Chapter 12.

ELECTROSTATICS

12.1 Electrons and protons are characterized by a property by virtue of which they exert forces of attraction and repulsion on one another. This property is called charge. On rubbing a body with a suitable material both acquire equal amounts of opposite charges due to transfer of some outer most loosely bound electrons from one body to the other the electric charge is, therefore, conserved. In this chapter we shall investigate quantitatively the force exerted by one charge over another, and the potential at a point in the neighborhood of charged body and charge storing devices etc.

12.2. Coulomb's Law

The first experiment to investigate quantitative law of force between localized charges was carried out by Charles Augustin de Coulomb in 1784. using a torsion balance. The results of the experiment can be stated in the form of a law called coulomb's law. The electric force between two static point charges varies directly to the product of charges with each charge and inversely with the square of the distance between them. If q_1 and q_2 are two point charges distance "r" apart as shown in fig. 12.1.

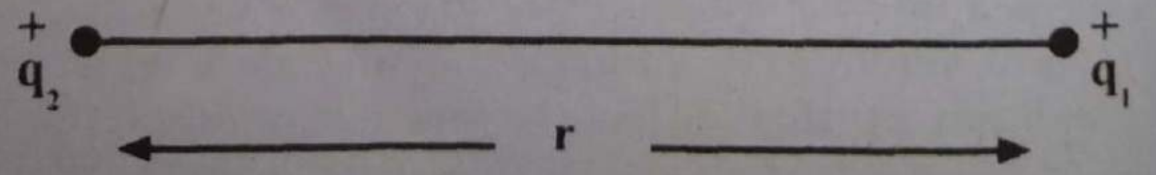


Fig. (12.1)

$$F = K\frac{q_1 q_2}{r^2} \quad \text{------------------ (12.1)}$$

Where K is the constant of proportionality and its value depends on the medium between the charges. The magnitude as well as the direction of force can be represented by the vector equation

$$\overline{F}_{12} = K\frac{q_1 q_2}{r^2}\hat{r}_{12} \quad \text{--------------(12.2)}$$

Where $\overline{F}_{12}$ is the force exerted by q_1 on q_2 and $\hat{r}_{12}$ is unit vector along the line joining the two charges from q_1 to q_2.

In case the charges are similar the force is that of repulsion and vice versa.

In SI units the unit of electric charge is coulomb.The unit coulomb which is defined as the amount of charge that flows through a given cross section of a wire in one second if there is a steady current of 1 ampere in the wire.The adoption of unit of current will be explained later.The measured value of K for free space is

$$K = 8.98755 \times 10^9 \ N.m^2.C^{-2}$$

In order to express other equations which will appear in the theory later in a simple form K is expressed in terms of 4π and another constant ϵ_0 called the permitivity of free space

i.e. $K = \frac{1}{4\pi\epsilon_0} = 9 \times 10^9 \ Nm^2C^{-2}$ the value of ϵ_0 is $8.85 \times 10^{-12} \ C^2/(Nm^2)$ i.e.

$$F = \frac{1}{4\pi\epsilon_0} \times \frac{q_1 q_2}{r^2} \quad \text{-------------------(12.3)}$$

The value of the constant of proportionality for a medium other than free space between the charges is less and it is written as

$$\frac{1}{4\pi\epsilon_0\epsilon_r} \text{ or } \frac{1}{4\pi\epsilon} \text{ where } \epsilon = \epsilon_0\epsilon_r \text{ is the}$$

permittivity of the medium

$$\epsilon_r = \frac{\epsilon}{\epsilon_0} \text{ is called relative permittivity of the medium.}$$

The force between two given charges, therefore, decreases by introducing a non conducting medium between them and eq. (12.3) can be written as

$$F = \frac{1}{4\pi\epsilon}\frac{q_1 q_2}{r^2} \quad \text{---------------------(14.4)}$$

The magnitude of charge on an electron is

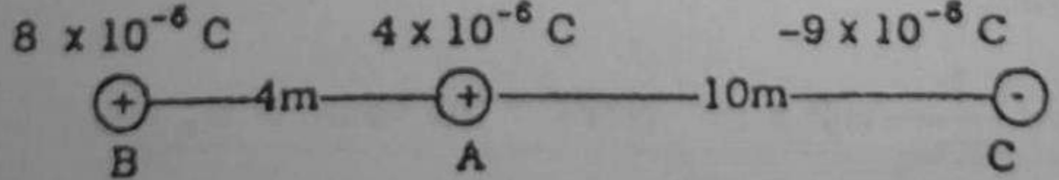
$e = 1.6 \times 10^{-19}$ C.

Thus charge of 1 coulomb is 6.25×10^{18} times the electronic charge.

Example 12.1

A point charge A of $+ 4 \times 10^{-6}$ Coulomb is placed on a line between two charges B of $+ 8 \times 10^{-6}$ C and C of $- 9 \times 10^{-6}$ C. The charge A is 4m from B and 10m from C. What is the force on A?

Solution.



Force on A due to B $= F_1 = K \dfrac{q_1 q_2}{r^2}$

$$= 9\times 10^{9} \frac{8\times 10^{-6} \times 4 \times 10^{-6}}{(4)^2} = \frac{9 \times 8 \times 4}{16} \times 10^{-3}$$

$= 18 \times 10^{-3}$ N, towards C

Force on A due to C = $F_2 = 9 \times 10^9 \times \frac{4 \times 10^{-6} \times (9) \times 10^{-6}}{(10)^2}$

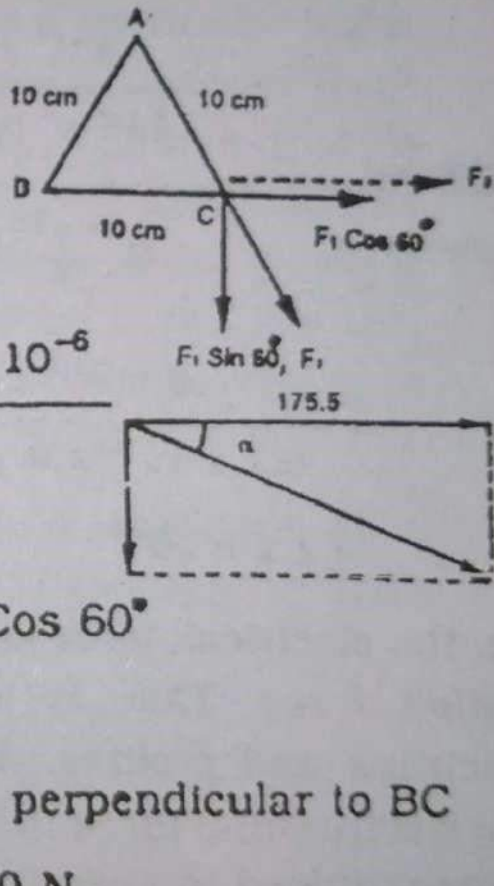
$$= \frac{9 \times 4 \times 9}{100} \times 10^{-3} = 3.24 \times 10^{-3} C, \text{ towards } C.$$

Since these two component forces are in the same direction, the resultant force is their arithmetic sum

$F_1 + F_2 = 21.24 \times 10^{-3}$ N, towards C.

Example 12.2.

Two charges of magnitudes +10 μ C and + 8 μ C are placed on the corners A and B of an equilateral triangle of sides 10 cm. Find the force on a charge of + 15 μ C placed at the third corner C.



Solution

Force on the charge at C due to charge at A = F_1

$$= 9 \times 10^9 \frac{10 \times 10^{-6} \times 15 \times 10^{-6}}{(.1)^2}$$

= 135 N along AC

Component of F_1 along BC = F_1 Cos 60°

= 135 x .5 = 67.5 N

Component of F_1 in the direction perpendicular to BC

= F_1 Sin 60° = 135 x .866 = 116.9 N.

Force on the charge at C due to charge at B along BC

$$= F_2 = 9 \times 10^9 \frac{8 \times 10^{-6} \times 15 \times 10^{-6}}{(.1)^2} = 108 N$$

∴Total force along BC= $F_1 \cos\theta + F_2 = 67.5 + 108 = 175.5$ N

∴Resultant force on the charge at C

$$= \sqrt{(175.5)^2 + (116.9)^2} = 210.86N$$

The resultant force is in a direction making an angle α with BC such that

$$\alpha = \tan^{-1} \frac{116.9}{175.5} = 33.67°.$$

Example 12.3

The distance between the electron and the proton of Hydrogen atom is about 5.3×10^{-11} m. Compare the electric and the gravitational forces between these two particles.

Solution.

$$\frac{F_{electrical}}{F_{gravitation}} = \frac{\frac{1}{4\pi\epsilon_0}\frac{e^2}{r_2}}{G\frac{m_e m_p}{r^2}}$$

$$= \frac{9 \times 10^9 \times (1.6 \times 10^{-19})^2}{6.7 \times 10^{-11} \times 9.1 \times 10^{-31} \times 1.7 \times 10^{-27}}$$

$$= 2.2 \times 10^{39}$$

i.e. the electrical force is 10^{39} times greater than the gravitation force. That is why gravitational force between electrons and protons of an atom is neglected and only the electrostatic force is taken to provide the centripetal force required to keep the electrons in rotation round the nucleus.

12.3 Electric Field

Before Maxwell and Faraday the forces that act between electric charges were thought of as direct and instantaneous interaction between them and it was called action at a distance.

Faraday conceived that the force between electric charges separated from one another is not a direct action at a distance but it is transmitted through intervening space whether that may be empty or occupied by matter.

An electric charge modifies the space around it such that any other charge brought in this region experiences force of electrical nature.

The modified space around the electric charges is called electric field In other words electric field is a region around a charge body in which another charge experiences an electric force.

(i) **Intensity of electric field**

The intensity of electric field at a particular point is greater if an electric charge experiences a greater force at that point. The difficulty is that the charge that is brought into the field will produce its own field and the originally existed field will be changed. In order to avoid it the field is supposed to be explored by a positive test charge + q_0 of very small magnitude (approaching to zero) so that the original field is not disturbed.

The force experienced by this test charge at a point per unit charge is the measure of intensity of field at that point in the direction of force

$$\vec{E} = \frac{\vec{F}}{q_0} \quad \text{-------------------(12.5)}$$

The electric intensity is directed from a positive charge and toward the charge in case of a negative charge.

The S.I. unit of electric field intensity is newton/coulomb. (NC^{-1}).

ii) Electric field Intensity near an isolated point charge q

Imagine a very small positive point charge q_0 placed at a distance r from point charge q_0 (fig.12.2)

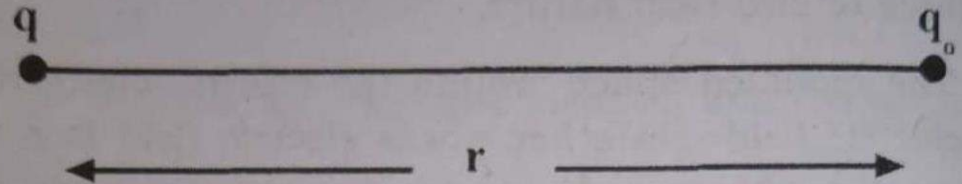


Fig. (12.2)

The magnitude of force on q_0 due to the charge q is

$$F = \frac{1}{4\pi\epsilon_0} \frac{q q_0}{r_2}$$

∴ Electric field Intensity

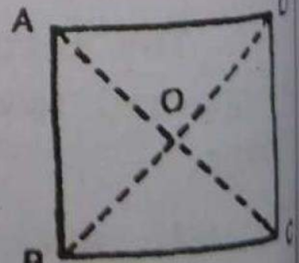
$$E = \frac{F}{q_0} = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2} \quad \text{------------------ (12.6)}$$

The direction of the intensity of electric field is that of the electric force.

Example 12.4

Charges each of +3 μ C are placed at three corners of a square whose diagonal is 6 cm long. Find the field intensity at the point of inter section of diagonals.

Solution



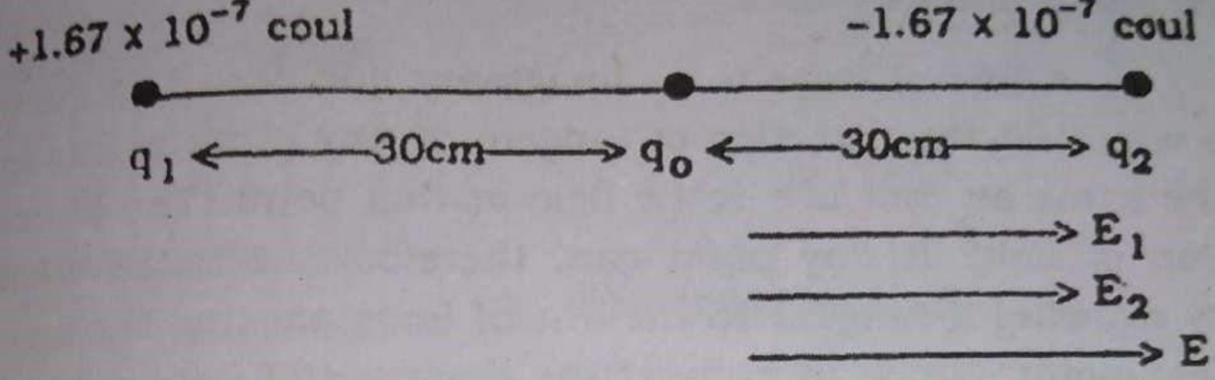
The fields at O due to charges at A and C mutually cancel each other being equal and opposite. The net field

$$E = 9 \times 10^9 \frac{3 \times 10^{-6}}{(3 \times 10^{-2})^2} = 3 \times 10^7 NC^{-1}$$

Example 12.5.

Find the electric intensity midway between the charges $+1.67 \times 10^{-7}$ coul and -1.67×10^{-7} coul separated by a distance of 60 cm

Solution



$$E = E_1 + E_2 = \frac{1}{4\pi\epsilon_0}\frac{q_1}{r_1^2} + \frac{1}{4\pi\epsilon_0}\frac{q_2}{r_2^2}$$

Since $q_1 = q_2$ and $r_1 = r_2$

$$E = 2\frac{1}{4\pi\epsilon_0}\frac{q_1}{r_1^2} = \frac{2 \times 9 \times 10^9 \times 1.67 \times 10^{-7}}{(.3)^2}$$

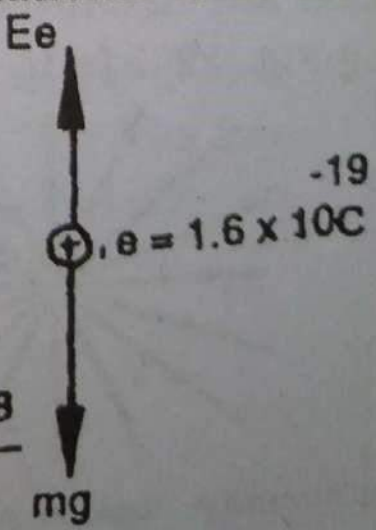
$$= 3.33 \times 10^4\ NC^{-1}$$

The field is directed along the line from q_1 to q_2.

Example 12.6.

Find the magnitude and direction of the electric field that will counter balance the gravitational force on a proton.

Solution



Let the field be E upwards i.e.

$$E.e = mg.$$

$$E = \frac{mg}{e} = \frac{1.67 \times 10^{-27} \times 9.8}{1.67 \times 10^{-19}}$$

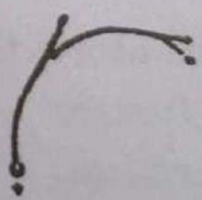
$= 1.02 \times 10^{-7}\ N\ C^{-1}$

12.4 Electric lines of Force

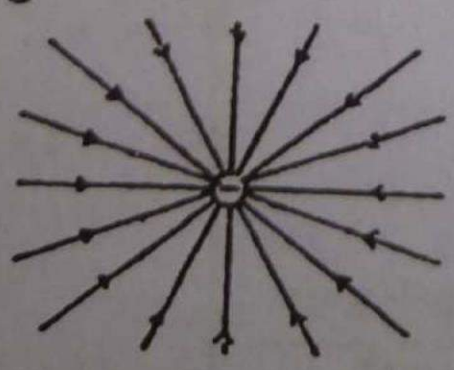
Electric field is a vector quantity and the field in space could be represented by associating a vector to every point of the field.

Faraday did not adopt this procedure and he introduced a novel method of visualizing the field by means of lines of force.

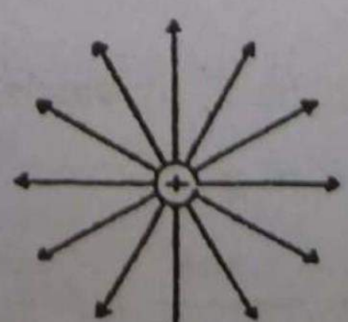
A line of force is an imaginary line drawn in such a way that the direction of tangent at any of its points is the same as that of electric field at that point. The direction of field at any point can, therefore, be determined by drawing a tangent to the line of force passing through that point. Since in general the direction of field varries from point to point the lines of force are usually curved. Lines of force do not intersect because the field cannot have two directions at a point. No lines of force originates or terminates in space surrounding the charge. Every line is a continuous line which originates on positive charge and terminates at negative charge. When we speak of an isolated charge it simply means that opposite charges on which the lines terminate or originate have large distances around it.



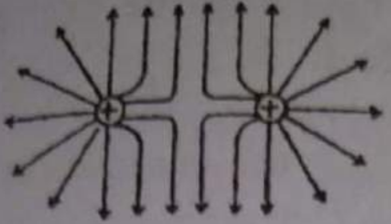
Electric fields due to different charge configurations are visualized by means of field lines in the following Fig. 12.3 (a to f)



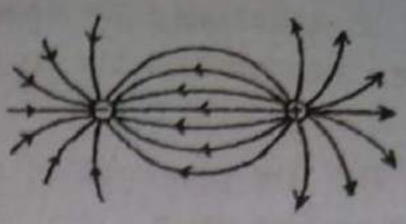
b) Around a negative pointcharge



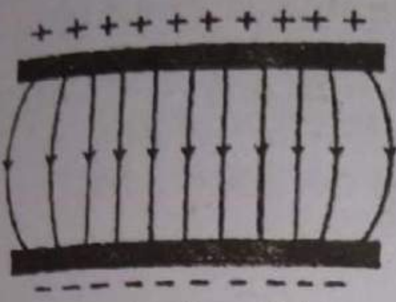
a) Around a positive point charge.



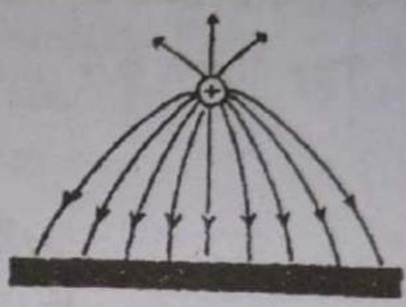
c) Around two like charges field at P is zero. (Neutral Point)



d) Around two unlike point charges



e) Between a pair of parallel oppositely charged conducting plates. Field is uniform in the space where lines of force are parallel metal plate.



f) Between a point charge and an oppositely charged metal plate

Fig.12.3 (a to f)

12.5 Electric Flux and Field Intensity

It is, of course, possible to draw a line of force through every point of an electric field but if this were done the whole space would be filled with lines and no individual line could be distinguished. By suitable limiting the number of lines in space the lines can be made to indicate the magnitude of field at a point. This is accomplished by spacing the lines in such a way that their number per unit area cutting a very small surface held perpendicular to the line of force at a given point gives the electric field at that point.

The total number of lines of force crossing a surface normally is called flux on that surface. The flux per unit area or flux density at any point gives the electric field at that point.

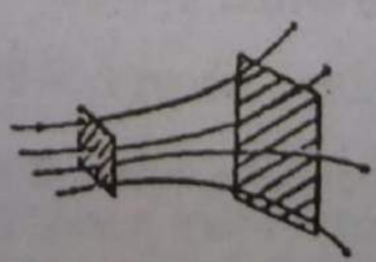


fig 12.4

The flux at a surface is determined by the product of flux density i.e electric field and the projection of it's area perpendicular to the field. or

By the product of area and the component of field normal to the area.

For a very small plane area ΔA such that the field at all of it's points is the same, the flux is given by:

The flux $\Delta\phi = \Delta A\,(E\cos\theta) = \overline{E}.\Delta\overline{A}$ -------- (12.7)

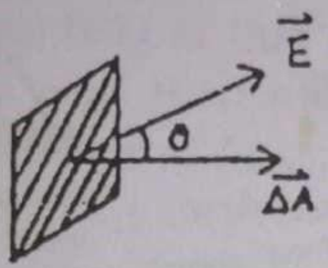
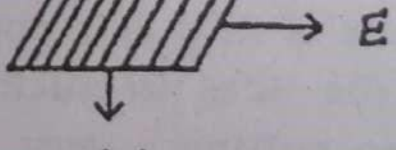


Fig: 12.5 (a)

Thus flux is a scalar product of electric intensity $\overline{E}$ and vector area ΔA. Although area is a scalar quantity, ΔA is taken as a vector quantity of magnitude equal to area and in the direction normal to it



(Fig 12.5(b)

1. The flux is positive when $\theta < 90°$
2. The flux is zero when $\theta = 90°$
3. The flux is negative when $\theta > 90°$
4. The flux is maximum when $\theta = 0°$

$= E\Delta A.$

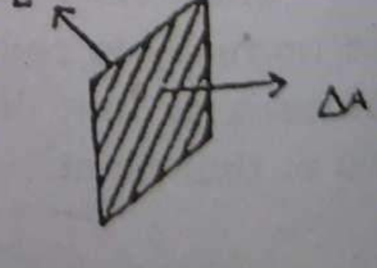


fig 12.5 (c)

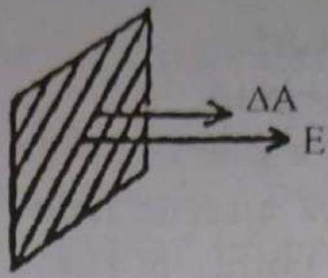


Fig 12.5 (d)

The convention that the electric field is equal to flux density at a point fixes the number of lines originating or terminating on a unit charge

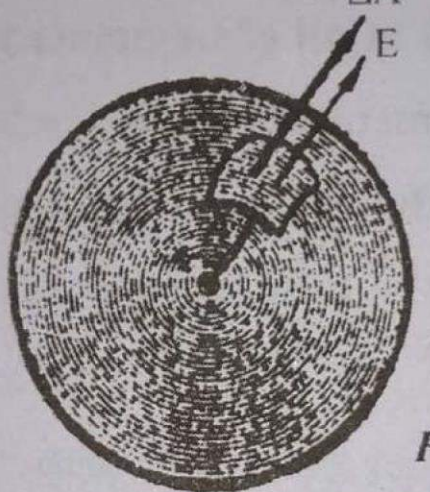


Fig 12.6

Consider an isolated point charge +q (fig .12.6). the lines of force from q will spread uniformly in space around it radially cutting the surface of an imaginary sphere drawn with the position of the charge as centre. normally at all portions. If r is the radius of sph re,

$$\text{Flux density} = \frac{\text{Number of lines Originating from q}}{\text{Surface area of sphere}}$$

The flux over the surface of sphere or the number of lines originating from $q = \Sigma E\Delta A = E\Sigma\Delta A = E\ 4\pi r^2$

$$\phi = \frac{1}{4\pi\epsilon_0}\ \frac{q}{r^2}\ 4\pi r^2 = q/\epsilon_0 \qquad \text{------------------ (12.8)}$$

We, therefore, see that if a unit charge is supposed to give rise to $\frac{1}{\epsilon_0}$ lines. the flux density at any point will give the intensity of field at that point.

12.6 Gauss's Law

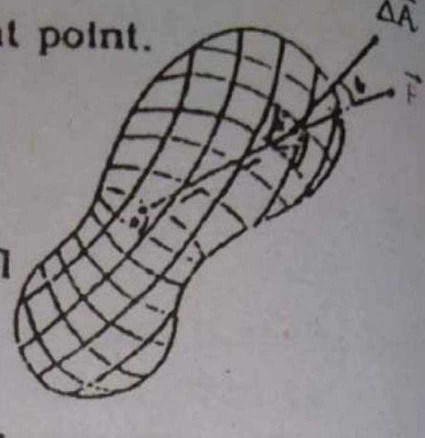


Fig 12.7

Consider a closed surface of any arbitrary size and shape surrounding a point charge q as shown in fig 12.7 electric field vectors at different points of the closed surface will have different magnitudes and directions. In order to determine flux over the closed surface, it is divided into very small parts, each part so small that it is almost plane and the field at all of its points is the same.

∴ Flux over an element of area $\Delta\phi = \overline{E}.\Delta\overline{A}$

and the flux over the whole surface, $\phi = \Sigma\overline{E}.\Delta\overline{A}$

which is equal to the total number of lines of force originating from q and crossing the closed surface normally i.e.

$$\phi = \Sigma\overline{E}.\Delta\overline{A} = \frac{q}{\epsilon_0}$$

Electric flux being the dot product of two vectors is a scalar quantity and hence if the surface encloses a number of scattered point charges

$q_1, q_2, q_3, \text{-------------------} q_n$, their fluxes can be added algebraically. The total flux ϕ due to all the point charges will be $\phi = \phi_1 + \phi_2 + \phi_3 + \text{-----} + \phi_n$

$$= \frac{q_1}{\epsilon_0} + \frac{q_2}{\epsilon_0} + \frac{q_3}{\epsilon_0} + \text{---------} + \frac{q_n}{\epsilon_0}$$

$$\phi = \sum_{r=1}^{n} q_r/\epsilon_0 \qquad \text{-------------------}(12.9)$$

This generalization is known as Gauss's law · The total outward flux over any closed hypothetical surface called Gaussian surface is equal to the total charge enclosed divided by ϵ_0 irrespective of the way in which the charge is distributed.

12.7 Applications of Gauss's Law

Gauss's law can be used to calculate the electric field only in those cases of charge distributions which are so symmetrical that by proper choice of a Gaussian surface the flux on it may possibly be evaluated. The following are few examples.

(a) Field of a uniform spherical surface charge at a distance r from its centre.

Let a charge q be uniformly distributed on the surface of a spherical Shell or that of a metal sphere (charge resides on the surface of metal).

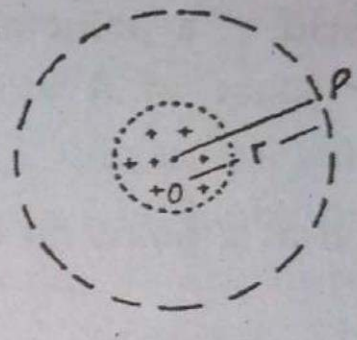


fig.12.8

Our object is to determine that field at a point P outside the shell at a distance r from the centre O.

The field could be determined using coulomb's law by adding vectorially the fields due to individual charge elements. The calculation is tedious because the fields vary both in magnitudes and directions. However, the field can be very easily determined using Gauss's law .

Imagine a Gaussian surface as that of a concentric sphere of radius r that contains the point P (fig.12.8) due to symmetrical charge distribution with respect to every point of Gaussian surface the field has the same magnitude every where on this surface and it is perpendicular to the surface at each point.

Flux over this Gaussian surface is

$$\phi = \Sigma E.\, \Delta A$$

$$= E \Sigma \Delta A = E\, 4\pi\, r^2$$

Applying Gauss's law.

$$E 4\pi r^2 = q/\epsilon_o$$

$$E = \frac{1}{4\pi\epsilon_o}\frac{q}{r^2} \quad \text{------------ (12.10)}$$

i) This shows that the electric field due to a uniform spherical surface charge distribution at an external point is the same as that produced by the same charge when concentrated at its centre.

ii) The field at a point on the surface is considered as the field at a point outside the charged surface but infinitely close to it at $r = a$ (a is the radius of charged shell)

∴Field at a point on the charged surface will be.

$$E = \frac{1}{4\pi\epsilon_o}\frac{q}{a^2}$$

This can also be expressed in terms of charge density δ (charge per unit area)

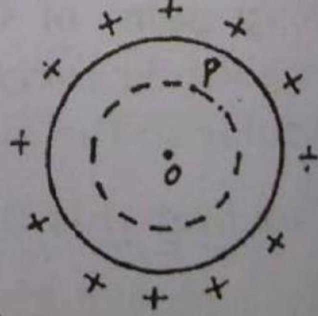
$$q = \sigma\, 4\pi a^2$$

$$\therefore E = \frac{1}{4\pi\epsilon_o}\frac{\sigma\, 4\pi a^2}{a^2} = \frac{\sigma}{\epsilon_o} \quad \text{---------------- (12.11)}$$

iii) In case if the point P is situated inside the charged surface (fig 12.9) the Gaussian surface passing through P immagined as in case 1 will enclose no charge and hence the flux is zero.

$$E = 4\pi r^2 = 0$$

$$E = 0$$



It means that there exists no field inside a spherical surface charge distribution

.fig 12.9

(b) Electric Intensity due to an infinite sheet of charge

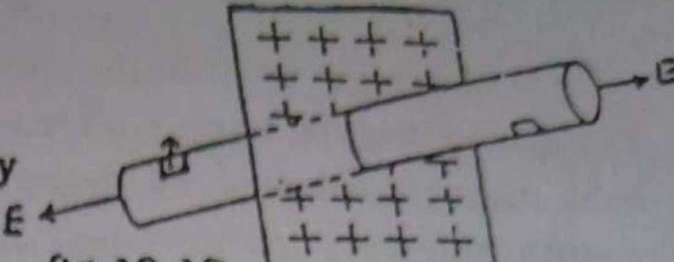


fig 12.10

A single thin sheet of charge can be prepared by depositing charge on one of a sheet of a non conducting material. In case of infinite plane the electric lines of force are perpendicular to the plane at all points.

The figure 12.10 shows a portion of a thin non conducting sheet of infinite size, one side of which is uniformly charged. Let the charge density be σ Cm^{-2}

A convenient Gaussian surface is that of a cylinder of cross section ΔA and length arranged to pierce the sheet, such that flat faces are each parallel to the sheet and are at distances r on either side of the sheet. The charge enclosed within the Gaussian surface is $\sigma \ \Delta A$.

The flux on the curved surface is zero because the angle between the field vector and area Vector of all elements of curved surface is 90°.

Let E be the electric field at a distance r from the sheet. The sum of flux on the two flat faces $\phi = E\Delta A + E\Delta A = 2E\Delta A$.

Applying Gauss's law

$$2E\Delta A = \frac{1}{\epsilon_0}\sigma \Delta A$$

$$E = \frac{\sigma}{2\epsilon_0} \quad \text{------------------(12.12)}$$

The field is independent of r showing that the field is the same at all points on each side of the plane.

If the charge is distributed evenly on both sides of the sheet each gives rise to a field $\frac{\sigma}{2\epsilon_0}$ and the net field at all points on either side will be $\frac{\sigma}{\epsilon_0}$

Although an infinite sheet cannot exist physically the derivation is useful as it yields correct results for real charged sheets of finite size if we consider the points not near to the edges and whose distances from the sheet is small.

(c) **Electric Intensity between two oppositely charged plates**

Consider two parallel metal plates separated by a small distance as compared to their size as shown in fig 12.11.

The plates carrying equal amounts of opposite charges will each have a charg density σ. Since the lines of force are parallel except near the edges each plate may be regarded to produce the same field as that produced by

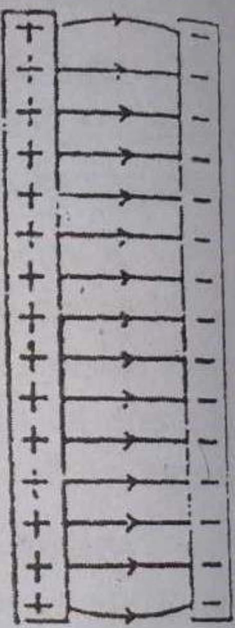


fig. 12.11

infinite charged sheets. The magnitude of electric field intensity at any point between the plates due to each plate is $\frac{\sigma}{2\epsilon_0}$ and along the same direction towards the -ve plate the net electric field at any point is

$$E = \frac{\sigma}{2\epsilon_0} + \frac{\sigma}{2\epsilon_0} = \frac{\sigma}{\epsilon_0}$$

$$\overline{E} = \frac{\sigma}{\epsilon_0}\hat{r}$$

$\hat{r}$ is a unit vector from +ve plate to the -ve plate.

12.8 Electric Potential

An electric charge experiences force at all points in an electric field and work has to be done by some external agency to move the charge from one point to the other against the electric force.

The situation is analogous to that of a mass raised to a higher altitude against the gravitational force. Since the work is recovered when the body descends it is considered to be stored as potential energy. We also know that this work is independent of the path along which the body is moved to that altitude.

Similarly the work done in moving a charge against the electric field is stored as electric potential energy. It is independent of the path adopted and hence the potential energy at a given location can be specified.

Work is done on the charge by the electric field when it moves back so that either it is accelerated or work is done on some external agency which prevents it from gaining speed.

Let a very small test charge q move from a point P to a point Q along any arbitrary path in the field as shown in fig.12.12

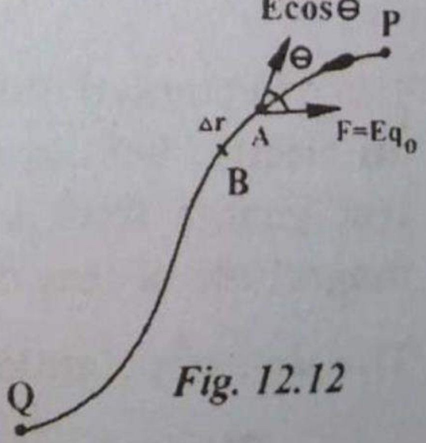


Fig. 12.12

In order to determine the work done the path is divided to small elements, each element being so small that it may be regarded as a straight line and the field at all of its points is the same.

The force on the charge at the point A is $F = Eq_0$

where E is the field at A in the direction shown in the diagram Work done in moving the test charge along an element from A to B $= Eq_o \overline{E} \cos\theta \, \Delta r = q_o \, E.\overline{\Delta r}$

This process is repeated right from P to Q taking care that the charge is moved very slowly always keeping it in electrostatic equilibrium i.e. when there is no net motion of charge within a conductor or on its surface, the conductor is said to be in electrostatic equilibrium.

$$\therefore \text{Total work} = \sum_{P}^{Q} q_o \, \vec{E} \, . \, \vec{\Delta r}$$

Hence the difference of potential energy of the test charge q_o at point Q and P

$$U_Q - U_P = q_o \sum_{P}^{Q} \vec{E} \, . \vec{\Delta r} \qquad \text{-----------------} (12.13)$$

Instead of dealing direct with potential energy of a charge it is useful to introduce the more general concept of potential energy per unit charge called potential difference between the points Q and P

$$V_Q - V_P = \frac{W_{P \to Q}}{q_o} = \sum_{P}^{Q} \overline{E}. \, \overline{\Delta r} \qquad \text{-----------------} (12,14)$$

Potential difference between two given points in an electric field is defined as the work done in moving a test charge from one point to the other divided by the magnitude of test charge.

The unit of potential difference is called Volt

The potential difference between two points in an electric field is 1 Volt if the work done per unit charge in moving the test charge between these points is 1 Joule

12.9 Absolute Potential at a point

Until now we have talked about the difference of potential between the points in the electric field. For defining absolute potential at a point there is a need for an arbitrary choice of zero of electric potential this can be understood by the fact that all heights are measured from sea level which is the conventional choice of the zero level. All temperatures on Celsius scale are measured above the temperatures of ice which is the conventional choice of zero temperature.

The zero reference potential is frequently taken as the potential of earth but for many other purposes the zero of potential is considered as the potential at a point greatly distant from all charges (at infinity)

The absolute potential at a point is the work done per unit charge when a test charge is moved from a point at infinity having zero potential to that point.

12.10. Electric Potential near an isolated Point Charge

Consider two points A and B in a straight line at distances r_A and r_B respectively from a point charge q as shown in fig.12.13. In order to obtain an expression for the potential difference between the point B and A the work needed to move the test charge from A to B per unit charge is to be determined

Since the electric field is varying from point to point the work is determined in steps, each so small that the field intensity within each step is nearly constant.In the first step the work done per unit charge.

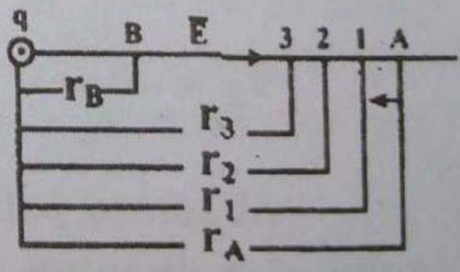


Fig. 12.13

$$\Delta W_1 = \frac{Eq_0(r_A - r_1)}{q_0} = -K\frac{q}{r^2}(r_A - r_1)$$

where r is neither r_A nor r_1 but an average distance between them from q_0. It makes things easier to take r as geometric mean of r_A and r_1

i.e $r = \sqrt{r_A - r_1}$

$$\therefore \Delta W_1 = \frac{Kq}{r_A r_1}(r_A - r_1) = -Kq\left(\frac{1}{r_1} - \frac{1}{r_A}\right)$$

In the next step

$$\Delta W_2 = -Kq\left(\frac{1}{r_2} - \frac{1}{r_1}\right)$$

$$\Delta W_3 = -Kq\left(\frac{1}{r_3} - \frac{1}{r_2}\right) \qquad \text{and so on}$$

$$\Delta W_n = -Kq\left(\frac{1}{r_B} - \frac{1}{r_n}\right)$$

Now

$$\Delta W = \Delta W_1 + \Delta W_2 + \cdots\cdots \Delta W_n = -Kq\left(\frac{1}{r_B} - \frac{1}{r_A}\right)$$

$$\therefore \text{Potential difference } V_B - V_A = Kq\left(\frac{1}{r_B} - \frac{1}{r_A}\right)$$

The absolute potential at the point B is obtained by considering the point A to be situated at infinity.

$$V_B = \frac{Kq}{r_B}$$

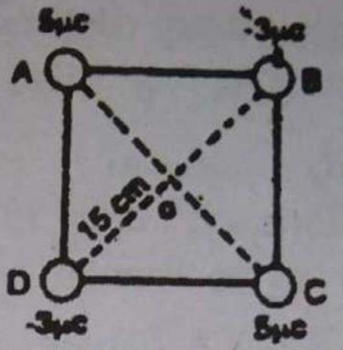
Absolute potential due to a point charge + q at a point at a distance r from it

$$V = -\frac{1}{4\pi\epsilon_0}\frac{q}{r} \qquad \text{.................. (12.15)}$$

Potential is a scalar quantity and the potentials at a point due to a number of charges are added algebraically.

Example12.7

Find the potential and field due to the charges placed at the ends of the diagonals of a square as shown in the diagram at the point of their intersection. Each diagonal is 30 cm long.



Solution

Potential at O due to charges at A and C

$$= \frac{9 \times 10^{9} \times 5 \times 10^{-6}}{.15} + \frac{9 \times 10^{9} \times 5 \times 10^{-6}}{.15}$$

$$= \frac{2 \times 9 \times 10^{9} \times 5 \times 10^{-6}}{.15}$$

$$= 6 \times 10^{5} \text{ Volts}$$

Potential at O due to charge at B and C

$$= \frac{-2 \times 9 \times 10^{9} \times 3 \times 10^{-6}}{.15} = -3.6 \times 10^{5} \text{ Volts}$$

Total Potential at $O = (6 - 3.6)\, 10^{6} = 2.4 \times 10^{5}$ Volts.

Electric Intensity at O is zero because the field at. A due to the charge at A is cancelled by the field due to charge at C. Similarly the field due to charge at B is cancelled by the field due to charge at D.

Example 12.8.

Find the velocity acquired by an electron in falling through a potential difference of 2000 Volts.

Solution.

$$Ve = \frac{1}{2} mV^{2}$$

$$V = \sqrt{\frac{2Ve}{m}} = \sqrt{\frac{2 \times 2000 \times 1.6 \times 10^{-19}}{9.1 \times 10^{-31}}}$$

$= 8.4 \times 10^7 \ ms^{-1}$

12.11. Relation between electric field and Potential

Consider two points a and b that are separated by a very small distance ΔS on a line of force AB as shown in fig. 12.14. The field is practically constant over the small distance ΔS. If a test charge $+ q_0$ is moved from a to b work is done by the electric field on the test charge.

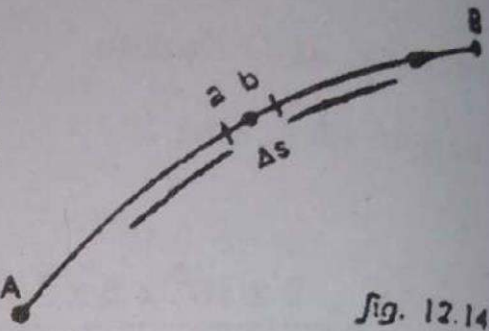


fig. 12.14

The work done by an outside agent in moving a positive charge against the field is conventionally taken + Ve and the work done by the field is taken as –Ve. The reason is that in this way positive work increases the potential energy of test charge.

∴ Work done by the field

$$\Delta W = - F \Delta S \quad = q_0 E \Delta S$$

Potential difference between the points a and b

$$\Delta V = \frac{\Delta W}{q_0} = - E \Delta S$$

$$E = - \frac{\Delta V}{\Delta S} \qquad \text{----------------- (12.16)}$$

$\frac{\Delta V}{\Delta S}$ is the rate of change of potential with respect to the distance and it is called potential gradient.

The relation 12.16 states that the electric Intensity at a point in an electric field is equal to the negative space rate of variation of potential at that point.

In case if the test charge is moved in a direction other than that along the line of force then $\Delta V = -E\cos\theta\ \Delta S$.

$$E\cos\theta = -\frac{\Delta V}{\Delta S}$$

In general we can say that the negative space rate of variation of potential at a point in any direction gives the electric field in that particular direction

Component of field along x– direction $E_x = -\dfrac{\Delta V}{\Delta x}$

similarly

$$E_y = -\frac{\Delta V}{\Delta y} \text{ and } E_z = -\frac{\Delta V}{\Delta Z}$$

Example 12.9

An electron is situated midway between two parallel plates 0.5 cm apart. One of the plates is maintained at a potential of 60 Volts above the other. What is the force on the electron.

Solution

$$F = Ee = -\frac{\Delta V}{\Delta S}e = -\frac{60}{5\times10^{-3}} \times (-1.6\times10^{-19})$$

$$= 1.92\times10^{-15}\ N$$

12.12 Electron Volt

Electron Volt is a unit of energy. In atomic Physics, the energies of accelerated fundamental particles are frequently expressed in terms of electron Volts.

Let a charge + q move under the influence of an electric field from a point A to another point B whose potential is lower by ΔV. The electric potential energy of the system is reduced by $q\Delta V$ because this much work has to be done by an external agent to restore the system to its original condition. The decrease of potential energy appear as kinetic energy of the particle.

$$q\Delta v = \frac{1}{2} mv^2 \qquad \text{-----------------} \quad (12.17)$$

It suggests that the energy can be expressed as the product of potential difference and charge. If we adopt the quantum of charge e as a unit of charge in place of coulomb we arrive at another unit of energy, the electron volt.

An electron volt is the energy required by an electron in falling through a potential difference of 1 volt.

1 electron- volt = (1 quantum of charge) (1 volt)

1 ev = 1.6×10^{-19}(Coulomb) (1 Volt)

= 1.6×10^{-19} Joules.

Example 12.10.

A electron acquires a speed of $10^6 ms^{-1}$ Find its energy in electron Volts

$$\text{Energy} = \frac{1}{2} mv^2 = \frac{1}{2} \times 9.1 \times 10^{-31} \times 10^{12} = 4.55 \times 10^{-19} J$$

$$= \frac{4.55 \times 10^{-19}}{1.6 \times 10^{-19}} eV = 2.84 \ eV$$

12.13 Equipotential Surfaces

The potential distribution in an electric field may be represented graphically by equipotential surfaces.

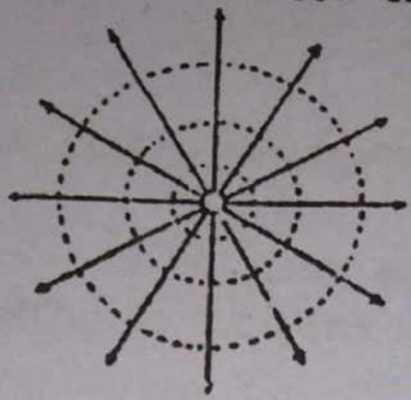
An equipotential surface is that, at all points of which the potential has the same value.Since the potential energy of a charged particle is the same at all points of a given equipotential surface it follows that no work is needed to move the charged particle over such a surface. Hence the surface through any point must be at right angles to the direction of the field at that point. The electric lines of force and equipotential surface are, therefore, mutually perpendicular.

A family of equipotentials can be sketched by drawing surfaces at right angles to the electric lines of force at each point.

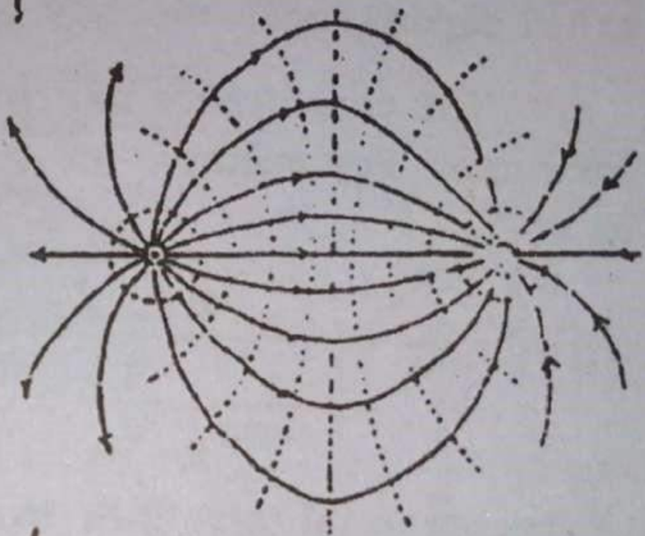
Equipotentials due to a few charge configurations are shown below.

Fig. 12.15

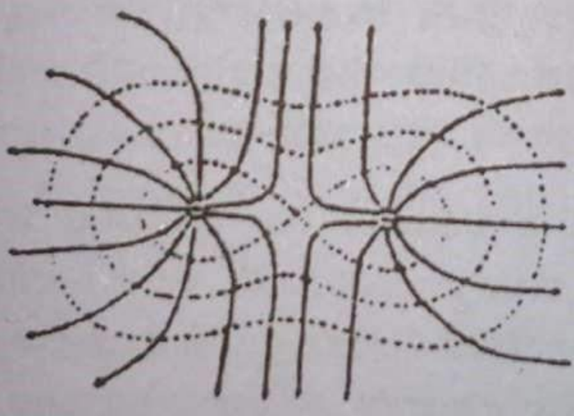
a)



b)



c)



Electric field lines due to different charge configuration.

12.14 Capacitance and Capacitors

The potential of a conductor depends on its own charge as well as the charges on the neighbouring bodies

For an isolated conductor

$q \propto V$

$q = CV$ .... (12.18)

C is a constant for a given conductor.

Electric charges generated by machines cannot be stored on a conductor beyond a certain limit as its potential rises to breaking value and the charge starts leaking to atmosphere.

$C = \frac{q}{V}$ measures how much charge should be placed on conductor to raise its potential to 1 Volt and it is taken as a measure of its capacity of holding electric charge called capacitance.

The capacitance depend. on the size and shape of conductor. For example for a spheric conductor of radius r

$$C = \frac{q}{V} = \frac{q}{\frac{1}{4\pi\epsilon_0}\frac{q}{r}} = 4\pi\epsilon_0 r \qquad \text{---------(12.19)}$$

i.e. proportional to radius. In general we can say for conductors of any shape that the greater the size of a conductor the greater is its capacitance.

The size of a conductor required to store huge amount of electric charge becomes very inconvenient and a device called capacitor is designed to have large Capacity of Storing electric charge without having large dimensions.

The principle of a capacitor is based on the fact that the potential of a conductor is greatly reduced without affecting the charge in it by placing another earth connected conductor or an oppositely charged conductor in its neighbourhood.

A system of two conductors separated by air or any insulating material forms a capacitor.

The conductors have equal and opposite charges and its capacitance is the ratio of the charge on one of the conductor to the potential difference between them.

The unit of capacitance is couls/Volt called farad.

Farad is a large unit and for practical purposes, convenient units are

micro farad $\mu F = 10^{-6}$ farads
pico farad $PF = 10^{-12}$ farads

12.15 Parallel Plate Capacitor

A very common and convenient type of capacitor is a parallel plate capacitor in which the conductors take the form of two plates parallel to each other and separated by a distance very small compared to the dimensions of the plates

Practically the entire field of such a capacitor is located in the region between the plates and it is uniform except at its outer boundary which is negligible when the plates are closer.

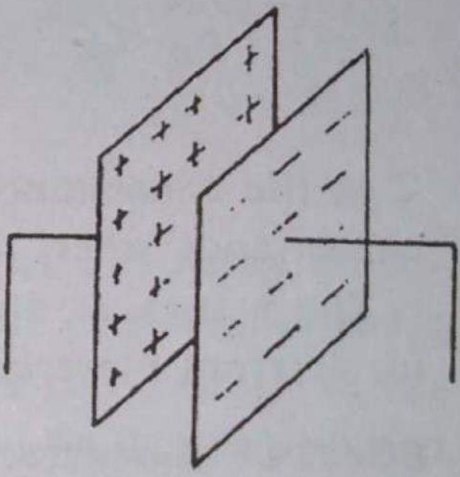


fig. 12.16

The charges on each plate are uniformly distributed on the inner sides of the plates due to attraction between opposite charges

Electric field $E = \dfrac{\sigma}{\epsilon_0}$

Potential difference $V = Ed = \dfrac{\sigma}{\epsilon_0} d$

where d is the distance between plates

Capacitance $C = \dfrac{q}{V} = \dfrac{A\sigma}{\sigma/\epsilon_0 d}$

$$C = \frac{\epsilon_0 A}{d}$$

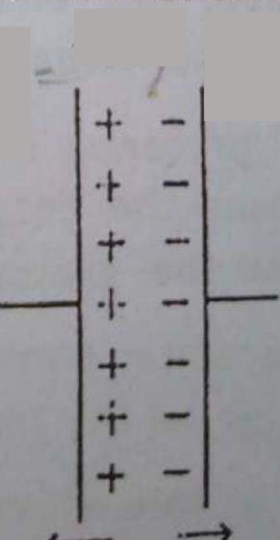


Fig.12.17

where A is the area of the plates.

If an insulating material completely fills the space between the plates

$$C = \frac{\epsilon A}{d} = \frac{\epsilon_r \epsilon_0 A}{d} \qquad \text{--------(12.20)}$$

The equation shows that the capacitance is directly proportional to the area of plates, inversely proportional to the distance between the plates and it is enhanced by ϵ_r if an insulating material called dielectric is introduced between the plates.

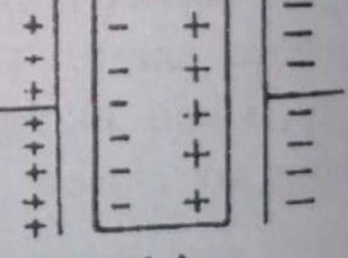
The relative permittivity or dielectric constant

$$\epsilon_r = \frac{\epsilon}{\epsilon_0} = \frac{C}{C_0} \qquad \text{--------(12.21)}$$

C is the capacitance when it has a dielectric. C_0 is the capacitance when the space between the plates is empty. The values of dielectric constants lie between 1—10 for different materials.

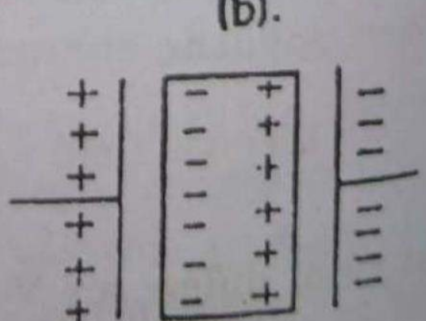
Effect of dielectric in a Capacitor

The electric field between the plates distorts the molecules of the dielectric. The molecules are polarized, one end becoming positive and other negative.



(a)

(b).



The presence of these charges decreases the potential difference between the plates (fig.12.18(a)).

A capacitor connected to a battery will accumulate more charge on the plates as shown in fig.12.18 (b).

In each case the dielectric increases the capaci-
...of the capacitor.

Example 12.11.

The parallel plates of an air filled capacitor are 1.0 mm apart. What must the plate area be if the capacitance is 1 farad ?

Solution

$$d = 1.0\ mm = 1 \times 10^{-3}\ m.$$

$$C = 1\ F \quad \epsilon_0 = 8.9 \times 10^{-12}\ C^2 N^{-1} m^{-1}$$

Solution

$$C = \frac{\epsilon_0 A}{d}$$

$$A = \frac{Cd}{\epsilon_0} = \frac{1 \times 10^{-3}}{8.9 \times 10^{-12}} = 1.1 \times 10^{8}\ m^2$$

12.16. Combinations of Capacitors

Capacitors of some fixed values are being manufactured. For a circuit the capacitance of a desired value can however, be obtained by suitable combination of capacitors. Capacitors can be combined in paralleis. series or both.

a) Parallel Combination.

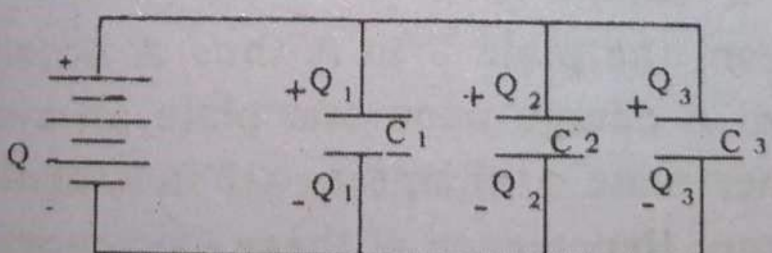


Fig: (12.19)

The figure (12.19) shows three capacitors connected in parallels. The upper plates of each capacitor are connected to a common terminal a and the lower plates to the other common terminal b.

A charge q given to the point a will divide itself to reside on the plates of individual capacitors according to their capacitance's such that

$$q = q_1 + q_2 + q_3$$

The potential difference V across each capacitor is that of the source

If C is the capacitance of the combination and C_1, C_2 and C_3 are the capacitance's of the capacitors joined in parallel

$$CV = C_1V + C_2V + C_3V$$

$$C = C_1 + C_2 + C_3 \qquad \text{------------------ (12.22)}$$

The capacitance of combination is equal to the sum of the capacitance's of individual capacitors. It is greater than the greatest individual one.

(b) Series Combination

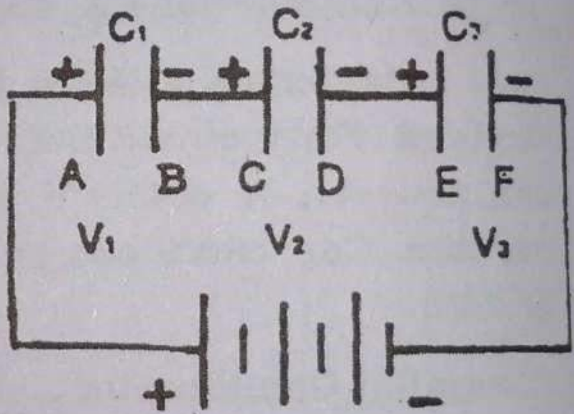


Fig. (12.20)

The figure shows three capacitors having the right-hand plate of one connected to the left hand plate of the next and so on connected in series. When a cell is connected across the ends of the system, a charge .q is transferred from the plate F to A thus A becomes positively charged. A charge upon one plate always attracts upon the other plate a charge equal in magnitude and opposite in sign. Hence each of these capacitors hold the same quantity of charge.

Let V be the potential difference across the combination. The potential differences across the individual capacitors V_1, V_2 and V_3 are such that their sum is equal to the applied potential difference V.

$$V = V_1 + V_2 + V_3$$

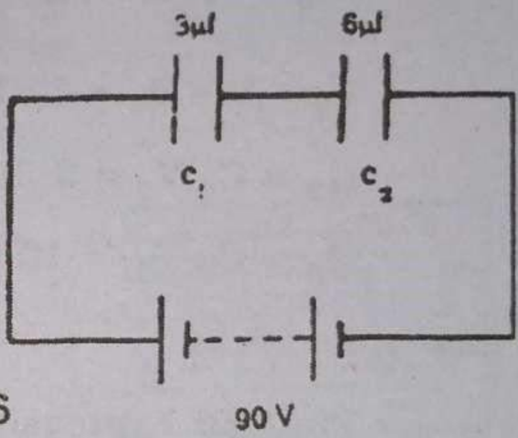
$$\frac{q}{C} = \frac{q}{C_1} + \frac{q}{C_2} + \frac{q}{C_3}$$

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_n} \quad \text{------------------(12.23)}$$

Thus to find the resultant capacitance of capacitors in series, we must add the reciprocals of their individual capacitance's. It will give the reciprocal of the resultant capacitance. The resultant is less than the smallest individual capacitance.

Example 12.12

Two capacitors C_1 (3μ f) and C_2 (6μ f) are in series across a 90 Volts d c.supply Calculate the charges on C_1 and C_2 and the p.d.across each.



Solution

Total Capacitance C is given by

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2}$$

$$C = \frac{C_1\, C_2}{C_1 + C_2} = \frac{3 \times 6}{9}$$

$= 2\ \mu f.$

The charges on C_1 and C_2 are the same and equal to q on C.

$$q = C\,V = 2 \times 10^{-6} \times 90 = 180 \times 10^{-6}\ \text{C}.$$

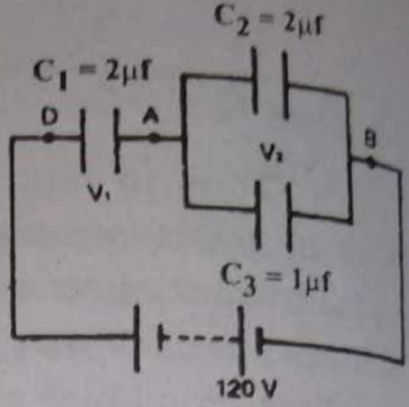
$$V_1 = \frac{q}{C_1} = \frac{180 \times 10^{-6}}{3 \times 10^{-6}} = 60 \text{ volts}$$

$$V_2 = \frac{q}{C_2} = \frac{180 \times 10^{-6}}{6 \times 10^{-6}} = 30 \text{ volts}$$

Example 12.13.

Find the charges on the capacitors connected as shown in the diagram across a 120 volts d.c. supply.

Solution.



Capacitance between A and B

$$C' = C_2 + C_3 = 3\,\mu f.$$

Overall Capacitance B to D

$$C = \frac{C_1\ C'}{C_1 + C'} = \frac{2 \times 3}{5} = 1.2\,\mu\, f.$$

Charge stored in this capacitance C

$$Q_1 = Q_2 + Q_3 = CV = 1.2 \times 10^{-6} \times 120 = 144 \times 10^{-6} \text{ Coul.}$$

$$V_1 = \frac{Q_1}{C_1} = \frac{144 \times 10^{-6}}{2 \times 10^{-6}} = 72 \text{ Volts}$$

$$V_2 = V - V_1 = 120 - 72 = 48 \text{ Volts}$$

$$Q_2 = C_2 V_2 = 2 \times 10^{-6} \times 48 = 96 \times 10^{-6} \text{ Coul.}$$

$$Q_3 = C_3 V_2 = 10^{-6} \times 48 = 48 \times 10^{-6} \text{ Coul.}$$

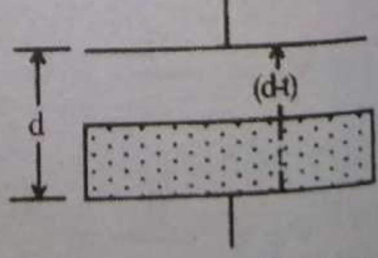
Example 12.14

Find an expression for the capacitance of a compound capacitor, the space between the plates of which is practically filled with a slab of dielectric.

Solution.

The capacitance of a parallel plate capacitor with dielectric completely filling the space between the plate is

$$C_d = \frac{\epsilon_r \epsilon_0 A}{d} = \frac{\epsilon_0 A}{\dfrac{d}{\epsilon_r}}$$



which is equivalent to the capacitance of an air capacitor with a distance $\dfrac{d}{\epsilon_0}$ between the plates.

It means that a thickness d of a dielectric is

equivalent to air thickness $\frac{d}{\epsilon_r}$

Hence the capacitance of a compound parallel plate capacitor with a slab of dielectric of thickness t partially filling the space between the plates.

$$C_d = \frac{\epsilon_0 A}{(d - t) + \frac{t}{\epsilon_r}}$$

(d-t) is the thickness of air space and the thickness t of

dielectric is equivalent of air space $\frac{t}{\epsilon_r}$

12.17 Different types of Capacitors

a) *Multiplate Capacitor:*

A multiplate capacitor consisting of large number of plates each of large area is designed to have large capacitance when N plates are used there are (N-1) individual capacitors in parallel.

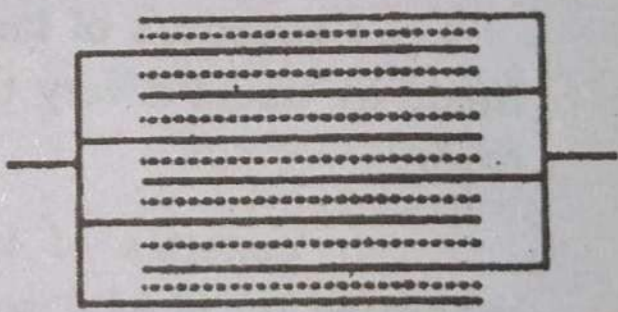
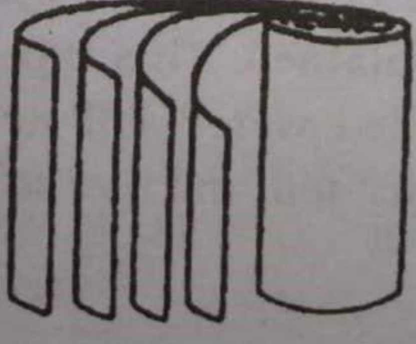


fig.12.21

In high grade capacitors mica is used as dielectric.

Inexpensive capacitors of capacitance upto 10 μF are usually made of alternate layers of tin or aluminium foil and waxed paper. These are frequently wound into rolls under pressure and sealed into moisture-resisting metal container.

fig.12 22



b) Variable Capacitor:

A variable capacitor of the kind used for tuning radio sets is shown in the diagram. It consists of two sets of semi circular aluminium or brass plates separated by air. One set of plates is fixed and the other is rotated by a knob to alter the effective area of the plates.

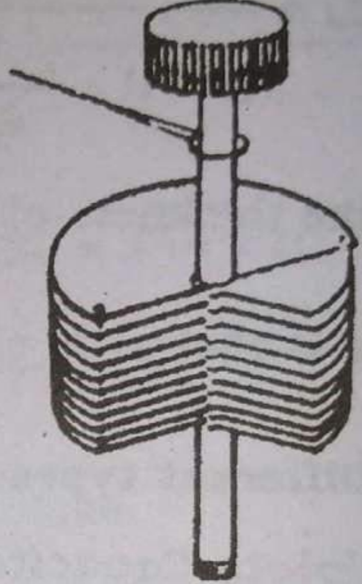


fig12.23

c) Electrolytic Capacitors.

Capacitors of large capacitance upto ·1000 μF are made by using a very thin insulating layer of aluminium oxide.

It consists of two sheets of aluminium equally separated by Muslin soaked in a special solution of ammonium borate. These are rolled up and sealed in an insulating container. Wires attached to the foil strips are then connected to an electric battery and a highly insulating thin film of aluminium oxide forms on the positive foil. A capacitor is thus formed in which the oxide film acts as the dielectric. Owing to the extreme thinness of the film, very large capacitance's which take up very little space may be obtained. This type of capacitor is used such that the Oxide-covered foil never become negative with respect to other foil and +ve and - ve terminals are marked.

12.1 Repulsion is the sure test of electrification. Explain.

12.2 Will a solid metal sphere hold a larger electric charge than a hollow sphere of the same diameter? Where does the charge reside in each case?

12.3 Explain why it is so much easier to remove an electron from an atom of large atomic weight than it is to remove proton?

12.4 Why it is not correct to say that potential difference is the work done in moving unit charge between the points concerned?

12.5 Why is it logical to say that potential of an earth connected object is zero? What can be said about the charge on earth?

12.6 Can an electric potential exist at a point in a region where the electric field is zero? Can the potential be zero at a place where the electric field intensity is not zero? Give examples to illustrate your reasoning.

12.7 An air capacitor is charged to a certain potential difference. It is then immersed in oil. What happens to its (a) capacitance (b) charge (c) Potential.

12.8 Two unlike capacitors of different potentials and charge are joined in parallel. What happens to their potential difference? How are their charges distributed? Is the energy of the system affected?

12.9 Four similar capacitors are connected in series and joined to a 36 Volts battery. The mid-point of group is earthed. What is the potential of the terminals of the group?

12.10 A point charge is placed at the centre of a spherical Gaussian surface. Is the flux changed:-

a) If the spherical Gaussian surface is re-place by a cube of the same volume.

b) If the sphere is replaced by a cube of $\frac{1}{10}$ of this volume.

c) If the charge is moved from the centre in the sphere.

d) If the charge is moved outside the sphere.

e) If a second charge is placed inside the sphere.

12.11 Four capacitors each of 2 μf are connected in such a way that total capacitance is also 2 μf. Show what combination gives this value.

12.12 A capacitor is charged by a battery. The battery is disconnected and a slab of some dielectric is slipped between the plates. Describe what happens to the charge, capacitance and potential difference.

12.13 Answer question 12.12 if the battery is not disconnected.

12.14 A capacitor is connected across a battery why does each plate receive a charge of the same magnitude? Will it be true even if the plates are of different sizes?

PROBLEMS.

12.1 Two unequal point charges repel each other with a force of 0.2 newtons when they are 10 cm apart. Find the force which each exerts on the other when they are

(a) 1 cm apart (b) 5 cm apart (Ans. 20N. 0.8N)

12.2 Two point charges of $+ 1\times 10^{-4}$ and $- 1 \times 10^{-4}$ coul are placed at a distance of 40 cm from each other. A Charge $+ 6 \times 10^{-5}$ coul is placed midway between them. What is the magnitude and direction of force on it.

(Ans. 2700×10^3 N towards -ve charge.)

12.3 Three point charges each of 4 μ.C. are placed at the three corners of a square of side 20 cm. Find the magnitude of the force on each.

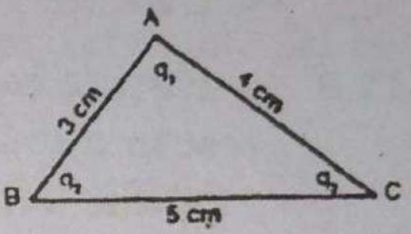
(Ans. 5.03 N. 5.04 N)

12.4 Three charges

$q_1 = + 7\times 10^{-6}$ C

$q_2 = - 4 \times 10^{-6}$ C and

$q_3 = - 5 \times 10^{-6}$ C are placed



at the vertices of a triangle as shown in the diagram.

The sides of the triangle measure 3,4 and 5 cm. Determine the magnitude and direction of the force on the charge q_1.

(Ans. 342.24 N. in the direction making an angle of 35.10° with BA)

12.5 Two small spheres, each having a mass of 0.1 gm, are suspended from the same point by silk threads each 20 cm long. The spheres are given equal charges and they are found to repel each other, coming to rest 24 cm apart. Find the charge on each.

(Ans. 6.86×10^{-8} C).

12.6 Two charges of $+ 2 \times 10^{-7}$ C and -5×10^{-7} C are placed at a distance of 50 cm from each other.

Find a point on the line joining the charges at which the electric field is zero.

(Ans. 86 cm away from 2×10^{-7} C charge)

12.7 What are the electric field and potential at the centre of a square whose diagonals are 60 cm. each when (a) charges each of 2μc are placed at the four corners. (b) charges of + 2μc are placed on the adjacent corners and − 4μc on other corners.

(Ans. (a) O. 2.4×10^{5} volts. (b) 8.5×10^{5} wc^{-1}, -1.2×10^{5} V)

12.8 A particle carrying a charge of 10^{-5} C starts from rest in a uniform electric field of intensity 50 Vm^{-1}. Find the force on the particle and the kinetic energy it acquires when it has moved 1.m.

(Ans. 5×10^{-4} N. 5×10^{-4} J)

12.9 A proton of mass 1.67×10^{-27} Kg and charge 1.6×10^{-19} C is to be held motionless between two horizontal parallel plates 10 cm apart. Find the Voltage required to be applied between the plates.

(Ans. 1.02×10^{-8} Volts)

12.10 A small sphere of weight 5×10^{-3} N is suspended by a silk thread 50 mm long which is attached to a point on a large charged insulating plane. When a charge of 6×10^{-8} C is placed on the ball the thread makes an angle of 30° with the vertical. What is the charge density on the plane.

(Ans. $\sigma = 8.5 \times 10^{-7}$ cm^{-2})

12.11 How many electrons should be removed from each of the two similar spheres each of 10 g so that electrostatic repulsion be balanced by gravitational force.

(Ans. $n = 5.4 \times 10^6$)

12.12 There is a potential difference of 150 volts between two conductors of a power line. A charge of 600 C is carried from one conductor to the other. What work is required? If the time necessary to transport the charge is 1.25 s how much power is used?

(Ans. 9×10^4 .J. 7.2×10^4 watts)

12.13 A metal sphere of 100 mm radius has a charge of 4.25×10^{-9} coul. What is the potential? (a) at its surface (b) at its centre.

What is the potential energy of a charge of 2.5×10^{-6} C at a point 150 mm from the centre of sphere?

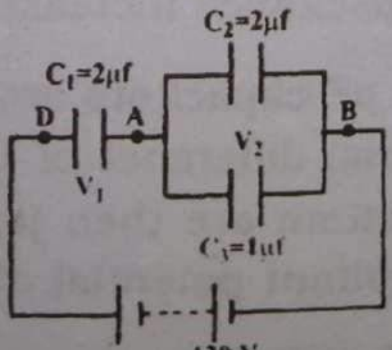
(Ans. 3.825×10^2 volts. 3.825×10^2 volts. 6.375×10^{-4} J)

12.14 An electron having an initial velocity of 10^{+3} cms^{-1} is directed from a distance of 1 mm at another electron whose position is fixed. How close to the stationery electron will the other approach.

(Ans: 0.533 mm)

12.15 Find the equivalent capacitance and charge on each of the capacitor shown in the diagram.

(Ans. 3 μf. q_1 = 10 μc. q_2= 20 μc. q_3= 30 μc)



12.16 Two capacitors of 2 μf and 8 μf are joined in series and a potential difference of 300 volts is applied. Find the charge and potential difference for each capacitor.

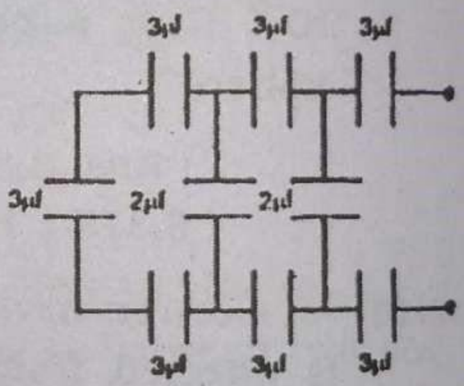
(Ans. $q_1 = q_2 = 4.8 \times 10^{-3}$ C. $V_1 = 240$ Volts, $V_2 = 60$ Volts)

12.17 A capacitor of 100 pF is charged to a potential difference of 50 volts. Its plates are then connected in parallels to another capacitor and it is found that the potential difference between its plates falls to 35 volts. What is the capacitance of the second capacitor.

(Ans. 42.85 pF)

12.18 Find the equivalent capacitance of the combination shown in the diagram.

(Ans:1μf)



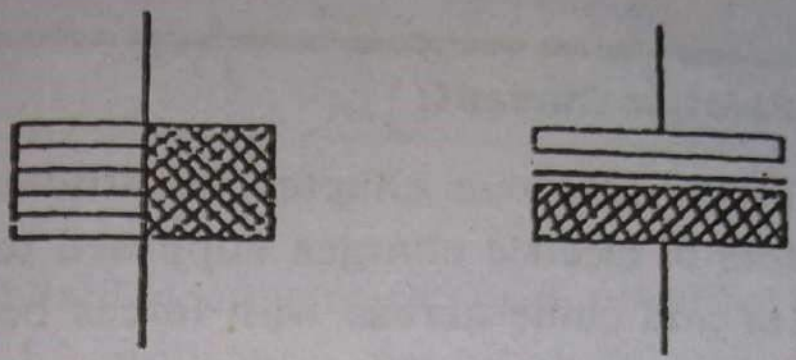
12.19 A parallel plate capacitor has plates 30 cm x 30 cm separated by a distance of 2 cm. By how much the capacitance changes if a dielectric slab of the same area but of thickness 1.5 cm is slipped between the plates. The dielectric constant of the material is 2.

(Ans. Capacitance increase by 2.39×10^{-11} farad)

12.20 Three 1.0 pF capacitors are charged separately to the potential difference of 100, 200 and 300 volts. The capacitors are then joined in parallels. What is the resultant potential difference

(Ans. 200 Volts)

12.21 Compare the capacitance's of two identical capacitors with dielectrics inserted as shown in the diagram. The dielectric constants are K_1 and K_2



$$\left(\text{Ans. } C_b/C_a = \frac{4K_1 K_2}{(K_1+K_2)^2}\right)$$

12.22 A capacitor of 10 μf and one of 20 μf are connected across batteries of 600 volts and 1000 volts respectively and then disconnected. They are then joined in parallels. What is the charge on each capacitor?

(Ans. 8.66×10^{-3} C. 17.32×10^{-3} C)

12.23 Attempt the problem 12.23 with the difference that the capacitors are joined in series after being charged. as before.

(Ans. $q_1 = q_2 = 1.066 \times 10^{-12}$C)