Unit 5

Work, Energy and Power

Teaching Periods 15 | Weightage % 10

Under-13 girls football tournament at Thar Coal Block II, Islamkot

In this unit student should be able to:

- Describe the concept of work in terms of the product of force F and displacement d in the direction of force (Work as scalar product of F and d).
- Distinguish between positive, negative and zero work with suitable examples.
- Calculate the work done from the force-displacement graph.
- Define work by variable force
- Calculate the work done from the force-displacement graph.
- Recall the concept of K.E
- Derive the equation of K.E by using W = **F.d**
- Recall the concept of potential Energy.
- Derive the equation of P.E from W = **F. d**.
- Show that the work done in gravitational field is independent of path.
- Calculate gravitational potential energy at a certain height due to work against gravity.
- Describe that the gravitational PE is measured from a reference level and can be positive or negative, to denote the orientation from the reference level.
- Use equations of absolute potential energy to solve problems.
- Explain the concept of escape velocity in term of gravitational constant G, mass m and radius of planet r.
- Express power as scalar product of force and velocity.
- Explain that work done against friction is dissipated as heat in the environment.
- State Work Energy theorem.
- Utilize work – energy theorem in a resistive medium to solve problems.
- State law of conservation of energy
- Explain Law of conservation of energy with the help of suitable examples.

5.1 Work:

Work is a very important physical concept Physics, it has usually different meaning as used in our daily life. In Physics work can be defined as, when a force is applied on a body it produces the displacement in a body in the direction of force, then we can say work is done on a body. There are some few important examples in which there occurs no work done, like If a person holds a bucket vertically in a hand and displaces it horizontally then in this case work by the gravity said to be zero, also if a person is circulating a body attached in a string, in which tension acts as a centripetal force, in this case work done on a body is also zero. In this chapter we will discuss about the answers of this questions.

5.1.1 Work done by a constant force:

If a force applied to an object does not change with respect to time, it is known as a constant force.

Work is measured by the product of the applied force and the displacement of the body in the direction of the force that is:

Work = (force). (displacement in the direction of the force)

If a force ***F*** acting on a body produces a displacement S in the body in the direction of the force (Fig. 5.1(a), then the work done by the force is given by:

W = F.s…… (5.1)

If the force F is making an angle θ with the direction of displacement of the body (Fig.5.1(b), then the work done is $w = (F\cos\theta)\,s = F\,s\cos\theta$ because $F\cos\theta$ is the component of F in the direction of displacement.

Note: The perpendicular component of force $F\sin\theta$ does no work on the cart.

Only horizontal component of force $F\cos\theta$ is responsible for work done on a body.

If a body moves through a displacement *s* while a constant force *F* acts on it in the same direction the work done by the force on the body is W = *Fs*.

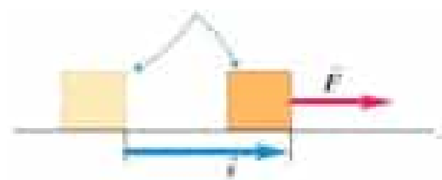


Fig: 5.1(a)

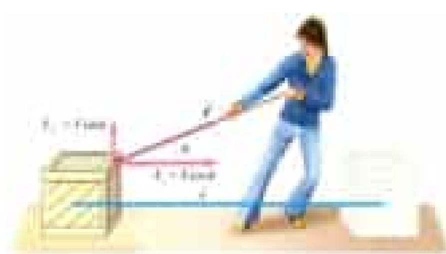


Fig: 5.1(b)

- Work can also be defined as the scalar or dot product of two parallel vectors force and displacement, since it follows the laws of scalar product.
- Work is a scalar physical quantity. The Joule (J) is the SI unit of energy and work.
- Joule is defined as the amount of work done when a force of one newton (1 N) is applied over a distance of one meter (1 m) in the direction of the force. Mathematically, 1 joule is equal to 1 newton-meter (N·m).

  OR

  the amount of energy transferred or expended to move an object.

5.1.2 Different cases of work done by constant force:

i) Positive work:

If Force and displacement are in parallel to each other as shown in figure 5.2a work done on a body will be maximum.

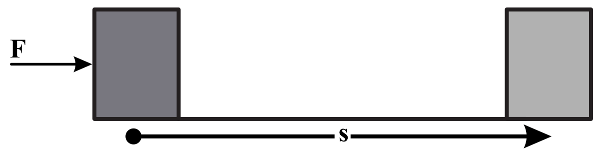


Fig: 5.2 (a)

W= Fs Cos θ $Cos(0)= 1$

W = Fs Cos 0

W = Fs

Example: Coolie pushing a load horizontally

ii) Zero work: If Force and displacement are mutually perpendicular to each other as shown in figure 5.2b work done on a body will be zero.

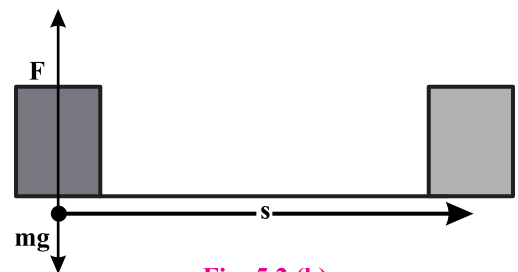


Fig: 5.2 (b)

W = Fs Cos θ $Cos(90) = 0$

W = Fs Cos 90

W = 0

Example:

1. Work done by a centripetal force is always zero, in this case tangential displacement and centripetal force are mutually perpendicular with each other hence work done on a body will be zero.
2. Coolie carrying a load on his head and moving horizontally with constant velocity. Then he applies force vertically to balance weight of body & its displacement is horizontal.

iii) Negative Work: If Force and displacement are in anti-parallel direction the work done on a body will be negative.

W = Fs Cos θ $Cos(180) = -1$

W = Fs Cos 180

W = - Fs

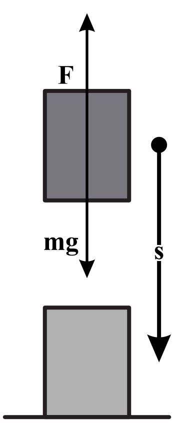


Fig: 5.2 (c)

Example: Work done on a body against gravitational field is negative, in this case a body is being displaced against the gravitational force as shown in figure 5.2c, hence displacement vector and force are anti-parallel to each other so work done on a body will be negative.

5.1.3 Work done by constant displacement time graph:

To calculate the work done by a force-displacement graph, you need to find the area under the graph. The area represents the work done.

If the force-displacement graph is a straight line as depicted in figure 5.3, the work done can be calculated using the formula:

Work = Force × Displacement

where the force is constant along the line.

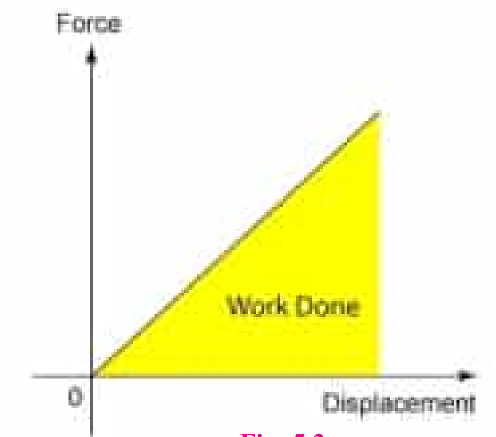


Fig: 5.3

However, if the graph is not a straight line as shown in figure 5.4 A, you will need to break the area under the graph into smaller shapes (e.g., rectangles, triangles) and calculate the area of each shape separately. Then, you sum up the areas of all the shapes to find the total work done.

Here's a step-by-step process to calculate the work done by a force-displacement graph when it's not a straight line:

Divide the graph into smaller shapes, such as rectangles or triangles, by drawing lines perpendicular to the displacement axis as represented in figure 5.4 B.

Calculate the area of each shape separately. For triangles, the area is given by the formula:

$$\text{Area} = \frac{1}{2} \times \text{Force} \times \text{Displacement}$$

$A = \frac{1}{2} \times F.d$ …..(5.2)

For rectangles, the area is given by the formula:

$\text{Area} = \text{Force} \times \text{Displacement}$

Calculate the area for each shape and write it down.

Sum up the areas of all the shapes to find the total work done.

$\text{Total Work} = \text{AreaI} + \text{AreaII} + \text{AreaIII}$

By following this process, we can calculate the work done by a force-displacement graph that is not a straight line.

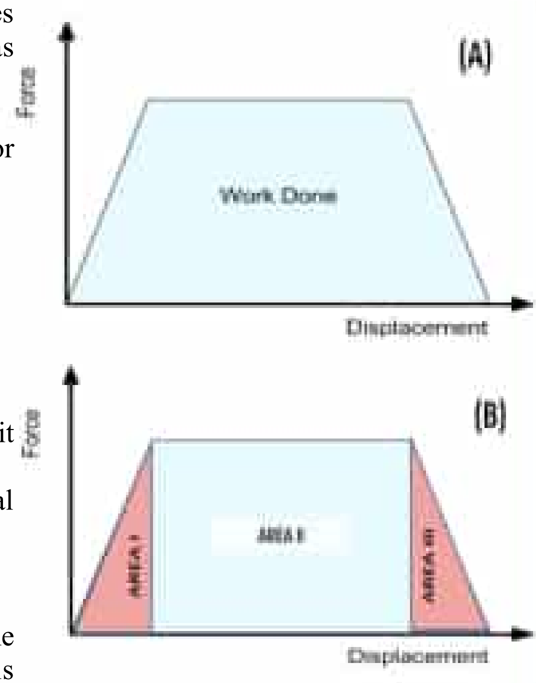


Fig: 5.4

Constant force displacement graph

Worked Example 5.1

Calculate the work done from the following force-displacemnt graph.

Solution:

Step 1: Write the known quantities and point out

Force F = 4N

Base of Area I = 4m

Base of Area II = 8m

Base of Area III =4m

Step 2: Write the formula and rearrange if necessary

Area II = Area of rectangle $=$ Force x displacement

Area I & III = Area of triangle = $\frac{1}{2} \times$ Force $\times$ Displacement

Total workdone $=$ Area I $+$ Area II $+$ Area III

Step 3: Put the value in formula and calculate

Total workdone $= W_T =$ Area I $+$ Area II $+$ Area III

$W_T = \frac{1}{2} \times \text{Force} \times \text{Displacement} + \text{Force x displacement} + \frac{1}{2} \times \text{Force} \times \text{Displacement}$

$W_T = \frac{1}{2} \times 4N \times 4m + 4N \text{ x } 8m + \frac{1}{2} \times 4N \times 4N$

W_T = 8N.m + 32 N.m + 8 N.m = 48N.m

W_T = 48 J

Self-Assessment Questions:

1. When does work have a positive value, and when does it have a negative value? Provide examples of each case.

2. In what units is work typically measured, and how are these units related to force and displacement?

5.2 Work done by variable force

Force varying with displacement

A variable force is a force that changes in magnitude or direction as a function of time, position, or any other relevant variable. Unlike a constant force, which remains unchanged, a variable force can have different values at different points or moments.

In this condition we consider the variable force to be variable for any elementary displacement ds as shown in figure 5.5, and work done in that elementary displacement is evaluated. Total work is obtained by integrating the elementary work from initial to final limits.

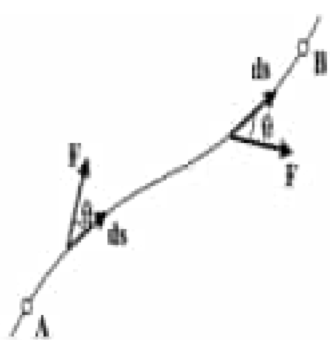


Fig: 5.5 Variable force

$\Delta W = \vec{F} \, . \, \overrightarrow{\Delta ds}$

$\Delta W = \sum \vec{F} \, . \, \overrightarrow{\Delta ds}$ …… (5.3)

Variable forces can arise in various situations and fields of study. Here are a few examples: Spring Force, Frictional Force etc.

Work done by variable force and its graphical calculation:

Consider a body covers displacement from x_i to x_f , when a variable force acts on it. To clarify the situation, we plot the graph between force and displacement covered by the body as shown in figure; **5.6** (a and b).

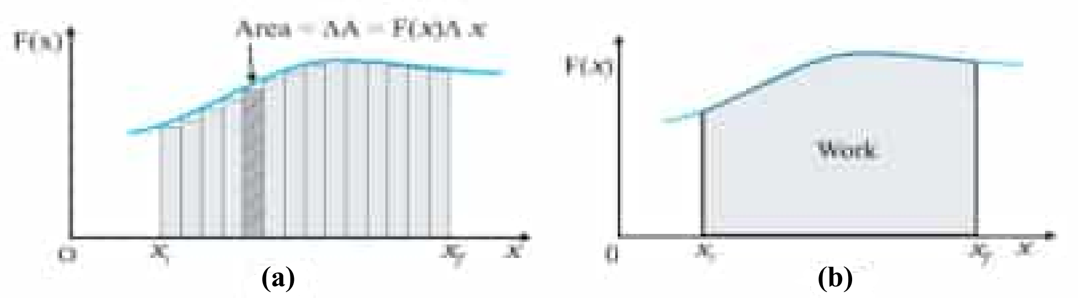


Fig: 5.6 (a and b) variable force displacement graph

the area under the covered line is representing the work done by the body. To calculate the work done by the body we divide the covered displacement into small segments $\Delta x_1, \Delta x_2, \Delta x_3 \ldots\ldots \Delta x_n$ and the corresponding forces for each segment are $\overrightarrow{F_{x1}}, \overrightarrow{F_{x2}}, \overrightarrow{F_{x3}} \ldots\ldots \overrightarrow{F_{xn}}$ as shown in figure 5.6.

We know the work done for each segment will be $W_1, W_2, W_3, \ldots \ldots W_n$.

The total work done in this case will be $W_T = W_1 + W_2 + W_3 + \cdots W_n = \sum_{i=1}^{n} W_i$

$$W_T = \overrightarrow{F_{1x}} . \overrightarrow{\Delta x_1} + \overrightarrow{F_{2x}} . \overrightarrow{\Delta x_2} + \overrightarrow{F_{x3}} . \overrightarrow{\Delta x_3} + \cdots + \overrightarrow{F_{xn}} . \overrightarrow{\Delta x_n} = \sum_{i=1}^{n} \overrightarrow{F_{xi}} . \overrightarrow{\Delta x_i}$$

$W_T = \sum_{i=1}^{n} \overrightarrow{F_{xi}} . \overrightarrow{\Delta x_i} = F_i cos\emptyset \Delta x_i \ldots$ (5.4)

Above equation represents the total work done by body when variable force acts on it.

Self-Assessment Questions:

1. How does the concept of work done by a variable force differ from work done by a constant force?
2. How can you represent a variable force graphically to analyze the work done?

5.3 Kinetic energy:

The faster the object moves, the greater is its kinetic energy when the object is stationary its kinetic energy is zero. (The energy possessed by a body by virtue of its motion called kinetic energy)

To find an expression for K.E of an object in motion, we must calculate the work done by the object. This work done must be equal to the change in K.E of the object

Suppose a force is applied on an object and it produces displacement in the direction of force along x-axis as depicted in figure 5.7

Hence work is done on a body which is stored in the form of kinetic energy in a body which is calculated as:

Work done by the body W= $\vec{F}.\vec{S} = FS\cos\theta$ $\quad \left[\begin{matrix}\theta = 0 \\ Cos(0) = 1\end{matrix}\right]$

= F.S $Cos\theta$

= F.S

W=F.S $\qquad \therefore (F = ma)$

$W = ma.S \longrightarrow (i)$

By using the 3rd equation of motion $(vf^2 - vi^2 = 2as)$ we take the value of s by introducing the initial conditions $(v_i = 0, v_f = v)$

$v^2 - 0 = 2\,as$

$s = \frac{V^2}{2a}$ put this value in equation (i)

$W = ma.\frac{V^2}{2a} = \frac{1}{2}mv^2$

$K.E = \frac{1}{2}mv^2$ ……(5.5)

This is an expression for Kinetic energy of a body.

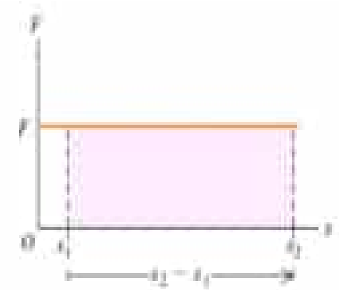


Fig: 5.7

Worked Example 5.2

A car with a mass of 1,200 kg is traveling at a velocity of 25 m/s. Calculate the kinetic energy of the car.

Solution:

Step 1:

Mass (m) = 1,200 kg

Velocity (v) = 25 m/s

Step 2:

The formula for kinetic energy is given by:

Kinetic Energy $K.E = \frac{1}{2}mv^2$

Step 3:

Substituting the given values into the formula:

$K.E = \frac{1}{2}(1200)(25)^2$

$=$ 375,000 Joules

Therefore, the kinetic energy of the car is 375,000 Joules.

Self-Assessment Questions:
1. How does the kinetic energy of an object change when its mass or velocity changes?
2. Can an object have a negative kinetic energy? Explain your answer.

5.4 Potential energy:

- How do energy concepts apply to the descending duck?
- We will see that we can think of energy as being stored and transformed from one form to another

When work is done on a body against any field/ force, an energy is stored in a body called potential energy. When a body of mass "m" is lifted to a height "h" against the gravitational force (mg), work is done on it. This work is stored in it in the form of gravitational potential energy. "The energy possessed by a body by virtue of its position is called potential energy P.E".

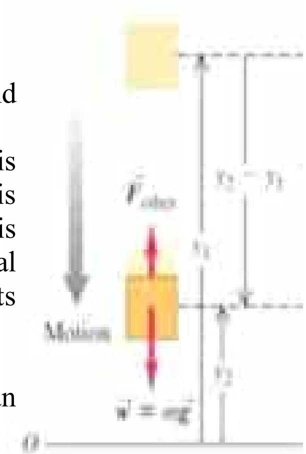


Fig: 5.8

Gravitational potential energy:

When work is done on a body against the gravitational force of an earth from y_1 level to y_2 level:

$$W_{grav} = Fs = -mg\,(y_1 - y_2)$$
$$W_{grav} = mgy_1 - mgy_2$$
$$W_{grav} = mgh \quad \ldots\ldots (5.6)$$

Hence Potential energy is stored in a body when work is done against gravitational force of an earth so:

P.E = mgh

Thus, potential energy of a body in the earth's gravitational field at a height "h" is "mgh" which is positive quantity with respect to earth's surface which is supposed to be the level of an arbitrary zero potential.

Self-Assessment Questions:
1. What factors determine the gravitational potential energy of an object in a given situation?
2. Can an object have negative potential energy? Explain your answer.

5.5 Work done against gravitational field:

Consider a body of mass 'm' which is taken every slowly to small height 'h' in the gravitational field such as that the acceleration of the body is zero. The work done in moving the body is given by:

$$\textit{Work done} = F_{ex}\,h = F_g\,h\,\cos\theta \ldots\ldots(5.7)$$

Where 'F_{ex}' is the external force applied on the body . Since the external force applied on the body and the displacement are along the same direction, therefore work done by external force 'W_{ex}' is given by :

$$W_{ex} = F_{ex}h \text{ ------------} (\cos 0^o = 1)$$

As the acceleration of the body is zero therefore magnitude of external force is equal to that of the force of gravity i.e.

$F_{ex} = mg$

Therefore $\quad W_{ex} = mgh \quad$ ----------(5.8)

Work done 'Wg' by the gravitational force 'Fg' is given by

$$W_g = \vec{F}_g . \vec{h} = F_g.h \text{ Cos } 180^o$$

$W_g = -F_g.h \quad \therefore \vec{F}_g$ and $\vec{h}$ are in opposite direction then angle between them is 180

Since $\quad F_g = mg$

$$W_g = -mgh$$

OR $\quad -W_g = mgh \quad$ ---------------(5.9)

Comparing eq (5.8) and (5.9)

$$W_{ex} = -W_g$$

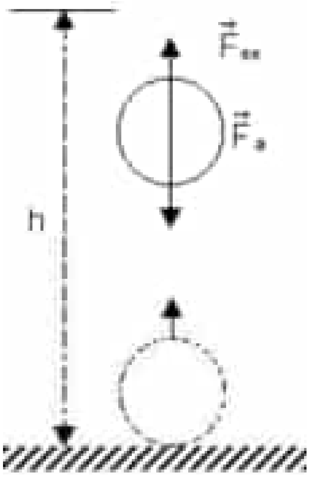


Fig: 5.9

By putting the value of Wg from eq. (5.9), we get

$$W_{ex} = -W_g = (-mgh) = mgh \text{ ......(5.10)}$$

This work done on the body by an external force against the gravitational force is stored in the form of potential energy and is known as gravitational potential energy represented by U_g

Therefore $\quad U_g = W_{ex} = -W_g = mgh$ ......(5.11)

This gravitational potential energy is the relative potential energy of the body with respect to some orbitary zero level..

Workdone in gravitational field is independent of path:

To prove the statement that the work done in the gravitational field is independent of path. Lets us take a closed triangular path ABC in gravitational field shown in Fig: for simplicity the base BC is taken perpendicular to the gravitational force "mg" initially the body is at A.

$W_{A\to B} = \vec{F}.\vec{S_1} = F.S_1 \text{ Cos}\alpha = Fh \quad \therefore S_1\cos\alpha = h$

$W_{A\to B} = mgh \quad \therefore F = mg$

$W_{B\to C} = \vec{F}.\vec{S_2} \quad \therefore \text{ Cos } 90^o = 0$

$W_{B\to C} = F.S_2\text{Cos } 90^o = 0$

$W_{C\to A} = \vec{F}.\vec{S_3} = F.S_3 \text{ Cos } (180 - \beta)$

$\to F.S_3 (-\text{Cos } \beta) \quad \therefore S_3\text{Cos } \beta = h$

$= -F.S_3\text{Cos } \beta \quad \therefore F = mg$

$= -mgh$

Thus, total work done along path A BCA

$= (mgh) + (0) - (mgh)$

$= 0$

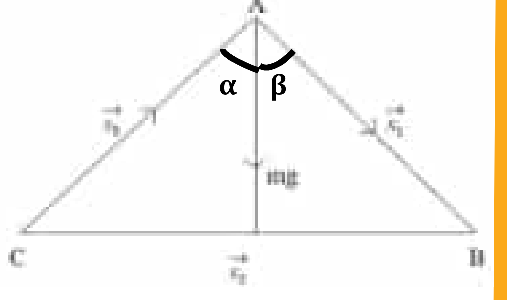


Fig: 5.10

Thus, either the body is moved from A to B and B to C or it is moved directly from A to C, in both the cases the work done is same, such type of field or force in which the work done is independent of path is called conservative field and in conservative field work done in a closed path is always zero.

Self-Assessment Questions:

1. When is the work done against a gravitational field positive, and when is it negative? Provide examples for each case.
2. How does the work done against gravity affect the potential energy of an object?

5.6 Absolute gravitational potential energy:

Absolute potential energy at a point is the amount of work done in moving a body from infinity to that point.

Expression for absolute gravitational potential energy:

Consider a body of mass 'm' at point A (1) in the gravitational field . If the body is lifted to a far point B ('n') in the gravitational field then work done in moving body can not be directly found using the formula:

Work done = Force x displacement

Because the gravitational force will not remain constant for such a large distance.

To overcome this difficulty, divide the distance between A and B in to large number of intervals each of width Δr . Δr is so small that the gravitational force through out this interval may be assumed to be constant.

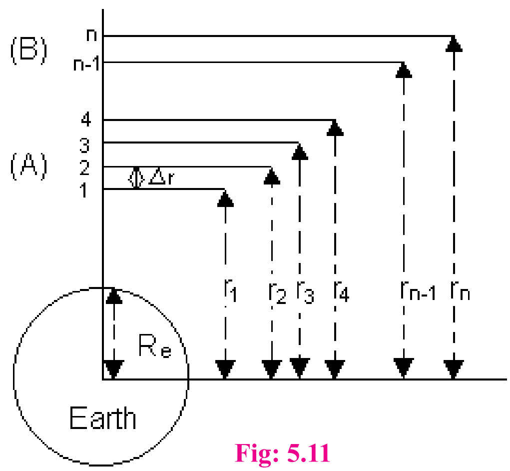


Fig: 5.11

If $\vec{F}$ be the gravitational force on the body at point 1 then magnitude is given by:

$$F_1 = \frac{GmM_e}{r_1^2}$$

Where G = Gravitational constant

Me = Mass of earth and

R_1 = Distance of point 1 from the center of the earth.

Similarly if r_2 be the gravitational force on the body at point 2 then its magnitude is given by:

$$F_2 = \frac{GmM_e}{r_2^2}$$

Where r_2 = Distance of point 2 from the centre of the earth.

The magnitude of the average force $\vec{F}$ acting through out the first interval is give by:

$$F = \frac{F_1 + F_2}{2}$$

$$F = \frac{\left(\frac{GmM_e}{r_1^2}\right) + \left(\frac{GmM_e}{r_2^2}\right)}{2} = \frac{1}{2}\left[\frac{GmM_e}{r_1^2} + \frac{GmM_e}{r_2^2}\right]$$

Or $$F = \frac{GmM_e}{2}\left[\frac{1}{r_1^2} + \frac{1}{r_2^2}\right] = \frac{GmM_e}{2}\left[\frac{r_2^2}{r_1^2} + \frac{r_1^2}{r_2^2}\right]$$

But $r_2 - r_1 = \Delta r$ ------------(i)

Therefore $r_2 = r_1 + \Delta r$-------------(ii)

Thus $$F = \frac{GmM_e}{2}\left[\frac{(r_1 + \Delta r)^2 + r_1^2}{r_1^2 r_2^2}\right]$$

$$F = \frac{GmM_e}{2}\left[\frac{r_1^2 + 2r_1\Delta r + (\Delta r)^2 + r_1^2}{r_1^2 r_2^2}\right]$$

As Δr is very small, therefore $(\Delta r)^2$ is negligible small

Therefore $$F = \frac{GmM_e}{2}\left[\frac{2r_1^2 + 2r_1\Delta t}{r_1^2 r_2^2}\right]$$

$$F = \frac{GmM_e}{r_1^2 r_2^2} \quad \text{----------(5.12)}$$

Work done in lifting the body from point '1' to '2' given by:

$$W_{1\rightarrow 2} = \vec{F} \cdot \vec{\Delta r} = F\Delta r \cos\theta$$

Since $\vec{F}$ and Δr are along the same direction.

Therefore $\theta = 0^o$ and $\cos 0^o = 1$

Therefore $W_{1\to 2} = F\Delta r$

By putting the values of 'Δr' and 'F' from eq (i) and (iii) , we get.

$$W_{1\to 2} = \frac{GmM_e}{r_1 r_2}(r_2 - r_1) = GmM_e\left(\frac{r_2 - r_1}{r_1 r_2}\right)$$

$$= GmM_e\left(\frac{r_2}{r_1 r_2} - \frac{r_1}{r_1 r_2}\right)$$

$$W_{1\to 2} = GmM_e\left(\frac{1}{r_1} - \frac{1}{r_2}\right) \ldots\ldots(5.13)$$

Similarly the work done in lifting the body from point 2 to 3 , 3 to 4 ------ and(n – 1) to n is given by :

$$W_{2\to 3} = GMm_e\left(\frac{1}{r_2} - \frac{1}{r_3}\right)$$

$$W_{3\to 4} = GMm_e\left(\frac{1}{r_3} - \frac{1}{r_4}\right)$$

$$W_{(n-1)\to n} = GMm_e\left(\frac{1}{r_{n-1}} - \frac{1}{r_n}\right)$$

The total work done in lifting the body from '1' to 'n' is given by:

$$W = W_{1\to 2} + W_{2\to 3} + W_{3\to 4} + \ldots\ldots W_{(n-1)\to 1}$$

Or $$W = GmM_e\left(\frac{1}{r_1} - \frac{1}{r_n}\right)$$

This work done is stored in the body as potential energy.

Thus P.E of the body at B with respect to point A

$$P.E = GmM_e\left(\frac{1}{r_1} - \frac{1}{r_n}\right)$$

The P.E of the body at point A with respect to point B is

$$\Delta U = -W$$

Or $$\Delta U = -G\,mM_e\left(\frac{1}{r_1} - \frac{1}{r_n}\right)$$

If the point 'B' lies at infinity then $r_n = \propto$ and $\frac{1}{\propto} = 0$

Therefore $\Delta U = -GmM_e\left(\frac{1}{r_1}\right)$

This P.E of the body at point 'A' is called Absolute potential energy.

$$\Delta U = (P.E)_{abs}$$

Therefore $$\boxed{(P.E)_{abs} = -\frac{GmM_e}{r_1}}$$

If 'R_e' be the radius of the earth then the absolute potential energy of the body at the surface of the earth is given y:

$$\boxed{(P.E)_{abs} = -\frac{GmM_e}{R_e} \quad \ldots\ldots(5.14)}$$

Absolute potential energy at certain height:

The absolute potential energy of the body at a certain height 'h' ($h \ll R_e$) above the surface of the earth is given by:

$$(P.E)_{abs} = -\frac{GmM_e}{R_e + h} = \frac{-GmM_e}{R_e\left(1 + \frac{h}{R_e}\right)}$$

$$(P.E)_{abs} = -\frac{GmM_e}{R_e} = \left(1 + \frac{h}{R_e}\right)^{-1}$$

Using Binomial theorem, we can write

$$\left(1 + \frac{h}{R_e}\right)^{-1} = 1 + \frac{(-1)}{1!}\frac{h}{R_e} + \frac{(-1)(-2)}{2!}\left(\frac{h}{R_e}\right) + \ldots$$

$$= 1 - \frac{h}{R_e} + \left(\frac{h}{R_e}\right)^2 + \ldots$$

Since h<< R_e. Therefore we can neglect the terms containing the higher powers of $\frac{h}{R_e}$

Therefore $\left(1+\frac{h}{R_e}\right)^{-1} = 1-\frac{h}{R_e}$

Thus $(P.E)_{abs} = \frac{GmM_e}{R_e} = \left(1-\frac{h}{R_e}\right)$……(5.15)

Worked Example 5.3

The mass of the earth is 5.98 $\times 10^{24}$ kg and the mass of the sun is 1.99 $\times 10^{30}$ kg, and the earth is 160 million km away from the sun, calculate the GPE of the earth.

Data:

$the\ mass\ of\ the\ Earth\ (m) = 5.98 \times 10^{24}\ kg\ and\ mass\ of\ the\ Sun\ (M)$

$M = 1.99 \times 10^{30}\ kg$

Solution:

Step 1: *The gravitational potential energy is given by*:

$$U = \frac{-GMm}{r}$$

Step 2: $U = \frac{6.673 \times 10^{-11} \times 5.98 \times 10^{24} \times 1.99 \times 10^{30}}{160 \times 10^{9}}$

$$U = 4963 \times 10^{30}\ J$$

Self-Assessment Questions:

1. In what scenarios is the absolute gravitational potential energy of an object considered zero, and why?
2. What happens to the absolute gravitational potential energy of an object as it moves higher or lower in a gravitational field?

5.7 Escape velocity:

Escape velocity on earth or any other planet is defined as the minimum velocity with which the body has to be projected vertically upwards from the surface of the earth or any other planet so that it just crosses the gravitational field of earth or of that planet and never return on its own.

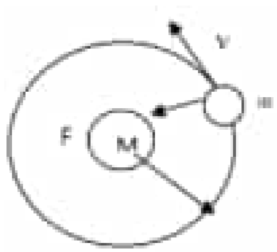


Fig: 5.12

Work is done at the cost of kinetic energy given to the body at the surface of earth. If Ves is the escape velocity of the body projected from the surface of earth. Then kinetic energy of the body. M and R are the mass and Radius of the earth respectively the body will escape out of the gravitational field.

$\frac{1}{2}$ m v^2 es = GMm/R

Ves = $\sqrt{2Gm/R}$ (i)

g= GM/R^2

Putting in equation (i) GmM = gR^2

Ves = $\sqrt{\frac{2gR^2}{R}}$

Ves = $\sqrt{2gR}$ ……(5.16)

The value of ves come out tube approximately 11.2 k ms^{-1}. The value of escape velocity depends upon mass and radius of the planet of the surface from which the body is to be projected, clearly the values of escape velocity of a body will be different for different planets.

5.8 Power:

Energy can be transferred from one object to another. If we are concerned about the measure of how fast energy is transferred, then the more energy transferred per second, the greater the power of the transfer process.

Power is the rate at which energy is transformed from one form to another or the work done per unit time.

$$\text{Power, P(watts)} = \frac{\text{Energy transfer or work done, W (joules)}}{\text{Time taken, t (seconds)}}$$

Where energy is transferred by a force doing work, the energy transferred is equal to the work done by the force. Therefore, the rate of transfer of energy is equal to the work per second. So, if the force does work W in time t, then

$$P = W/t$$

The power is the total work done divided by the total time interval.

$$P_{av} = \frac{\text{Total work}}{\text{Time interval}}$$

Let,

$$\text{Total work} = \Delta W$$

$$\text{Total work} = \Delta t$$

$$P_{av} = \frac{\Delta W}{\Delta t}$$

$$= \vec{F}\frac{\Delta W}{\Delta t}$$

$$= \vec{F}\frac{\Delta s}{\Delta t}$$

$$P_{av} = \vec{F}.\vec{V}_{av}$$

or

$P_{av} = FV\ Cos\Theta$……(5.17)

Therefore the power can also be defined as the Scalar/dot product of force and velocity.

- It's a scalar physical quantity and follows the laws of scalar product.
- It's fundamental unit is watt.
- Its dimensions are $M\ L^2\ T^{-3}$

DO YOU KNOW?

1 watt = 1N . 1m .s^{-1}
Its units are Erg/sec (C.G.S) foot – pound/sec (F.P.S) and Joule/sec = watt (M.K.S or S.I)
1 erg = 10^{-2} Watt
1 Ft.lb/sec = 1.356 Watt
1 Ft.lb/sec = 1.82 X 10^{-3} hp
1hp = 746 watt
½ hp = 373 watt

5.8.2 Work done against friction is dissipated as heat in the environment:

No system is perfect. Whenever there is a change in a system, energy is transferred and some of that energy is dissipated.

A rise in temperature is caused by the transfer of wasteful energy in mechanical processes. The energy is dissipated into the system.

In a mechanical system, energy is dissipated when two surfaces rub together. Work is done against friction which causes heating of the two surfaces – so the internal (thermal) energy store of the surfaces increases and this is then transferred to the internal energy store of the surroundings.

There are many electrical appliances that are used in the home to transfer electrical energy to other useful forms. Every system will waste some energy, and so the useful and wasted energy can always be identified.

Appliance	Useful energy	Wasted energy
Electric kettle	Energy that heats the water.	Internal (thermal) energy heating the kettle. Infrared radiation transferred to the surroundings.
Hairdryer	Internal (thermal) energy heating the air. Kinetic energy of the fan that blows the air.	Sound radiation. Internal (thermal) energy heating the hairdryer. Infrared radiation transferred to the surroundings.
Lightbulb	Light radiation given out by the hot filament.	Infrared radiation transferred to the surroundings.
TV	Light radiation that allows the image to be seen. Sound radiation that allows the audio to be heard.	Internal (thermal) energy heating the TV set. Infrared radiation transferred to the surroundings.

Devices can be made to reduce the energy that they waste or 'dissipate' to the surroundings. One example is lubrication being used to reduce the friction between moving parts of a machine. This reduces the thermal energy transferred.
For systems that are designed to transfer thermal energy, the wasteful dissipation of thermal energy to the surroundings can also be reduced. This is often done by using thermal insulation, for example, making a kettle from plastic, which is a thermal insulator.

Self-Assessment Questions:
1. How does escape velocity relate to the kinetic energy and gravitational potential energy of an object?
2. How does increasing power affect the rate at which work is done or energy is transferred?

5.9 Work energy theorem:

It states that total work done on the body is equal to the change in kinetic energy. (Provided body is confined to move horizontally and no dissipating forces are operating).

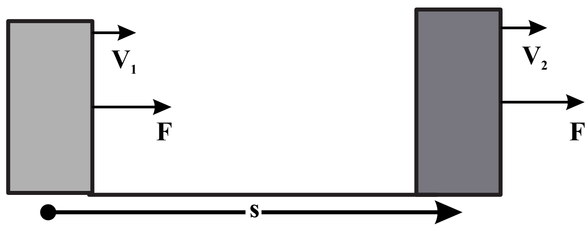


Fig: 5.13

Consider a body of mass m moving with initial velocity v_1 after travelling through displacement s its final velocity becomes V_2 under the effect of force F.
As we know that

$$2as = V_2^2 - V_1^2$$

$$a= = \frac{V_2^2 - V_1^2}{2s}$$

hence external force acting on the body is

$$F= ma$$

$$F = m\ \frac{V_2^2 - V_1^2}{2s}$$

Therefore, work done on body by external force is

$$W = \vec{F}.\vec{S}$$

$$W = m\frac{V_2^2 - V_1^2}{2s}\quad s.\ Cos(0)$$

$$W = ½\ mv_2^2 - ½\ mv_1^2$$

$$W = K.E_2 – K.E_1$$

$$W = \Delta K \quad .....(5.18)$$

Worked Example 5.4

A person riding their bike has a mass of 120 kg; they are riding at 10m/s. suddenly a dog crosses the road and to avoid hitting the dog the bicyclist brakes applying a braking force of 500 N for a distance of 10 meters. What is the final velocity of the bicyclist when they stop braking?

Step 1: *Identify the mass of the object. The mass is* 120 *kilograms.*

Step 2: *Identify the initial velocity.*

The initial velocity is $10 m/s$.

Step 3: *Identify or calculate the work done on the object.*

The force on the object is 500 *newtons over a distance of* 10 *meters.*

Since the force is a braking force it is resisting the motion of the object making the work done negative.

$$\boldsymbol{W} = \vec{\boldsymbol{F}} \times \boldsymbol{\Delta x} = -500 \times 10 = -5{,}000 Joules$$

Step 4: *Identify or calculate the initial energy of the object.*

Using the kinetic energy formula.

$$K.E_{initial} = \frac{1}{2} m v_{initial}^2$$

$$K.E_{initial} = \frac{1}{2}(120)(10)^2 = 6{,}000 Joules$$

Step 5: *Add the result from Step* 4 *with the result from Step* 3.

$$6{,}000\ Joules + (-5{,}000\ Joules) = 1{,}000\ Joules$$

Step 6: *Using the result from Step* 5 *equate it to the equation of kinetic energy and solve for velocity to receive the final velocity of the object.*

$$K.E_{final} = 1{,}000 Joules$$

$$\frac{1}{2} m v_{final}^2 = 1{,}000$$

$$\frac{1}{2}(120) v_{final}^2 = 1{,}000$$

$$v_{final} = \sqrt{16.7} = 4.1 m/s$$

The final velocity of the bicyclist when they stop braking is $4.1 m/s$.

Self-Assessment Questions:

1. Explain how positive work and negative work contribute to changes in an object's energy according to the work-energy theorem.
2. Is the work-energy theorem valid only for conservative forces, or does it apply to non-conservative forces as well? Explain.

5.10 Transformation of energy:

Energy can neither be created nor destroyed. It can only be transformed from one form to another. A loss in one form of energy is accompanied by an equal increase in the other forms of energy. The total energy remains constant'.

According to Einstein's mass energy relation:

$E = mc^2$, energy can be converted into mass and mass can be converted to energy. Pair production is the example of conversion of energy in to lass

On the other hand Nuclear fission and Fusion are examples of conversion of mass in to energy.

5.10.1 Law of conservation of energy:

Consider a body of mass 'm' is placed at a point from the ground at certain height h.

P.E of the body at P = mgh

K.E of the body at P = 0

Total energy of the body at P = K.E + P.E = 0 + mgh

Total energy at P = mgh --------(i)

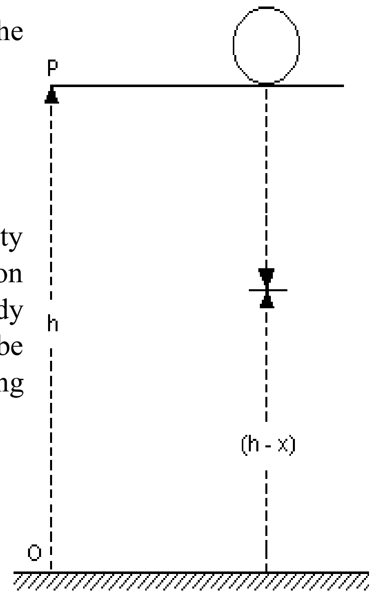


Fig: 5.14

If the body is allowed to fall freely under the action of gravity then its P.E will go on decreasing whole its K.E will go on increasing just before hitting the ground the P.E of the body will be minimum or zero while K.E of the body will be maximum. If 'v' be the velocity of the body just before hitting the ground then K.E of the body = ½ mv^2

The velocity of the body can be found by formula:

$$2gh = V_f^2 - V_i^2$$

Where V_i = initial velocity at 'P' = 0

V_f = final velocity at O= v

Therefore $2gh = V^2 - 0^2$

$V^2 = 2gh$

$$\text{K.E at O} = \frac{1}{2}m \times 2gh = mgh$$

P.E at O (near the ground) = 0

Total energy of the body at point 'O' = mgh ---------(ii)

If V be the velocity of the body at point 'Q' then

$$\text{K.E at Q} = \frac{1}{2}mv^2$$

P.E at Q = mgh (h-x) = mgh – mgx

V can be found by formula

$$2gh = V_f^2 - V_i^2$$

Where $V_i = 0$, $V_f = V$ and $h = x$

Therefore $2gx = (V)^2 - 0^2$

$$V^2 = 2gx$$

Therefore K.E at Q = $\frac{1}{2} m \times 2gx = mgx$

Total energy at Q = K.E + P.E

$= mgx + mgh - mgx$

Total energy at Q $= mgx$ ------------(iii)

From equation (i) ,(ii) and (iii) it can be seen that the total energy of the body remains constant every where provided there is no force of friction involved during the motion of the body.

If there is some force of friction acting on the body then a friction of P.E. is lost in doing work against the force of friction, Thus ,

Total energy = K.E + P.E + Loss of energy or work done against force of friction.

Examples of Conservation of energy:

1. When we switch on an electric bulb, we supply electrical energy to it which is converted into heat and light energies, i.e.

   Electrical Energy = Heat energy + Light energy

2. Fossil fuels e.g. coal and petrol is stores of chemical energy. When they burn, chemical energy is converted in to heat energy i.e.

   Chemical Energy = Heat energy + Losses

3. The heat energy present in the steam boiler can be used to derive a steam engine, Here heat energy is converted into kinetic (mechanical energy), i.e.

   Heat Energy = Mechanical energy + Losses

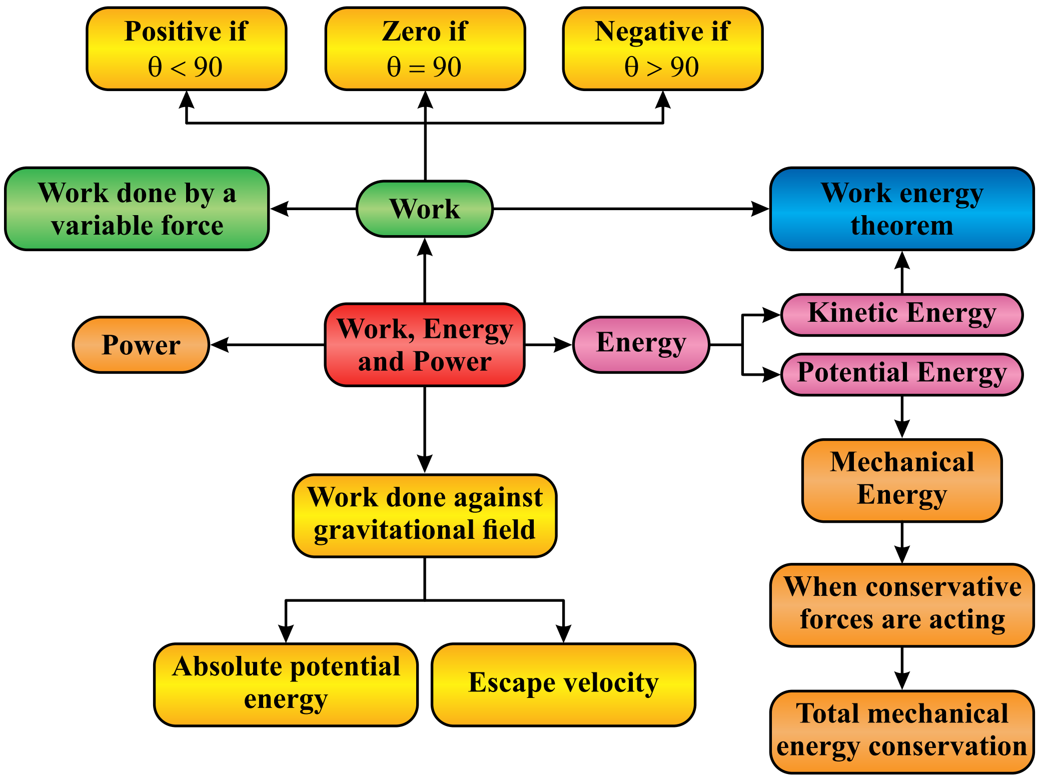
4. In rubbing our hands we do mechanical work which produces heat, i.e.

   Mechanical Energy = Heat energy + Losses

SUMMARY

- Work is said to be done on a body, when a force is applied on a body and it produces displacement in a body in the direction of force.
- If force and displacement are parallel to each other work done will be positive.
- If force and displacement are perpendicular to each other work done will be zero.
- If force and displacement are anti Parallel to each other work done on a body will be negative.
- The energy possessed by a body due to its position/configuration is called potential energy.
- When work is done on a body against the gravitational force of an earth, an energy is stored in a body called gravitational potential energy.
- The work required to lift a body from a certain point in the gravitational field to infinity is called absolute potential energy.
- Minimum velocity required a body so that it emerges out from the gravitational field of an earth called escape velocity.
- The value of escape velocity of earth is 11.2 km/s.
- The time rate of doing work is called power.
- The work done in a closed path is always zero, this field is called conservative field.
- Electric field, gravitational field and magnetic field are conservative fields.
- Frictional force/ self-adjusting force is a non-conservative force in nature.
- Work done by centripetal force is always zero.
- The scalar/dot product of force and velocity of a body is called Mechanical Power.
- The ratio of output to input is called efficiency.
- Energy can neither be created nor destroyed but transform from one from to another form.

CONCEPT MAP


Positive if θ < 90
Zero if θ = 90
Negative if θ > 90
Work done by a variable force
Work
Work energy theorem
Power
Work, Energy and Power
Energy
Kinetic Energy
Potential Energy
Mechanical Energy
Work done against gravitational field
When conservative forces are acting
Absolute potential energy
Escape velocity
Total mechanical energy conservation

EXERCISE

Section (A): Multiple Choice Questions (MCQs)

1. Work done by centripetal force is always:
   (a) Maximum (b) Minimum
   (c) Zero (d) None of these
2. A body of mass 5 kg is moving with a momentum of 10 kg m/s . A force of 0.2 N acts on it in the direction of motion of the body for 10 seconds. The increase in kinetic energy is:
   (a) 2.8 J (b) 3.2 J (c) 3.8 J (d) 4.4 J
3. The kinetic energy of a light and a heavy object is same. Which object has maximum momentum?
   (a) Light object (b) Heavy object
   (c) Both have same momentum (d) N.O.T
4. Two bodies of mass 1 kg and 2 kg have equal momentum. Then the ratio of their kinetic energies is:
   (a) 2 :1 (b) 3:1 (c) 1 :3 (d) 1:1
5. A body fallen from height h . After it has fallen a height h/2 , its will possess :
   (a) only potential energy (b) only kinetic energy
   (c) half potential half kinetic energy (d) more kinetic and less potential
6. Which of the following quantity can be calculated by multiplying force and velocity ?
   (a) acceleration (b) power (c) torque (d) work
7. The minimum velocity given to an object so that it emerges out from the gravitational field of earth is about ________________________ :
   (a) 11.2 km/s (b) 15.3 km/s (c) 5 km/s (d) 9.8 km/s
8. When one joule of work is done on a body in one second, power of body is said to be :
   (a) One watt (b) 0.5 watt (c) zero (d) 100 watt
9. The absolute potential energy of an object depends on:
   (a) The object's mass and height (b) The object's mass and speed
   (c) The object's shape and size (d) The object's color and temperature
10. The escape velocity of a planet depends on which of the following factors?
   (a) The mass of the planet only
   (b) The radius of the planet only
   (c) Both the mass and the radius of the planet
   (d) The density of the planet

Section (B): Structured Questions

CRQs:

1. How does work relate to the transfer of energy?
2. How is power related to work and time?
3. What is the difference between average power and instantaneous power?

4. How does gravitational potential energy change with height and mass?
5. What is the law of conservation of energy?
6. How does energy efficiency play a role in various energy transformations?
7. What is the work-energy theorem and how is it expressed mathematically?
8. How is the work done by a variable force calculated?

ERQs:

1. How the work done in gravitational field is independent of path
2. Calculate gravitational potential energy at a certain height due to work against gravity.
3. Describe that the gravitational PE is measured from a reference level and can be positive or negative, to denote the orientation from the reference level.
4. Show that the work done in gravitational field is independent of path
5. Define work by variable force. Calculate the work done from the force-displacement graph.

Numericals:

1. A man pulls a trolley through a distance of 10 m by applying a force of 50 N which makes an angle of 60^o with the horizontal. Calculate the work done by the man? **(Ans: 250 J)**
2. A 100 kg man runs up a long flight of stairs in 9.8 second. The vertical height of the stair is 10 m. Calculate its power? **(Ans: 1000 Watts)**
3. When an object is thrown upwards. It rises to a height 'h'. How high is the object in terms? of h, when it has lost one third of its original kinetic energy? **(Ans: h/3)**
4. A 70 kg man runs up a hill through a height of 3 m in 2 seconds.
   (a) How much work does he do against gravitational field?
   (b) What is the average power output? **(Ans: 2060 J, 1030 Watt)**
5. A neutron travels a distance of 12 m in a time interval of 3.6×10^{-4} sec. Assuming its speed was constant, Find its kinetic energy? Take the mass of neutron 1.7×10^{-27} kg. **(Ans: 5.78 e V)**
6. A stone is thrown vertically upwards and can reach to a height of 10m, find the speed of stone, when it is just 2m above the ground? **(Ans: 12.9 m/s)**
7. The potential energy of a body at the top of a building is 200 Joules when it is dropped its kinetic energy just before striking the ground is 160 Joules, find the work done against the air resistance? **(Ans: 40 J)**
8. Find the energy equivalent of 1 gram? **(Ans: 9×10^{13} J)**
9. A 1Kilowatt motor pump, pumps the water from the ground to a height of 10 m. Find, how much litres of water it can pump in one hour? **(Ans: 3.6×10^4 litre)**
10. A rocket of mass 2kg is launched in air, when it attains height of 15m the 400 Joules of its chemical fuel burns. Find the speed of rocket at maximum height? **(Ans: 10 m/s)**
11. A motor pumps the water at the rate 500 gram/minute to the height of 120 m. If the motor is 50% efficient then how much input electric power is needed? **(Ans: 20 watt)**