Unit 2

Kinematics

Teaching Periods 8 **Weightage %** 5

Pride of Pakistan, Muhammad Asif is a Pakistani Professional snooker player. He is two times winner of the amateur IBSF World Snooker Championship.

In this unit student should be able to:

- Describe a vector and its representation.
- Describe the Cartesian coordinate system.
- Resolve a vector into two perpendicular components.
- Describe vector nature of displacement.
- Analyze and interpret patterns of motion of objects using displacement-time graph, velocity-time graph acceleration-time graph.
- Determine the instantaneous velocity of an object moving along the same straight line by measuring the slope of displacement-time graph.
- Derive equation of uniformly accelerated motion.
- Solve the problems.
- Understand projectile motion.
- Calculate height, range and time of flight using equations of projectile motion.

Even a person without a background in physics has a collection of words that can be used to describe moving objects. Words and phrases such as going fast, stopped, slowing down, speeding up, and turning provide a sufficient vocabulary for describing the motion of objects. In physics, we use these words and many more. We will be expanding upon this vocabulary list with words such as **distance, displacement, speed, velocity, and acceleration**. As we will soon see, these words are associated with physical quantities that have different definitions. The physical quantities that are used to describe the motion of objects can be divided into two categories. The quantity is either a **vector** or a **scalar**.

Scalar Quantities:

All those physical quantities which can be specified by a magnitude and a proper unit are known as "Scalar Quantities".

Scalar quantities do not need direction for their description. Scalar quantities are added, subtracted, multiplied or divided by the simple rules of algebra.

Examples:

Work, electric flux, volume, viscosity, density, power, temperature and electric charge etc.

Vectors Quantities:

All those physical quantities having both magnitude and direction with proper unit and also obeys the Vector Algebra are known as "Vector Quantities".

We can not specify a vector quantity without mention its direction.

Examples:

Displacement, Velocity, acceleration, force, momentum, etc.

Representation of Vectors:

Vector quantities can be represented in two ways

- Analytical Or Symbolic representation
- Graphical representation

Analytical Or Symbolic representation:

In analytical method vectors are denoted by a letter with arrow or bold letters such as:

$\vec{A}, \vec{B}$, or $\boldsymbol{a}, \boldsymbol{b}, \boldsymbol{A}, \boldsymbol{C}$

Graphical representation:

In graphical method vectors are denoted by a line segment with arrow, the starting point of line is called tail and the ending point of line having arrow is known as head of vector. The length of line showing the magnitude of given vector as shown in figure2.1

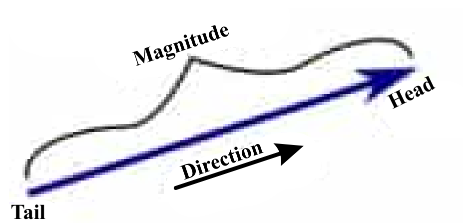


Fig: 2.1 Representation of vector

Addition of vectors by Head to Tail method (Graphical Method):

Head to Tail method or graphical method is one of the easiest methods used to find the resultant vector of two of more than two vectors.

Consider two vectors $\vec{A}$ and $\vec{B}$ acting in the directions as shown in figure 2.2:

Head to tail rule is a method of vector addition in which tail of second vector is connected by head of first vector. All vectors are connected in this way.

Finally, from tail of first vector to the head of last vector we will draw a vector called resultant vector.

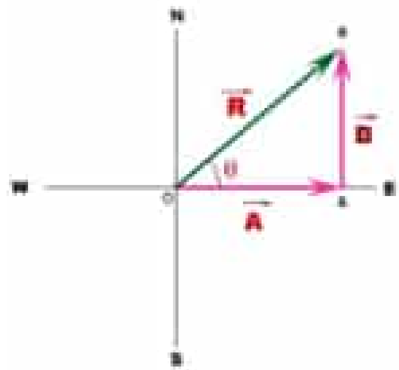


Fig: 2.2
Addition of vectors A and B

Mathematically for resultant vector 'R'

$$\overline{OB} = \overline{OA} + \overline{AB}$$

$$\vec{R} = \vec{A} + \vec{B} \;......(2.1)$$

Addition of Vectors:

Properties of vector addition:

Commutative law of vector addition:

Consider two vectors $\vec{A}$ and $\vec{B}$. Let these two vectors represent two adjacent sides of a parallelogram. We construct a parallelogram OACB as shown in the diagram. The diagonal OC represents the resultant vector $\vec{R}$.

$$\vec{R} = \vec{A} + \vec{B}$$

$$\vec{R} = \vec{B} + \vec{A}$$

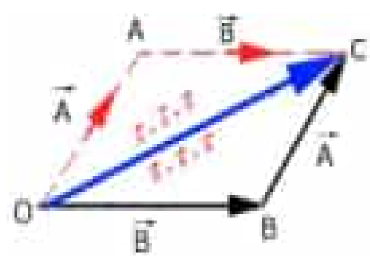


Fig: 2.3

Therefore

$$\vec{A} + \vec{B} = \vec{B} + \vec{A} \;......(2.2)$$

This fact is referred to as the commutative law of vector addition. it shows that the order in which vectors are added has no physical significance as shown in figure 2.3.

Associative law of vector addition:

The law states that the sum of vectors remains same irrespective of their order or grouping in which they are arranged.

Consider three vectors $\vec{A}$, $\vec{B}$ and $\vec{C}$

Applying "head to tail rule" to obtain the resultant of $(\vec{A}+\vec{B})$ and $(\vec{B}+\vec{C})$ Then finally again find the resultant of these three vectors:

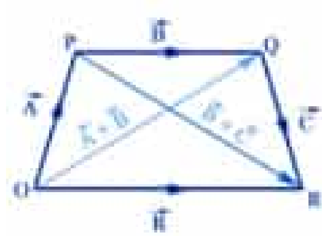


Fig: 2.4

$\overline{OR} = \overline{OP} + \overline{PR}$ Or

$\vec{R} = \vec{A} + (\vec{B} + \vec{C})$ ⟶ *(i)*

and

$\overline{OR} = \overline{OQ} + \overline{QR}$

$\vec{R} = (\vec{A} + \vec{B}) + \vec{C}$ ⟶ *(ii)*

Thus from eq. (i) and (ii)

$\vec{A} + (\vec{B} + \vec{C}) = (\vec{A} + \vec{B}) + \vec{C}$ ...... **(2.3)**

This fact is known as the Associative Law of Vector Addition as shown in figure 2.4.

Magnitude of resultant vector:

Magnitude or resultant vector can be determined by using either Cosine law or Sine law.

Resultant by cosine law $R = \sqrt{A^2 + B^2 - 2AB\cos < OAB}$ **...... (2.4)**

Resultant by sine law $\frac{A}{Sin\alpha} = \frac{B}{Sin\beta} = \frac{R}{Sin\theta}$ **...... (2.5)**

Multiplication and division of vector by number:

A vector can be multiplied and divided by a number using simple algebraic rules. In case of multiplication/division by positive number (non zero) only changes the magnitude of given vector. If number is negative then the change comes in the direction of vector also as shown in figure 2.5 (a) and (b).

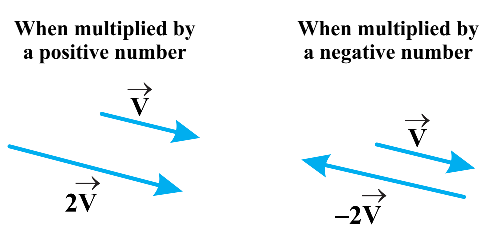


Figure 2.5 (a) and (b): multiplication of vector by positive and negative number

DO YOU KNOW?

The product of a vector $\vec{V}$ by a scalar quantity (m) follows the following rules:

- **(m) $\vec{V}$ = $\vec{V}$ (m)** ... commutative law of multiplication.
- **m(n$\vec{V}$) = (mn) $\vec{V}$** ... associative law of multiplication .
- **(m + n) $\vec{V}$ = m $\vec{V}$ + n $\vec{V}$** ... distributive law of multiplication.

Self-Assessment Questions:

1. What happens to the direction of a vector when it undergoes scalar multiplication?
2. If vector A has components (3, -2) and vector B has components (-1, 5), what are the components of A + B?

Cartesian coordinate system:

Cartesian coordinate system is a set of three mutually perpendicular lines (X-axis, Y-axis and Z-axis) with common initial point called origin used to find out location of any point as shown in figure 2.6.

Such as you are presenting data on a line graph, or simply
finding the location of a car park on a map of a National Park, you will need to have an understanding of coordinates.

There are many coordinate systems among them Cartesian or Rectangular coordinate system is one of most used and easy to understand.

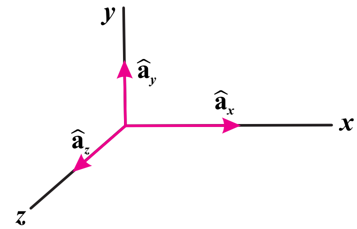


Fig: 2.6 Cartesian coordinates

In a rectangular coordinate system, vectors can be classified into different types based on their characteristics and properties. Here are some commonly defined types of vectors in relation to a rectangular coordinate system:

Unit vector:

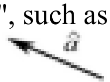
"A unit vector is defined as a vector in any specified direction whose magnitude is unity i.e. 1. A unit vector only specifies the direction of a given vector. "

A unit vector can be determined by dividing the vector by its magnitude.

For example, unit vector of a vector A is given by:

$$\hat{a} = \frac{\vec{A}}{A} \text{ ...... (2.6)}$$

The symbol is usually a lowercase letter with a "hat / cap / circumflex", such as:



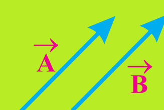
(Pronounced "a-hat")

In three dimensional coordinate system unit vectors $(\hat{\imath}, \hat{\jmath}, \hat{k})$ having the direction of the positive X-axis, Y-axis and Z-axis are used as unit vectors. These unit vectors are mutually perpendicular to each other as shown in fig: 2.7.

DO YOU KNOW?

Equal Vectors

Two vectors are considered equal if they have the same magnitude and direction



$\vec{B} = \vec{A}$

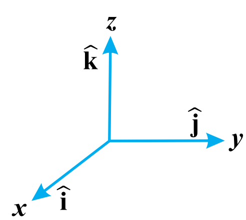


Fig: 2.7 Unit vectors i, j, k

Free vector:

A free vector can be moved or translated without changing its essential characteristics, such as magnitude and direction.

It is represented by an arrow and is not attached to any specific point in space. Consider a free vector $\vec{A}$ changing its position in XYZ plane without changing its direction and magnitude is an example of free vector as shown in fig: 2.8

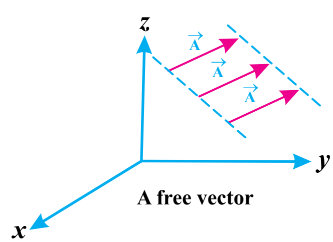


Fig: 2.8 free vector

Position vector:

A Vector that indicates the position of a point in a coordinate system is referred to as **position vector.**

Suppose we have a fixed reference point O, then we can specify the position the position of a given point P with respect to point O by means of a vector having magnitude and direction represented by a directed line segment OP, this vector is called position vector and represented by $\vec{r}$ as shown in figure 2.9.

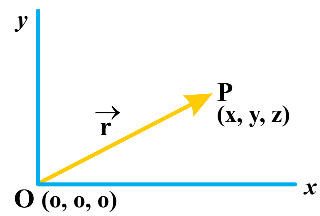
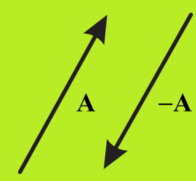


Fig: 2.9 position vector

These are some of the types of vectors commonly encountered in a rectangular coordinate system. Understanding these types and their properties can be helpful in various mathematical and physical applications involving vectors.

DO YOU KNOW?

two vector equal in magnitude but opposite in direction are known as **negative vector** of each other.



a null vector is a resultant vector of two equal vectors acting in opposite directions.

$$\vec{O} = \vec{A} + (\overrightarrow{-A}) = |0|$$

Resolution of vector:

The process of splitting a vector into rectangular components is called "RESOLUTION OF VECTOR"

We can resolve a vector into three components. Such as **x-component, y-component, z-component along the three axis of coordinates system respectively.**

These components are called rectangular components of vector.

Method of resolving a vector into rectangular components:

Consider a vector $\vec{V}$ acting at a point making an angle θ with positive X axis. Vector $\vec{V}$ is represented by a line OA as shown in fig: 2.10. From point A draw a perpendicular AB on X-axis. Suppose OB and BA represents two vectors. Vector OA is parallel to X-axis and vector BA is parallel to Y-axis. Magnitude of these vectors are V_x and V_y respectively. By the method of head to tail we notice that the sum of these vectors is equal to vector $\vec{V}$.

Thus V_x and V_y are the rectangular components of vector $\vec{V}$

V_x = Horizontal component of $\vec{V}$ along x-axis

V_y = Vertical component of $\vec{V}$ **along y-axis**

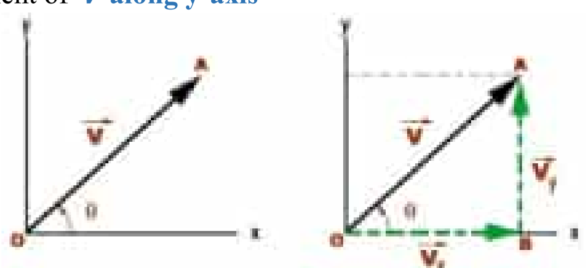


Fig: 2.10

Magnitude of horizontal component

Consider right angled triangle

$$Cos\ \theta = \frac{\overline{OB}}{\overline{OA}}$$

$$\overline{OB} = \overline{OA}\ Cos\ \theta$$

$$\boxed{V_x = V\ Cos\ \theta}$$

Magnitude of vertical component

Consider right angled triangle

$$Sin\ \theta = \frac{\overline{BA}}{\overline{OA}}$$

$$\overline{BA} = \overline{OA}\ Sin\ \theta$$

$$\boxed{V_y = V\ Sin\ \theta}$$

Direction of the Vector

$$Tan\ \theta = \frac{\overline{BA}}{\overline{OB}}$$

$$Tan\ \theta = \frac{V_y}{V_x}$$

Addition of vectors by rectangular components method:

Consider two vectors $\vec{V_1}$ and $\vec{V_2}$ making angles θ_1 and θ_2 with +ve x-axis respectively.

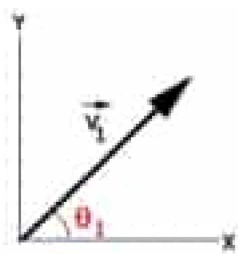


Fig: 2.11 (a)

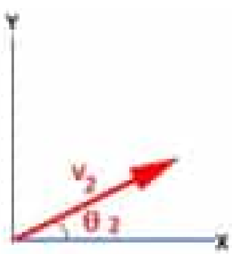


Fig: 2.11 (b)

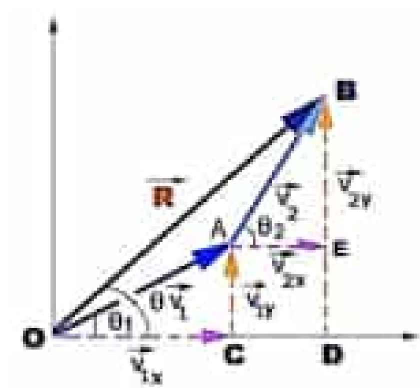


Fig: 2.11 (c)

Resolve vector $\vec{V_1}$ into two rectangular components $\vec{V_{1x}}$ and $\vec{V_{1y}}$ as shown in fig: 2.11 (c).
Magnitude of these components are:

$$V_{1x} = V_1\ Cos\ \theta_1$$

and

$$V_{1y} = V_1\ Sin\ \theta_1$$

Resolve vector $\overrightarrow{V_2}$ into two rectangular components V_{2x} and V_{2y} as shown in figure 2.11(c)
Magnitude of these components are:

$$V_{2x} = V_2 \, Cos\, \theta_2$$

and

$$V_{2y} = V_2 \, Sin\, \theta_2$$

Now move vector $\overrightarrow{V_2}$ parallel to itself so that its initial point (tail) lies on the terminal point (head) of vector $\overrightarrow{V_1}$ as shown in the Fig: 2.11 (c).
Representative lines of $\overrightarrow{V_1}$ and $\overrightarrow{V_2}$ are OA and OB respectively. Join O and B which is equal to resultant vector of $\overrightarrow{V_1}$ and $\overrightarrow{V_2}$

Resultant vector along X-axis can be determined as:

$$\overline{OD} = \overline{OC} + \overline{CD}$$

$$\overline{OD} = \overline{OC} + \overline{AE} \quad \therefore \overline{CD} = \overline{AE}$$

$$\overrightarrow{R_x} = \overrightarrow{V}_{1x} + \overrightarrow{V_{2x}}$$

$$\overrightarrow{R_x} = V_1 \, Cos\, \theta_1 + V_2 \, Cos\, \theta_2 \; \ldots\ldots(2.7)$$

Resultant vector along Y-axis can be determined as:

$$\overline{DB} = \overline{CA} + \overline{EB}$$

$$\overline{OD} = \overline{DE} + \overline{EB} \quad \therefore \overline{CA} = \overline{DE}$$

$$\overrightarrow{R_y} = \overrightarrow{V}_{1y} + \overrightarrow{V_{2y}}$$

$$\overrightarrow{R_y} = V_1 \, Sin\, \theta_1 + V_2 \, Sin\, \theta_2 \ldots\ldots(2.8)$$

Now we will determine the magnitude of resultant vector by using the Pythagoras' theorem.

$$Hypotenous^{\,2} = Base^2 + Perpendicular^2$$

$$|\overrightarrow{OB}|^2 = |\overrightarrow{OD}|^2 + |\overrightarrow{DB}|^2$$

$$R^2 = R_x^2 + R_y^2$$

$$R = \sqrt{R_x^2 + R_y^2}$$

$$R = \sqrt{(V_1 \, Cos\, \theta_1 + V_2 \, Cos\, \theta_2)^2 + (V_1 \, Sin\, \theta_1 + V_2 \, Sin\, \theta_2)^2} \ldots\ldots(2.9)$$

Finally the direction of resultant vector will be determined.
Again in the right angled triangle $< DOB$:

$$tan < DOB = \frac{\overline{DB}}{\overline{OD}}$$

$$tan < DOB = tan\theta = \frac{R_y}{R_x}$$

$$\theta = tan^{-1}\left(\frac{R_y}{R_x}\right) \ldots\ldots(2.10)$$

Where θ is the angle that the resultant vector makes with the positive X-axis. In this way we can add a number of vectors in a very easy manner. This method is known as *addition of vectors by rectangular components* ***method.***

Self-Assessment Questions:
1. Define a unit vector and its significance in vector representation.
2. What is a position vector, and how is it used in Cartesian coordinates?

Product of two vectors:

There are two types of vector product which can be classified as
- *Scalar product (Dot product)*
- *Vector product (Cross product).*

Scalar or dot product:

The scalar product of two vectors $\vec{A}$ *and* $\vec{B}$ is written as $\vec{A}.\vec{B}$ and is defined as,

"When two parallel vectors are multiplied, their resultant quantity will be a scalar, this is called scalar or dot product."

$$\vec{A}.\vec{B} = AB\cos\theta \quad \text{......(2.11)}$$

Where A and B are the magnitudes of vectors A and B and θ is the angle between them.

For physical interpretation of dot product of two vectors A and B, these are first brought to a common origin.

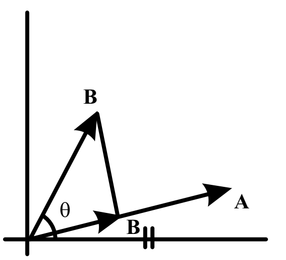


Fig: 2.12

Then, $\vec{A}.\vec{B} = (A)\,(projection\ of\ B\ on\ A)$

As shown in fig: 2.12.

$$\vec{A}.\vec{B} = A\,(magnitude\ of\ component\ of\ B\ in\ the\ direction\ of\ A)$$

$$\vec{A}.\vec{B} = A\,(B\cos\theta) = AB\cos\theta$$

Similarly $\vec{B}.\vec{A} = B\,(A\cos\theta) = BA\cos\theta$

We come across this type of product when we consider the work done by a force F whose point of application moves a distance d in a direction making an angle θ with the line of action of F, as shown in fig: 2.13.

$$Work\ done = Force\ x\ Displacement$$

$$Work\ done = Fd\cos\theta$$

$$\boldsymbol{F.d = Fd\cos\theta = work\ done}$$

Where θ is the angle between $\vec{F}$ and $\vec{d}$

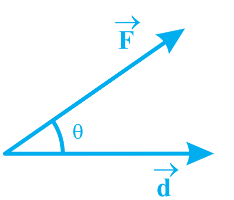


Figure 2.13

Worked Example 2.1

Find $\vec{a}.\vec{b}$ *when* $\vec{a} = \hat{\imath} - 2\hat{\jmath} + \hat{k}$ *and* $\vec{b} = 3\hat{\imath} - 4\hat{\jmath} - 2\hat{k}$

Step 1:

$\vec{a} = \hat{\imath} - 2\hat{\jmath} + \hat{k}$

$\vec{b} = 3\hat{\imath} - 4\hat{\jmath} - 2\hat{k}$

$\vec{a}.\vec{b} = (\hat{\imath} - 2\hat{\jmath} + \hat{k}).(3\hat{\imath} - 4\hat{\jmath} - 2\hat{k}) = 3 + 8 - 2$

Step 2: $\vec{a}.\vec{b} = 9$

$\therefore\ \hat{\imath}.\hat{\jmath} = \hat{k}.\hat{\jmath} = \hat{\imath}.\hat{k} = 0$

$\therefore\ \hat{\imath}.\hat{\imath} = \hat{\jmath}.\hat{\jmath} = \hat{k}.\hat{k} = 1$

Characteristics of scalar product:

$$\vec{A}.\vec{B} = AB\cos\theta$$
$$\vec{B}.\vec{A} = BA\cos\theta$$

It can be used to find the angle between two vectors.

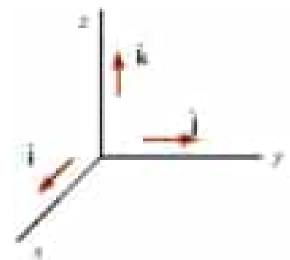
$$\boldsymbol{A.B} = \boldsymbol{B.A}$$

- The order of multiplication is irrelevant. In other words scalar product is commutative.
- The scalar product of two mutually perpendicular vectors is zero, hence these vectors are also called as Orthogonal vectors.

$$\boldsymbol{A.B} = AB\cos 90^0 = 0 \qquad since\ \cos 90^0 = 0 \text{ as shown in fig:2.14}$$

- In case of unit vectors î , ĵ and k̂, since they are mutually perpendicular, therefore,

$$\hat{\imath}.\hat{\jmath} = \hat{\jmath}.\hat{k} = \hat{k}.\hat{\imath} = 0$$



- The scalar product of two parallel vectors is equal to the product of their magnitudes. Thus, for parallel vectors $(\theta = 0^0)$

$$\boldsymbol{A.B} = AB\cos 0^0 = AB since\ \cos 0^0 = 1$$

- In case of unit vectors

$$\hat{\imath}.\hat{\imath} = \hat{\jmath}.\hat{\jmath} = \hat{k}.\hat{k} = 1$$

- And for antiparallel vectors $(\theta = 180^0)$

$$\boldsymbol{A.B} = AB\cos 180^0 = -AB\ since\ \cos 0^0 = -1$$

- The self-product of a vector A is equal to square of its magnitude.

$$\boldsymbol{A.A} = AA\cos 0^0 = A^2$$

- Scalar product of two vectors A and B in terms of their rectangular components

$$\boldsymbol{A.B} = (A_x\hat{\imath} + A_y\hat{\jmath} + A_z\hat{k}).(B_x\hat{\imath} + B_y\hat{\jmath} + B_z\hat{k})$$
$$\boldsymbol{A.B} = \boldsymbol{A_xB_x + A_yB_y + A_zB_z}$$

Vector or cross product:

The vector product of two vectors A and B, is a vector which is defined as

"When two perpendicular vectors are multiplied, their resultant quantity will be a vector, this is called vector or cross product."

$$\boldsymbol{A \times B = AB\sin\theta\,\hat{n}} \text{ ......(2.12)}$$

Where n̂ is a unit vector perpendicular to the plane containing A and B as shown in fig: 2.15. its direction can be determined by right hand rule. For that purpose, place together, the tails of vectors A and B to define the plane of vectors A and B. the direction of the product vector is perpendicular to this plane. Rotate the first vector A into B through the smaller of the two possible angles and curl the fingers of the right hand in the

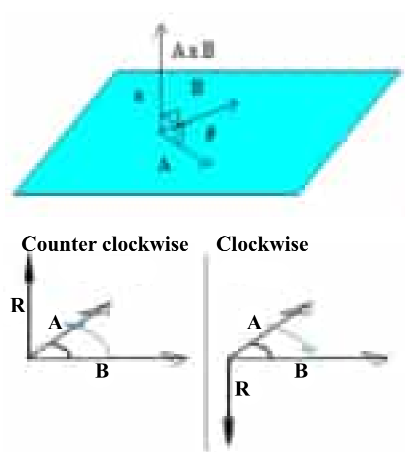


Fig: 2.15

direction of rotation, keeping the thumb upright. The direction of the product vector will be along the upright thumb, as shown in the fig: 2.15. because of this direction rule, B x A is a vector opposite in sign to A x B as shown in fig: 2.16. Hence,

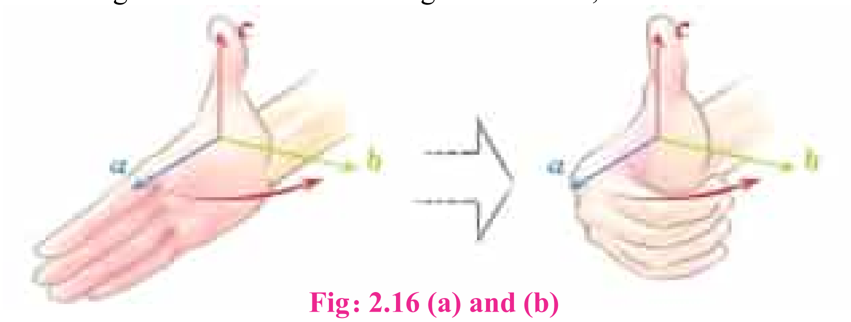


Fig: 2.16 (a) and (b)

Characteristics of cross product:

- Since **A** x **B** is not the same as B x A, the cross product is non-commutative.

$$\boldsymbol{A} \times \boldsymbol{B} = -\boldsymbol{B} \times \boldsymbol{A}$$

- The vector product is associative i.e. if m is a scalar, then

$$(m\,\boldsymbol{A}) \times \boldsymbol{B} = \boldsymbol{A} \times (m\,\boldsymbol{B})$$
$$= m\,(\boldsymbol{A} \times \boldsymbol{B})$$

- Vector product is distributive over the addition i.e.

$$\boldsymbol{A} \times (\boldsymbol{B} + \boldsymbol{C}) = \boldsymbol{A} \times \boldsymbol{B} + \boldsymbol{B} \times \boldsymbol{C}$$
$$(\boldsymbol{A} \times \boldsymbol{B}) + \boldsymbol{C} = \boldsymbol{A} \times \boldsymbol{C} + \boldsymbol{B} \times \boldsymbol{C}$$

- The cross product of two perpendicular vectors has maximum magnitude

$$\boldsymbol{A} \times \boldsymbol{B} = \boldsymbol{AB\,sin90^0\,\hat{n}} \quad since\ sin90^0 = 1$$
$$= \boldsymbol{AB\,\hat{n}}$$

- The cross product of two parallel vectors is null vector, because for such vectors $\theta = 0^0\ or\ 180^0.\ Hence$

$$\boldsymbol{A} \times \boldsymbol{B} = \boldsymbol{AB\,sin0^0\,\hat{n}} \qquad sin0^0 = 0, sin180^0 = 0$$
$$\boldsymbol{A} \times \boldsymbol{B} = \boldsymbol{AB\,sin\,180^0\,\hat{n}}$$
$$\boldsymbol{A} \times \boldsymbol{B} = 0$$

  As a consequence

$$\boldsymbol{A} \times \boldsymbol{A} = 0$$
$$Also$$
$$\boldsymbol{\hat{\imath}\,x\,\hat{\imath}} = \boldsymbol{\hat{\jmath}\,x\,\hat{\jmath}} = \boldsymbol{k\,x\,k} = 0$$

- In case of unit vectors, since they form a right handed system and are mutually perpendicular.

$$\boldsymbol{\hat{\imath}} \times \boldsymbol{\hat{\jmath}} = \boldsymbol{\hat{k}},$$
$$\boldsymbol{\hat{\jmath}} \times \boldsymbol{\hat{k}} = \boldsymbol{\hat{\imath}},$$
$$\boldsymbol{\hat{k}} \times \boldsymbol{\hat{\imath}} = \boldsymbol{\hat{\jmath}}$$

Cross product of two vectors A and B in terms of their rectangular components is:

$$\boldsymbol{A} \times \boldsymbol{B} = (A_x\hat{\imath} + A_y\hat{\jmath} + A_z\hat{k}) \times (B_x\hat{\imath} + B_y\hat{\jmath} + B_z\hat{k})$$
$$\boldsymbol{A} \times \boldsymbol{B} = (A_yB_z - A_zB_y)\,\hat{\imath} + (A_zB_x - A_xB_z)\,\hat{\jmath} + (A_xB_y - A_yB_x)\,\hat{k}$$

Cross product or vector product can be written as,

$$\boldsymbol{A} \times \boldsymbol{B} = \begin{vmatrix} i & j & k \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

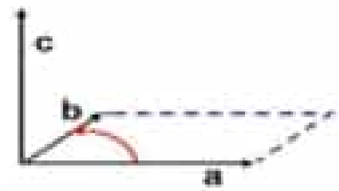


Fig:2.17 cross product of A×B

The magnitude of $A \times B$ is equal to the area of the parallelogram formed with A and B as two adjacent sides as shown in fig: 2.17.

Examples of vector product:

When a force F is applied on a rigid body at a point whose position vector is r from any point of the axis about which the body rotates, then the turning effect of the force, called the torque

$$\vec{\tau} = \vec{r} \times \vec{F}$$

Lets consider the Force of 5N is acts perpendicularly on edge of door to open it, the distance from the axis is 2m. calculate the torque produced.

$$\vec{\tau} = 2m \times 5N$$
$$\vec{\tau} = 10Nm$$

The force on a particle of charge q and velocity v in a magnetic field of strength B is given by vector product.

$$\vec{F} = q(\vec{v} \times \vec{B})$$

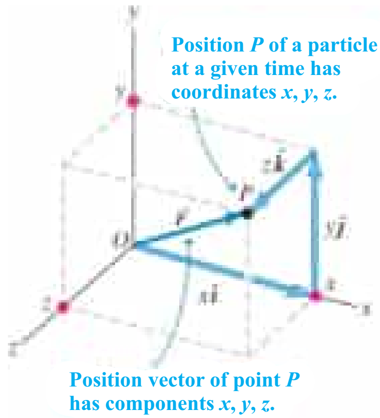
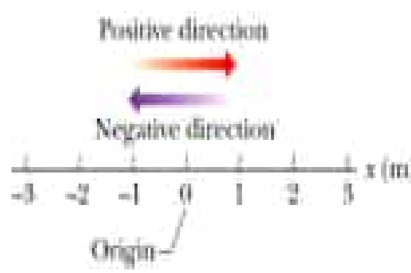


Fig: 2.18

Position Vector Or Displacement vector:

A vector $\vec{r}$ **which is directed towards the point P in rectangular coordinate system** is known as position or displacement vector. The position vector can be written in terms of its components as shown in fig:2.18.

$$\vec{r} = r_x\hat{\imath} + r_y\hat{\jmath} + r_z\hat{k}$$



Self-Assessment Questions:

1. If two vectors A and B are parallel to each other, what is the value of their cross product?
2. If vector A has components (2, -3, 5) and vector B has components (-1, 4, 2), what is their dot product A · B?

Speed and velocity:

Speed is a measurement of how fast an object moves relative to a reference point. It does not have a direction and is considered a magnitude or scalar quantity. So we can also consider the speed as the magnitude of velocity. Speed can be figured by the formula:

Speed = Distance/Time

or

s = d/t

- The direction of $\vec{V}_{av}$ is the same as the displacement $\Delta\vec{r}$.
- The standard unit for speed is m/s.
- Dimensional formula of speed is $[LT^{-1}]$.

There are different types of speed Such as:

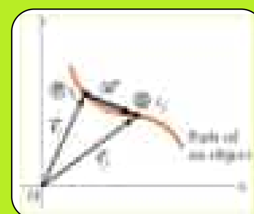
DO YOU KNOW?

Two dimensions

the position of an object is described by its position vector $\vec{r}(t)$ always points to particle from origin. Displacement can be measured as

$$\overrightarrow{\Delta r} = \vec{r_2} - \vec{r_1}$$

$$\overrightarrow{\Delta r} = \Delta x\hat{i} - \Delta y\hat{j}$$



Average speed:

The average speed of an object is greater than or equal to the magnitude of the average velocity over a given interval of time. The two are equal only if the path length is equal to the magnitude of the displacement.

Uniform Speed:

If an object covers equal distances in equal intervals of time than the speed of the moving object is called uniform speed. In this type of motion, position – time graph is always a straight line.

Instantaneous speed:

Instantaneous speed is the speed of an object at any particular moment in time. It is different from average speed because average speed is the total distance divided by total time.

In this measurement, the time $\Delta t \rightarrow 0$.

Instantaneous speed $$v = \lim_{\Delta t \to 0} \frac{\Delta s}{\Delta t} \quad ......(2.13)$$

Velocity:

When an object is in motion, its position changes with time. But how fast is the position changing with time and in what direction? To describe this, we define the quantity average velocity. Average velocity is defined as the change in position or displacement (Δx) divided by the time intervals (Δt), in which the displacement occurs**:**

The rate of change of displacement of an object in a particular direction with respect to time is called velocity.

Velocity =Displacement /Time

Velocity is a vector quantity its SI unit is meter per second (m/s). Its dimensional formula is $[L\,T^{-1}]$.

Displacement-time graphs:

In physics graph is very powerful tool to find out the visually relation between two quantities.

Displacement-time graphs show how the displacement of a moving object changes with time as shown in Fig: 2.19.

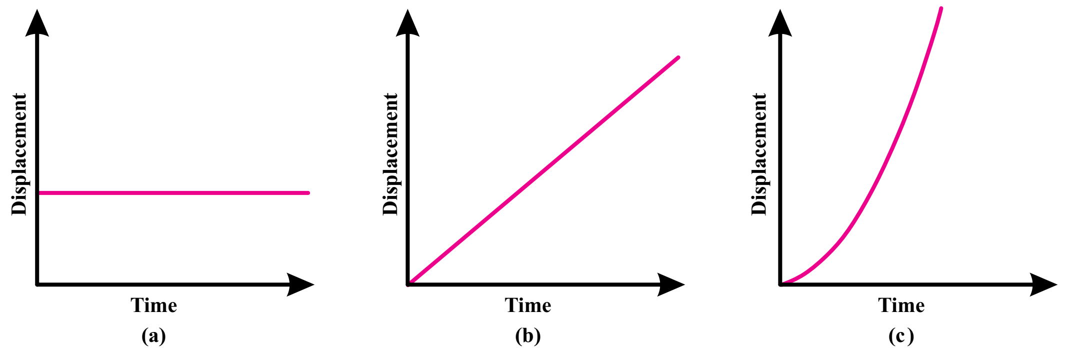


Figure 2.19 (a) Zero velocity (b) Uniform velocity (c) Variable velocity

Average velocity:

It is the total displacement covered by a body divided by total time taken as the graph of average velocity given in the fig:2.20 (a) and (b).

- If the instantaneous velocity of a body becomes equal to the average velocity, then body is said to be moving with uniform velocity.
- Mathematically average velocity of a body can be written as:

Average velocity $\vec{v}_{avg} \equiv \dfrac{\Delta \vec{r}}{\Delta t}$

$$\vec{v}_{avg} = \frac{\Delta x}{\Delta t}\hat{i} + \frac{\Delta y}{\Delta t}\hat{j} = v_{avg,x}\hat{i} + v_{avg,y}\hat{j}$$

Instantaneous velocity is the speed of an object at any particular moment in time. In this measurement, the time $\Delta t \rightarrow 0$. $\boldsymbol{v}$ is tangent to the path in x-y graph;

$$\vec{v} \equiv \lim_{t\rightarrow 0} \vec{v}_{avg} = \lim_{t\rightarrow 0} \frac{\Delta \vec{r}}{\Delta t} = \frac{d\vec{r}}{dt} \quad \ldots\ldots(2.14)$$

$$\vec{v} = \frac{d\vec{r}}{dt} = \frac{dx}{dt}\hat{i} + \frac{dy}{dt}\hat{j} = v_x\hat{i} + v_y\hat{j}$$

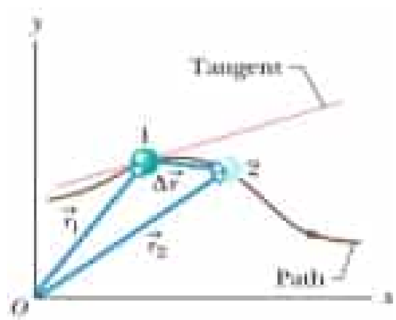


Fig: 2.20 (a)

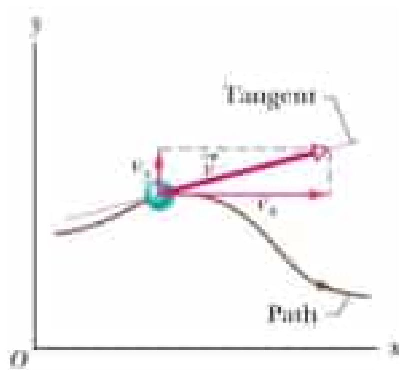


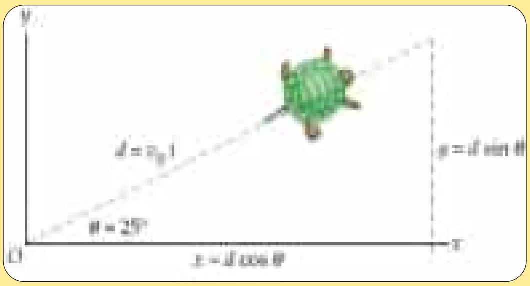
Fig: 2.20 (b)

Worked Example 2.2

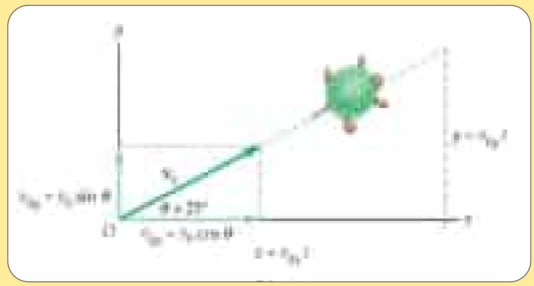
A turtle starts at the origin and moves with the speed of v_0=10 cm/s in the direction of 25° to the horizontal.

(a) Find the coordinates of a turtle 10 seconds later.

(b) How far did the turtle walk in 10 seconds?



(a)



(a)

Step 1: you can solve the equations independently for the horizontal (x) and vertical (y) components of motion and then combine them!

$$\vec{v}_0 = \vec{v}_x + \vec{v}_y$$

- X components:

$$v_{0x} = v_0 \cos 25^\circ = 9.06 \text{ cm/s} \qquad \Delta x = v_{0x}t = 90.6 \text{ cm}$$

- Y components:

$$v_{0y} = v_0 \sin 25^\circ = 4.23 \text{ cm/s} \qquad \Delta y = v_{0y}t = 42.3 \text{ cm}$$

Step 2:

- Distance from the origin:

$$d = \sqrt{\Delta x^2 + \Delta y^2} = 100.0 \text{ cm}$$

Acceleration:

Acceleration can be defined as the change in velocity with respect to time.

Acceleration = Change in velocity /time taken

- It is a vector quantity, Its SI unit is meter/ sec^2 (m/s^2).
- Its dimension is [L T $^{-2}$].
- It may be positive, negative or zero.

Positive Acceleration:

If the velocity of an object increases with time, its acceleration is positive.

Negative Acceleration:

If the velocity of an object decreases with time, its acceleration is negative. The negative acceleration is also called retardation or deceleration graph of retardation is given in the fig: 2.21.

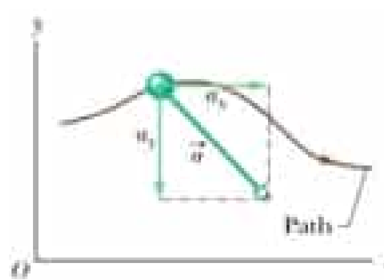


Fig: 2.21

(i) Uniform acceleration: A body is said to have uniform acceleration if magnitude and direction of the acceleration remains constant during particle motion.

Note: If a particle is moving with uniform acceleration, this does not necessarily imply that particle is moving in straight line. e.g. Projectile motion.

(ii) Non-uniform acceleration: A body is said to have non-uniform acceleration, if magnitude or direction or both, change during motion.

Average & Instantaneous Acceleration:

Average acceleration:

The direction of average acceleration vector is the direction of the change in velocity vector as

$$\vec{a}_{avg} \equiv \frac{\Delta\vec{v}}{\Delta t}$$

$$\vec{a}_{avg} = \frac{\Delta v_x}{\Delta t}\hat{i} + \frac{\Delta v_y}{\Delta t}\hat{j} = a_{avg,x}\hat{i} + a_{avg,y}\hat{j}$$

Instantaneous acceleration:

$$\vec{a} \equiv \lim_{t\to 0}\vec{a}_{avg} = \lim_{t\to 0}\frac{\Delta\vec{v}}{\Delta t} = \frac{d\vec{v}}{dt} \qquad \vec{a} = \frac{d\vec{v}}{dt} = \frac{dv_x}{dt}\hat{i} + \frac{dv_y}{dt}\hat{j} = a_x\hat{i} + a_y\hat{j} \qquad \ldots\ldots(2.15)$$

Instantaneous acceleration is defined as. The ratio of change in velocity during a given time interval such that the time interval goes to zero.

- The magnitude of the velocity (the speed) can change
- The direction of the velocity can change, even though the magnitude is constant
- Both the magnitude and the direction can change

On $V_x - t\ graph$ the slope of the tangent is the instantaneous acceleration for a particle.

Equations of Motion For Uniform Acceleration:

Every motion can be described in terms of displacement, distance, velocity, acceleration, time. The relations between these quantities are known as the equations of motion. In case of uniform acceleration, there are three equations of motion. Hence, these equations are used to derive the components like displacement(s), velocity (initial and final), time(t) and acceleration(a). Therefore they can only be applied when acceleration is constant and motion is in a straight line. The three equations are,

$$V_f = V_i + at$$

$$V_f^2 = V_i^2 + 2as$$

$$S = V_i t + ½at^2$$

Where displacement (s), initial velocity (V_i), final velocity (V_f), acceleration (a) and time (t).

- Equations of kinematics are valid for uniform acceleration.

Derivation of Equation of Motion:

The equations of motion can be derived using the following methods:

- *Derivation of equations of motion by Simple Algebraic Method*
- *Derivation of equations of Motion by Graphical Method*

In the previous classes you have learn to derive these equation by algebraic methods, In this sections, the equations of motion are derived by graphical method

Derivation of First Equation of Motion by Graphical Method:

The first equation of motion can be derived using a velocity-time graph for a moving object with an initial velocity of u, final velocity v, and acceleration a.

In the figure 2.22,

The velocity of the body changes from A to B in time t at a uniform rate.

BC is the final velocity and OC is the total time t.

A perpendicular is drawn from B to OC, a parallel line is drawn from A to D, and another perpendicular is drawn from B to OE (represented by dotted lines).

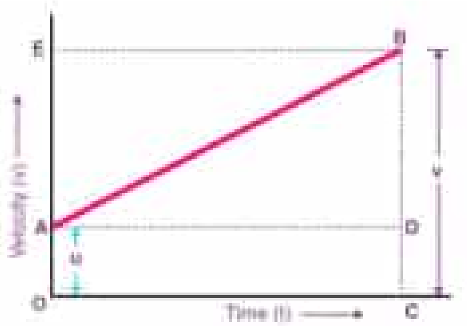


Fig: 2.22

$Following\ details\ are\ obtained\ from\ the\ graph\ above$:

$The\ initial\ velocity\ of\ the\ body,\ u\ =\ OA$

$The\ final\ velocity\ of\ the\ body,\ v\ =\ BC$

$From\ the\ graph, we\ know\ that$

$BC\ =\ BD\ +\ DC$

$Therefore,\ v\ =\ BD\ +\ DC$

$v\ =\ BD\ +\ OA\ (since\ DC\ =\ OA)$

$Finally,$

$v\ =\ BD\ +\ u\ (since\ OA\ =\ u)$ ………….(i)

$Now, since\ the\ slope\ of\ a\ velocity-time\ graph\ is\ equal\ to\ acceleration\ a,$

$So,$

$a\ =\ slope\ of\ line\ AB$

$a\ =\ BD/AD$

$Since\ AD\ =\ AC\ =\ t, the\ above\ equation\ becomes$:

$BD\ =\ at$ ……….(ii)

$Now, combining\ Equation\ (i)\ \&\ (ii), the\ following\ is\ obtained$:

$v\ =\ u\ +\ at$

Derivation of Second Equation of Motion by Graphical Method

From the fig: 2.23, we can say that

$Distance\ travelled\ (s) = Area\ of\ figure\ OABC$

$= Area\ of\ triangle\ ABD + Area\ of\ rectangle\ OADC$

$s = (1/2AB \times BD) + (OA \times OC)$

$Since\ \boldsymbol{BD} = \boldsymbol{EA}, the\ above\ equation\ becomes$

$s = (1/2AB \times EA) + (u \times t)$

$As\ \boldsymbol{EA} = \boldsymbol{at}, the\ equation\ becomes\ because\ V = at;\ EA\ represent\ the\ velocity\ (V)$

$s = 1/2 \times at \times t + ut$

$by\ rearranging, the\ equation\ becomes$

$s = ut + 1/2at^2$

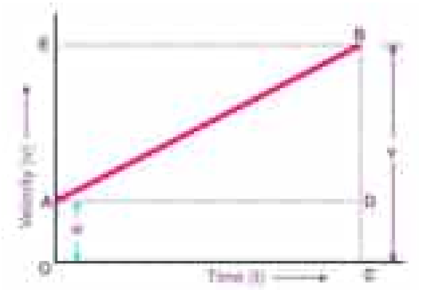


Fig: 2.23

Derivation of Third Equation of Motion by Graphical Method

From the fig: 2.24, we can say that The total distance travelled, s is given by the Area of trapezium OABC.

Hence,

$S = ½\ (Sum\ of\ Parallel\ Sides) \times Height$

$S = ½\ (OA + CB) \times OC$

$Since,\ OA = u,\ CB = v, and\ OC = t$

$The\ above\ equation\ becomes$

$S = ½\ (u + v) \times t$

$Now, since\ t = (v - u)/\ a$

$The\ above\ equation\ can\ be\ written\ as:$

$S = ½\ ((u + v) \times (v - u))/a$

$Rearranging\ the\ equation, we\ get$

$S = ½\ (v + u) \times (v - u)/a$

$S = (v^2 - u^2)/2a$

$Third\ equation\ of\ motion\ is\ obtained\ by\ solving\ the\ above\ equation:$

$v^2 = u^2 + 2aS$

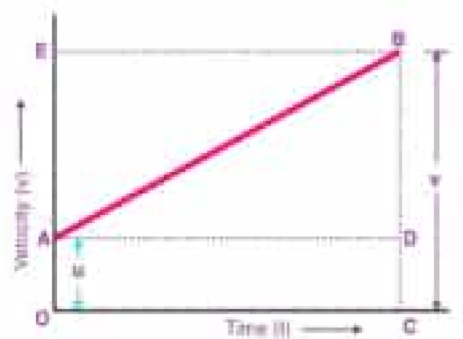


Fig: 2.24

Self-Assessment Questions:

1. If the displacement-time graph is a straight line, what does it indicate about the object's motion?
2. What does the area under a velocity-time graph represent?

Worked Example 2.3

A Car with an initial speed of 1 m/sec was in motion for 10 minutes, and then it came to a stop, the velocity right before it stopped was 5 m/sec. What was the constant acceleration of the car?

Solution:

Step 1: Write the known quantities and point out quantities to be found.

$Initial\ Velocity = 1\ m/sec$

$Final\ Velocity = 5\ m/sec$

Time for which the car was in motion $= 10$ mint

Acceleration $= ?$

Step 2: Write the formula and rearrange if necessary

Using First equation of motion,

$v = u + at$

Step 3: Put the value in formula and calculate

$5 = 1 + a \times (10 \times 60)$

$a \times 600 = 4$

$a = 4/600$

$a = 0.0066\ m/sec^2$

Worked Example 2.4

A cycle covered 2 km in 8 minutes and the initial velocity of the cycle was 1 m/sec. Find the acceleration that the cycle had in its motion.

Solution:

Step 1: Write the known quantities and point out quantities to be found.

Displacement covered $= 2km$

Total Time Taken $= 8minutes = 8 \times 60 = 480$ seconds.

Initial Velocity $= 1\ m/sec$

Using Second equation of motion to find the acceleration of the cycle,

Step 2: Write the formula and rearrange if necessary

Second Equation of motion, $S = ut + 1/2(at^2)$

Step 3: Put the value in formula and calculate

$2000 = 1 \times 480 + 1/2(a \times 480^2)$

$2000 = 480 + 115200a$

$1520 = 115200a$

$$a = 0.0139 m/sec^2$$

Projectile Motion:

In this universe we see different objects motion in different dimensions, some are moving along a linear path, like a car travelling along a rectilinear path and some are moving along a circular path/track. If a cricketer hits a ball which is placed on the ground, this ball will follow a curved path and will hit the ground, also if a missile is fired then we see it will always follow a curved path which are the examples of two dimensional motion. In this section we will be able to know the answers of these questions that what affects the motion of bodies which is responsible for the curved Path motion of bodies.

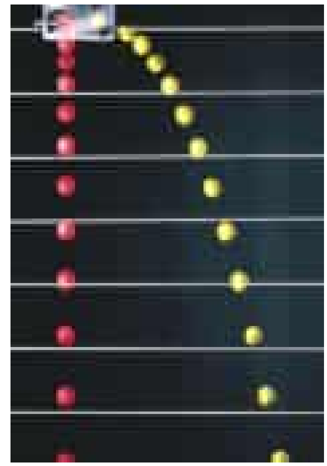


Fig: 2.25

Projectile motion:

- The motion of an object in a plane under the influence of force of gravity of earth.
- Gravitational force of earth is responsible for the Projectile motion and the curved path followed by a projectile is called its trajectory.

Assumptions for projectile motion:

It is easy to analyze the projectile motion if following assumptions are in consider:

- 1. The value of acceleration due to gravity is considered as constant throughout the projectile motion and it is always directed downwards.
- 2. The effect of air resistance is negligible.
- 3. Projectile motion is not affected due to rotation of an earth.

Average velocity vector:

The red ball is dropped at the same time that the yellow ball is fired horizontally as shown in fig: 2.25. Photos are separated by equal time intervals, Δt. At any time the two balls have the same: y,. The yellow ball has equal Δx, and v_x during each time Δt. Projectile motion is analyzed to two motions: horizontal motion with constant velocity and vertical motion with constant acceleration

The x and y motion in projectile motion:

The motion of body in x, y plan is shown in fig: 2.26

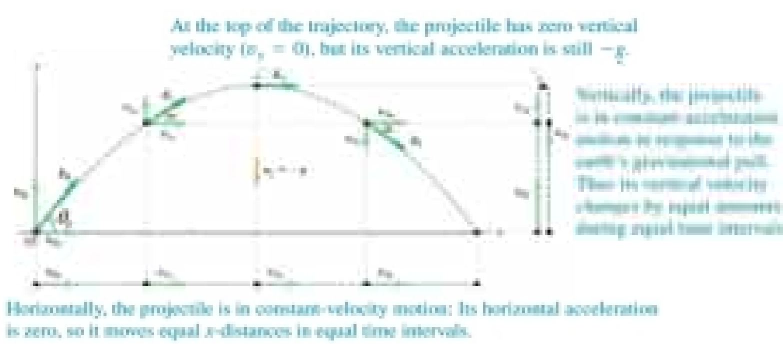


Fig: 2.26

$ax = 0$	$ay = -g$
Horizontal	**Vertical**
$v_x = v_{0x} + a_x t$	$v_y = v_{0y} + a_y t$
$x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2$	$y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$
$v_x^{\ 2} = v_{0x}^{\ 2} + 2a_x(x - x_0)$	$v_y^{\ 2} = v_{0y}^{\ 2} + 2a_y(y - y_0)$

v_{0x} is constant. $v_{0x} = v_0 \cos\theta_0$ ……(2.16)

v_{0y} changes continuously. $v_{0y} = v_0 \sin\theta_0$ ……(2.17)

After considering all the conditions the Equations will be

$$v_x = v_{0x} \qquad v_y = v_{0y} - gt$$
$$x = x_0 + v_{0x}t \qquad y = y_0 + v_{0y}t - \tfrac{1}{2}gt^2$$

Time taken by projectile to reach the maximum height:

If we consider t is the time required to achieve the maximum height by a projectile then it can be found by using 1st equation of motion.

$$V_{fy} = V_{iy} + a_y t$$
$$0 = V_o \text{Sin}\theta - gt$$
$$V_o \text{Sin}\theta = gt$$
$$t = \frac{V_o \text{Sin}\theta}{t}$$

Total time of flight of projectile:

The time during which projectile remains in air is called its "Total time of flight" (T). from the definition we know that total time of flight should be the sum of time requires to reach maximum height t_h and time required to hit back the ground t_g. So therefor

$$T = t_h + t_g$$
$$T = 2t$$

Thus $T = \frac{2V_o \text{Sin}\theta}{g}$ ----------(2.18)

$\therefore\ t_h = t_g = t$

$\therefore\ t = \frac{V_o \text{Sin}\theta}{g}$

Maximum height reached by the projectile:

The maximum vertical distance covered by the projectile is called maximum height. It is denoted by 'H' the maximum height reached by the projectile can be found by using formula:

$$Y = V_{iy}\, t + \frac{1}{2} a_y t^2$$

$$H = V_o \text{Sin}\theta \left(\frac{V_o \text{Sin}\theta}{g}\right) + \frac{1}{2}(-g)\left(\frac{V_o \text{Sin}\theta}{g}\right)^2$$

$$H = \frac{v_o^2 \sin^2\theta}{g} - \frac{v_o^2 \sin^2\theta}{2g}$$

$$H = \frac{v_o^2 \sin^2\theta}{2g} \quad \ldots\ldots(2.19)$$

$$\therefore V_{iy} = V_{oy} = V_o \text{Sin}\theta$$

$$\therefore t = \frac{V_o \text{Sin}\theta}{t}$$

$$\therefore a_y = -g$$

Range of the projectile:

The horizontal distance covered by the projectile between point of projection and point of return to level of projection is called Range of the projectile and is represented by 'R' and can be found by formula.

$X = V_{ox}\, T$

Let's consider $X = R$; $V_{ox} = V_o \text{Cos}\theta$

and $T = \frac{2V_o \text{Sin}\theta}{g}$

After putting all these values Now

$$R = V_o \text{Cos}\theta \left(\frac{2V_o \text{Sin}\theta}{g}\right)$$

$$R = v_o^2 \left(\frac{2\text{Sin}\theta\text{Cos}\theta}{g}\right) \qquad \therefore \quad 2\text{Sin}\theta\ \text{Cos}\theta = \text{Sin}\ 2\theta$$

$$R = \frac{v_o^2\ \text{Sin}\ 2\theta}{g} \ldots\ldots(2.20)$$

Projectile Motion at Various Initial Angles:

From the mathematically relation of range we observe that V_o and g are constants, so maximum range of the projectile can be achieved only when Sin 2θ becomes maximum (Sin 2θ=1). as we know that $\text{Sin}90^0 = 1$; this relation shows that the maximum range can be achieved only projectile will be launched at angle of 45^0.

Taking $R = R_{max}$ and $\theta = 45^o$

$$R_{max} = \frac{v_o^2 \ \text{Sin}\ 2(45^0)}{g}$$

$$R_{max} = \frac{v_o^2 \ \text{Sin}\ 90^o}{g}$$

$$R_{max} = \frac{V_0^{\ 2}}{g} \ \ldots\ldots(2.21)$$

Thus for the maximum horizontal range, the angle of projectile should be 45°.
Complementary values of the initial angle result in the same range

- The heights will be different The maximum range occurs at a projection angle of 45°

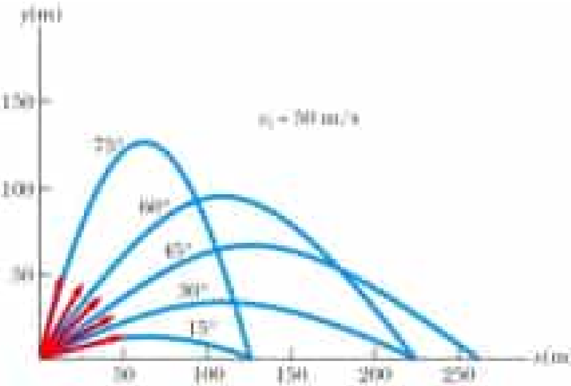


Fig: 2.27 range covered at different angles

The range of different projected bodies at different angle is shown in fig:2.27.

Self-Assessment Questions

1. When an object is projected horizontally, how does its vertical velocity change over time?
2. Can the range of a projectile be increased by increasing its initial velocity? Explain your answer.

Worked Example 2.5

A body is projected with a velocity of $20ms^{-1}$ at the angle of 50 degree to the horizontal plane. Find the time of flight of the projectile.

Solution:

Step 1: Write the known quantities and point out quantities to be found.

Initial Velocity Vo $= 20ms^{-1}$

angle $\theta = 50°$

$g = 9.8ms^{-2}$

Step 2: Write the formula and rearrange if necessary

Formula for time of flight is,

$$\mathbf{T} = \frac{\mathbf{2V_oSin\theta}}{\mathbf{g}}$$

Step 3: Put the value in formula and calculate

$T = 2 \times 20 \times \sin 50°/9.8$

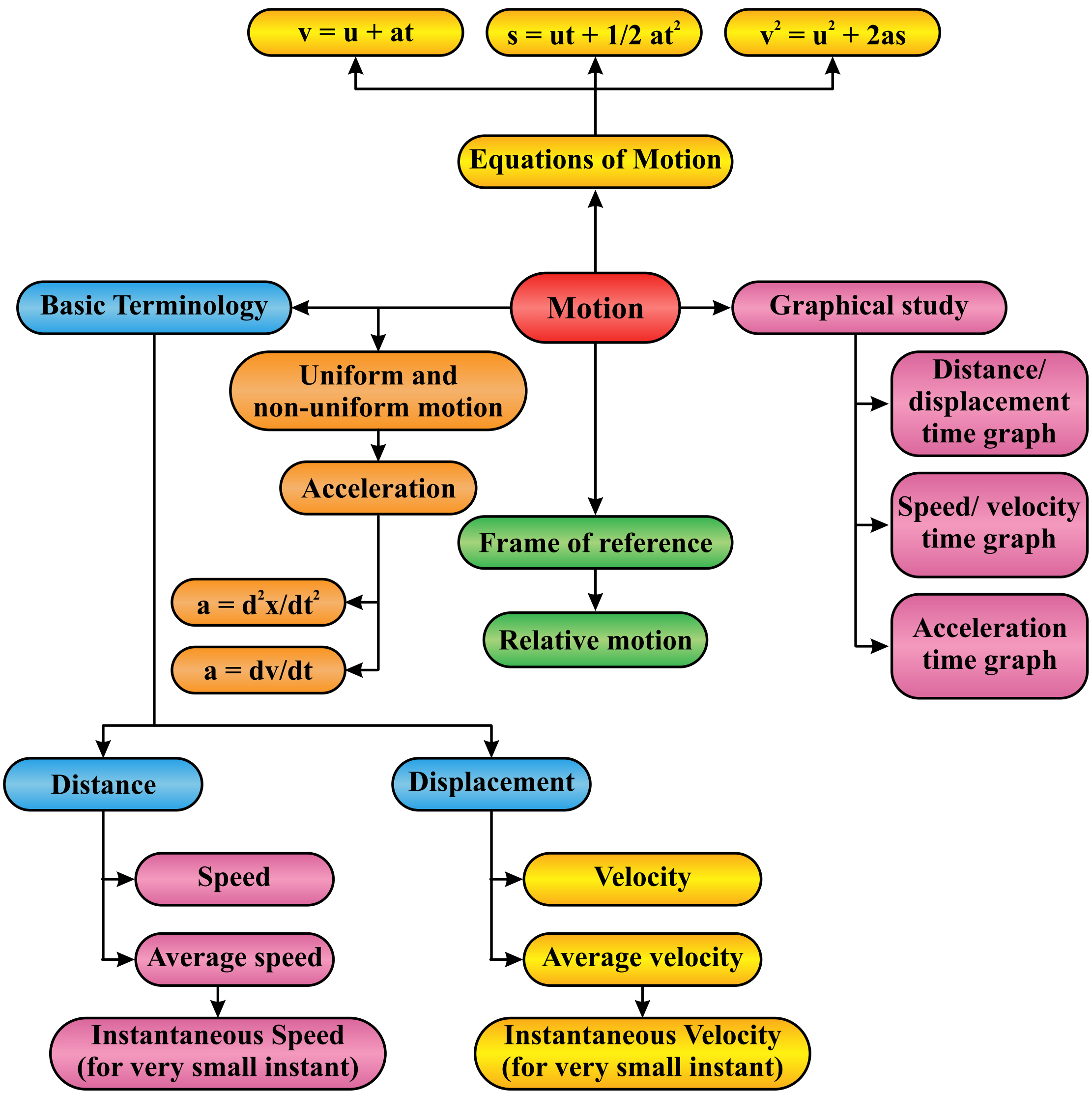
$T = 2 \times 20 \times 0.766/9.8$

$T = 30.64/9.8$

$\mathbf{T = 3.126\ sec}$

Therefore time of flight is 3.126 second.

CONCEPT MAP


v = u + at
s = ut + 1/2 at^2
v^2 = u^2 + 2as
Equations of Motion
Basic Terminology
Motion
Graphical study
Uniform and non-uniform motion
Distance/ displacement time graph
Acceleration
Speed/ velocity time graph
Frame of reference
a = d^{2}x/dt^2
Relative motion
Acceleration time graph
a = dv/dt
Distance
Displacement
Speed
Velocity
Average speed
Average velocity
Instantaneous Speed (for very small instant)
Instantaneous Velocity (for very small instant)

SUMMARY

- Study of motion without considering the forces involved is called Kinematics.
- Location of an object in space or co-ordinates system is known as Position.
- Change in position of an object in particular direction is called Displacement.
- Total path length traveled by an object is called Distance.
- Magnitude of the velocity vector; how fast an object moves is known as Speed.
- Velocity is the Rate of change of position; includes direction.
- Rate of change of velocity; can be positive or negative is called Acceleration.
- Uniform Motion is the Constant speed and direction.
- Changing speed or direction is known as Non-Uniform Motion.
- Velocity of an object at a specific moment in time is known as Instantaneous Velocity.
- Average Velocity is the Total displacement divided by the total time.
- Equations of Motion are the Mathematical expressions linking displacement, velocity, acceleration, and time for uniform and constant acceleration motion.

$$V_f = V_i + at$$

$$V_f^2 = V_i^2 + 2as$$

$$S = V_i t + \frac{1}{2}at^2$$

- Deceleration is the negative acceleration; the object slows down.
- Retardation is also known as deceleration.
- Motion under the influence of gravity alone, with no other forces acting on the object is known as Free Fall.
- Projectile Motion is the motion of an object under the influence of gravity, moving horizontally and vertically.
- Time of flight is the total time taken by a projectile to reach the ground.
- Horizontal distance covered by a projectile is known as Range.
- The maximum range of projectile can be obtained at the angle of 45^o.

EXERCISE

Section (A): Multiple Choice Questions (MCQs)

1. To get a resultant displacement of 10m, two displacement vectors of magnitude 6m and 8m should be combined:
   (a) Parallel (b) Antiparallel
   (c) At an angle of 45^0 (d) Perpendicular to each other
2. The velocity of a particle at an instant is 10 m/s and after 5sec the velocity of particle is 20m/s. The velocity 3 sec before in m/s is:
   (a) 8 (b) 4
   (c) 6 (d) 7
3. A ball is thrown upwards with a velocity of 100 m/s. It will reach the ground after:
   (a) 10 sec (b) 20 sec
   (c) 5 sec (d) 40 sec
4. Two projectiles are fired from the same point with the same speed at angles of projection 60^0 and 30^0 respectively. Which one of the following is true?
   (a) The range will be same (b) Their maximum height will be same
   (c) Their landing velocity will be same (d) their time of flight will be same
5. The ratio of numerical values of average velocity and average speed of a body is always:
   (a) Unity (b) Unity or less
   (c) Unity or more (d) Less than unity
6. If the average velocities of a body become equal to the instantaneous velocity, body is said to be moving with:
   (a) Uniform acceleration (b) Uniform velocity
   (c) Variable velocity (d) Variable acceleration
7. At the top of a trajectory of a projectile, the acceleration is:
   (a) maximum (b) minimum
   (c) zero (d) g
8. At what angle the range of projectile becomes equal to the height of projectile?
   (a) 65^0 (b) 45
   (c) 76^0 (d) 30^0
9. The angle at which dot product becomes equal to the cross product is:
   (a) 65^0 (b) 45^0
   (c) 76^0 (d) 30^0
10. If the dot product of two non-zero vectors vanishes; the vectors will be:
   (a) in the same direction (b) opposite direction to each other
   (c) perpendicular to each other (d) zero

CRQs:

1. Is the range equivalent to the horizontal distance in the projectile motion, or do they have distinct meaning? Explain
2. If air resistance is taken into account in case of projectile motion, then what parameters of projectile are influenced?
3. What are some real-life examples of projectile motion? Can motion of an aero plane be considered as projectile motion? Explain
4. Where do we use the concept of speed and velocity in our daily life? Can you give some examples?
5. Can an object have an initial velocity of zero and still experience uniformly accelerated motion

ERQs:

1. What are three equations of uniformly accelerated motion and how they are derived?
2. How does the horizontal component of velocity in projectile motion behaves?
3. What is the significance of the area under a velocity-time graph in the context of accelerated motion?
4. How do the initial launch angle and velocity affect projectile motion?
5. Explain the concept of independence between horizontal and vertical motion in projectile motion

Numericals:

1. A helicopter is ascending at the rate of 12 m/s. At a height of 80 m above the ground, a package is dropped. How long does the package take to reach the ground? **(Ans: 5.4 sec)**
2. Two tug boats are towing a ship each exerts a force of 6000 N, and the angle between two ropes is 60^0. Calculate the resultant force on the ship? **(Ans:10392 N)**
3. A car starts from rest and moves with a constant acceleration. During the 5^{th} second of its motion, it covers a distance of 36 meters. Calculate:
   (a) acceleration of the car
   (b) the total distance covered by the car during this time. **(Ans: $8 m/s^2$, 100 m)**
4. Show that the range of projectile at complementary angles are same with examples?
5. At what angle the range of projectile becomes equal to the height of projectile? **(Ans: 76^0)**
6. A marter shell is fired at a ground level target 500 m distance with an initial velocity of 90 m/s. What is the launch angle? **(Ans: 71.4^0)**