Chapter. 13

CURRENT ELECTRICITY

13.1. Electric Current:

In the previous chapter we studied the behavior and effects of electric charges supposed to be in stationary states and came across with forces between charges, the electric potential. From this chapter onwards, we shall remain concerned with the study of the properties and effects of electric charges when they are in motion. This field is known as electrodynamics.

The first thing which comes to our mind in the study of electricity is the generation of electric current, which we shall show as the rate of motion of charges in a conductor.

The best conductors are metals, particularly silver, copper, gold, aluminium. In these metals, the electric charges can flow very easily. In metallic conductors, there is always present a large number of free electrons actually detached from their parent atoms which constitute a kind of electron gas. These free electrons are capable of moving freely in random directions in the interatomic space of the metal. In insulators, the electrons are rigidly bound to the atoms and so cannot move and hence no current can flow through them.

In the absence of an electric field across the conductor, the free electrons have a thermal velocity at normal temperature which is of the order of a million metre per second. The velocities of these electrons are randomly directed and in effect the number of electrons moving in one direction is just equal to the number moving in the opposite direction to maintain the neutral state of the conductor.

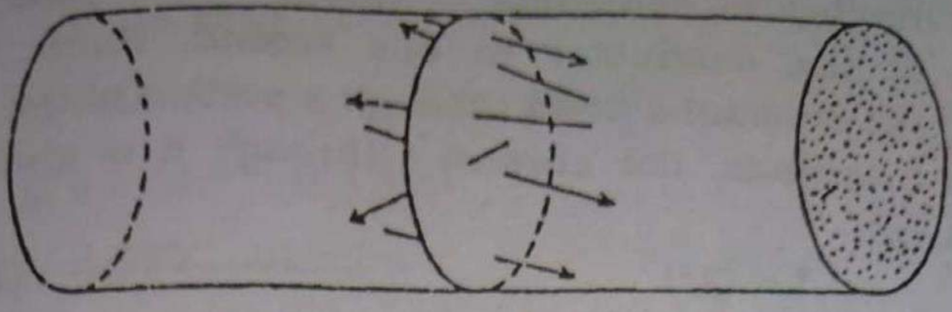


fig. 13.1

On establishing an electric field between the two ends of a wire by applying a potential difference, these free electrons having a negative charge on them, experience a force in the opposite direction to the field and hence acquire an acceleration in that direction. These free electrons which, in the absence of the electric field were moving towards the positive terminal of the battery are now accelerated while those moving away from the said terminal are retarded, the net effect is that an additional component of velocity towards the positive terminal is superimposed on the free electrons over their thermal velocity after they suffered certain number of collisions with the atoms.This additional component of velocity due to the electric field is known as drift velocity. The magnitude of this velocity is of the order of 0.01 metre per second. This drift velocity in the free electrons is really responsible for the generation of electric current in the wire. Thus we see that electric current in a conductor is due to the flow of electrons.

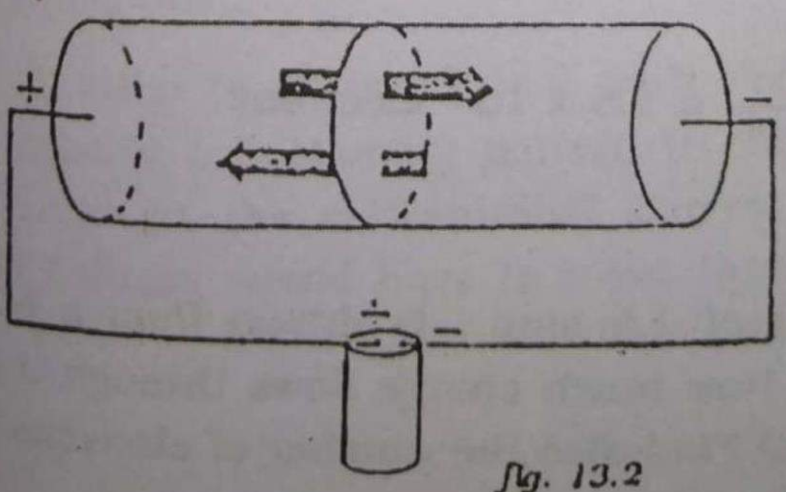


fig. 13.2

We define the strength of current in a conductor as the number of coulombs of charge which pass any section of the conductor in one second. Hence if a charge of Q coulombs flows through a section of the wire in time t seconds, the current I through it is given by the relation.

$$I = Q/t \qquad \text{-------------------(13.1)}$$

The unit of current is one coulomb per second. This unit in S.I. is called an "ampere", after the name of the French scientist Andre Marie Ampere.The smaller units of current are milliampere (mA) = 10^{-3} ampere and micro ampere (μ A) = 10^{-6} ampere.

Example 13.1

A current of 2.4 amp. is flowing in a wire. How many electrons pass a given point in the wire in one second if the charge on an electron is 1.6×10^{-19} coulomb.

<u>Reasoning:</u> If we know the total charge passing the point in 1 sec., we can divide that by the charge on an electron and thereby obtain the number of electrons.

Solution:

Since I is the number of coulombs passing a point in the wire in 1 sec. we have that 2.4 coulomb of charge flows that point in 1 second.

But each electron carries a charge 1.6×10^{-19} C. so that number of electrons passing per second is

$$\frac{2.4}{1.6\times 10^{-19}} = 1.5 \times 10^{19} \text{ electrons}$$

Example 13.2

A current of 1.6 amp is drawn from a battery for 10 minutes. How much charge flows through the circuit in this time? Find also the number of electrons flowing during this time.

Solution:

$I = 1.6$ amp.

$t = 10$ minutes $= 600$ second

Now $Q = I \times t = 1.6 \times 600$

$= 960$ coulombs.

$$\text{Number of electrons} = \frac{960}{1.6 \times 10^{-19}} = 6 \times 10^{21}$$

13.2. DIRECTION OF A CURRENT:

There are two equivalent ways of describing the same current as a flow of negative charge in one direction or an equal flow of positive charge in the opposite direction. When a current is set up in a wire by connecting its ends respectively to the two terminals of battery, the electrons move along the circuit from the negative terminal to the positive terminal of the battery. This we call as the electronic current.

After the discovery of electron, the electric current was shown due to the flow of electrons and hence the direction of electric current should have been reversed to be that of electronic current but we know that a negative charge moving in one direction is in effect, equivalent to a positive charge moving in the opposite direction in a wire, therefore, the old belief about the direction of current from positive to the negative terminal of the battery has been retained.

In order to differentiate it with electronic current, it has become a customary pattern to consider the electric current as the conventional current in which the positive charges would have to move through the circuit from a point of higher potential to a point at lower potential.

13.3. ELECTRIC RESISTANCE AND OHM'S LAW.

It is a common fact that flow of a fluid in a medium under some applied force experiences some sort of friction or resistance in its path. The electric current we have defined as the flow of electrons in the conductor, it is logical to assume that these electrons would come across some sort of resistance during their flow.

We assigned the strength of the current due to the drift velocity of the electrons which depends upon the strength of the electric field. These free electrons during their course of motion towards the positive terminal of the battery experience collisions with the atoms of the conductors which in turn reduces or some times destroys the velocity gained due to the accelerations.The current I should be proportional to the potential difference V between the two ends of the conducting wire.

or $I \propto V$

$$I = KV \qquad \text{------------------(13.2)}$$

where K is a constant known as the conductance of the material of the wire.

It is customary to speak of the resistance R of the wire instead of the conductance K, which is the reciprocal of the conductance i.e.

$$R = \frac{I}{K}$$

or equation 13.2 becomes

$$I = \frac{V}{R} \qquad \text{------------------(13.3)}$$

This equation is known as Ohm's law, discovered by the German scientist George Simon Ohm who found that a linear relationship existed between current and potential difference, for which the graph is a straight line as shown in fig. 13.3

We define Ohm's law for metallic conductors as "The current through a conductor is directly proportional to the potential difference between the ends of the conductor, provided that physical conditions remain the same". Thus

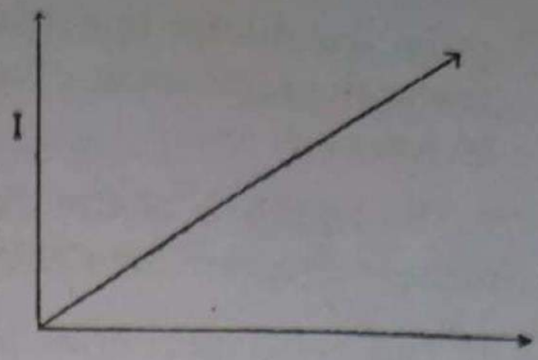


fig 13.3

$$\frac{V}{I} = R, \text{ a Constant}$$

or Resistance = Volts per ampere

The S.I. unit of resistance is the Ohm. Shown as Ω. (The capital Greek letter Omega).

Therefore one ohm is the resistance of a conductor through which a current of one ampere passes when a potential difference of one volt is maintained across the end of the conductor. Large resistances are more conveniently expressed in Kilo Ohm ($K\Omega = 10^3$ Ohm) or Mega-ohm ($M\Omega = 10^6$ Ohm) and small resistances in milliohm ($m\Omega = 10^{-3}$ Ohm) or micro Ohm ($\mu\Omega = 10^{-6}$ Ohm).

It is important to mention that Ohms law is valid only for metallic resistance at a given temperature and for steady currents.

Example:- 13.3

A light bulb has resistance of 150 Ω, find the current in it when it is connected to a 225 volt source.

Solution:

From Ohms law $I = V/R$

$$= \frac{225 \text{ V}}{150} = 1.5 \text{ A.}$$

13.4 RESISTIVITY

From Ohm's law, we saw that the resistance of a given conductor is constant under certain conditions. Actually the resistance of a conductor depends upon certain factor, such as

i) The length L of the conductor; the longer the conductor, the greater should be its resistance.

ii) The cross sectional area. A of the conductor; thicker wires have less resistance, and

iii) The material of which the conductor is made. Under the same conditions, wire of different metals show different values of resistances.

From the above conditions we see that resistance of a conductor or wire is directly proportional to its length and inversely proportional to its cross sectional area. That is

$$R \propto L/A$$

or

$$R = \rho \frac{L}{A} \qquad \text{----------(13.4)}$$

The constant of proportionality ρ(Greek letter rho) is known as the resistivity or specific resistance of the material which means that resistivity is a property of the material.

Since $\rho = \frac{RA}{L}$, therefore the S.I. unit of resistivity is Ohm metre (Ω.m).

By definition, the resistivity of a material is the resistance of a wire of the material per unit of its length and per unit area of its cross section, in other words the resistance of a cube of unit length.

Dependence of resistivity upon temperature:

The electrical resistance of most metals increases with the increase in temperature, because atoms sitting on their sites in metal start vibrating more violently about their mean position due to increase in temperature. This increases the probability of collision of free electrons with them which ultimately affects the drift velocity of free electrons for a given applied voltage. As we have seen earlier that the electric current depends upon the drift velocity of the free electrons, so with rising temperature, the decrease in the drift velocity is attributed to the decrease in the electric current or the increase in the electrical resistance for a given applied voltage.

Experimentally it has been observed that the change in the resistance of a metallic conductor with the change in the temperature of the conductor is nearly linear over a wide range of temperature above and below $0^\circ C$. Let us suppose that the resistance of a wire at $0^\circ C$ is R_o and at a higher temperature $t^\circ C$, it is R_t. The change in resistance is $R_t - R_o$ for a change in temperature $(t^\circ - 0^\circ)$ C. If we denote the changes in resistance and temperature respectively by ΔR and Δt, then

$$\Delta R \propto R_o \,.\, \Delta t$$

$$= \alpha R_o \Delta t$$

Where α is the constant of proportionality

or $R_t - R_o = \alpha R_o . \Delta t$

or $$\alpha = \frac{R_t - R_o}{R_o - \Delta t} \qquad \text{-------------------(13.5)}$$

Thus the constant α gives the fractional change in resistance per unit resistance per Kelvin change in temperature. It is known as the temperature Coefficient of resistance or of resistivity.

Since resistivity ρ is directly proportional to the resistance of a metal. We can thus derive that

$$\alpha = \frac{\rho_t - \rho_o}{\rho_o \times \Delta t}$$

where ρ_t and ρ_o refer to resistivities at $t^\circ C$ and $0^\circ C$ respectively

or $\rho_t = \rho_o (1 + \alpha . \Delta t)$ ....(13 6)

The following table mentions the values of ρ and α all at $0^\circ C$

Example 13.4

Find the potential difference across two ends of a copper wire to maintain a steady current of one ampere. The length of the wire is one metre, radius of cross-section is 0.25 cm.

Solution

The resistance of the copper wire will be given by the formula.

$$R = \rho \frac{L}{A}$$

$$L = 1 \text{ m}$$

$$\rho = 1.54 \times 10^{-8} \, \Omega - \text{m}$$

$$r = 0.25 \text{ cm} = 0.0025 \text{ m};$$

$$A = \pi r^2 = ?$$

$$A = \frac{22}{7} \times (0.0025)^2 \text{ m}^2$$

$$= 19.6 \times 10^{-6} \text{ m}^2$$

Hence $R = 1.54 \times 10 \times \dfrac{1}{19.6 \times 10^{-6}} = 0.000785 \, \Omega$

and the potential difference required will be

$$V = IR$$

$$= 1.0 \text{ A} \times 0.000785 \, \Omega$$

$$V = 0.000785 \text{ Volts}$$

Table 13.1

Resistivities ρ and Temperature Coefficients α. All at 0°C.

Material	ρ (Ω.m)	α (0°C^{-1})
Silver	1.52×10^{-8}	0.0038
Copper	1.60×10^{-8}	0.0039
Gold	2.27×10^{-8}	0.0034
Aluminium	2.63×10^{-8}	0.0040
Tungsten	5.0×10^{-8}	0.0045
Iron	11.0×10^{-8}	0.0052
Platinum	11.0×10^{-8}	0.00392
Constantan (Cu 60% + Ni 40%)	49×10^{-8}	0.00001
Nichrome (Ni 60% + Fe 24% + Cr 16%)	100×10^{-8}	0.00004
Manganin (Cu 84% + Mn 12% + Ni4%	44×10^{-8}	0.00000
Carbon	35×10^{-6}	-0.0005
Germanium	0.60	-0.048
Silicon	2300	-0.075
Wood	10^{13}	
Glass	$10^{10}-10^{14}$	
Mica	$10^{11}-10^{15}$	

Examples: 13.5.

A rectangular bar of iron is 2 by 2 cm in cross section and is 40 cm long. what is the resistance of the bar when ρ for iron is 1.1×10^{-7} Ω m.

Solution:

We know that $R = \rho \dfrac{L}{A}$

$$= \frac{1.1 \times 10^{-7} \times 0.40}{(0.02 \times 0.02)} = \frac{4.4 \times 10^{-8}}{4.4 \times 10^{-4}}$$

$$= 1.1 \times 10^{-4}\ \Omega$$

Example 13.6.

What would be the resistance of the above bar at 500°C.?

Solution:

We have from the table that $\alpha = 5.2 \times 10^{-3}\ C^{-1}$

$$\frac{R_t - R_o}{R_o} = \alpha\,(t - o)$$

$$\frac{R_t - 1.1 \times 10^{-4}}{1.1 \times 10^{-4}} = 5.2 \times 10^{-3}\ (500 - 0)$$

from which

$$R_t - 1.1 \times 10^{-4} = 1.1 \times 10^{-4} \times 5.2 \times 10^{-3} \times 500$$

$$= 2.86 \times 10^{-4}\ \Omega$$

or $\quad R_t = (2.86 + 1.1) \times 10^{-4}\ \Omega$

$$= 3.96 \times 10^{-4}\ \Omega$$

The resistance of the bar is roughly four times higher at this temperature than it was at 0^{o}C

Example: 13.7.

The resistance of a platinum resistance thermometer is 200.0 Ω at 0°C and 257.6Ω when immersed in hot bath. What is the temperature of the bath when α for platinum is 0.00392°C^{-1}?

Solution:

$$\frac{R_t - R_o}{R_o \times t} = \alpha$$

or $t = \dfrac{R_t - R_o}{\alpha \times R_o} = \dfrac{257.6 - 200.0}{0.00392 \times 200}\ ^{\circ}C$

$\dfrac{57.6}{0.784}\ ^{\circ}C = 73.5^{\circ}C$

Example: 13.8.

A water heater draws 30 A from a 220 volts power source 15 meters away. What is the minimum cross section of the copper wire that can be used if the voltage is not to be lower than 210 volts at the heater?

Solution:

The voltage drop is (220 – 210) = 10 V and the resistance that corresponds to this drop when the current 30A is

$$R = \frac{V}{I} = \frac{10V}{30A} = 0.33\Omega$$

The total length of the wire involved is twice the distance between the source and the heater, so the length of the wire will be 2× 15m = 30 m.

From the table ρ for copper is $1.6 \times 10^{-8}\ \Omega$ m.

$$\text{Cross-sectional area } A = \frac{\rho.L}{R} = \frac{1.6 \times 10^{-8} \times 30 \times 30}{10} m^2$$

$$= 1.44 \times 10^{-6}\ m^2$$

$$= 1.44\ mm^2$$

13.5. COMBINATION OF RESISTORS.

Quite often we find an electric circuit containing a large number of elements such as resistors, capacitors or batteries inter connected together in a complicated manner. We call such circuits a network.

For the cases of resistors, we consider here few types of networks.

fig.(i)

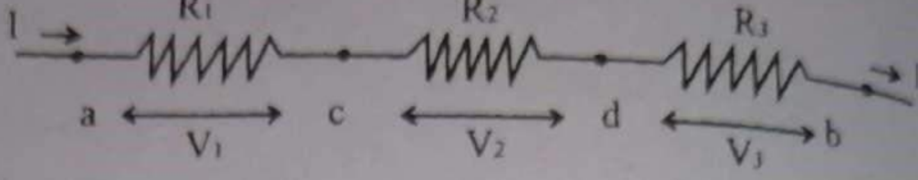


fig.(ii)

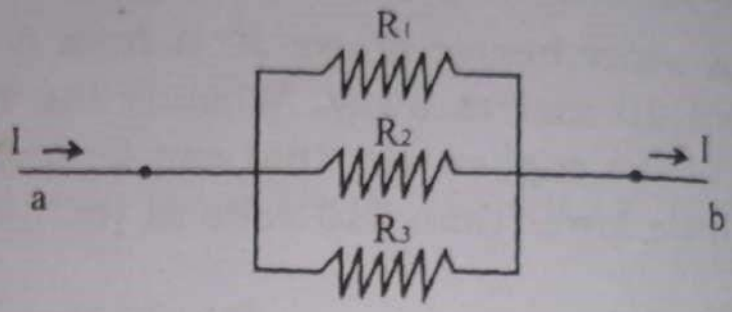


fig.(iii)

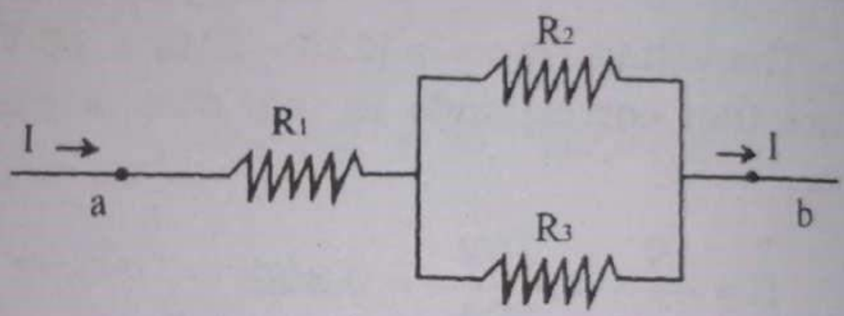


fig.(iv)

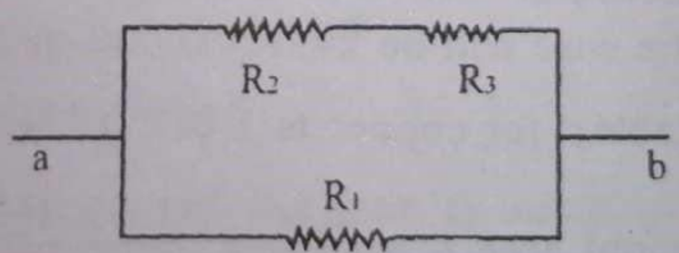


Fig. (13.4) (i to iv)

The above figures 13.4 (i to iv) show four different ways in which three resistors R_1, R_2 and R_3 might be connected to form a network. In fig.13.4 (i) the resistor are joined end to end providing a single path to the current between the points a and b. Such an arrangement of resistors is called series connection. Any number of resistors can be so joined in which the same current will flow through each resistor.

The resistors in fig. 13.4 (ii) are said to be joined in parallel between the points a and b. In this arrangement, each resistor provides an alternate path to the current between the end points a and b which means that current I is divided at the point x into three different paths in the respective resistors and ultimately rejoins at the end point b. In fig 13.4 (iii) resistors R_2 & R_3 are joined in parallel with one another and this combination is in series with resistor R_1 while in fig.13.4 (iv) resistors R_2 and R_3 are in series and this combination is in parallel with R_1.

We can find a single resistor equivalent in value which could replace a combination of resistors in any given network keeping the potential difference and the current unaltered in the circuit. If any one of the network in the given figures were replaced by its equivalent resistance R we could safely write.

$$V_{ab} = R\,I \qquad \text{------------------ (13.7)}$$

where V_{ab} is the potential difference between the points a and b and I is the current in the circuit.

(1). **Resistances in Series:**

Referring to figure 13.4 (1) . The current I in the series combination is the same in each resistor when the potential difference between the points a and b is V_{ab} :

As $V_1 = IR_1$, $V_2 = IR_2$ and $V_3 = IR_3$

As $V_{ab} = V_1 + V_2 + V_3$

$$IR = IR_1 + IR_2 + IR_3$$

$$R = R_1 + R_2 + R_3 \qquad \text{-------------------(13.8)}$$

Hence $R = R_1 + R_2 + R_3$ from eq: 13.8. The equivalent resistance of any number of resistors in series equals the sum of the values of individual resistances.

(2). Resistances in Parallel

In this connection, the potential defferences between the terminals of each resistors must be the same and equal to V_{ab}, while the current I at the point x (in fig.13.4(II)) is divided into three resistors R_1, R_2 and R_3 as I_1, I_2 and I_3 respectively

From Ohm's law

$$I_1 = \frac{V_{ab}}{R_1},\ I_2 = \frac{V_{ab}}{R_2} \text{ and } I_3 = \frac{V_{ab}}{R_3}$$

But $I_1 + I_2 + I_3 = I$

or

$$I = \frac{V_{ab}}{R_1} + \frac{V_{ab}}{R_2} + \frac{V_{ab}}{R_3}$$

$$\frac{I}{V_{ab}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

Since $\frac{I}{V_{ab}} = \frac{1}{R}$ we have

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \qquad \text{-----------(13.9)}$$

that is the sum of the reciprocals of individual resistances connected in parallel is equal to the reciprocal of the equivalent resistance. Since we have defined the conductance of a material as the reciprocal of its resistance, hence we can say that in the parallel combination of resistances, the sum of the individual conductance is equal to the equivalent conductance in the network.

The equivalent resistances of network in fig.13.4. (iii) and (iv) can be found by the same method by considering them as combinations of series and parallel arrangements. Thus in fig. 13.4. (iii), the combination of R_2 and R_3 in parallel is first replaced by its equivalent resistance R_4 which then forms a series combination with R_1 giving $R = R_1 + R_4$ and in fig.13.4. (iv) the combination of R_2 and R_3 in series forms a parallel combination with R_1 hence R can be determined.

Example:13.8.

A battery of 6 volts is connected to two resistors of 3Ω and 2Ω joined together in series. Find the current through the circuit and the potential drop across each resistor.(fig.13.5.)

Solution:

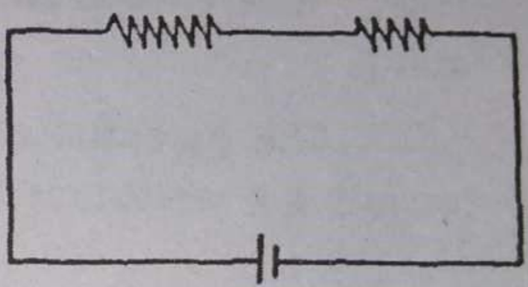


Fig.13.5

Since the resistors are joined in series, the equivalent resistance R will be equal to 3Ω + 2Ω = 5Ω. Therefore the current *I* through the circuit will be

$$\frac{6v}{5\Omega} = 1.2\ A$$

The voltage drop across 3Ω resistor

$$V = IR = 1.2A \times 3\Omega = 3.6\ V$$

& the voltage drop across 2Ω resistor

$$V_2 = IR = 1.2A\times 2\Omega = 2.4\ V$$

So that $V_1 + V_2 = 3.6V + 2.4V = 6V$ which is the voltage of the battery.

Example:13.9.

Find the equivalent resistance in the given circuit and also the current I_1, I_2 and I_3

Solution:

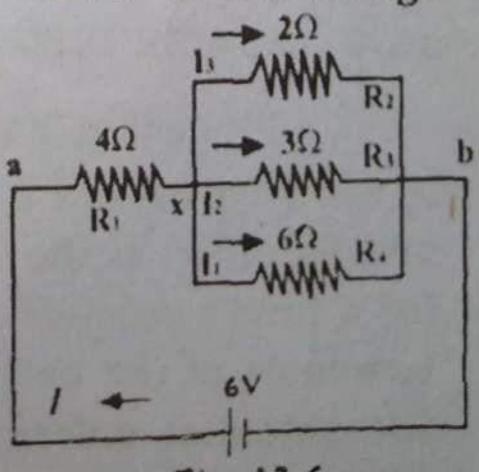


Fig. 13.6

For the parallel Combination of R_2, R_3 and R_4 the equivalent resistance R_5 is given by

$$\frac{1}{R_5} = \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4} = \frac{1}{2} + \frac{1}{3} + \frac{1}{6} = 1$$

or $R_5 = 1\Omega$

This R_5 is now in series with R_1 hence

$$R = R_1 + R_5 = 4\Omega + 1\Omega = 5\Omega$$

The current I in the circuit $= \frac{V}{R} = \frac{6v}{5\Omega} = 1.2A$

Current I_1 can be calculated if we know the potential difference existing between the two end points x and b.

The potential difference between points a and x including a resistance of

$$4\Omega = IR = 1.2 \text{ A} \times 4\Omega = 4.8 \text{ v.}$$

Therefore the potential difference between points a and b will be

$$6V - 4.8V = 1.2V$$

or the current I_1 across a resistance of $6\Omega = \frac{1.2V}{6\Omega} = 0.2A$

Similarly $I_2 + I_3$ will be found as 0.4 A and 0.6A respectively so that $I_1 + I_2 + I_3 = 0.2A + 0.4A + 0.6A = 1.2A$ which is total current from battery to the circuit.

13.6 POWER DISSIPATION IN RESISTORS

Suppose a battery is connected across a resistor R which produces a potential difference of V across its end fig. 13.7. If the current I flows through this resistor for a time t seconds, the charge transported between the terminals of the battery is given by $Q = I \times t$. Since V is the potential difference causing the transfer of charge, therefore the work done in transfering charge $Q = QV$. This work is done at the expense of the potential energy of the charges as they pass through the resistor. This loss of potential energy is converted into vibrational energy of the atoms to which the electrons collide during their motion and thus the energy lost by the electrons is gained by the atoms of the conductor (resistor) in the form of heat.

Hence the heat developed in the resistor in t second is QV We define power as the rate of doing work i.e the work done or energy spent per unit time, hence the power dissipated as heat due to electric current in the resistor is given by

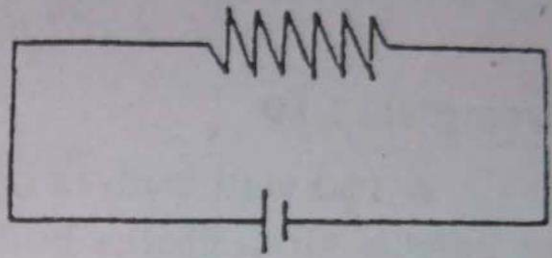


fig.13.7.

$$\text{Power} = \frac{\text{Work done}}{\text{time}}$$

$$P = \frac{QV}{t} = VI \text{----------} 13.9.$$

as the current $I = Q/t$

This is the general relation for power delivered from a source of electric current operating at a voltage V. From ohms law equation (13.9) can be converted in terms of resistance R by changing I or V.

Hence we have

$$P = VI = I^2R = V^2/R \qquad \text{------------------} (13.10)$$

Recalling that volt is joule per coulomb and current is coulomb per second the unit of power is

$$\frac{\text{joule}}{\text{coulomb}} \times \frac{\text{coulomb}}{\text{second}} = \frac{\text{joule}}{\text{second}}$$

Which is called watt.

Higher values of power are expressed in kilowatt = 1000 watts or megawatt = 10^6 watts.

If a current I flows steadily through a resistor R for a time t, the total heat energy supplied to the resistor is given by heat energy = Power x time = V.I.t = I^2Rt = V^2 Rt joules.

Usually the energy supplied by electric current through its generating stations is measured in terms of a unit known as Kilowatt-hour (KWh) which is the energy delivered by the current in one hour when it supplies energy at the rate of 1000 joule per second i.e.

$$1\text{ KWh} = 1000\text{ Joule/Sec} \times 1\text{ hour}$$

$$= 1000\text{ J/s} \times 3600\text{s}$$

$$= 36 \times 10^5\text{ Joules}$$

Example:13.10

A 100 watt bulb is operated by 240 volts. What is the current through the bulb?

Solution:

Since $P = VI$

$100\text{ watt} = 240\text{ V} \times I$

$$I = \frac{100\text{ watts}}{240\text{V}} = 0.416\text{ A}$$

Example: 13.11.

An electric Kettle of 1000 watts rating boils a certain quantity of water in 8 minutes. How much heat has been generated for boiling this water.

Solution:

Heat = Power x time

$$= 1000\text{ watts} \times 8 \times 60\text{ second}$$

$$= 48 \times 10^4\text{ Joules}$$

Example: 13.12.

How much current is drawn by a half horse power electric motor operated from 240 V source of electricity? Assume that the efficiency of motor is 80%.

Solution:

One horse power = 746 watts

$\frac{1}{2}$ horse power = 373 watts

The power input to motor = IV = 240 x 0.8

The out put of the motor = 373 watts.

$$I = \frac{\text{Power out Put}}{\text{Efficiency} \times \text{volt}} = \frac{373 \text{ W}}{0.8 \times 240}$$

$$= 1.95 \text{ A}$$

13.7. ELECTROMOTIVE FORCE:

We have seen that an electric field is needed to maintain a current in a conductor,When this electric current passes through a resistor. it dissipate energy which is transformed into heat. Thus to sustain a current in a conductor, some source of energy is needed, so that it could continuously supply power equal to that which is dissipated as heat in the resistor. The strength of such a source is known as electromotive force. Usually the potential difference that exists between the two terminals of a battery or any source of electrical energy when it is not connected to any external circuit is called its electromotive force, simply read as E.M.F and is represented by E.

As charges pass through a source of electrical energy i.e a cell or a battery, work is done on them, the electromotive force may be defined as the work done per coulomb on the charges.

The e.mf of an automobile storage battery is 12 V, which means that 12 joules of work is done on each coulomb of charge that passes through the battery. In the case of a battery, chemical energy is converted into electrical energy by means of the work done on the charges in transit through it; in a generator, mechanical energy is converted into electrical energy and in a thermocouple, heat energy is converted into electrical energy and so on.

The e.m.f of an electrical source bears a relationship to its power output analogous to that of applied force to mechanical power in a machine, which is the reason for its name.

Let us consider a simple circuit in which a resistor R is connected by leads of negligible resistance to the terminals of a battery (fig.13.8.)

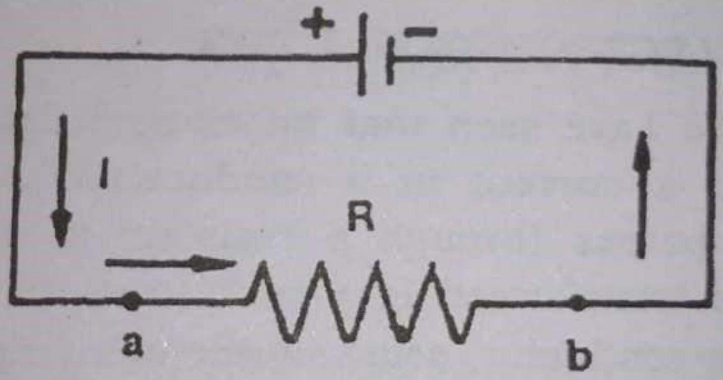


Fig. 13.8.

A current I will flow through the resistor in a direction from a to b, the potential V_a being higher than potential V_b. The same current will flow through the battery from its negative terminal to the positive terminal. The battery is made of some electrolyte and electrodes for the production of e.m.f. and hence when this current flows the battery, it encounters some resistance by the electrolyte present between its two electrodes. This resistance is known as the internal resistance of the battery.

Thus the current in the circuit from Ohm's law is given by the relation

$$I = \frac{E}{R + r}$$, where r is the internal resistance

$$E = IR + Ir \quad \text{------------------(13.11)}$$

Here IR is the voltage to drive the current I through the external resistor R and Ir is" lost voltage" driving current I through the internal resistance of the battery denoting IR. the potential difference between the two terminals of the battery by V we have

$$V = E - Ir \quad \text{------------------ (13.12)}$$

This shows that the potential difference between the terminals of a battery drops when it delivers a current. However when no current is drawn, there is no potential drop across the internal resistance so that the terminal potential difference is equal to its e.m.f. i.e.

$$V = E \quad \text{------------------ (13.13)}$$

In effect, we can say that the internal resistance of a battery governs the maximum current it can supply.

Example:13.13.

A storage battery whose e.m.f is 12 V and whose internal resistance is 0.2Ω is to be charged at rate of 20 A. What applied voltage is required ?

Solution:

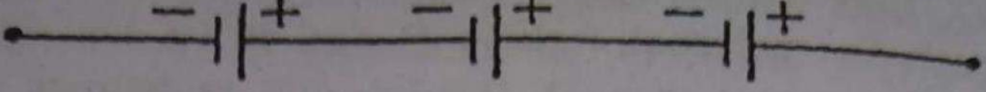
The applied Voltage V must exceed the battery e.m.f E by the amount Ir to provide the charging current I. hence.

$$V = E + Ir = 12\ V + (20A)\ (0.2\Omega)$$

$$= 12\ V + 4\ V$$

$$= 16\ V.$$

When the e.m.f of a single cell is too small for a particular application, two or more can be connected in series (fig.13.9)



(fig.13.9)

The e.m.f of the set is sum of e.m.f of the individual cells and the internal resistance of the set is the sum of individual resistances

$$E - series = E_1 + E_2 + E_3 \text{ ------------ } = E_n$$

$$r - series = r_1 + r_2 + r_3 \text{ ------------- } = r_n$$

A familiar example of such an arrangement is the use of lead acid cells in series to make a 12V battery of a car

When the e.m.f of a battery or cell is sufficient but its capacity is too small, two or more batteries or cells can be connected in parallel to give more current. The total current is the sum of the currrent delivered from the individual battery or cell (fig.13.10).

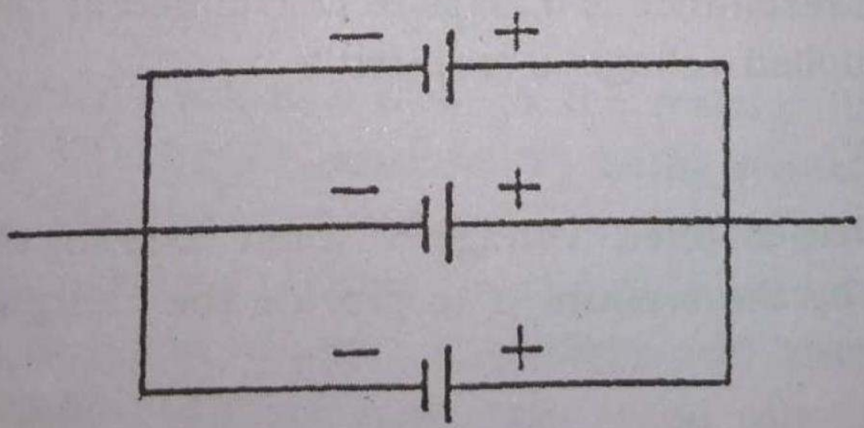


fig.13.10

QUESTIONS:

13.1 Electrons leave a dry cell and flow through a lamp bulb back to the cell. Which terminal, the positive or the negative is the one from which electrons leave the cell? In which direction is the conventional current.

13.2 Both p.d. and e.m.f. are measured in volts. What is the difference between these concepts?

13.3 Can you construct two wires of the same length, one of copper and one of iron, that would have the same resistance at the same temperature.

13.4 Why does the resistance of a conductor rises with the rise in temperature?

13.5 Why is heat produced in a conductor due to flow of electric current?

13.6 When a metal object is heated, both its dimensions and its resistivity increase. Is the increase in resistivity likely to be a consequence of the increase in length?

13.7 It is sometime, said that an electrical appliance, " uses up" electricity. What does such an appliance actually use in its operation?

13.8 Do bends in a wire affect its resistance?

13.9 Resistances of 10Ω . 30Ω and 40Ω are connected in series.If the current in 10Ω resistance is 0.1A, what is the current through the others?

13.10 Ten resistances of different value are connected in parallel. If the p.d.across one of them is 5V, what is the p.d. across the remaining nine resistors?

13.11 For a given potential difference V, how will the heat developed in a resistor depend on its resistance R ? will the heat be developed at a higher rate in a larger or a smaller R?

13.12 Is there any electric field inside a conductor carrying an electric current?

13.13 How does the current flowing in a conductor depend on the number of mobile charge carriers per unit length? On their average velocity? On the charge per carrier?

13.14 (a) What is the equivalent resistance of three 5Ω resistors connected, (i) in series (ii) in parallel.

(b) If a potential difference of 60V is applied across series connection, what is the current in each resistor?

13.15 Can the terminal voltage of a battery be zero?

13.16 Why is the internal resistance of a cell not constant?

PROBLEMS

13.1 A certain battery is rated at 80 ampere hour. How many coulomb of charge can this battery supply

(Ans: 2.88×10^5 C).

13.2 A silver wire 2 m long is to have a resistance of 0.5Ω. What should its diameter be

(Ans: 2.78×10^{-4}m).

13.3 A current of 6A is drawn from a 120 V line. What power is being developed? How much energy in joule and in Kilowatt is expended if the current is drawn steadily for one week.

(Ans: 720 watt. 120.83 kwh, 4.35×10^8 J)

13.4 Currents of 3A and 1.5A flow through two wires, one that has a potential difference of 60V across its ends and another that has a potential difference of 120 V across its ends. Compare the rate at which energy pass through each wire.

(Ans: 1)

13.5 A wire carries a current of 1 A. How many electrons pass a point in the wire an each second? (6.3×10^{18} electrons).

13.6 An electric drill rated at 400W is connected to a 240V power line. How much current does it draw?

(Ans: 1.67A)

13.7 Resistors of 20Ω, 40Ω, 50Ω are connected in parallel across a 50V power source. Find the equivalent resistance of the set and the current in each resistor?

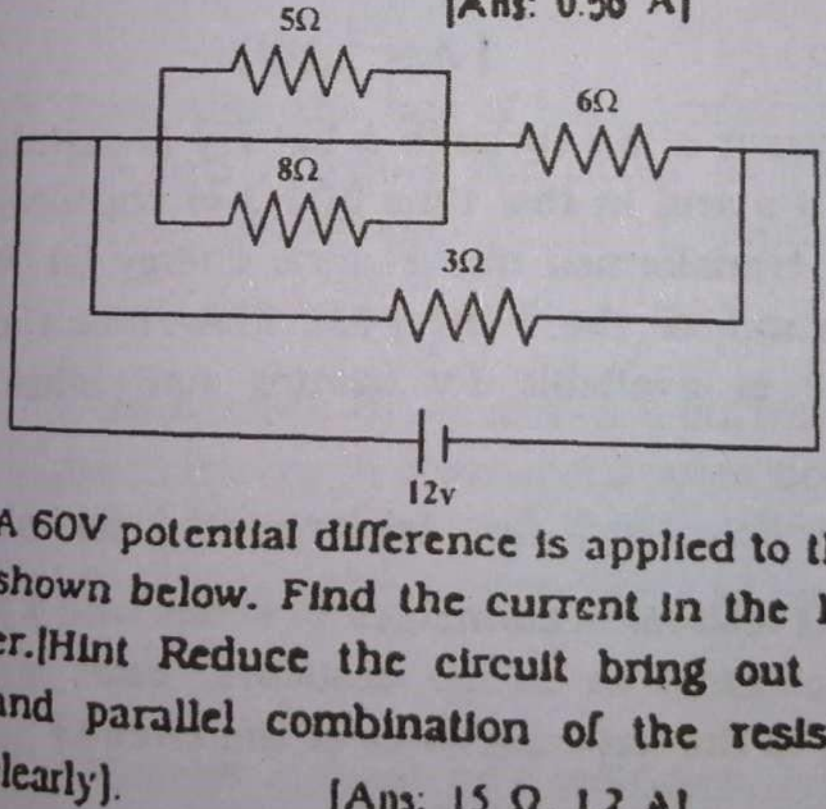
[Ans: 10.5 Ω $I_1 = 2.5A, I_2 = 1.25A, I_3 = 1A$]

13.8 (a) Find the equivalent resistance of the network shown below

[Ans: 2.18 Ω]

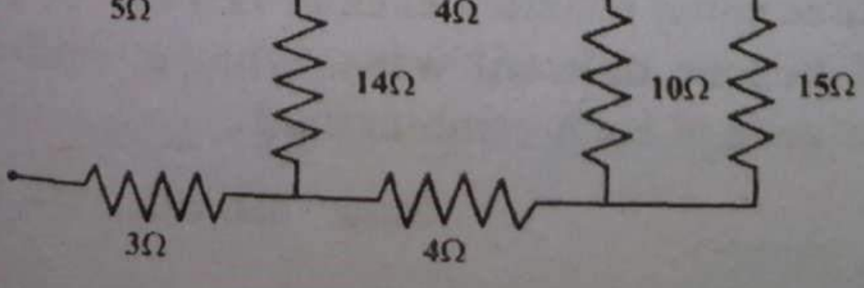
(b) What is the current in 8Ω resister if the potential difference of 12V is applied to the network?

[Ans: 0.56 A]



13.9 A 60V potential difference is applied to the circuit shown below. Find the current in the 10Ω resister. [Hint Reduce the circuit bring out the series and parallel combination of the resistors more clearly].

[Ans: 15 Ω. 1.2 A]



13.10 A source of what potential difference is needed to charge a battery of 20 V e.m.f and internal resistance of 0.1Ω at a rate of 70A.

[Ans: 27V]

13.11 A battery of 24 V is connected to a 10Ω load and a current of 22 A flows. Find the internal resistance of the battery and its terminal voltage.

(Ans: 0.9 Ω, 22 V)

13.12 A 40 Ω resistor is to be wound from platinum wire 0.1 mm in diameter. How much wire is needed?

[Ans: 2.85m]

13.13 The battery of a pocket calculator supplies 0.35A at a p.d. of 6V. What is the power rating of the calculator?

[Ans: 2.1W]

13.14 A current of 5A through a battery is maintained for 30 s and in this time 600 J of chemical energy is transformed into electric energy (a) What is the e.m.f of the battery? (b) How much electric power is available for heating and other uses?

(a) [Ans: 4V], (b) 20W)

13.15 A 12Ω resistor is connected in series with a parallel combination of 10 resistors, each of 200Ω. What is the net resistance of the circuit?

(Ans: 32Ω]

13.16 Three equal resistors each of 12Ω can be connected in four different ways. What is equivalent resistance of each combination?

[Ans: 4Ω, 8Ω, 18Ω, 36Ω]

13.17 Find the resistance at 50 °C of a copper wire 2 mm in diameter and 3 m long.

[Ans: 0.0184 Ω]

13.18 The resistance of a tungsten wire used in the filament of a 60w bulb is 240Ω when the bulb is hot at a temperature of 2020 °C what would you estimate its resistance at 20 °C

[Ans: 25.4 Ω]

13.19 A water heater that will deliver 1 kg of water per minute is required. The water is supplied at 20 °C and an output temperature of 80 °C is desired. What should be the resistance of the heating element in the water if the line voltage is 220V?

[Ans: 11.5Ω]

13.20 Prove that the rate of heat production in each of the two resistors connected in parallel are inversely proportional to the resistances.

[Ans: $P = \frac{V^2}{R}$; $P \propto \frac{1}{R}$]

13.21 A 240V cloth dryer draws a current of 15A. How much energy in Kwh and Joules does it use in 45 minutes operation and how much will be the cost at the rate of Rs.1.45 per unit of electric energy?

[Ans: 2.7 kwh, 9.72×10^6 J, Rs: 39]

13.22 A resistor is made by winding on a spool a 40 m length of constantan wire of diameter 0.8 mm. Calculate the resistance of the wire at a) 0 °C. b) 50 °C. Assume ρ at 0 °C .

(Ans: 49×10^{-8} Ω — m. (a) 38.99 Ω (b) 39.01 Ω)