Unit 7 Fluid Dynamics

Teaching Periods 9 **Weightage %** 6

Wind energy form at Gharo: Energy driven through air.

In this unit student should be able to:

- Describe that real fluids are viscous fluids.
- Describe that viscous forces in a fluid cause a retarding force on an object moving through it.
- Explain how does the magnitude of the viscous force on an object moving in fluid depend on the size and velocity of the object.
- Apply Stokes' law to derive an expression for the terminal velocity of spherical body falling through a viscous fluid.
- Use the equation of terminal velocity to solve problems.
- Define the terms: steady (streamline or laminar) flow, incompressible flow and non-viscous flow as applied to the motion of an ideal fluid.
- Explain that at a sufficiently high velocity, the flow of viscous fluid undergoes a transition from laminar to turbulence conditions.
- Describe that the majority of practical examples of a fluid flow and resistance to motion in fluids involve turbulent rather than laminar conditions.
- Identify that the equation of continuity is a form of the principle of conservation of mass.
- Solve problems by using the equation of continuity.
- Describe that the pressure difference can arise from different rates of flow of a fluid (Bernoulli's Effect).
- Interpret and apply Bernoulli Effect in the: filter pump, Venturi meter, atomizers, flow of air over an aero foil and in blood physics.

Fluid Dynamics:

In our daily life, we observe the motion of fluids i.e. gases and liquids through pipes, ducts and passage ways. Examples in our daily life are – water streams from a fire hose, blood courses through our veins. Why does rising smoke curl and twist? How does a nozzle increase the speed of water emerging from nose? How does the body regulate blood flow? The physics of fluids in motion -fluid dynamics-allows us to answer these and many other queries.
Aircraft move through the air, when a fluid is in motion or an object move through a fluid, the pressure within fluid varies with velocity. The forces generated by this pressure difference were explained in 1852 by the German physicist Gustav Magnus. He resolved the problems of, why projectiles spinning about an axis other than their direction of motion will curve off course. Such spinning is played technically by bowler through bowling as swing of ball. Air dynamics is used in throwing and kicking the ball in cricket and football respectively. These phenomenon plays vital role science and technology to make our games exciting.

7.1Fluid Friction:

7.1.1 Real Fluids are viscous fluids:

Viscous fluids are fluids that resist deformation and flow. They have a high resistance to shear or flow and exhibit internal friction between adjacent layers of the fluid. This internal friction is responsible for the viscosity of the fluid.
Viscous fluids can be found in many aspects of our daily lives. Here are a few examples:

Honey or Syrup: Honey and syrups are examples of highly viscous fluids. When you pour honey or syrup from a container, it flows slowly and tends to stick to the spoon or container. The resistance to flow and the slow pouring rate are due to the high viscosity of these fluids.

Motor Oil: Motor oil used in engines also exhibits viscosity. It is designed to have a certain viscosity to provide lubrication and minimize friction between engine parts. High-viscosity motor oil is used in engines with larger clearances, while low-viscosity oil is used in engines with smaller clearances.

Paint or Varnish: Paints and varnishes also exhibit viscosity. When you apply paint to a surface, it adheres and spreads according to its viscosity. High-viscosity paints are thicker and tend to form thicker layers, while low-viscosity paints flow more easily and may result in thinner coatings.
These examples highlight the behavior of viscous fluids in different situations. Viscosity plays a crucial role in various aspects of our daily lives, from cooking to transportation to manufacturing processes. Understanding and controlling the viscosity of fluids is essential in numerous industries and scientific fields.

7.1.2Viscous force in a fluid:

Viscous force is an opposition between layers of a fluid. Viscous forces in a fluid are proportional to the rate of change of velocity of fluid's layers. The viscosity of a fluid serves as the proportionality constant.

Suppose a viscous fluid between two plates in which one plate is stationary and other is moveable as shown in (figure 7.1).

Fig: 7.1
vicous fluid between two plates

The fluid is directly in contact with each plate is held to surface by the adhesive force between the molecules of the liquid and those of the plates. Thus, the upper surface of the fluid moves with the same speed v as the upper plate, whereas the fluid in contact with the stationary plate remains stationary.

The stationary layer of fluid retards the flow of the layer just above it, which in turn retards the flow of next layer, and so on. Thus, the velocity varies continuously from 0 to v through the fluid. The increase in velocity divided by the distance over which change is made – equal to $\frac{v}{l}$ is called the ***velocity gradient*** which is defined as '***the rate of change of velocity with distance normal to the direction of flow of the layers of the fluid with respect to object passing through the fluid'***.

Viscosity is a fluid's resistance to flow. Fluids resist the relative motion of the immersed objects through them as well as to the motion of layers with differing velocities within them.

For a given fluid , it is required force F which is proportional to the area of fluid in contact with each plate 'A', and to the speed , '*v*', and is inversely proportional to the separation 'l', between the plates as shown in figure 7.1.

$$F \propto v\,\frac{A}{l}$$

For more viscous fluid ,the greater force is the required.

Hence, the proportionality constant for this equation is defined

as the coefficient of viscosity η which is intrinsic property of the fluid.

$$F = \eta A\,\frac{v}{l} \quad \text{---------------------} \quad (7.1)$$

$$\eta = \frac{F/A}{v/l}$$

The SI unit for η is $N \cdot s/m^2$ = Pa. s (Pascal. Second). In CGS system, its unit is called dyne. s/cm^2, which is called a Poise (P). The temperature has a strong effect; the viscosity of liquids such as motor oil decreases rapidly as temperature increases, while for gases it increases.

Table 7.1: Coefficients of Viscosity

Fluid (Temperature in ^{0}C)	Coefficient of viscosity, η (Pa. s)
Water (0^0)	1.8×10^{-3}
(20^0)	1.0×10^{-3}
(100^0)	0.3×10^{-3}
Whole Blood (37^0)	$\approx 4.0\times10^{-3}$
Blood Plasma (37^0)	$\approx 1.5\times10^{-3}$
Engine Oil (30^0)	2.0×10^{-1}
Glycerin (20^0)	1.5×10^{-1}
Air (20^0)	1.8×10^{-5}
Hydrogen (0^0)	9.0×10^{-6}
Water Vapor (100^0)	1.3×10^{-2}

Self-Assessment Questions:

1. What is viscosity, and how does it relate to fluid friction?
2. Describe the relationship between fluid velocity and fluid friction.

7.2.1 Derive an expression for terminal velocity of spherical body:

Stokes' law describes the drag force experienced by a small spherical object moving through a viscous fluid. The drag force acting on the object is directly proportional to its velocity and the viscosity of the fluid, and it is given by the equation:

$$\text{Drag Force} = 6\pi\eta rv \ \ldots \ldots (7.2)$$

where:
Drag Force is the force experienced by the object due to the fluid drag (measured in newtons, N).
η (eta) is the dynamic viscosity of the fluid (measured in Pascal-seconds, Pa·s or N·s/m²).
r is the radius of the spherical object (measured in meters, m).
v is the velocity of the object relative to the fluid (measured in meters per second, m/s).
The terminal velocity of a spherical body occurs when the drag force equals the gravitational force acting on the object. At terminal velocity, the net force on the body is zero, resulting in constant velocity.
Considering the gravitational force given by:

$$\text{Gravitational Force} = (4/3)\pi r^3 \rho g$$

where:
ρ (rho) is the density of the spherical body (measured in kilograms per cubic meter, kg/m³).
g is the acceleration due to gravity (measured in meters per second squared, m/s²).
At terminal velocity, the drag force is equal to the gravitational force, so we can equate the two expressions:

$$6\pi\eta rv = (4/3)\pi r^3 \rho g$$

Simplifying the equation:

$$6\eta v = (4/3) r^2 \rho g$$

Dividing both sides by 6η:

$$v = (2/9)(r^2 \rho g)/\eta \ \ldots\ldots (7.3)$$

This equation gives us the expression for the terminal velocity of a small spherical body moving through a viscous fluid according to Stokes' law. The terminal velocity is directly proportional to the square of the radius of the body (r^2), the density of the body (ρ), and the acceleration due to gravity (g), and inversely proportional to the dynamic viscosity of the fluid (η).

Self-Assessment Questions:
1. What factors affect terminal velocity?
2. Does the mass of an object affect its terminal velocity?

Worked Example 7.1

Calculate the terminal velocity of a raindrop of radius 0.2 cm. (Density of water 1000 kg/m^3 and that of air 1 kg /m^3).

Solution:

Step 1:

r = 0.2 cm

V_t = ?

Step 2:

$$v_t = \frac{2gr^2}{9\eta}\,(\rho - \sigma)$$

Step 3:

$$v_t = \frac{2\times9.8\times(0.2^{-3})^2\times999}{9\times10^{-3}}$$

V_t = 8.7 m/s

7.3 Fluids in Motion:

Fluids can move or flow in many ways. Water may flow smoothly and slowly in a quiet stream or violently over a water fall. The air may form a gentle breeze or a raging tornado. To deal with such diversity, it is necessary to classify some of the basic types of fluid flows.

7.3.1 Types of Fluids:

In ***steady flow*** the velocity of the fluid particles at any instant is constant as time passes. Every particle passing through certain point has the same velocity whereas at another location the velocity may vary, or as in river, which usually flows fastest near its center and slowed near its banks.

Streamline flow:

When a fluid flows slowly as shown along a pipe, the flow is said to be steady and lines are called streamlines. In figure (a) 7.2, they are parallel to the walls of the pipe. A streamline is a line drawn in the fluid such that a tangent to the stream – line at any point is parallel to the fluid velocity at that point. The fluid velocity can vary (in both magnitude and direction) from point to point along a streamline, but at any given point, the velocity is constant in time, as required by the condition of steady flow. Such steady flow of fluid is called ***streamline flow OR Laminar flow***

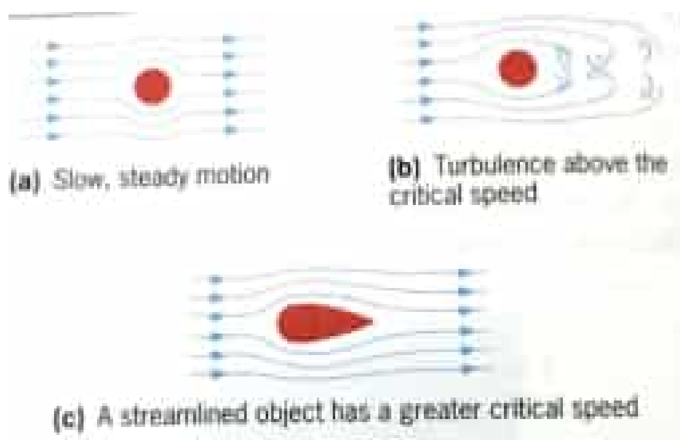


Fig: 7.2 types of fluid flow

Laminar flow refers to the smooth, orderly movement of a fluid in which layers of the fluid slide past each other in parallel. In laminar flow, the fluid moves in well-defined streamlines without any significant mixing or turbulence. This type of flow is characterized by a low Reynolds number, indicating a relatively slow and viscous flow.

Key characteristics of laminar flow include:

- Streamline Flow
- Smooth Velocity Profile
- Low Mixing and Diffusion
- Low Shear Stress
- Predictable Flow Behavior

Laminar flow is commonly observed in situations with low flow rates, small pipe diameters, and high fluid viscosity. It can be found in applications such as certain laboratory experiments, microfluidic devices, and some industrial processes that require precise control of fluid motion.

Unsteady flow:

Unsteady flow refers to the flow of a fluid that changes with time. In unsteady flow, the properties of the fluid, such as velocity, pressure, and density, vary at different locations and change over time. This is in contrast to steady flow, where the fluid properties remain constant at any given location.

Unsteady flow can occur in various fluid systems and is often associated with dynamic or transient conditions. Some examples of situations that involve unsteady flow include:

Water waves, Pulsatile blood flow, Pipe filling and draining, Turbulent flow etc.

Understanding unsteady flow is crucial in various engineering applications, such as the design of pipelines, pumps, turbines, and environmental fluid dynamics. It allows engineers and scientists to study the transient behavior of fluids and analyze the effects of time-varying flow conditions on system performance and stability

Incompressible fluid flow:

Mostly liquids are incompressible during which density of fluid remains constant even with varying pressure. In contrast, gases are highly compressible. However, there are certain situations in which the density of a flowing gas remains constant enough that the flow can be considered incompressible.

Non-viscous fluid flow:

An incompressible, non-viscous fluid is called an ***ideal fluid***, with zero viscosity flows in an unhindered manner with no dissipation of energy. Although no real fluid has zero viscosity at normal temperatures, some fluids have negligibly small viscosities.

7.3.2 Transition from Laminar to turbulent flow:

In laminar flow, the streamlines of a fluid follow smooth paths. In contrast, for a fluid in turbulent flow, vortices form, detach and, propagate. When viscous or laminar flow exceeds certain limit is said to be transforming from laminar to turbulent. Just look at cigarette smoke undergoes a transition from laminar to turbulent flow. What is the criterion that determines whether flow is laminar or turbulent?

In 19th century Reynolds investigated the conditions that would give turbulence in the flow of a fluid. ***Reynolds number***, R_e, which is ***the ratio of the typical inertial force to the viscous force and thus is a pure dimensionless number.*** The inertial force has to be proportional to the density, ρ, and the typical velocity of the fluid, $\overline{v}$, because of rate of change of momentum i.e. $F = \frac{dp}{dt}$. The viscous force is proportional to the viscosity 'η' and inversely proportional to the characteristic length scale 'L' over which the flow varies. For flow through a pipe with a circular cross-section, this length scale is the diameter of the pipe, L= 2r. Thus, the formula for calculating the Reynolds number is

DO YOU KNOW?

Make a small hole near the bottom of an open tin can. Fill the can with water, which then proceeds to spurt from the hole. If you cover the top of the can firmly with the palm of your hand, the flow stops.

$$\text{Kinematics Viscosity} = \frac{\text{Dynamic Viscosity}}{\text{Fluid density}}$$

$$\text{Rynollds no} = \frac{\text{Fluid velocity} \times \text{Internal diameter}}{\text{Kinematic Viscosity}}$$

hy

$$R_e = \frac{\rho\, \overline{v} L}{\eta} \;\ldots\ldots(7.4)$$

The velocity of liquid flow is given by

$$V = \frac{R_e\, \eta}{2\rho} \;\ldots\ldots(7.5)$$

V= Speed of fluid
r = radius
η= Viscosity
ρ=fluid density
R_e = Reynolds Number

The Reynolds number is important in analyzing the type of flow, when there is substantial velocity gradient. The flow is determined by using following standards:

. **Laminar** when *Re < 2300*
. **Transient** when *2300 < Re < 4000*

Transient flow, is flow where the flow velocity and pressure are changing with time. When changes occur to a fluid system such as the starting or stopping of a pump, closing or opening a valve, or changes in tank levels, then transient flow conditions exist: otherwise the system is steady state.

. **Turbulent** when *4000 < Re*

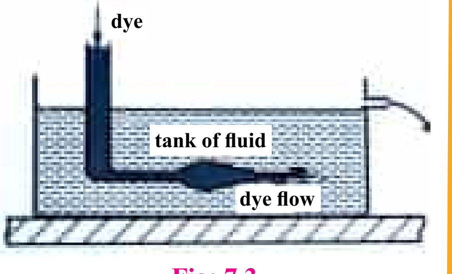
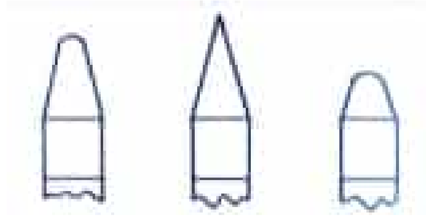


Fig: 7.3

The flow of liquid can easily be demonstrated with the apparatus shown in figure 7.3. A dye flows from the tube into the tank and by altering the pressure head the flow can be made either turbulent or laminar. As you can see in figure 7.3, the flow becomes turbulent when the line ceases to be straight.

The streamlining of bodies are most important in the design of cars, submarines and nose cones of aircrafts and rockets, since a reduction in drag can reduce vibration and also save large amounts of fuel. Figure 7.4 shows the best shapes for rocket cones for the ***subsonic***, ***supersonic*** and ***hypersonic*** flight respectively.



subsonic *supersonic* *hypersonic*

Fig: 7.4

Worked Example 7.2

The volume rate of an air conditioning system to be 3.84×10^{-3} m^3/s. The air is sent through an insulated, round conduit with a diameter of 18 cm. This calculation assumed laminar flow. (a) Was this a good assumption? (b) At what velocity would the flow become turbulent?

Solution:

Step 1:

Volume rate = 3.84×10^{-3} m^3/s.

η of air = 0.0181 m Pa.s

ρ of air = 1.23 kg/m^3

diameter = 18 cm

Velocity =?

Step 2:

Volume rate = Av

$3.84\times10^{-3} = \pi\,(0.09)\,v$

v = 0.15 m/s

Step 3:

$$R_e = \frac{2\rho v r}{\eta}$$

$$R_e = \frac{2\times(1.23)\times0.15\times0.09}{0.0181\times10^{-3}}$$

R_e = 1835

Since the Reynolds number is 1835 < 2000, the flow is laminar and not turbulent. The assumption that the flow was laminar is valid.

Step 4:

To find the maximum speed of the air to keep the flow laminar, consider the Reynolds number

$$R_e = \frac{2\rho \quad r}{\eta} \le 2000$$

$$v = \frac{2000(0.0181\times10^{-3})}{2(1.23)(0.09)}$$

$$v = 0.16\ m/s$$

Significance:

When transferring a fluid from one point to another, it desirable to limit turbulence. Turbulence results in wasted energy, as some of the energy intended to move the fluid is dissipated when eddies are formed. In this case, the air conditioning system will become less efficient once the velocity exceeds 0.16 m/s, since this is the point at which turbulence will begin to occur.

7.3.3 Fluid flow and resistance to motion in fluids involve turbulent rather than laminar conditions:

In practical examples of fluid flow and resistance to motion in fluids, turbulent conditions are more prevalent than laminar conditions. Turbulent flow is a type of fluid motion characterized by chaotic and irregular movement of fluid particles, resulting in whirlpool, fluctuations in velocity and pressure. On the other hand, laminar flow is smooth, ordered, and occurs in layers without any significant mixing or swirling.

Several factors contribute to the prevalence of turbulent flow in practical scenarios:

High Reynolds Numbers: Turbulent flow is more likely to occur at higher Reynolds numbers, which is a dimensionless parameter that measures the ratio of inertial forces to viscous forces in the fluid. In many real-world applications, such as in industrial processes, transportation, and natural phenomena like rivers and atmospheric flows, the Reynolds numbers are often large enough to induce turbulent behavior.

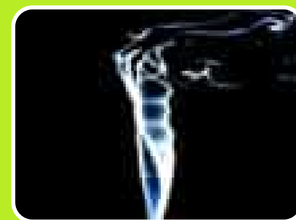
Rough Surfaces: When fluid flows over rough surfaces, such as in pipes, channels, or around objects, it can lead to turbulent flow due to the disruption of the fluid layers. Turbulence enhances the mixing of fluid components and affects the resistance to motion.

High Velocities: High flow velocities can promote turbulent behavior, especially in situations where the fluid encounters sudden changes in cross-sectional area or flow direction as shown in figure b.

Agitation and Stirring: In industrial processes, mixing, and agitation systems, turbulent flow is deliberately induced to ensure efficient mixing of substances and heat transfer.

Pressure Gradients: Rapid changes in pressure along the flow path can lead to turbulent flow patterns, as the fluid tries to adjust to the varying conditions.

DO YOU KNOW?



Smoke rises smoothly for a while and then begins to form swirls and eddies. The smooth flow is called laminar flow, whereas the swirls and eddies typify turbulent flow. Smoke rises more rapidly when flowing smoothly than after it becomes turbulent, suggesting that turbulence poses more resistance to flow.

Fig: 7.5

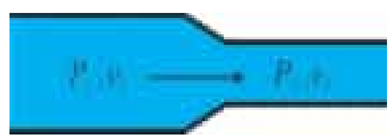


Fig: 7.6

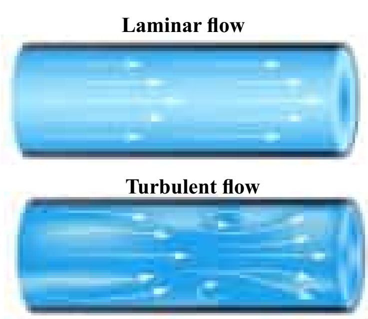


Fig: 7.7

Examples of practical situations involving turbulent flow include:
- Airflow around vehicles, airplanes, and wind turbines as depicted in figure a.
- Fluid flow in pipes, especially in scenarios with high flow rates or rough internal surfaces as shown in figure c.
- Rivers and streams, where the irregularities in the bed and banks induce turbulence.
- Ocean currents and waves, which often involve turbulent motion.
- Mixing processes in chemical reactors, industrial tanks, and bioreactors.
- Combustion in engines and furnaces, where turbulent mixing of fuel and air improves combustion efficiency.

While laminar flow can occur in specific situations, such as slow and smooth flow in small tubes or in some laboratory setups, turbulent flow dominates in most real world fluid flow applications due to its complex and dynamic behavior, which significantly influences resistance to motion and various other fluid phenomena.

7.4 Equation of Continuity:

Suppose a steady laminar flow of a fluid through an enclosed tube or pipe as shown in figure the speed of the fluid varies with the diameter of the tube variation. The mass flow rate which follows the ***law conservation of mass*** is defined as **the mass Δm of fluid that passes at given point per unit time Δt**:

mass flow rate $= \frac{\Delta m}{\Delta t}$ ……(7.6)

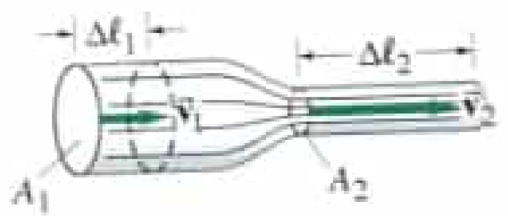


Fig: 7.8

In figure 7.8, the volume of a fluid passing through area A_1 in a time Δt is $A_1\Delta l_1$, where Δl_1 is the distance the fluid moves in time Δt. The velocity of fluid passing through A_1 is $v_1 = \Delta m_1/\Delta t$. Then the mass flow rate:

$$\frac{\Delta m_1}{\Delta t} = \frac{\rho_1\ \Delta V_1}{\Delta t}$$
$$= \frac{\rho_1\, A_1\, \Delta l_1}{\Delta t} = \rho_1\, A_1 v_1$$

Where $\Delta V_1 = A_1\Delta l_1$ is the volume of mass Δm. Similarly, through A_2, the flow rate is $\rho_2 A_2 v_2$. Since no fluid flows in or out the sides of the tube, the flow rates through A_1 and A_2 must be equal.

$$\frac{\Delta m_1}{\Delta t} = \frac{\Delta m_2}{\Delta t} \quad \text{……(7.7)}$$

and

$$\rho_1\, A_1 v_1 = \rho_2\, A_2 v_2$$

This is called the equation of continuity.

If the fluid is incompressible which is an excellent approximation for liquids under most circumstances, then $\rho_1 = \rho_2$, the equation of continuity is

$A_1 V_1 = A_2\, V_2$ (ρ is constant) ……(7.8)

The product A*V* is the volume rate of flow which is defined as ***the volume of fluid passing a given point per second.***

$$\frac{\Delta v}{\Delta t} = A\frac{\Delta l}{\Delta t} = Av \text{ ……(7.9)}$$

Its SI unit is m^3/s.

The product of area and velocity is describing that where the cross-sectional area is large, the velocity is small, and where the area is small, the velocity is large.

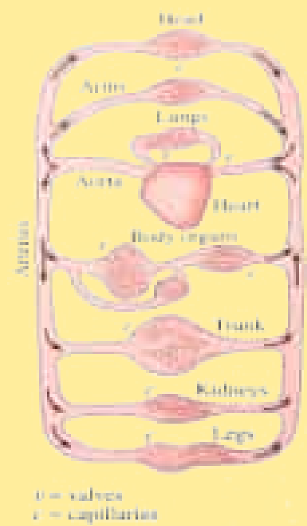
Self-Assessment Questions:
1. State the equation of continuity in terms of fluid flow.
2. How does the equation of continuity relate to conservation of mass?
3. How does the velocity of a fluid change when it flows through a constricted pipe?

Worked Example 7.3

The radius of aorta is about 1.2 cm, and the blood passing through it has a speed of about 40 cm/s. A typical capillary has a radius of about 4×10^{-4} cm, and blood flows through it at a speed of about 5×10^{-4} m/s. Estimate the number of capillaries that are in the body.

Step 1:

radius of aorta = 1.2 cm
speed in = 40 cm/s
Capillary radius = 4×10^{-4} cm
Speed out = 5×10^{-4} m/s
No of capillaries =?



Let A_1 be the area of the aorta and A_2 be the area of all the capillaries through which blood flows. Then $A_2 = N\pi r_{cap}^2$, where $r_{cap} = 4\times10^{-4}$ cm is the estimated average radius of one capillary. From equation of continuity, we have

$$v_2 A_2 = v_1 A_1$$

$$v_2 \, N \, r_{cap}^2 = v_1 \; r_{aorta}^2$$

$$N = \frac{v_1}{v_2}\,\frac{r_{aorta}^2}{r_{cap}^2}$$

$N = (0.40 \text{ m/s} / 5\times10^{-4})\,(1.2\times10^{-2} \text{ m} / 4\times10^{-6} \text{ m})^2$

$N = 7\times10^9$

on the order of billion capillaries.

Assume that blood density is same from aorta to capillaries. By equation of continuity, the volume flow rate in the aorta must equal the volume flow rate through all the capillaries. The total area of all the capillaries is given by the area of a typical capillary multiplied by the total number N of capillaries.

7.5 Bernoulli's Principle:

When a fluid is in motion the pressure within the fluid flow varies with velocity of the fluid for streamlined flow. This pressure variation is a consequence of Bernoulli's theorem proposed in 1740. Bernoulli's principle states that *the velocity of a fluid is high, the pressure is low, and the velocity is low, the pressure is high.*

DO YOU KNOW?

When a truck moves very fast, it created a low pressure area, so dusts are being pulled along in the low pressure area. If we stand very close to railway track in the platform, when a fast train passes us, we get pulled towards the track because of the low pressure area generated by the sheer speed of the train.

7.5.1 Bernoulli's Equation:

Bernoulli's equation deals with energy conservation law for the steady-state flow of incompressible fluids, such as water. It relates the energy of the fluid in terms of its pressure, velocity, and height.

Bernoulli's equation can be derived from the first principle using the law of conservation of energy. According to energy conservation principles, energy can neither be created nor destroyed. Therefore, during streamline flow, the total mechanical energy remains constant. A few assumptions need to be made before deriving the equation.

Assumptions

- The flow must be steady and streamline.
- The fluid is incompressible the density should remain constant at all points during the flow.
- There are no viscous forces in the fluid, and friction is negligible.

Consider a pipe whose diameter and elevation change as a fluid passes through it. Consider a small mass of the fluid with density ρ that flows from point 1 to 2 as shown in figure 7.9. The work done by a force F on the fluid to displace it by an infinitesimal distance Δx is given by,

$$W = F\Delta x$$

Therefore, at points 1 and 2, the work done are,

$$\Delta W_1 = F_1 \Delta x_1$$
$$\Delta W_2 = F_2 \Delta x_2$$

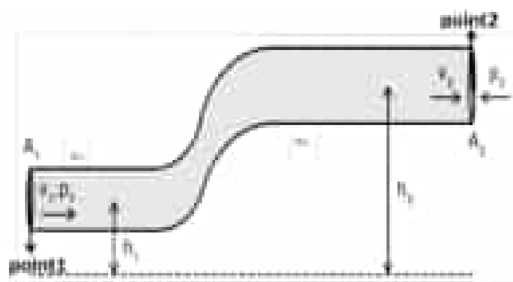


Fig: 7.9 fluid passthrough pipe

Total work done when the fluid moves 1 to 2 is,

$$\Delta W = \Delta W_1 - \Delta W_2$$
$$\text{or}, \Delta W = F_1 \Delta x_1 - F_2 \Delta x_2$$

Force is the product of pressure and area $(F = pA)$, and the volume is the product of length and cross – sectional area $(V = Ax)$. Therefore,

$$\Delta W = p_1A_1\Delta x_1 - p_2A_2\Delta x_2 = p_1\Delta V - p_2\Delta V = (p_1 - p_2)\Delta V$$

Now, when the fluid moves from point 1 to 2, there is a change of kinetic energy.

$$\Delta K.E = \frac{1}{2} m_2v_2^2 - \frac{1}{2} m_1v_1^2 = \frac{1}{2}\rho\Delta V v_2^2 - \frac{1}{2}\rho\Delta V v_1^2 = \frac{1}{2}\rho\Delta V(v_2^2 - v_1^2) \text{ ......(7.10)}$$

Similarly, the change in potential energy is given by,

$$\Delta U = mgh_2 - mgh_1 = \rho\Delta V g(h_2 - h_1) \text{ ......(7.11)}$$

The work done in moving the fluid is the sum of the change in kinetic and potential energies.

$$\Delta W = \Delta K.E + \Delta U$$

$$\text{or, } (p_1 - p_2)\Delta V = \frac{1}{2}\rho\Delta V(v_2^2 - v_1^2) + \rho\Delta V g(h_2 - h_1)$$

$$\text{or, } p_1 - p_2 = \frac{1}{2}\rho(v_2^2 - v_1^2) + \rho g(h_2 - h_1)$$

$$p_1 + \frac{1}{2}\rho v_1^2 + \rho g h_1 = p_2 + \frac{1}{2}\rho v_2^2 + \rho g h_2$$

$$\text{or, } p + \frac{1}{2}\rho V^2 + \rho g h = \text{constant}$$

This is Bernoulli's equation.

Therefore

$P + ½\rho V^2 + \rho g h = \text{Constant}$ ......(7.12)

The height h is measured from some convenient reference level, for example at the surface of the liquid. The equation is really a statement of conservation of energy per unit volume in the fluid**.**

- $½\rho V^2 + \rho g h$ is total pressure.
- $\rho g h$ is static pressure.
- $½\rho V^2$ is dynamic pressure.

Worked Example 7.4

Water leaves the jet of a horizontal hose at 10 m/s, if the velocity of water within the hose is 0.4 m/s, calculate the pressure P within the hose. (Density of water 1000 kg/m^3 and atmospheric pressure is 100000 Pa).

Solution:

Step 1:

$V_1 = 10$ m/s

$V_2 = 0.4$ m/s

P = ?

Density of water = 1000 kg/m^3

Atmospheric Pressure = 100000 Pa

Step 2:

Here $h_1 = h_2$, so Bernoulli's equation becomes

$P_1 + ½\rho V_1^2 = P_2 + ½\rho V_2^2$

Step 3:

$100000 + ½ \times 1000 \times 100 = P_2 + ½ \times 1000 \times 0.16$

$P_2 = 1.5 \times 10^{5}$ Pa

7.5.2 Applications of Bernoulli's Principle:

Bernoulli's principle, also known as the Bernoulli effect, describes the relationship between fluid velocity and pressure in a flowing fluid. It states that in an ideal, incompressible, and non-viscous fluid, the sum of the fluid's kinetic energy (velocity) and potential energy (pressure) remains constant along a streamline. This principle has various practical applications, including those mentioned in your question:

Filter Pump: In a filter pump, Bernoulli's principle is applied to increase the pressure of the fluid passing through the pump. By reducing the cross-sectional area of the pump's outlet, the fluid's velocity increases according to the principle, leading to a decrease in pressure (as kinetic energy increases). This pressure drop helps draw fluid into the pump and through the filter medium, facilitating the filtration process.

Fig: 7.10 Filter pump

Venturi Meter: A Venturi meter is a device used to measure the flow rate of a fluid in a pipe. It consists of a gradually narrowing tube (Venturi tube) inserted in the pipe. As the fluid flows through the narrowing section, its velocity increases according to Bernoulli's principle, and the pressure decreases. By measuring the pressure difference between the narrowest section and the wider parts of the pipe, the flow rate can be determined.

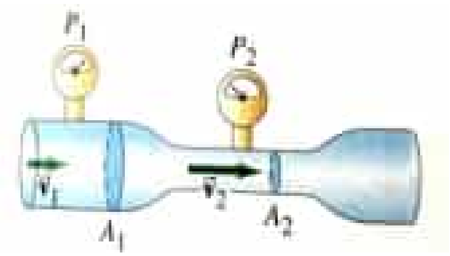


Fig: 7.11 Venturi meter

Atomizer: Atomizers are devices used to convert liquids into fine sprays or mists. The application of Bernoulli's principle is crucial in this process. As the liquid passes through a small nozzle in the atomizer, its velocity increases, leading to a decrease in pressure according to the principle. This decrease in pressure facilitates the breakup of the liquid into tiny droplets or a fine spray.

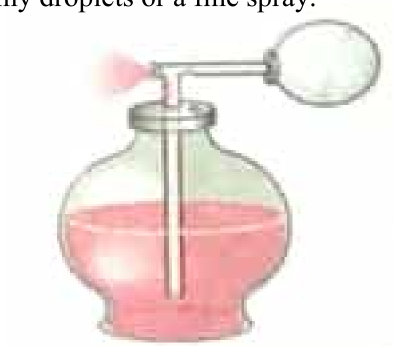


Fig: 7.12 Atomizer

DO YOU KNOW?

Why does smoke go up a chimney?

It's partly because hot air rises as its less dense and therefore buoyant. When wind blows across the top of a chimney, the pressure is less there than inside the house. Hence, air and smoke are pushed up the chimney by the higher indoor pressure. Even on an apparently still night there is usually enough ambient air flow at the top of a chimney to assist upward flow of smoke.

Flow of Air over an Aerofoil: When air flows over an aerofoil (such as the wing of an aircraft), the shape of the aerofoil causes the air to travel faster over the top surface compared to the bottom surface. According to Bernoulli's principle, the air pressure decreases over the top surface due to the increased velocity, creating a pressure difference that results in lift, allowing the aircraft to stay airborne

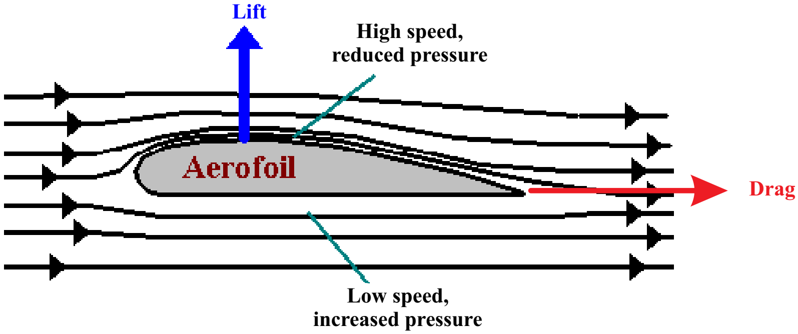


Fig: 7.13 An aerofoil

Blood Physics: In the circulatory system, Bernoulli's principle is applicable to the flow of blood through blood vessels, particularly in areas of constriction or stenosis. When the diameter of a blood vessel narrows, the blood velocity increases, leading to a decrease in pressure, which can have clinical implications in conditions such as atherosclerosis or stenosis.

It's essential to note that while Bernoulli's principle is an excellent theoretical tool for understanding fluid behavior in these applications, real-world fluids may have additional complexities, such as viscosity and compressibility, which need to be considered for precise calculations and analyses.

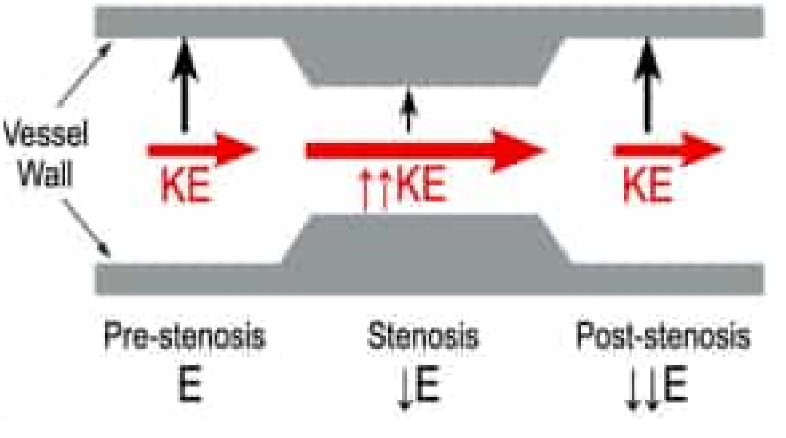
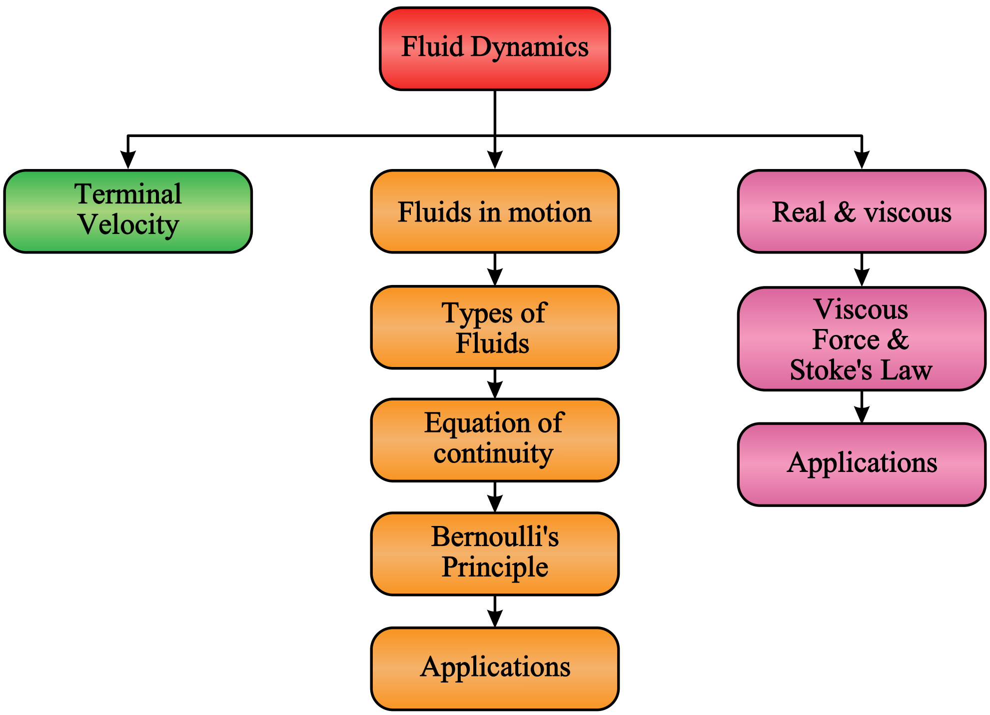


Fig: 7.14
Stenosis or pre stenosis of blood vessels walls

SUMMARY

- The physics of fluids in motion is Fluid Dynamics:
- A frictional force between adjacent layers of fluid as the layers move past one another is called Viscosity:
- The rate of change of velocity with distance normal to the direction of flow of the layers of the fluid with respect to object passing through the fluid is known as Velocity Gradient:
- When fluid resistance of a falling object equals its weight, the net force is zero and no further acceleration occurs is called Terminal Velocity:
- The velocity of the fluid particles at any is constant as time passes is called Steady flow: Whenever the velocity at a point in the fluid changes as time passes, such flow becomes turbulent is known as unsteady flow:
- An incompressible, non-viscous fluid is called an ***ideal fluid***,
- The sliding of layers of fluid parallel and smoothly to direction of flow is Laminar Flow:
- The ratio of the typical inertial force to the viscous force and thus is a pure dimensionless number is called Reynolds Number:
- The velocity of a fluid is high, the pressure is low, and the velocity is low, the pressure is high is known as Bernoulli's Principle:

CONCEPT MAP


Fluid Dynamics
Terminal Velocity
Fluids in motion
Real & viscous
Types of Fluids
Viscous Force & Stoke's Law
Equation of continuity
Applications
Bernoulli's Principle
Applications

EXERCISE

Section (A): Multiple Choice Questions (MCQs)

1. For an incompressible fluid, the flow rate is
   a) equal for all surfaces.
   b) constant throughout the pipe.
   c) greater for the larger parts of the pipe.
   d) none of the above

2. Bernoulli's principle states that for horizontal flow of a fluid through a tube, the sum of the pressure and energy of motion per unit volume is
   a) increasing with time
   b) decreasing with time
   c) constant
   d) varying with time

3. Which of the following is associated with the law of conservation of energy in fluids?
   a) Archimedes' principle
   b) Bernoulli's principle
   c) Pascal's principle
   d) equation of continuity

4. As the speed of a moving fluid increases, the pressure in the fluid
   a) increases
   b) remains constant
   c) decreases
   d) may increase or decrease, depending on the viscosity

5. If the cross-sectional area of a pipe decreases, what happens to the fluid velocity?
   a) Increases
   b) Decreases
   c) Remains the same
   d) Depends on the fluid density

6. A sky diver falls through the air at terminal velocity. The force of air resistance on him is
   a) half his weight
   b) equal to his weight
   c) twice his weight
   d) Cannot be determined from the information given.

7. Wind speeding up as it blows over the top of a hill
   a) Increases atmospheric pressure there.
   b) decreases atmospheric pressure there.
   c) doesn't affect atmospheric pressure there.
   d) equal's atmospheric pressure.

8. A fluid is undergoing "incompressible" flow. This means that:
   a) the pressure at a given point cannot change with time
   b) the velocity at a given point cannot change with time
   c) the velocity must be the same everywhere
   d) the pressure must be the same everywhere
   e) the density cannot change with time or location

9. A fluid is undergoing steady flow. Therefore:
   a) the velocity of any given molecule of fluid does not change
   b) the pressure does not vary from point to point
   c) the velocity at any given point does not vary with time
   d) the density does not vary from point to point

10. The equation of continuity for fluid flow can be derived from the conservation of:
   a) energy
   b) mass
   c) volume
   d) pressure

Section (B): Structured Questions

CRQ's:

1. What is difference between streamline and turbulent flow?
2. Would a drinking straw work in space where there is no gravity? Explain.
3. Why do airplanes take off into wind?
4. Describe terminal velocity in liquids.
5. Discuss the significance of Reynolds number.
6. State Bernoulli's principle.
7. Give two applications of Bernoulli's principle.
8. 'Fluid flow is turbulent rather than laminar', support this statement.
9. Discuss importance of Stokes law.
10. Justify spin of ball in Bernoulli's principle.

ERQ's:

1. Derive equation of continuity. Also show its physical significance.
2. Derive Bernoulli's equation.
3. Discuss viscous force in fluids.
4. Define fluid dynamics and explain its significance in the study of fluids. How does it differ from fluid statics?
5. Discuss the concept of Reynolds number and its significance in fluid dynamics. Explain how Reynolds number relates to the transition between laminar and turbulent flow.

Numericals:

1. Two spherical raindrops of equal size are falling through air at a velocity of 0.08 m/s. If the drops join together forming a large spherical drop, what will be the new terminal velocity? **(0.13 m/s)**

2. Calculate the viscous drag on a drop of oil of 0.1 mm radius falling through air at tis terminal velocity. (Viscosity of air = 1.8×10^{-5} Pa. s; density of oil = 850 kg / m^3) **(3.48×10^{-8} N)**

3. What area must a heating duct have if air moving 3.0 m/s along it can replenish the air every 15 minutes in a room of volume 300 m^3. Assume air density remains constant. **(0.11 m^2, 0.33 m)**

4. Water circulates throughout a house in a hot-water heating system. If the water is pumped at a speed of 0.50 m/s through a 4.0 cm diameter pipe in the basement under a pressure of 3.0 atm, what will be the flow speed and pressure in a 2.6 cm diameter pipe on the second floor 5.0 m above? Assume the pipes do not divide into branches. **(1.2 m/s, 2.5 atm)**

5. What is the volume rate of flow of water from a 1.85 cm diameter faucet if the pressure head is 12 m? **(4.6×10^{-3} m^3/s)**

6. The stream of water emerging from a faucet 'neck down' as it falls. The cross-sectional area is 1.2 cm^2 and 0.35 cm^2. The two levels are separated by a vertical distance of 45 mm as shown in figure. At what rate does water flow from the tap? **(34 cm^3 / s)**

7. Water leaves the jet of a horizontal hose at 10 m/s. If the velocity of water within the hose is 0.40 m/s, calculate the pressure within the hose. Density of water is 1000 kg/m^3 and atmospheric pressure is 100000 Pa. **(1.5×10^5 Pa)**

8. What is the maximum weight of an aircraft with a wing area of 50 m^2 flying horizontally, is the velocity of the air over the upper surface of the wing is 150 m/s and that the lower surface 140 m/s ? Density of air is 1.29 kg/m^3 **(9.3×10^4 N)**

9. A liquid flows through a pipe with a diameter of 0.50 m at a speed of 4.20 m/s . What is the rate of flow in L/min? **(49500 L/min)**

10. Calculate the average speed of blood flow in the major arteries of the body, which have a total cross-sectional area of about 2.1 cm^2. Use the data of example **(13.6 cm/s)**