

Some organ pipes are closed at one end and open at the other (Fig.12.18). The closed end is a *pressure anti-node*; the air at closed end meets a rigid boundary, so there is no limit that how far the pressure can deviate from atmospheric pressure. The closed end is also a *displacement node* since the air near it cannot move beyond that rigid surface. Some wind instruments like shehnai and clarinet are effectively pipes closed at one end.

Using displacement the fundamental mode of vibration has a **node** at close end, and **anti-node** at the open end. Since, it is mentioned earlier that the

Fig: 12.18 Standing waves in a closed pipe at one end, for First, Second and third harmonics.

Remember that

Two thin organ pipes of the same length one open at both ends and one closed at one end do not have the same fundamental wavelength (frequency). Since, the wavelength of closed pipe is twice as large compared to open pipe at both ends, therefore a frequency half as large.

distance between a node and anti-node shall always be λ/4 for fundamental mode.
Hence;

First Harmonic:

$$L = \frac{\lambda}{4} \quad \text{or} \quad \lambda = 4L$$

which is twice as large as the wavelength ($2L$) of fundamental mode of vibration in a pipe of same length, open at both ends.

Since; $f_1 = \frac{v}{\lambda_1} = \frac{v}{4L}$

Second Harmonic:

If the air column vibrates for second harmonic or first overtone, then from figure.12.21

$$L = \frac{\lambda_2}{2} + \frac{\lambda_2}{4} = \frac{3\lambda_2}{4}$$

Since; $\lambda_2 = \frac{4L}{3}$ $\qquad f_2 = \frac{3v}{4L}$

Or, $\boldsymbol{f_2 = 3 f_1}$

Similarly for third harmonic or second overtone

$$L = \frac{5\lambda_3}{4}$$

And $\lambda_3 = \frac{4L}{5}$

and $\boldsymbol{f_3 = 5 f_1}$

Note that the standing wave frequencies for closed pipe at one end are odd integral multiple of the fundamental frequency.

$$f_n = \frac{v}{\lambda_n} = \frac{nv}{4L} = nf_1 \qquad n = (1, 3, 5\ldots\ldots) \qquad \ldots\ldots (12.27)$$

Self-Assessment Questions:

1. A pipe has a length of 2.46 m (a) Determine the frequency of first harmonic and the first two overtones, if the pipe is open at both ends, take v = 344 ms^{-1}. Determine the frequencies of fundamental mode and first two overtones for closed pipe.
2. Is the vibration of a string in a piano, guitar, or violin a sound wave? Explain.

12.3 Doppler Effect of Sound:

The Doppler effect is a phenomenon in physics that describes the perceived change in frequency of a wave (such as sound or light) when the source of the wave or the observer is in relative motion. It is named after the Austrian physicist Christian Doppler, who first described it in 1842.

In the case of sound waves, the Doppler effect occurs when there is relative motion between the source of the sound and the observer. The key characteristic of the Doppler effect is that the frequency of the sound waves appears higher (or lower) to the observer depending on the direction of motion.

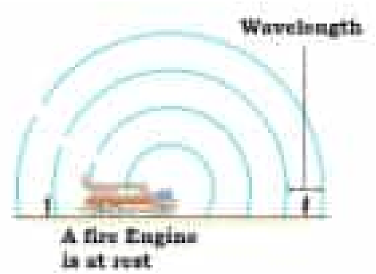


Fig: 12.19
When the fire engine is at rest the wavelength of the sound is the same in front of and behind the truck and then frequency heard by the listener is. $f = v/\lambda$

The Doppler effect can be observed in everyday situations, such as when a vehicle with a siren passes by. As the vehicle approaches, the sound of the siren is heard at a higher pitch (higher frequency) than its actual source frequency. When the vehicle moves away, the sound of the siren is heard at a lower pitch (lower frequency) than its actual source frequency.

12.3.1 Change in Observed Frequency due to the Relative Motion of Source or Listener or Both:

A source emits a sound wave at frequency f, with velocity v and wavelength λ which means that wave crests (regions of maximum amplitude, indicated by circles in (Fig. 12.20) leave the source spaced by a time interval

$$T = 1/f.$$

Moving Source:

Source is moving towards the observer at rest:

If the source is moving at velocity v_s ***toward a stationary observer*** on the right, figure.12.20 shows that the wavelength, the distance between the crests is smaller to the right. Figure.12.20 (b) shows that at the instant that crest 4 is emitted, crest 3 has travelled outward a distance vT_s from point 3. During the same time interval, the source has advanced a distance v_sT_s. The wavelength λ` as measured by the observer on the right is the distance between crests 4and 3:

$\lambda' = vT_s - v_sT_s$

The frequency at which the crests arrive at the observer is the observed wave frequency f'. The observed time period T' between the arrival of two crests is the time it takes sound to travel a distance $(v\text{-}v_s)T_s$:

$$T' = \frac{(v - v_s)T_s}{v}$$

The observed frequency is $f' = \frac{1}{T_o} = \frac{v}{(v-v_s)} \text{ x } \frac{1}{T_s}$

Dividing numerator and denominator by v and substituting $f_s = \frac{1}{T_s}$ yields

$$f' = \left(\frac{v}{v-v_s}\right) \text{ x } f_s \qquad \text{...... (12.28)}$$

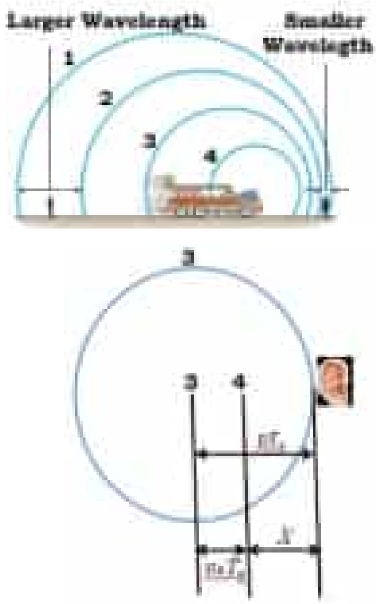


Fig: 12.20 (a) A Fire engine is moving to the right at speed v_s while it blows its siren. The siren emits wave crests at position 1, 2, 3 and 4. Each wave crest moves outward in all directions, from the point at which it was emitted, at speed v. (b) The observed wavelength λ` is the distance between the wave crests.

This shows that the observed frequency is higher than the source frequency, when the source moves in the same direction as the wave towards the observer.

Source is moving away from the observer at rest:

If the source is moving at velocity v_s ***away from the stationary observer*** on the left, then according to figure.12.20, the wavelength, the distance between the crests is larger to the left. The wavelength λ` as measured by the observer on the left in (Fig.12.20) is the distance between crests 3 and 4:

$\lambda' = vT_s + v_sT_s$

Hence; the observed frequency by the stationary listener shall be less than the source frequency, given as

$$f' = \left(\frac{v}{v+v_s}\right) \text{x}\, f_s \qquad \ldots\ldots (12.29)$$

Moving Observer:

Observer is moving towards the source at rest:

A stationary source emits a sound wave at frequency f_s and wavelength $\lambda_s = v/f_s,$ where v is the speed of sound. An observer moving towards the stationary source with speed v_o would observe a shorter time interval between crests. Just as crest 1 reaches the observer, the next crest 2 is at distance λ_s ahead. Crest 2 catches up with the observer with time T' earlier than the time T_s, when the distance vT' the wave crest 2 travels toward the observer is equal to wavelength of sound minus the distance the observer travels towards the wave crest 2 as shown in figure 12.21.

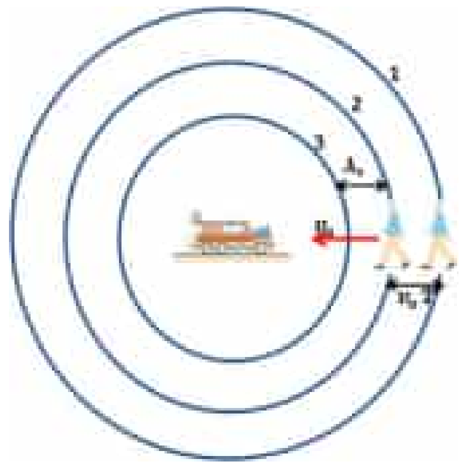


Fig: 12.21
An observer is moving towards the stationary source with speed v_o. The observed frequency shall be greater than the source frequency.

$$vT' = \lambda_s - v_oT'$$

$$\lambda_s = vT' + v_oT'$$

$$\lambda_s = (v + v_o)\, T' \qquad \ldots\ldots (12.30)$$

Given that $f' = \frac{1}{T'}$ and $\lambda_s = v/f_s$ replacing T$'$ and λ_s in equation 12.31

$$f' = \left(\frac{v + v_o}{v}\right) f_s \qquad \ldots\ldots (12.31)$$

Observer is moving away from the source at rest:

Now consider an observer moving away from the source at velocity v_o. He observes a longer time interval between crests. Just as crest 1 reaches the observer, the next crest 2 is a distance λ_s away. Crest 2 catches up with the observer at a time T' later when the distance the wave crest travels toward the observer is equal to the distance the observer travels away from the wave crest plus the wavelength (Fig.12.22)

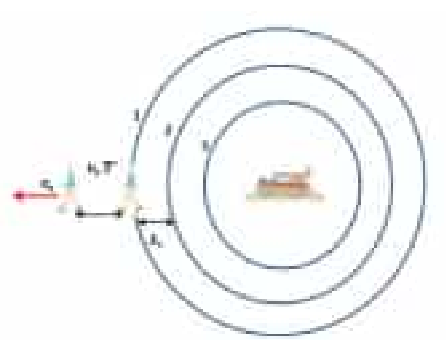


Fig: 12.22
An observer is moving away from the stationary source with speed v_o. The observed frequency shall be lower than the source frequency.

$$vT' = v_oT' + \lambda_s$$

or

$$(v - v_o)\, T' = \lambda_s$$

$$(v - v_o)\, T' = v/f_s$$

$$f' = \left(\frac{v - v_o}{v}\right) f_s \qquad \ldots\ldots (12.32)$$

An observer moving away from the source detect a frequency lower than the f_s.

Motion of Both Source and Observer:

Since the relative change in the frequency (pitch) of sound with respect to listener is dependent upon the motion of source of sound or listener.

If the **source and listener both moves towards each other** then we combine the two Doppler's shifts. First consider the relative change in frequency with respect to stationary listener as the source is approaching towards the listener. Let f_L be the frequency detected by the listener.

$$f_L = \left(\frac{v}{v-v_s}\right) f_s \qquad \text{...... (12.33)}$$

If at some moment the listener starts moving towards the approaching source with velocity $\boldsymbol{v}_o$, then the detected frequency f_o shall be

$$f_o = \left(\frac{v+v_o}{v}\right) f_L \qquad \text{...... (12.34)}$$

Substituting the value of f_L yields

$$\boldsymbol{f_o} = \left(\frac{v+v_o}{v-v_s}\right) \boldsymbol{f_s} \qquad \text{...... (12.35)}$$

In case the **source and listener are moving away from each other** then the observed frequency shall be

$$\boldsymbol{f_o} = \left(\frac{v-v_o}{v+v_s}\right) \boldsymbol{f_s} \qquad \text{...... (12.36)}$$

Worked Example 12.3

A source of sound and listener are moving towards each other with velocities which are 0.5 times and 0.2 times the speed of sound respectively. If the frequency of emitting sound is 2000 Hz, calculate the percentage change in the frequency with respect to the listener.

Solution:

Step 1: Write the known quantities and point out the quantities to be found.

Speed of source; $v_s = 0.5v$

Speed of listener; $v_o = 0.2v$

$f = 400\text{Hz}$

Required: $f_o = ?$

Step 2: Write the formula and rearrange if necessary. $f_o = \left(\frac{v+v_o}{v-v_s}\right) f_s$

Step 3: Put the values in the formula and calculate.

$f_o = \left(\frac{v+0.2v}{v-0.5v}\right) x\ 2000$ taking v common

$f_o = \left(\frac{1.2}{0.5}\right) x\ 2000 =$ **4800 Hz**

Percentage in Frequency = $\left(\frac{f_o - f_s}{f_s}\right) x\ 100\% = \left(\frac{4800-2000}{2000}\right) x\ 100\% =$ **140%**

Shock Waves:

For a plane moving slower than sound, the wave crests in front of it are closer together due to the plane's motion. An observer to the right would measure the frequency increases. (Fig.12.24a)

For a plane moving at the speed of sound the wave crests pile up on top of each other; they move to the right at the same speed as the plane , so they can't get ahead of it. The wall of high pressure air is called sound barrier. An observer to the right would measure the wavelength of zero-zero distance between waves crests therefore and infinite frequency. (Fig.12.24b)

If the source moves at the speed greater than the speed of sound, figure 12.24 c shows that the wave crests pile on top of one another to form cone-shaped shock waves, which travel outward in the direction indicated. There are two principal shock waves formed, one starting at the nose of the lane and one at the tail. The sound of the shock is referred to the Sonic boom.

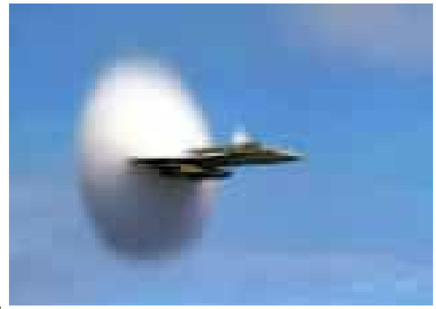


Fig: 12.23
A fighter plane breaking the sound barrier

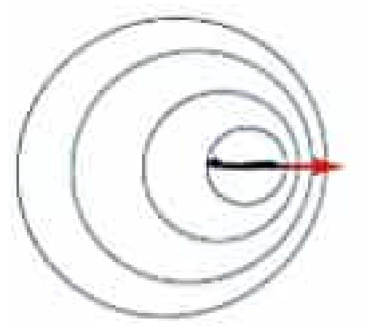


Fig: 12.24 (a)

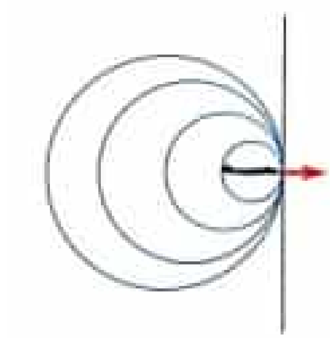


Fig: 12.24 (b)

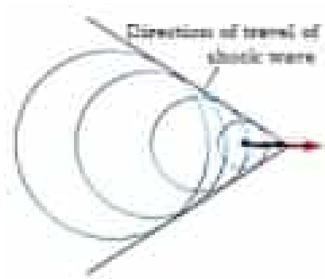


Fig: 12.24 (c)

Fig: 12.25
The variation in pressure at a point on the ground as a supersonic plane flies overhead.

The pressure variation shown in Fig 12.25 is the sonic boom caused by the air plane flying at the constant velocity greater that the speed of the sound. The variation is called the N –shaped (because the pressure graph is shaped like the letter N).

12.3.2 Applications of Doppler:

Echolocation is a valuable navigation tool for various life forms on our planet. While oil birds and cave swiftlets use audible sound waves for echolocation (detectable by humans), dolphins, whales, and most bats utilize ultrasound (20 Hz to 200 KHz) instead.

Bats and dolphins gain an advantage by sensing the Doppler shift between emitted and reflected waves, allowing them to locate and track fast-moving prey effectively.

The Doppler Effect for light is vital in ASTRONOMY. Analyzing light emitted by elements in distant stars reveals wavelength shifts compared to the same element's light on Earth. These shifts, known as red shifts, indicate motion of the stars. Redshift observations have provided evidence for expanding universe cosmological theories, suggesting the universe evolved from a great explosion billions of years ago in a relatively small region of space.

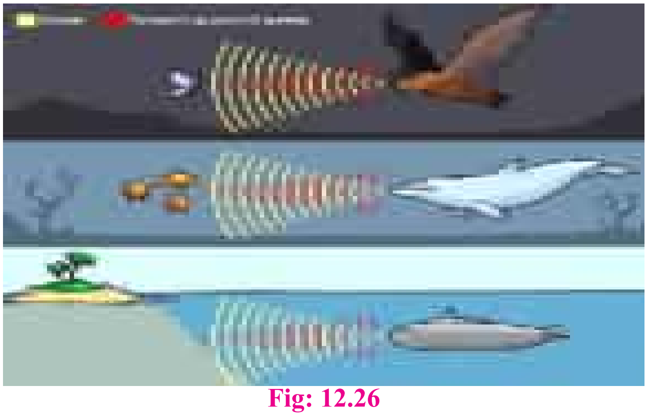


Fig: 12.26
The use of Doppler effect by Bat, Dolphin and a submarine for echolocation.

SONAR devices emit ultrasonic pulses (Fig.12.27) to determine distances. The time delay between pulse emission and reflection helps measure seafloor distance. For low-range submarine detection, the Navy uses low-frequency active (LFA) sonar with audible sound (100 to 500 Hz) instead of ultrasonic. Air guns use to study the earth's interior structure and locate oil underground.

VOR (Very High Frequency Omnidirectional Range) is a guiding system at airports that directs incoming aircraft to the airport's location. Modern airports like Quaid-e-Azam International Airport Karachi and Islamabad Airport use Doppler VOR systems (Fig: 12.28). The electromagnetic signal used operates in the VHF range (30 MHz-300MHz).

RADARS (Radio Detection and Ranging) serve civil and military purposes at airports and air bases, detecting aircraft presence by evaluating range. Doppler radar,

DO YOU KNOW?

The speed of the supersonic plane is often given as **Mach Number,** name for Austrian Physicist **Ernst Mach** (1838-1916).

The Mach number is the ratio of the speed of the plane to the speed of sound.

A plane flying at Mach 3.3 where the air temperature is 11° C is moving at 3.3 x 338 m/s= 1100 m/s with respect to the air. The Pressure variation for the Concorde is 93Pa above and below the atmospheric pressure at a speed of Mach 2 and at a height 15.8 km; the noise level heard from the Concorde's boom is usually 120 dB directly under flight path.

Fig: 12.27 A ship with a sonar device to locate the depth of the seafloor.

Fig: 12.28 A VOR system installed at airport maintain a highly reliable, safe, and efficient ground-based navigation system.

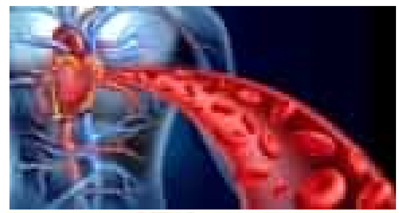
based on the Doppler Effect, is used to determine aircraft speed and direction. It is also crucial in weather forecasting, showing storm location and wind velocity.

RADAR SPEED TRAPS or Speed (Radar) guns use by traffic police to measure the speed of an automobile or use in Cricket matches the measure the speed of a ball deliver by the bowler. A reflected electromagnetic signal is received from the automobile by the radar gun

Fig: 12.29
A radar system and a radar trap

12.3.3 Doppler's Effect and Cardiovascular Problems:

Ultrasound is also used to examine organs such as heart, liver, gallbladder, kidneys, breasts and eyes, and to locate tumors. It can be used to diagnose various heart conditions and to assess damage after a heart attack. Ultrasound can show movements, so it is used to assess heart valve function and to monitor blood flow in large blood vessels. Since, ultrasound provides real-time images. It is sometimes used to guide procedures such as biopsies, in which a needle is used to take a sample from an organ or tumor for testing.



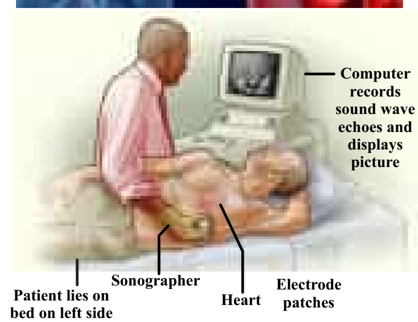


Fig: 12.30 Ultrasound examination of Heart. The computer image is called **echocardiogram**.

Doppler ultrasound is a newer technique that is used to examine blood flow. It can help reveal blockage to blood flow, show the formation of plaque in arteries, and provide detail information on the heartbeat of the patient. In ultrasonic imaging, ultrasound waves (> 20,000 Hz) rather than sound waves of of audible frequencies (20Hz to 20,000Hz) are used. Waves with small wavelengths diffract less around the same obstacle than do waves with larger wavelengths. The frequencies used in imaging are typically in the range of 1 to 15 MHz, which means that the wavelengths in human tissue are in the range of 0.1 to 1.5 mm. As a comparison if sound waves at 15 KHz were used, the wavelength inside the body would be 10 cm. Higher frequencies give better resolution but at expense of less penetration; sound waves are absorbed within a distance of 500λ in tissue.

Self-Assessment Question:

1. The source and observer of a sound wave are both at rest with respect to ground. The wind blows in the direction from source to observer. Is the observed frequency Doppler shifted? Explain.

DO YOU KNOW?

Why are sound waves used rather than electromagnetic waves like X-rays?

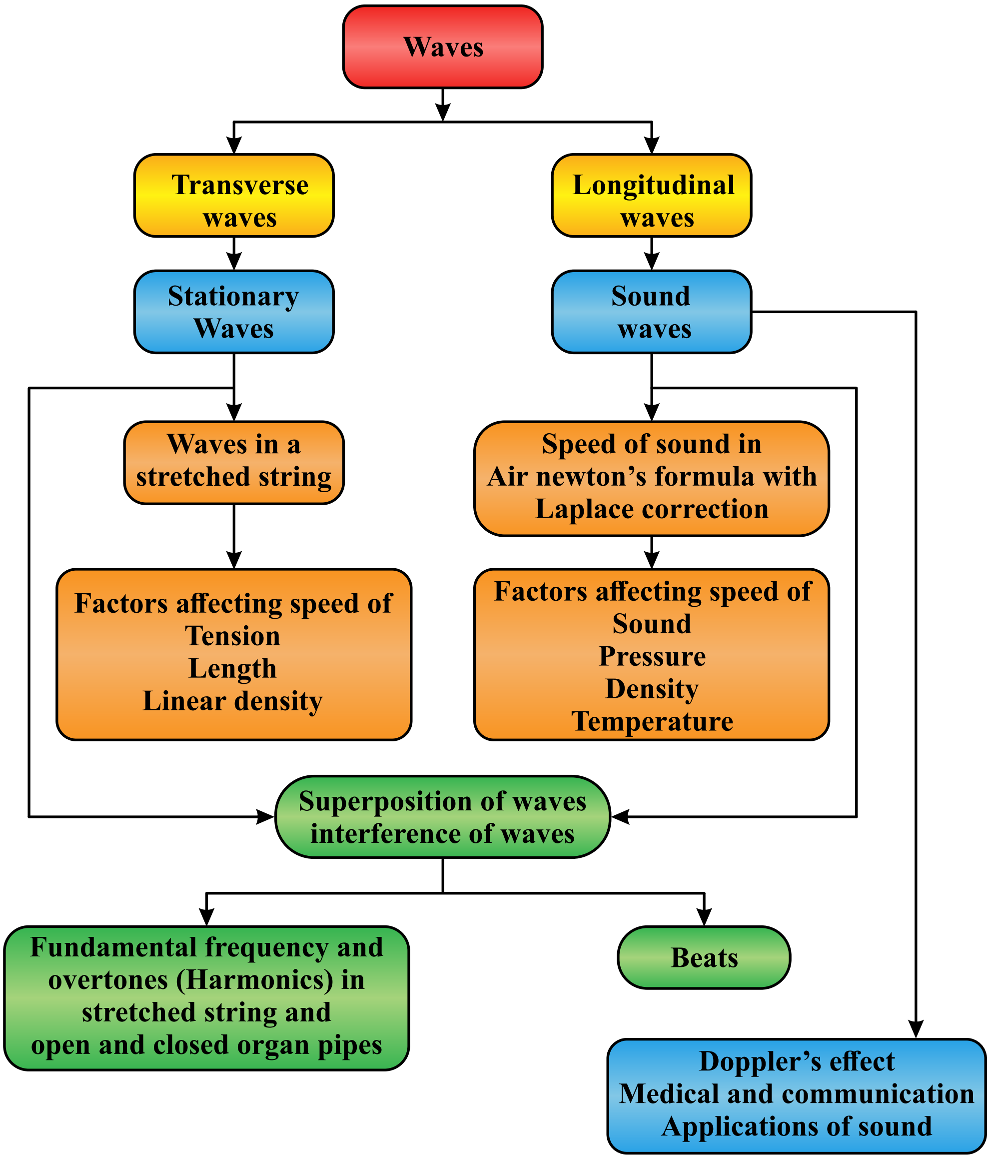
X-rays are damaging to tissue, especially to rapidly growing fetal tissue. After decades of use, ultrasound has no known adverse effects.

SUMMARY

- A sound wave can be described either by gauge pressure p, which means the pressure fluctuation above, and below the ambient atmospheric pressure, or by displacement s on each point in the medium from its un-displaced position.
- Humans with excellent hearing can hear frequencies from 20 Hz to 20 KHz. The term infrasound and ultrasound are used to describe sound waves with frequencies below 20Hz and above 20 KHz respectively.
- The speed of sound in a fluid is $v = \sqrt{\frac{B}{\rho}}$.
- The speed of sound in an ideal gas at any absolute temperature T can be found if is known at one temperature: $v = v_o\sqrt{\frac{T}{T_o}}$, where the speed of sound at absolute temperature T_o is v_o.
- The speed of sound in air at 0°C or 273 K is 332 m/s.
- For sound waves travelling along the length of the thin solid rod, the speed is approximately $v = \sqrt{\frac{Y}{\rho}}$ (thin solid rod).
- In a standing sound wave in a thin pipe, an open end is the pressure node and the displacement anti-node; a closed end is the pressure anti-node and a displacement node.
- The principle of superposition; when two or more waves overlap, the net disturbance at any point is the sum of the individual disturbances due to each wave.
- A harmonic travelling wave can be described by $y(x,t) = A\sin(kx - \omega t)$.
- For a pipe open at both ends, $\lambda_n = \frac{2L}{n}$, $f_n = \frac{nv}{2L} = nf_1$, where n = 1,2,3,……
- For a pipe closed at one end, $\lambda_n = \frac{4L}{n}$, $f_n = \frac{nv}{4L} = nf_1$ where n = 1,3,……
- When two sound waves are closed in frequency the superposition of the two produces a pulsation called beats. $f_{beats} = \Delta f$.
- Doppler Effect, if v_s and v_o are the velocities of the source and the observer, the observed frequency is $f_o = \left(\frac{1 \pm \frac{v_o}{v}}{1 \mp \frac{v_s}{v}}\right) f_s$.

  where v_s and v_o are positive in the direction of the propagation of the wave and the wave medium is at rest.

CONCEPT MAP


Waves
Transverse waves
Longitudinal waves
Stationary Waves
Sound waves
Waves in a stretched string
Speed of sound in Air newton's formula with Laplace correction
Factors affecting speed of Tension Length Linear density
Factors affecting speed of Sound Pressure Density Temperature
Superposition of waves interference of waves
Fundamental frequency and overtones (Harmonics) in stretched string and open and closed organ pipes
Beats
Doppler's effect Medical and communication Applications of sound

EXERCISE

Section (A): Multiple Choice Questions (MCQs)

1. The speed v of a wave represented by $y = A\sin(\omega t - kx)$ is:
   a) k/ω b) ω/k c) ωk d) 1/ωk
2. Two sound waves are $y = A\sin(\omega t - kx)$ and $y = A\cos(\omega t - kx)$. The phase difference between the two waves is:
   a) π/2 b) π/4 c) π d) 0
3. If v_a, v_h and v_m are the speeds of sound in air, hydrogen gas, and a metal at the same temperature, then:
   a) $v_a > v_h > v_m$ b) $v_m > v_h > v_a$ c) $v_h > v_m > v_a$ d) $v_h > v_a > v_m$
4. The speed of sound in air at STP is 332 m/s. If the air pressure becomes double at the same temperatue, the speed of sound become:
   a) 1382 m/s b) 664m/s c) 332 m/s d) 166 m/s
5. How does the speed of sound v in air depend on the atmospheric pressure P:
   a) $v \propto P^0$ b) $v \propto P^{-1}$ c) $v \propto P^2$ d) $v \propto P^1$
6. The speed of sound in a gas is proportional to:
   a) square root of isothermal elasticity b) square root of adiabatic elasticity
   c) isothermal elasticity d) adiabatic elasticity
7. The length of a pipe closed at one end is L. In the standing wave whose frequency is 7 times the fundamental frequency, what is the closest distance between nodes?
   a) $\frac{1}{14}$ L b) $\frac{1}{7}$ L c) $\frac{2}{7}$ L d) $\frac{4}{7}$ L
8. A 620 Hz frequency song of a ice cream trolley approaches with speed v to a boy standing at the door of his house is heard with frequency f_1. If the trolley is stopped and the boy approaches to the ice cream trolley with same speed v; the boy now hears the sound with frequency f_2. choose the correct statement:
   a) $f_1 = f_2$; both are greater than 620Hz b) $f_2 > f_1 >$ 620Hz
   c) $f_1 = f_2$; both are lesser than 620Hz d) $f_1 > f_2 >$ 620Hz.
9. The speed of sound in a gas in which two waves of wavelength 50 cm and 50.4 cm produce 6 beats per second is:
   a) 338 m/s b) 350 m/s c) 378 m/s d) 400 m/s
10. The speed of a wave in a medium is 760 m/s. If 3600 waves are passing through a point in the medium in 2 minutes, then its wave length is:
   a) 13.85 m b) 25.3 m c) 41.5 m d) 57.2 m

CRQs

1. Why sound can not travel through vacuum.
2. On what factors speed of sound depends on
3. What are the conditions for the interference of waves
4. Is the energy of a wave is maximum or minimum at nodes?
5. Why is it possible to understand the words spoken by two people at the same time?

ERQs:

1. Define a sound wave and explain its nature as a longitudinal wave. Discuss the key properties of sound waves, such as frequency, wavelength, amplitude, and speed of propagation
2. Discuss the concept of the Doppler Effect in sound waves. Explain how the motion of a source or observer affects the perceived frequency and pitch of a sound wave. Provide examples to illustrate the Doppler Effect in daily life.
3. What is standing wave? How they are produced? Also elaborate the concept of nodes and antinodes in standing waves. Explain their locations and the relationship between node spacing and wavelength.
4. Define the Doppler Effect and Derive the Doppler Effect equation for sound waves in terms of the relative velocity between the source, observer, and the speed of sound.
5. Define standing waves and explain how they are formed.
6. Discuss the concept of harmonics in standing waves. Explain how harmonics are formed and their relationship with the fundamental frequency.

Numericals:

1. The equation of a wave is $y(x, t) = 3.5 \sin\{\frac{\pi}{3.0}x - 66t\}$ cm where t is in seconds and x and y both are in cm. Find (a) the amplitude and (b) the wavelength of this wave.
   (a) 3.5 cm (b) 6.0 cm
2. Why is it that your own voice sounds strange to you when you hear it played back on a tape recorder, but your friends all agree that it is just what you r voice sounds like?
3. Why the speed of sound in solids is much faster than the speed of sound in air?
4. An increase in pressure of 100 k Pa causes a certain volume of water to decrease by 5×10^{-3} percent of its original volume. Find (a) Bulk modulus of water. (b) What is the speed of sound in water? **(a) 2000 MPa (b) 1400 ms^{-1}**
5. A uniform string of length 10.0 m and weight 0.25 N is attached to the ceiling. A weight of 1.00 kN hangs from its lower end. The lower end of the string is suddenly displaced horizontally. How long does it take the resulting wave pulse to travel to upper end? Neglect the weight of string in comparison to hanging mass). **(16 msapprox)**
6. A travelling sine wave is the result of the superposition of two other sine waves with equal amplitudes, wavelengths, and frequencies. The two component waves each have amplitude 5.00 cm. if the resultant wave has amplitude 6.69, what is the phase difference φ between the component waves? **(96^o)**

7. In order to decrease the fundamental frequency of a guitar string by 4.0%, by what percentage should you reduce the tension? **(7.8%)**
8. A string 2.0 m long is held fixed at both ends. If a sharp blow is applied to the string at its centre, it takes 0.050 s for the pulse to travel to both ends of the string and return to the middle. What is the fundamental frequency of oscillation for this string? **(10Hz)**
9. A sound source of frequency f_o and an observer are located at a fixed distance apart. Both the source and observer are at rest. However, the propagation medium (through which the sound waves travel at speed v) is moving at a uniform velocity v_m in an arbitrary direction. Find the frequency detected by the observer giving physical explanation.
10. A train sounds its whistle while passing by a railroad crossing. An observer at the crossing measures a frequency of 219 Hz as the train approaches the crossing and a frequency of 184 Hz as the train leaves. The speed of the sound is 340ms^{-1}. Find the speed of the train and frequency of its whistle. **v_T= 29.5 ms^{-1} f_o = 200 Hz**