Chapter 14.

MAGNETISM AND ELECTRO-MAGNETISM

It has been known from centuries that a magnet exerts force on another magnet from a distance. The phenomenon is similar to that of force between electric charges or gravitational force between material particles.

14.1. Magnetic field due to Current

In 1819 Christian Oersted, Professor of Physics at Copenhagon discovered that a pivoted magnetic needle is deflected from its normal north - south direction when even a current bearing wire is held parallel to it as if the current behaves as a magnet.

The force that acts on the magnetic needle can be described by visualizing the needle as being situated in the field created by electric currents through conductors or magnets. This is called magnetic field of Induction (Magnetic field would be a more suitable name but it has been acquired for historical reason by another magnetic quantity to be discussed in higher classes).

In 1820 Ampere observed that two current bearing conductors exert forces on each other and suggested that the magnetic condition of magnets is caused by currents within the body of magnets. These currents may now be identified with the motion of electrons in the atoms of the magnetic material.

When electric charges are at rest they exert electrostatic forces of attraction or repulsion on each other. When the charges are in motion they still exert these electrostatic forces but, in addition, magnetic forces appear because of motion. Isolated moving positive or negative charges create both electric and magnetic fields but an electric current through a conductor produces only a magnetic field because the electric field of moving electron is neutralized by the field of fixed protons in the conductors.

A magnetic field is a region in which a force is experienced on a moving charge or a magnet.

This force depends upon.

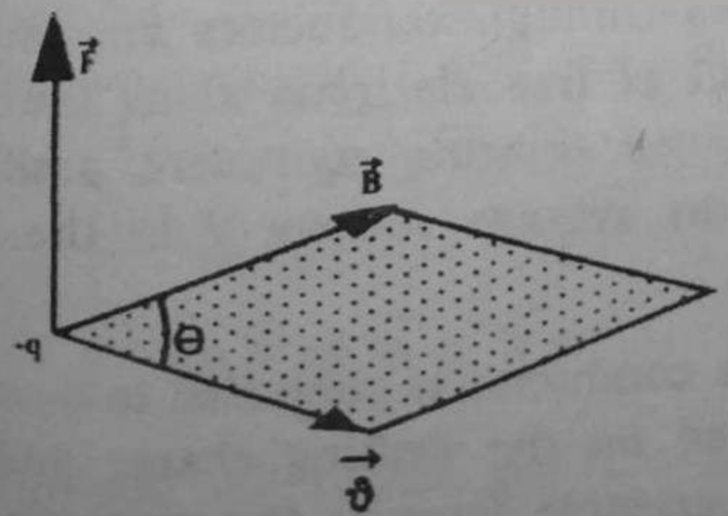
1. The magnitude of charge q.
2. The speed of the moving charge ϑ.
3. The magnetic field of Induction B.

The magnetic Induction B is a Vector quantity defined from the relation.

$$\vec{F} = q(\bar{\vartheta} \times \bar{B}) \qquad \text{(14.1)}$$

The direction of force is perpendicular to both the direction of motion of charge and the direction of B given by cross product rule.

It also follows that the force on the charge in the field is zero either when the charge is stationary or moving along the direction of B.



The magnitude of B is given by

$$B = \frac{F}{q\vartheta \sin\theta} \quad \text{-------------------(14.2)}$$

A unit magnetic field of Induction is said to exist at a point where the force per unit charge experienced by a positive test charge, moving with a velocity of $1 ms^{-1}$ in the direction perpendicular to the field is 1 newton.

$$\text{Unit of } B = \frac{\text{newton}}{\text{coul} \times \text{metre/second}} = \frac{\text{newton}}{\text{ampere} \times \text{metre}}$$

It is called tesla (T).

Example 14.1.

A proton enters a uniform magnetic field of Induction B = 0.300 Tesla in a direction making an angle of 45° with the direction of field. What will be the magnitude of force if the velocity of proton is $10^4 ms^{-1}$?

Solution:

$$F = q\,V\,B \sin\theta = 1.6 \times 10^{-19} \times 10^4 \times 0.3 \times \sin 45°$$

$$= 3.4 \times 10^{-16}\ N$$

14.2. Force on a Current Carrying Conductor in a Uniform Magnetic Field.

Currents through conductors are caused by the directional drift of free electrons along the conductors. Conventionally we imagine equivalent positive charges drifting with an average velocity V in the direction of current.

When a conductor is subjected to a magnetic field force is exerted on the drifting charge and hence the conductor experiences force in the magnetic field of Induction.

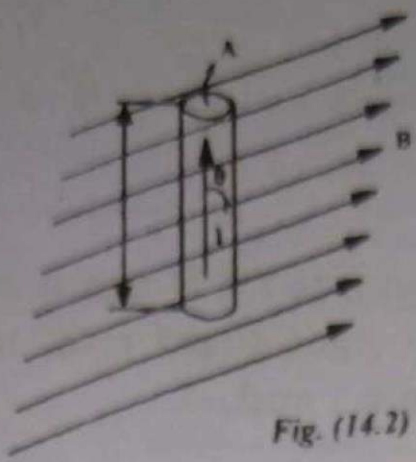


Fig. (14.2)

Consider a linear conductor, of length L and carrying a current I, be subjected to a uniform magnetic field of Induction B which makes an angle θ with direction of current as shown in Fig.14.2.

If there are n number of free electrons per unit olume the total moving charge

$q = n A \ell e$

where A is the area f cross section of conductor.

The force on the conductor

$$\vec{F} = q(\vec{\vartheta} \times \vec{B}) = n A \ell e \ (\vec{\vartheta} \times \vec{B})$$

This expression can be changed in terms of the current by a little mathematical manipulation. Consider the length of conductor, a Vector in the direction of $\vec{\vartheta}$ which can be written $\hat{a}\,\ell$ where $\hat{a}$ is a unit Vector in the direction of $\vec{\vartheta}$. Similarly $\vec{\vartheta}$ can be written as $\hat{a}\,\vec{\vartheta}$ where $\vec{\vartheta}$ is the magnitude of drift velocity.

$$\vec{F} = n A \ell e \,(\hat{a}\vec{\vartheta} \times \vec{B})$$

$$\vec{F} = n A ev \,(\hat{a}\,\ell \times \vec{B})$$

$$\vec{F} = n A e\vartheta \,(\vec{\ell} \times \vec{B})$$

The average drift velocity $\vartheta = \dfrac{\ell}{t}$ where t is the time taken by q to cross the length of the conductor.

$$\vec{F} = \frac{nAe\ell}{t}(\vec{\ell} \times \vec{B}) = \frac{q}{t}(\vec{\ell} \times \vec{B})$$

$$\vec{F} = I(\vec{\ell} \times \vec{B}) \qquad \text{..................} \quad (14.3)$$

The magnitude of force is $I \ell B \sin\theta$ and the direction of force is given by right hand rule. It is in the direction perpendicular to both I and B.

$$B = \frac{F}{I \ell \sin\theta}$$

Example 14.2.

A steady current of 25 A is passing through a horizontal power line 50 m in length held between two poles in the north south direction. The earth's magnetic field is 10^{-4} tesla at that place and angle of dip is 60°, find the force on the wire.

Solution:

Angle of dip is the angle between the direction of earth's magnetic field and the horizontal in the magnetic meridian.

$$\vec{F} = I(\vec{\ell} \times \vec{B}) = I\ell B \sin\theta$$

$$= 25 \times 50 \times 10^{-4} \times \sin 60^\circ$$

$$= 0.108\ N$$

14.3. Torque on a Current Carrying Rectangular Coil placed in a Magnetic Field

In Fig. 14.3 a rectangular coil is suspended in a uniform magnetic field and the plane of the coil is parallel to the field. A current I is flowing round the coil in the direction shown.

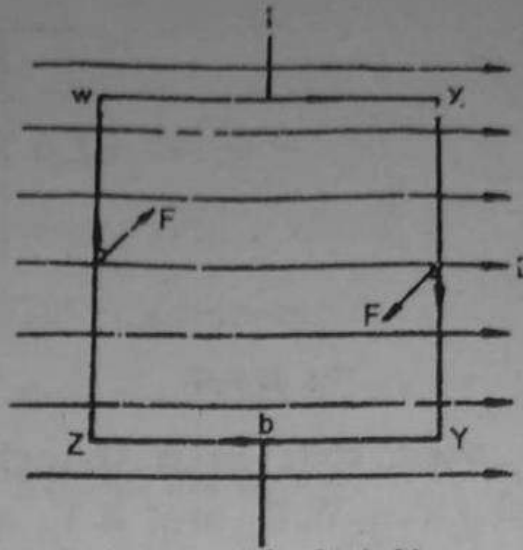


Fig (14.3)

The vertical side WZ of the coil experiences a force F which is directed perpendicularly into the paper. There is an equal and opposite force on xy. Since the plane of the Coil is parallel to the field there is no force on either WX or ZY. the forces on WZ and XY constitute a couple whose torque τ is given by

$$\tau = F b$$

where b is the width of the coil. The directions of the current in WZ and XY are each at 90° to the magnetic field, and therefore from equation 14.3

$$F = B I \ell \sin 90^\circ . = BI\ell$$

where ℓ is the length of each vertical side of the coil.

$$\therefore \tau = B I \ell b = B I A. \qquad \text{Where } A = \text{Area of coil.}$$

For a coil of N turns.

$$\tau = B I A N$$

As soon as the coil turns under the influence of the torque it ceases to be parallel to the field. Figure 14.4 shows the situation of the coil when it is at some angle α to the field.

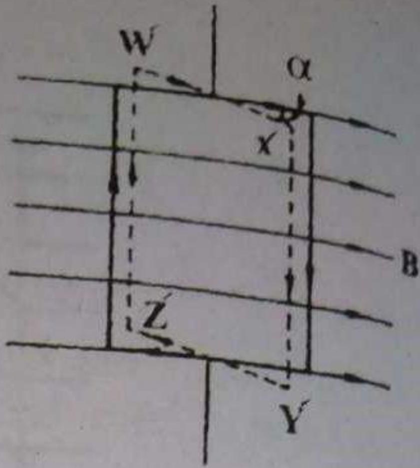


Fig. (14.4)

It can be seen that even though the Coil has turned its vertical sides W'Z' and X'Y' are still perpendicular to the field. The forces acting on vertical sides therefore have the same magnitude and the same direction as they had before the coil turned. However the separation of the forces alter so that the torque τ has a reduced value given by $\tau = F\, b \cos\alpha$

Therefore in general

$$\tau = B I A n \cos\alpha \qquad \text{-------------------(14.4)}$$

For values of α other then zero the field exerts forces on W'X' and Z'Y' but these forces are always parallel to the axis about which the Coil is turning in opposite directions and therefore make no contribution to the torque. The formula is also valid for circular Coil.

Example 14.3.

A 50 turn rectangular Coil of wire 4 cm by 5 cm is suspended in a vertical plane.

A horizontal uniform magnetic field of 2.0 $\frac{N}{Am}$ is maintained in the plane of Coil. Compute the torque experienced by the coil when a current of 0.3 amp flows through it

Solution:

N = 50

A = 4 x 5 = 20 cm = 0.002 m^2

$\alpha = 0$ B = 2

Torque

$$\tau = B\,I\,A\,N \cos\alpha$$

$$= 2 \times .3 \times .002 \times 50 \times 1$$

$$= 0.06 \text{ newton-metre}$$

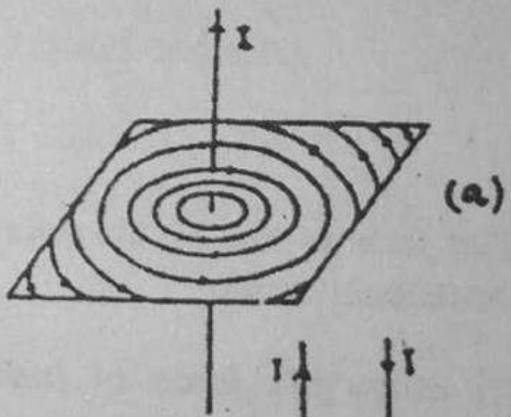
14.4. Magnetic flux and flux density

Magnetic field of Induction in a region can be visualized by magnetic lines of induction just as electric field was represented by electric lines of force.

Lines of induction are defined in the same way as electric lines are defined. Unlike electric lines of force the magnetic lines of induction are endless and continuous lines in the field region and can be traced using a small compass needle. Figure 14.5. Shows the lines of Induction in a few cases.

(a) Magnetic lines of Induction around a long Straight Current carrying wire in a plane perpendicular to the wire.

The lines are concentric circles with their centres at the wire. If the wire is grasped by right hand with the thumb in the direction of current, the fingers encircle the conductor in the direction of field. This is called right hand grip rule.



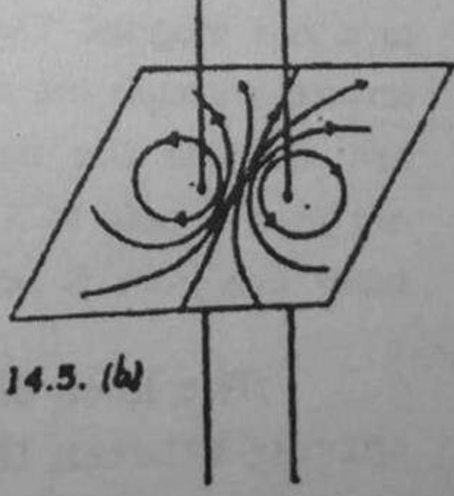
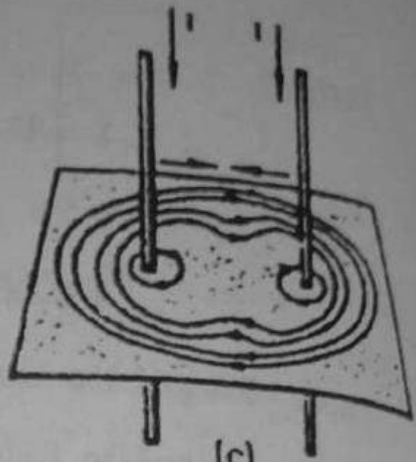


Fig 14.5. (b)

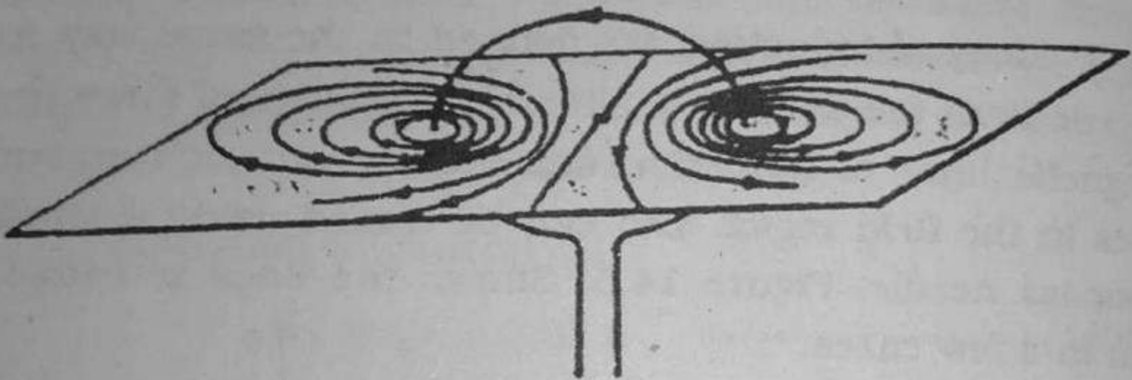
(b) Lines of Induction due to two long parallel wires carrying current in opposite directions.

(c) Lines of Induction due to two long parallel wires carrying currents in the same direction.

(d) Lines of force of Induction due to a circular loop. The lines enter one face which is the South pole emerge from other face which is North pole. On looking at a face if the current appears clock wise the face is South pole.



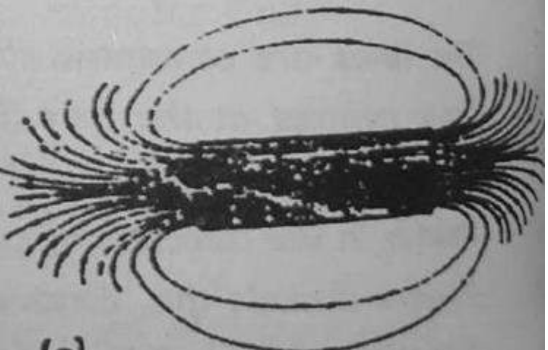
(c)



(d)

The face is the North pole if on looking on it. the current is anticlock wise.

(e) Lines of force of Induction due to a bar magnet. The lines are continuous through the material of magnet. Outside the magnet the lines are from N pole to S pole but inside there are from S pole to N pole.



(e)

The lines of Induction are to be drawn with some spacing between them so that they may be distinguished as separate lines. It is so manipulated that the number of lines per unit area passing through a very small surface held normal to the lines at a point is equal to the magnitude of B at that point.

The magnetic flux over a surface is defined in the same way as electric flux. It is the number of magnetic lines of induction crossing the surface normally.

The magnetic flux over a small surface at every point of which the field is the same is

$$\Delta \phi_m = \vec{B} \quad \Delta\vec{A} \qquad \text{-------------(14.5)}$$

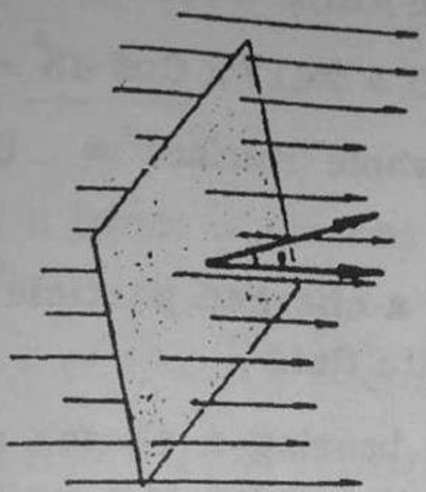


Fig. (14.6)

Magnetic flux is expressed in the unit called weber.

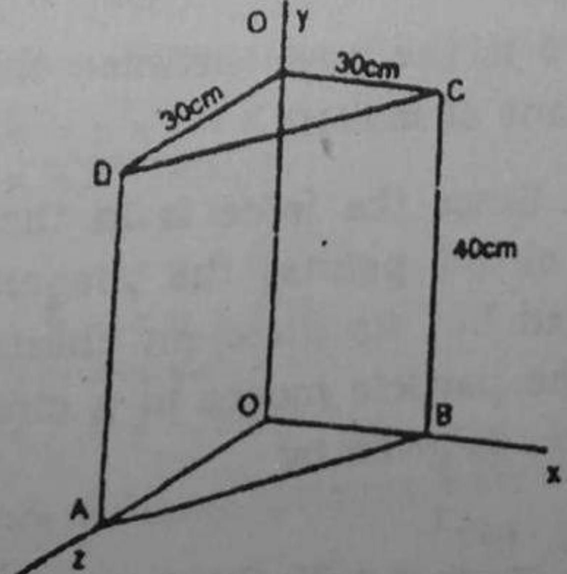
Since the magnetic field of induction is the number of lines passing normally per unit area it is the flux density and its units are Weber m^{-2}.

$$B = \frac{\Delta \phi_m}{\Delta A_n} \qquad \text{------------------ (14.6)}$$

Example 14.4.

Find the magnetic flux across the Surface of triangular prism shown in the diagram if the flux density is 2 tesla along X-axis

$$\phi_m = BA \cos \theta$$



a) **Flux across the face**

$$OADO' = B\,A\,\text{Cos}\,\theta = 2 \times (.3 \times .4) = 0.24$$

$$DC = \sqrt{30^2 + 30^2} = \sqrt{1800} = 42.42\ \text{cm} = .42\ \text{m}$$

$\therefore$ area of the face ABCD = $.42 \times .4 = 0.17\ m^2$

Flux across ABCD = $2 \times .17 \times \text{Cos}\ 45^\circ = 0.24$

Flux over the whole Surface = $0.24 + 0.24$ = 0.48 Webers.

14.5 (I) Force on a charged particle moving on a magnetic field .

When a particle bearing a charge q and moving with a velocity v enters the region of a uniform magnetic field of Induction. B it is acted upon by a force

$$F = q\,(\vec{\vartheta} \times \vec{B}) = q\,VB\,\text{Sin}\,\theta$$

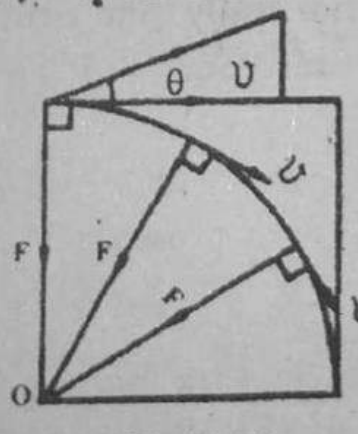


Fig. 14.7

when θ is the angle between the plane of the field and the plane of motion.

Since the force is in the direction perpendicular to ϑ at all points, the magnitude of ϑ remains unchanged but its direction changes from point to point and the particle moves in a circular path. The centripetal force is given by

$$\frac{m\vartheta^2}{r} = q\,\vartheta B\,\text{Sin}\,\theta$$

$$r = \frac{m\vartheta}{q\ B\ \mathrm{Sin}\ \theta} \qquad \text{------------(14.7)}$$

where r is the radius of the circular arc and m is the mass of the particle.

In case if the magnetic field is perpendicular to the plane of motion.

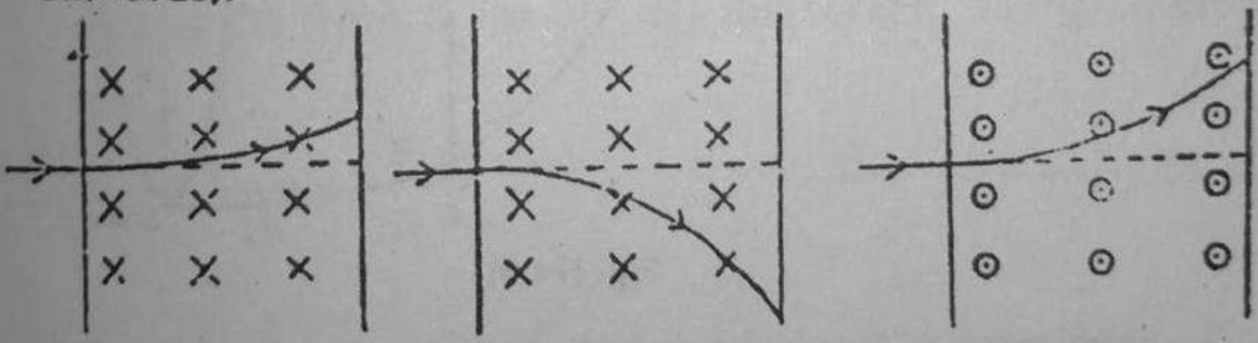
$$r = \frac{m\vartheta}{qB}$$

For a beam of protons $F = e\ (\vec{\vartheta}\ \times\ \vec{B})$ and for a beam of electrons

$$F = -\,e(\vec{\vartheta}\ \times\ \vec{B}) = e(\vec{\vartheta}\ \times \vec{B})$$

The figure 14.8 illustrates the direction of deflection in a few cases when the plane of field is perpendicular to the plane of motion.

(It is a convention that a cross (x) indicates a field directed inwards of the page and a dot (.) means a direction outwards).



a) Trajectory of a Beam of Protons.

b) Trajectory of a Beam of Electrons.

c) Trajectory of a Beam of Electrons.

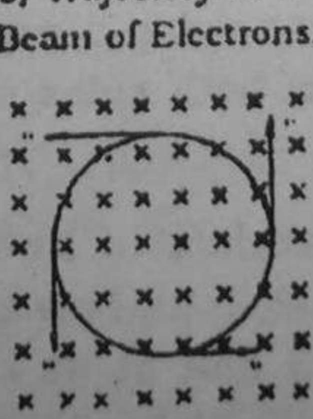


Figure 14.8 (d). Trajectory of a Beam of electron when the field exists in an extended region.

(ii) Determination of Charge to Mass ratio of an electron.

The above knowledge was utilized by Sir J.J. Thomson to determine $\frac{e}{m}$ of an electron.

The apparatus consists of a highly evacuated pear shaped glass bulb into which several metal electrodes are sealed. (fig.14.9).

Electrons are produced by heating a tungsten filament by passing a current through it.

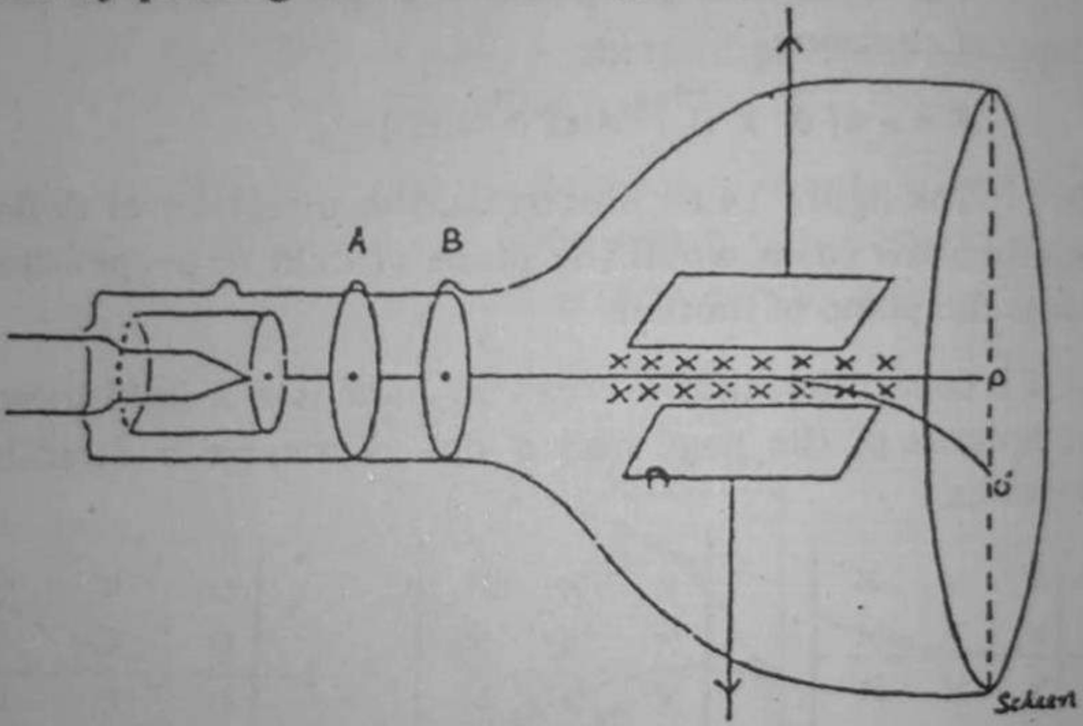


Fig. (14.9)

The electrons moving side ways are also directed towards the screen by applying a negative potential on a hollow cylinder open on both sides surrounding the filament.

Electrons are accelerated by applying a potential difference of above 1000 Volts between the filament and the metal disc A with a hole at its centre. A further potential difference of 500 Volts is applied between the discs A and B. This arrangement focuses the electron beam to the hole of the disc B from where it further proceeds in a straight line. If the total potential difference between the filament and the disc B is V, the velocity acquired by the electrons is given by the energy equation.

$$\frac{1}{2} m v^2 = V_e$$

$$v = \sqrt{\frac{2V_e}{m}} \quad \text{------------------(14.9)}$$

The beam strikes the screen coated with Zinc Sulphide after passing through the middle of two horizontal metal plates and a spot of light is produced at "O" on the screen where the beam strikes and its position is noted.

A magnetic field of induction B is produced along y-axis by two identical current carrying circular coils placed on either side of the tube at the position of plates. The force due to magnetic field on moving electrons makes them move in a circular path and the light spot shifts from 0 to 0′, on the screen. Using equation 14.7

$$\frac{e}{m} = \frac{v}{rB} \quad \text{------------------ (14.10)}$$

$\frac{e}{m}$ can be computed from this expression if

the radius of the circular arc in which the beam moves in the field region is determined. The radius is calculated from the shift of light spot.

A better method of determining. V is as under:

An electric field E is produced between the plates by applying a suitable potential difference to exert a force Ee on the electrons opposite to that due to magnetic field. The potential difference V_1 is so adjusted that the two fields neutralize each others effects and the spot comes back to its original position O

$$E = \frac{V_1}{d}, \text{ where } d = \text{distance between plates.}$$

$$Ee = e\vartheta B$$

$$\vartheta = \frac{E}{B} \qquad \text{------------------(14.11)}$$

Substituting in eq 14.10

$$\frac{e}{m} = \frac{E}{rB^2} \qquad \text{------------------(14.12)}$$

$$e/m = 1.75888 \times 10^{11}\ Ckg^{-1}$$

As $e = 1.60207 \times 10^{-19}\ C$

$$m = \frac{1.60207 \times 10^{-19}}{1.75888 \times 10^{11}} = 9.1084 \times 10^{-31}\ kg$$

14.6. Ampere's Law

Ampere's Law is some what analogous to Gauss's Law of electrostatics and it helps to determine the magnetic field of induction in a few cases of current configurations.

Consider first a long straight wire carrying a current, I in the direction shown in the fig. 14.10 the lines of force are concentric circle with their common centre on the wire. Hence the magnetic fields at all points on a curve taken in form a circle round the wire is tangential and of the same magnitude. Biot and Savart experimentally found that the magnitude of the field depends directly on twice the current I and inversely on the distance r from the conductor.

$$B \propto \frac{2I}{r}$$

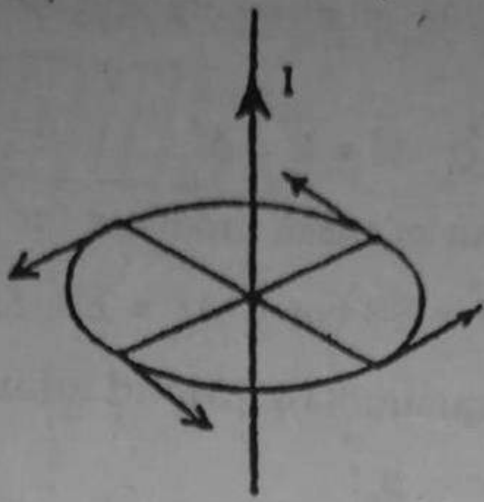


Fig.(14.10)

The constant of proportionality is written as $\frac{\mu_o}{4\pi}$

and its value is 10^{-7}. μ_o is called permeability of free space.

$$B = \frac{\mu_o}{4\pi} \times \frac{2I}{r}$$

$$B = \frac{\mu_o I}{2\pi r} \qquad \text{-----------------(14.13)}$$

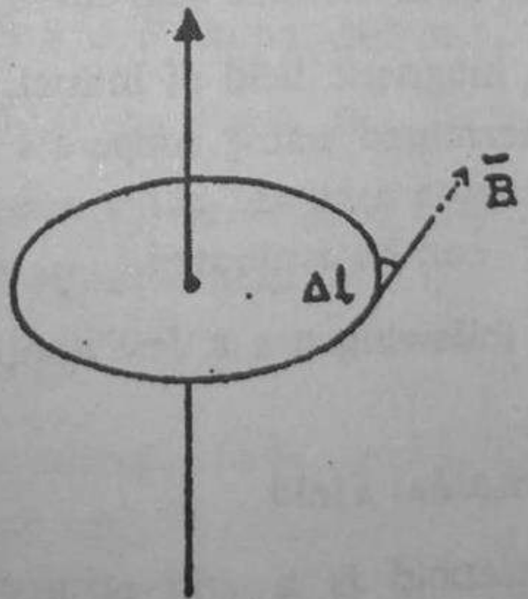


Fig.(14.11)

Again let the circle be divided into small elements each of length $\Delta \ell$. Multiplying the length of each element by the tangential component of field which is in the direction of $\Delta \ell$ for all elements and for this special case we get

$$B \cos 0^\circ \, \Delta \ell = \vec{B} \cdot \Delta \ell$$

The sum of these products for all the elements is

$$\Sigma \vec{B} \cdot \Delta \vec{\ell} = \Sigma B \cos 0^\circ \, \Delta \ell = \Sigma B \, \Delta \ell = B \, \Sigma \, \Delta \ell$$

using equation(14.13)and taking $\Delta \ell = 2\pi r$ we get

$$\Sigma \vec{B} \cdot \Delta \vec{\ell} = \frac{\mu_o I}{2\pi r} \times 2\pi r = \mu_o I \qquad \text{----------(14.14)}$$

This relation is called Ampere's Law and it is true for a closed curve of any shape taken in the magnetic field because the distance of the element from the conductor is not involved in this expression. The law states that the sum of the products of the tangential component of magnetic field of induction and the length of an element of a closed curve taken in the magnetic field is μ_o times the current which passes through the area bounded by this curve. If the closed curve is taken in the magnetic field such that it encloses no current then $\Sigma \vec{B} \cdot \Delta \vec{\ell} = 0$. as is the case with curve C_2 in the Fig.14.11.

The magnetic field of Induction due to a current can be determined using Ampere's Law only if we can possibly imagine a closed curve around which the quantity $\Sigma \vec{B} \cdot \Delta \vec{\ell}$ can be evaluated.

The following are a few applications of Ampere's Law.

a) Solenoidal Field

A Solenoid is a coil of insulated Copper Wire wound on a long cylinder with close turns.

Except at the ends the lines of magnetic Induction are fairly parallel and closely packed inside the solenoid indicating that the field is strong and uniform in the middle portion of the solenoid. Outside the solenoid the lines are widely separated and the field is weak.

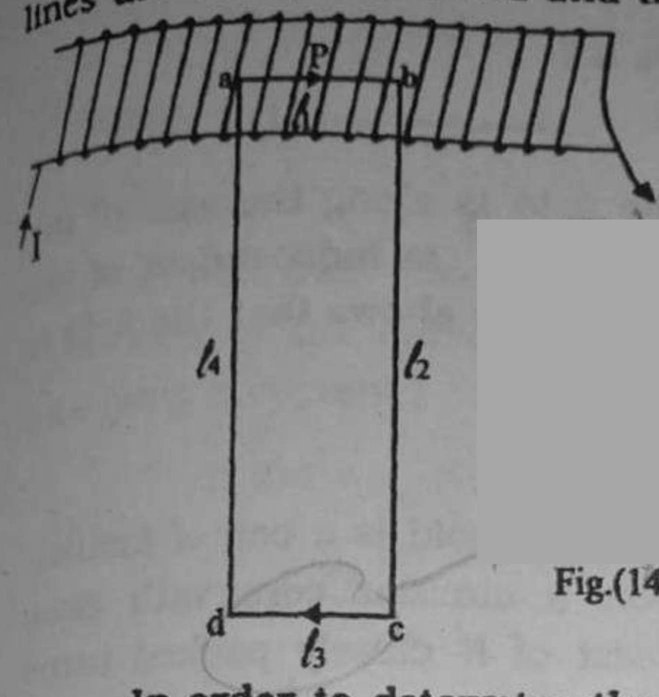


Fig.(14.12)

In order to determine the magnetic Induction at a point P on the axis of Solenoid well inside it, imagine a rectangular loop abcd with the side ab on the axis and the side cd far away where the field is zero as shown in fig. 14.12. It is divided into four elements ℓ_1, ℓ_2, ℓ_3 and ℓ_4. By Ampere circuital law

$$\sum_{\ell=1}^{4} (\Delta\vec{\ell}.\vec{B})_\ell = \mu_o \times \text{current enclosed}$$

Inside the solenoid $\vec{B}$ is parallel to ℓ_1 so

$$(\Delta\vec{\ell}_1.\vec{B})_1 = \ell_1 B \cos 0^\circ = \ell_1 B$$

The field outside the solenoid is very small, it can be neglected and put equal to zero.

$$\therefore (\Delta\vec{\ell}_3.\vec{B})_3 = 0$$

As $\vec{B}$ is perpendicular to ℓ_2 and ℓ_4 inside the solenoid, the field is zero i.e.

$$(\Delta\vec{\ell}_2.\vec{B})_2 = (\Delta\vec{\ell}_4.\vec{B})_4 = 0$$

$$\therefore \sum_{\ell=1}^{4} (\Delta\vec{\ell}.\vec{B})_\ell = \ell_1 B = \mu_o \times \text{current enclosed.}$$

If there are n turns per unit length of the solenoid, and each turn carries a current I, the current enclosed by the loop abcda will be $n \ell_1 I$.

$$\therefore \ell_1 B = \mu_0 n \ell_1 I$$

i.e $\quad B = \mu_0 nI \quad$ ------------------ (14.15)

The direction of the field is along the axis of the solenoid. Eq.(14.15) shows that $\vec{B}$ is independent of the position within the solenoid which shows that the field is uniform within a long solenoid.

b) **Toroidal Field**

A toroid or a circular solenoid is a coil of insulated Copper wire wound on a circular core with close turns. Let the toroid consist of N closely packed turns and carry a current I.

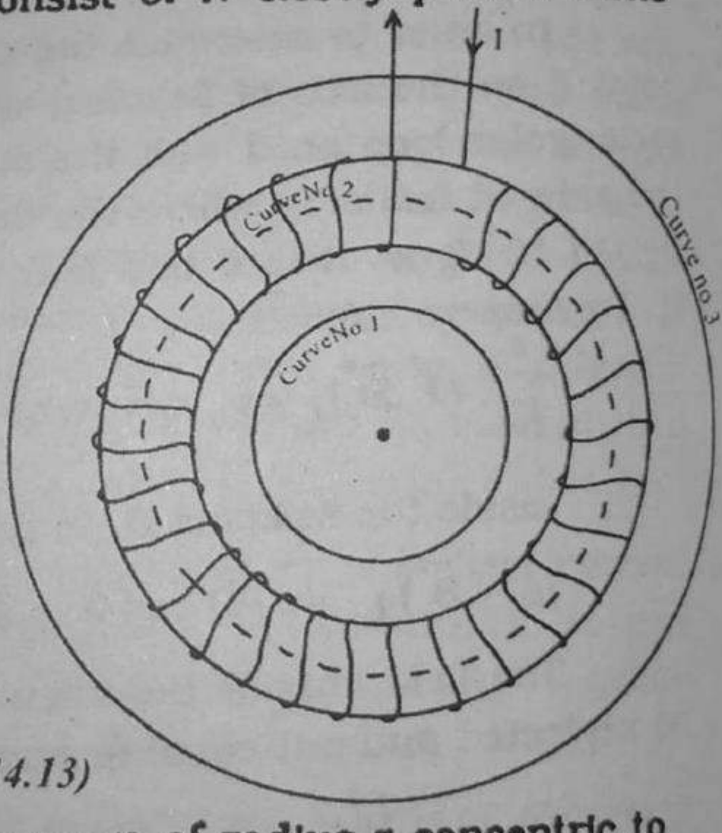


Fig. (14.13)

Imagine a circular curve of radius r concentric to the core as shown in Fig 14.13.

It is evident from symmetry that the field at all points of the curve must have the same magnitude and it should be tangential to the curve at all points.

$$\therefore \Sigma \vec{B} \cdot \vec{\Delta \ell} = \Sigma B \Delta \ell \cos 0^\circ = B \Sigma \Delta \ell = B 2\pi r$$

Let us consider the following cases:-

1. If the circular path (marked 1) is outside the core on the inner side of the toroid it encloses no current.

$$B\,2\pi r = 0$$

$$\therefore B = 0$$

2. If the circular path (marked 2) is within the core the area bounded by the curve will be threaded by N turns each carrying a current I

$$B\,2\pi r = \mu_o NI$$

$$\therefore B = \frac{\mu_o NI}{2\pi r} \qquad \text{------------------}(14.16)$$

which is the same as that at the centre of the solenoid.

3. If the circular path (marked 3) is out side the Core on the outer side of toroid the area bounded by the curve will be threaded by each turn twice but in opposite directions and the algebraic sum of all the currents is zero

$$B = 0.$$

A toroid, therefore, produced a uniform magnetic field of induction which is confined in the space occupied by the core.

Example 14.5.

There is a current of 25 A in a long straight wire. What is the flux density at a point 3 cm from the wire.

Solution:

$$B = \frac{\mu_o I}{2\pi r} = \frac{4\pi \times 10^{-7} \times 25}{2\pi \times .03} = 1.67 \times 10^{-4}\,T$$

Example 14.6.

A toroid has 3000 turns. The inner and outer diameter are 22 cm and 26 cm. Calculate flux density inside the core when there is a current of 5.0 amp.

Solution:

Mean Radius = 12 cm

$$B = \frac{\mu_0 NI}{2\pi r} = \frac{4\pi \times 10^{-7} \times 3000 \times 5}{2\pi \times .12} = 0.025\ T$$

14.7. Electromagnetic Induction

In the year following the discovery by Oersted that an electric current produces magnetic field of Induction, the question before the scientists was whether a magnetic field can, in some way, induce an electric current. Continuous attempts were made for about ten years to detect currents in the Coils by subjecting them to strong magnetic fields but the answer was that such an effect apparently does not exist.

In 1830 Joseph Henry in the United States and a year later Faraday in England independently observed that an emf is set up in a coil placed in a magnetic field when ever the flux through the coils changes. The effect is called electro-magnetic Induction and if the coil forms a part of a closed circuit, the induced emf causes a current to flow in the circuit.

Experiments show that the magnitude of emf depends on the rate at which the flux through the coil changes. It also depends on the number of turns N on the coil and it is useful to define a quantity called flux linkage as being the product of the number of turns and the flux through the coil.

Flux linkage = $N\phi$ ------------------(14.17)

It is not necessary that a conductor is in the form of a coil in order for it to be able to acquire an emf. Emf can be induced in a straight conductor when ever it is caused to cut across magnetic Induction lines.

The magnetic flux through a circuit can be changed in a number of different ways:-

1). By changing the relative position of the coil with respect to a magnet or current bearing solenoid.

2). By changing current in the neighbouring coil or by changing current in the coil itself.

It should be noted that early attempts were bound to fail because the coil has resistance and energy is necessary to force a current through it. No energy is expended when the coil is lying motionless in the field.

Laws of Electromagnetic Induction.

A detailed investigation of magnetic field of Induction leads to two laws known as Faraday's laws.

1. An emf is induced in a coil through which the magnetic flux is changing. The emf lasts so long as the change of flux is in progress and becomes zero as soon as the flux through the coil becomes constant.

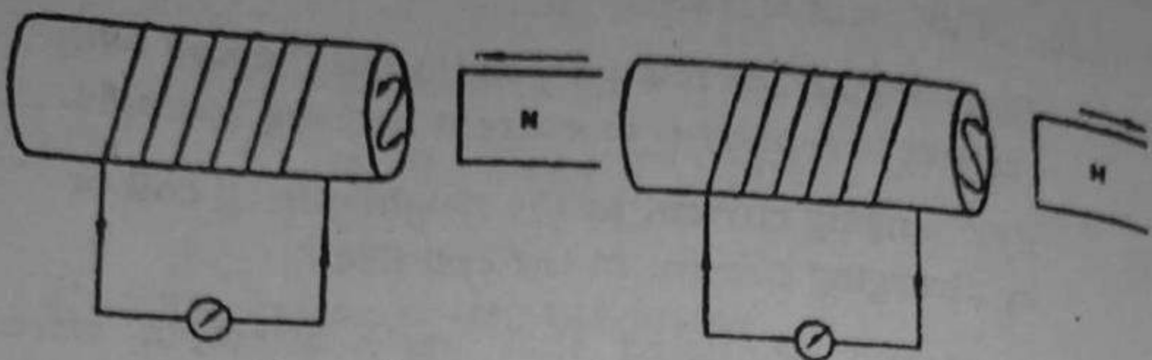
2. The magnitude of induced emf depends only upon the number of turns and the time rate of change of flux linked with the circuit. It can be expressed as

$$\xi = -\frac{d}{dt}(N\phi) \qquad \text{------------------(14.18)}$$

The negative sign is introduced for the reason to be explained in the next article.

14.8. Lenz's Law

The directions of induced current as shown in fig.(14.14) was carefully studied by Lenz and the results were generalized most elegantly into a rule in 1835 called Lenz's Law.



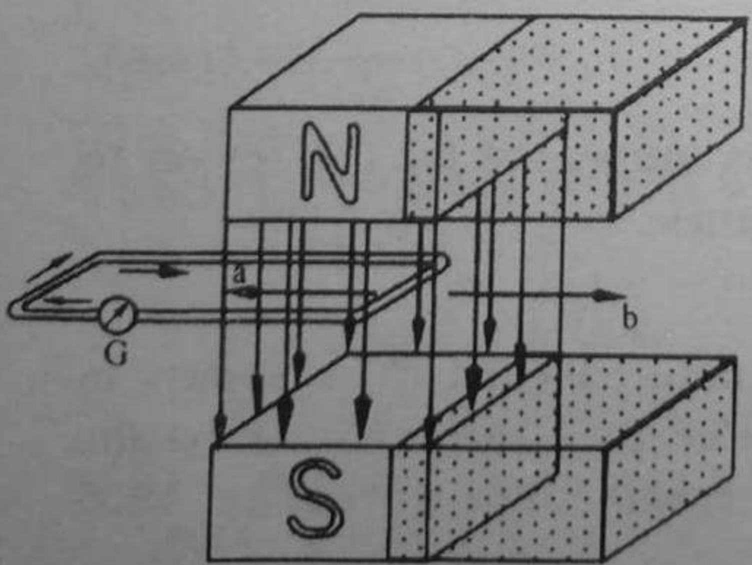
(a) fig. (14.14) (b)

The law states that the induced current always flows in such a direction as to oppose the change which is giving rise to it. This is why a negative sign is introduced in equation (14.18).

It can be explained with the help of following examples.

a). When the N pole of a bar magnet is approaching the face of the coil it becomes a North face by the induction of current in anticlockwise direction to oppose the forward motion of the magnet. Fig. 14.15.

b). When the N pole of the magnet is receding the face of the coil becomes a south pole due to a clockwise induced current to oppose the backward motion.



The direction of the induced current is such that the magnetic force on the wires is always opposite to the applied force.

Fig. (14.15)

c). Let a wire ab of length ℓ which makes a part of a circuit be pulled towards left. According to Lenz's Law this motion should be opposed by the current induced. The current must produce a force towards right.

Example.14.7.

A coil of 600 turns is threaded by a flux of 8×10^{-5} webers. If this flux is reduced to 3×10^{-5} webers in 0.015s, what is the average induced emf?

Solution:

$$\xi = -N \cdot \frac{\Delta\phi}{\Delta t} = \frac{-600\,(8-3) \times 10^{-5}}{.015} = -2.0 \text{ Volts}$$

14.9. (a) Self - Induction.

A coil through which a current is flowing has an associated magnetic field. If, for any reason, the current changes, then so too does the magnetic flux and an emf is induced in the coil. Since this emf has been induced in the coil by a change in the current through the same coil, the process is known as Self Induction.

In accordance with Lenz's Law, the emf opposes the change that has induced it and it is therefore known as a back emf.

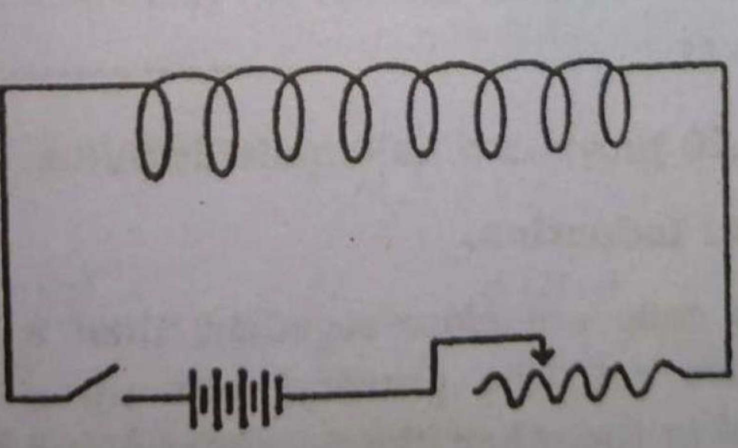


Fig. 14.16.

If the current is increasing, the back emf opposes the increase. If the current is decreasing, it opposes the decrease. The measure of the ability of a coil to give rise to a back emf is known as the Self-Inductance of the coil. It is defined by

$$\xi = -L\frac{\Delta I}{\Delta t} \qquad \text{---------------- (14.19)}$$

where

ξ = the back emf induced in the coil in Volts

L = the Self-Inductance of the coil. The unit of self Inductance is henry.

$\frac{\Delta I}{\Delta t}$ = the rate of change of current in the coil amp. s^{-1}

The Self Inductance of a coil is 1 henry if the current varying through it at the rate of 1 ampere per second induces a back emf of 1 Volt.

The value of L depends on the dimensions of the coil, the number of turns and the permeability of the core material.

If ϕ is the flux through a coil of N turns when it is carrying a current I.

$$\xi = -\frac{\Delta(N\phi)}{\Delta t} = -L\frac{\Delta I}{\Delta t} = -L\frac{\Delta(LI)}{\Delta t}$$

or $\quad N\phi = LI \qquad$ ------------------(14.20)

Equation 14.20 provides an alternate definition of L.

(b) **Mutual Induction.**

If two coils are close together, then a changing current in one coil (the primary) sets up a changing magnetic field in the other (the Secondary) and so induces an emf in it. The effect is known as mutual Induction.

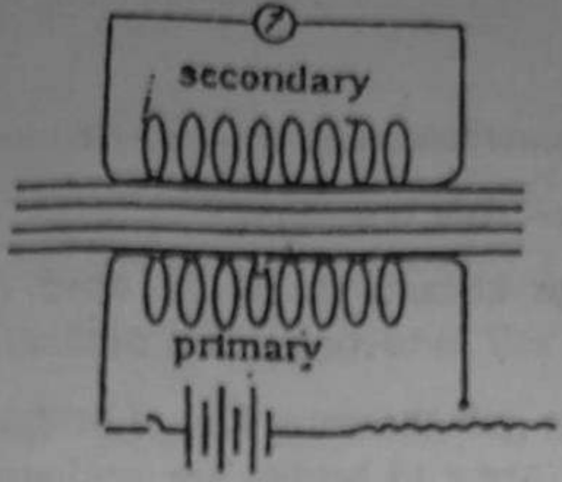


Fig. (14.17)

The mutual Inductance, M, of the pair of coils is defined by

$$\xi_2 = -M \frac{\Delta I_1}{\Delta t} \qquad \text{-----------------(14.21)}$$

Where ξ_2 is the back emf induced in the Secondary coil.

$\frac{\Delta I_1}{\Delta t}$ = the rate of change of current in the primary.

M = the mutual Inductance of the pair of coils M has the same value no matter which of a given pair of coil is taken to be the primary. Its unit is also henry.

If the rate of change of flux - linkage in the Secondary is $\frac{\Delta(N_2 \phi_2)}{\Delta t}$ as a result of the current in the primary changing at the rate $\left(\frac{\Delta I_1}{\Delta t}\right)$ then

$$\xi_2 = -\frac{\Delta(N_2\phi_2)}{\Delta t} = -M\frac{\Delta I}{\Delta t} = -\frac{\Delta(MI_1)}{\Delta t}$$

where N_2 is the number of turns in the secondary coil

It follows that

$$N_2 \phi_2 = M I_1. \qquad \text{-----------------(14.22)}$$

This equation provides an alternative definition of M.

(c) **Non Inductive Winding.**

In bridge circuits such as used for resistance measurements, self Inductance is a nuisance.

When the galvanometer key of bridge is closed the currents in the arms of bridge are re-distributed unless the bridge happens to be balanced. While the currents are being re-distributed these are changing and self Induction delays the reading of a new equilibrium. Thus the galvanometer deflection at the instant of closing the key does not correspond to steady state which the bridge will eventually reach. It may, therefore, be misleading.

To minimize their self Inductance, coils of the bridge and resistance boxes are so wound as to set up extremely small magnetic fields. The wire is doubled back on itself before being coiled up as shown in the figure. Such a coil is said to be non-Inductive.

In this type of winding, current flows in opposite directions in the double-wires and consequently, the magnetic flux set up by one wire is neutralized by that due to the outer wire. Hence Self Induced emf's will not be produced when the current through the circuit changes

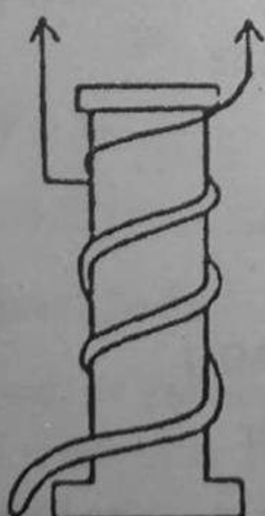


fig. (14.18)

Non Inductive coil

(d) **Motional EMF.**

When, a conductor is moved across a magnetic field, a potential difference appears across its ends. This potential difference is known as motional emf.

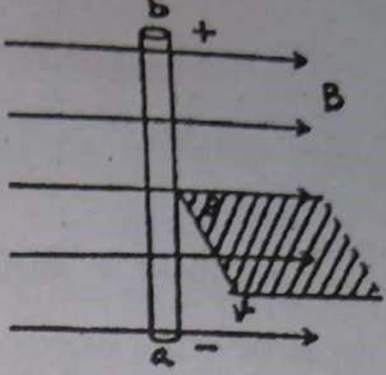


fig.(14.19.)

Consider a wire of length of ℓ moving across the magnetic field of Induction $\vec{B}$ with a velocity $\vec{v}$ as shown in the figure 14.19. Each free electron of the conductor is moving with the conductor and thus experiences a force

$$\vec{F} = -e(\vec{v} \times \vec{B}) = e(\vec{B} \times \vec{v}) \quad \text{from b to a}$$

The electrons go on accumulating at the end a leaving the end b with a positive charge till the force of electric field balances the force due to the motion of conductor. Thus a potential difference is set up from b to a.

Let the total charge that flows be q

$\therefore$ Potential difference = work done by unit charge

$$V = \frac{F\ell}{q} = \frac{q\,v\,B\,\text{Sin}\,\theta\ell}{q} = vB\ell\sin\theta.$$

If the conductor is moving at right angles to the field $\theta = 90°$.

$$V = vB\ell \qquad \text{------------------(14.23)}$$

Example14.8.

A pair of adjacent coils has a mutual Inductance of 1.5 henrys. If the current in the primary changes from 0 to 20 ampere in 0.050 s what is the average induced emf in the Secondary? If the Secondary has 800 turns what is the change of flux in it?

Solution:

$$\xi_S = M\frac{\Delta I_1}{\Delta t} = 1.5 \times \frac{20}{.050} = 600 \text{ Volts}$$

$$\xi_S = N_S\frac{\Delta\phi}{\Delta t}$$

$$600 = 800 \times \frac{\Delta\phi}{0.050}$$

$$\Delta\phi = 0.038 \quad \text{Webers.}$$

Example 14.9.

A circuit in which there is a current of 5 amp is changed so that the current falls to zero in 0.1 s. If an average emf of 200 volts is induced, what is the self inductance of the circuit.

Solution:

$$\xi = L\frac{\Delta I}{\Delta t}$$

$$200 = L\frac{5}{0.1}$$

$$L = 4 \text{ henrys.}$$

Example: 14.10

The flux density B in a region is 0.5 weber/ m^2 directed vertically up wards. Find the emf induced in a straight wire 5 cm long perpendicular to B when it is moved across the field in a direction at an angle of 60° with the horizontal with a speed of 100 cm. s^{-1}

Solution:

Since the angle between the direction of motion and horizontal is 60°, the angle between the field and motion is 30°.

Induced emf = $B v \ell \sin\theta$

$= 0.5 \times 1 \times .05 \times .5$

$= 0.0125$ Volts

14.10. Alternating Current Generator (Dynamo).

An electric generator is a device to convert mechanical energy into electrical energy .

The principle of the generator is that an emf is induced in the coil due to changing magnetic flux linkage when it is rotated between the poles of a magnet. The essential parts of an alternating current generator are:-

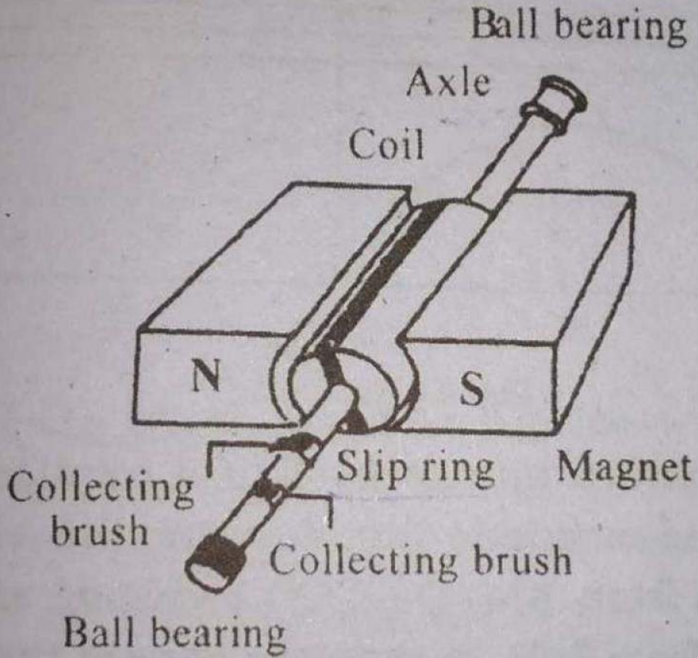


Fig. (14.20)

1. **Field Magnet.** It is strong permanent horse-shoe magnet, which produces a strong and uniform magnetic field of Induction $\vec{B}$ between its poles.
2. **Armature.** It is a soft iron cylinder mounted on an axle which rotates on ball bearings thus rotating the cylinder between the poles of the magnet. A coil of Insulated copper wire of large number of turns is wound on the cylinder in the grove cut length wise as shown in the figure.
3. **Slip Rings and Collecting Brushes.** The ends of

the coil are joined to two separate copper rings fixed on the axle. Two carbon brushes remain pressed against each of the rings which form the terminals of the external circuit.

Let the coil of an area A with number of turns N start rotating clockwise from the position marked .1. in the figure 14.21

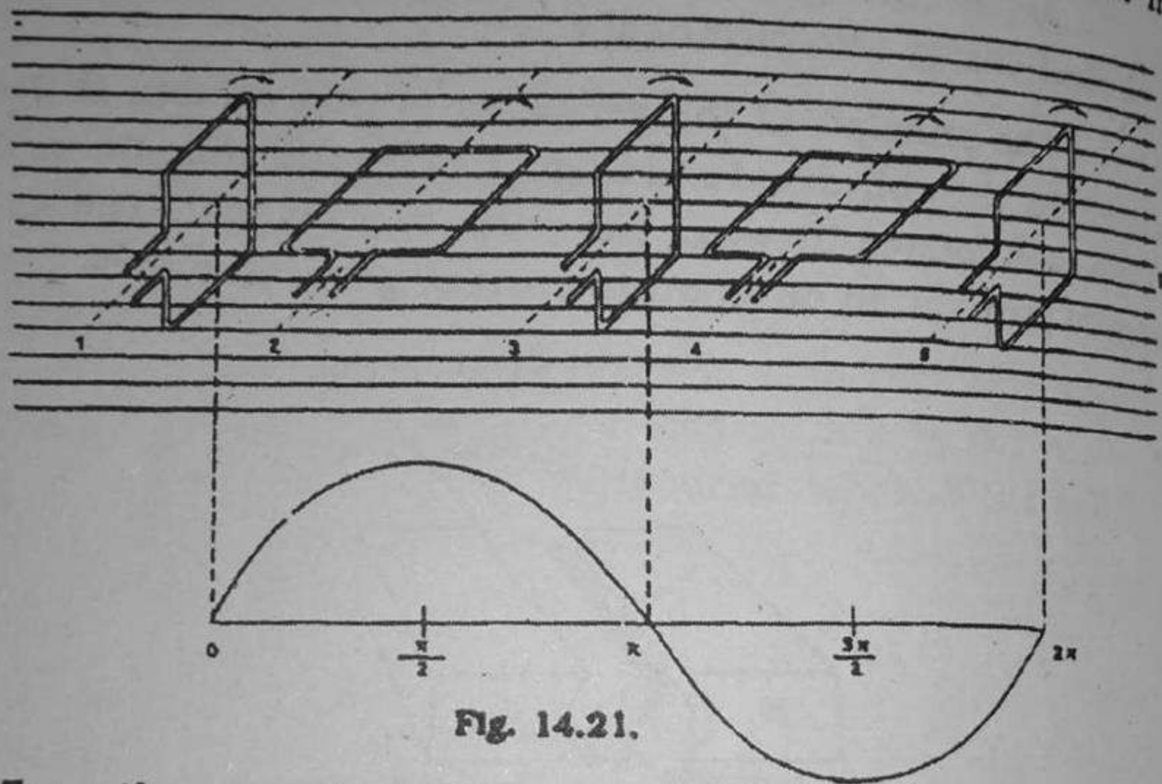


Fig. 14.21.

When the armature rotates by half a rotation from position 1 to 3. The magnetic flux entering the left hand side face decreases from BAN-O-(-BAN) inducing an emf clockwise (as seen from left) in order to oppose the cause.

At positions 1 and 3 the coil is rotating for sometime almost parallel to the lines of magnetic flux and the rate of change of flux and hence the induced emf is zero. The emf is maximum at positions 2 and 4 of the coil where the rate of change of flux is maximum.

During the second half rotation from position 3 to 5 the emf is in the reverse direction because the flux through the face under consideration increases from -(BAN)-O-BAN. Thus an alternating emf is generated. At thermal power house the armature of the generator is rotated by steam turbines and at hydroelectric power house it is set into rotation by the turbines driven by water fall.

A Motional emf $(\vec{v} \times \vec{B})$ $N\ell$ is setup in each of the sides ab and cd in opposite directions when the coil is rotated because these sides are moving in opposite sense with respect to the magnetic field.

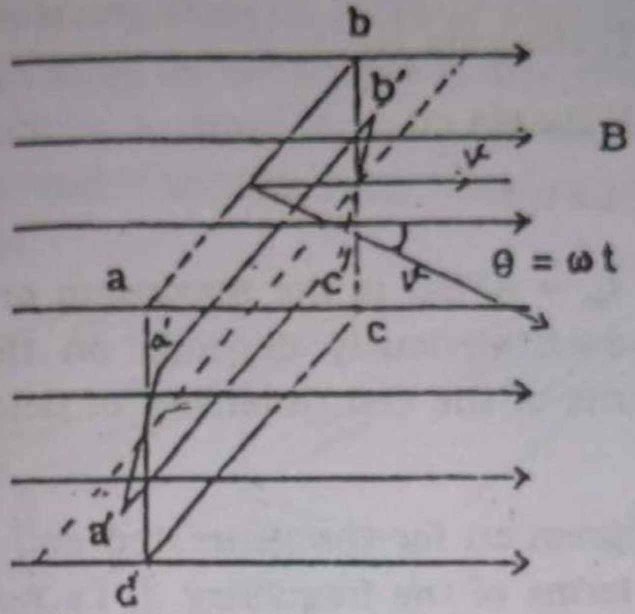


Fig. (14.22)

Since the other two sides are moving in the same sense with respect to field. The emfs are induced in the same direction and cancel each other. Total emf in the coil when it is at the position a'.b'.c'.d'

$\xi = 2\,vBN\ell \sin \omega t$

v = Linear Velocity

B = magnetic field of Induction

N = number of turns

ℓ = Length of the coil

ω = angular velocity

t = time in which the coil moves from the position abcd to a' b' c' d'

Since each particle of the sides ab and cd rotates in a circle of radius equal to half the width of the coil b

$$v = \frac{b}{2}\omega$$

$$\xi = 2\frac{b}{2}\omega B N \ell \sin \omega t$$

$$\xi = (b\ell) NB\omega \sin \omega t$$

$$\xi = A N B\omega \sin \omega t$$

$$\xi = \xi_o \sin \omega t$$

where $\xi_o = ANB\omega$ is the maximum or peak value of the emf which obviously depends on the area and number of turns of the coil. Intensity of field and speed of rotation.

The expression for the generated emf can also be expressed in terms of the frequency .f. i.e number of rotation per second

$$\xi = \xi_o \sin 2\pi ft \qquad \text{----------------- (14.24)}$$

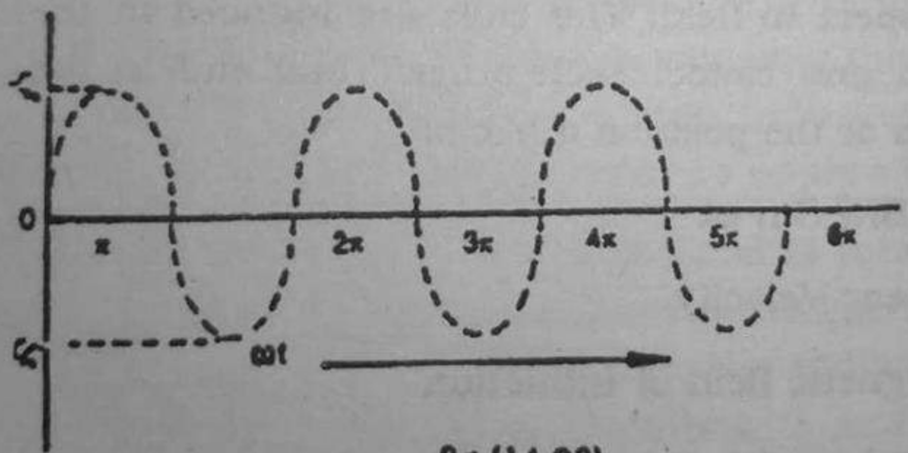


fig (14.23)

Any small generator employing a permanent magnet is commonly called a magneto and it is used in ignition system of petrol engines. motor bikes and motor boats, etc.

The field magnets of large generators are electromagnets and these generators are called alternators. The performance of A.C generator is more satisfactory when the armature is stationary and the field magnet rotates around the armature. Stationary armature is called stator and rotating magnet rotor.

14.11. D.C. Generator.

By replacing the slip rings of an A.C. generator by a simple split ring, or commutator, the generator can be made to produce a direct current through the external circuit. A generator modified for this function is called d.c. generator.

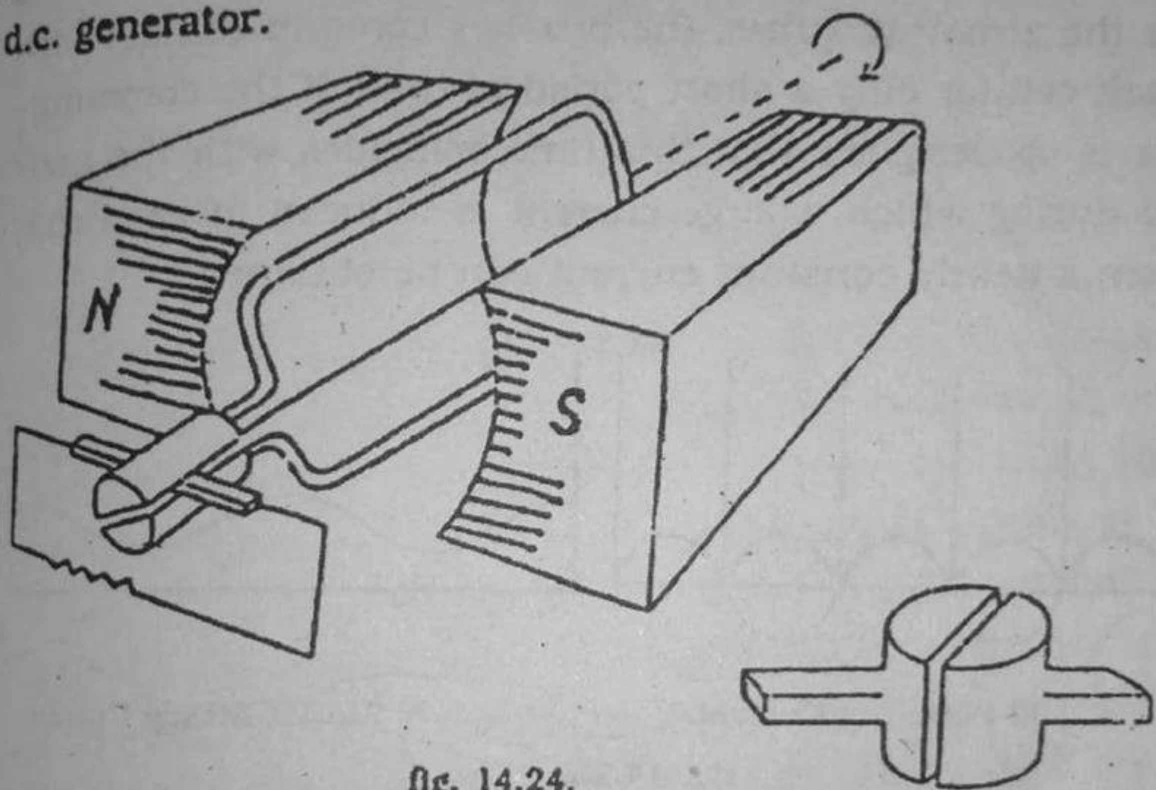
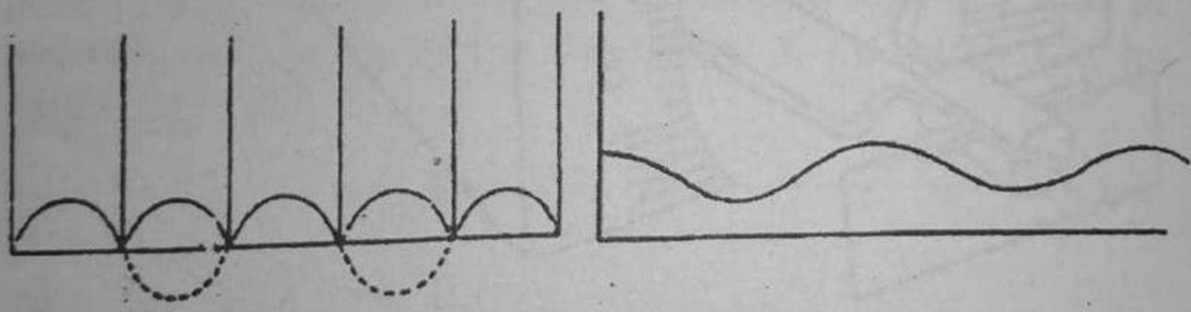


fig. 14.24.

Fig. 14.24 shows the armature coil of this generator after some time as it rotates in a clockwise direction from its vertical position. The induced emf increases gradually from zero to a maximum during the first quarter of the revolution and decreases to zero during the second quarter of the revolution.

As the coil rotates past its next vertical position, an emf is induced in the coil in the reverse direction just in the case of an A.C generator. But at this moment, the two halves of the split ring exchange contact with the brushes so that the direction of induced emf in the external circuit remains unaltered. This emf again increases from zero to a maximum and then falls to zero when the coil is back in the initial vertical position.

The variation of the emf with the rotation of the coil is shown in fig. 14.25. Although the current is unidirectional, it fluctuates from zero to maximum. Such a current is called pulsating current. To obtain a steady current, a number of coils are mounted at different angles round the armature and the commutator is divided into a corresponding number of segments with each coil connected to two diametically opposite segments. Thus, as the armature turns, the brushes come in contact with each coil for only a short period of time. If the commutator is so designed that this time coincides with the period during which a large current is induced in each coil, then a nearly constant current can be obtained.



a) Pulsating Current.   b) Almost Steady Current.

Fig. 14.25.

14.12. Electric Motor.

An electric motor is a device which converts electrical energy into mechanical energy.

A simple D.C. motor is shown in fig 14.26. It follows from article 14.3 that when a current is passed through a coil capable of rotation in a magnetic field of induction, it experiences a couple.

τ = BIAN cosα. To rotate the coil in anticlockwise direction.The couple becomes zero when the face of the coil becomes perpendicular to the field. If the coil turns

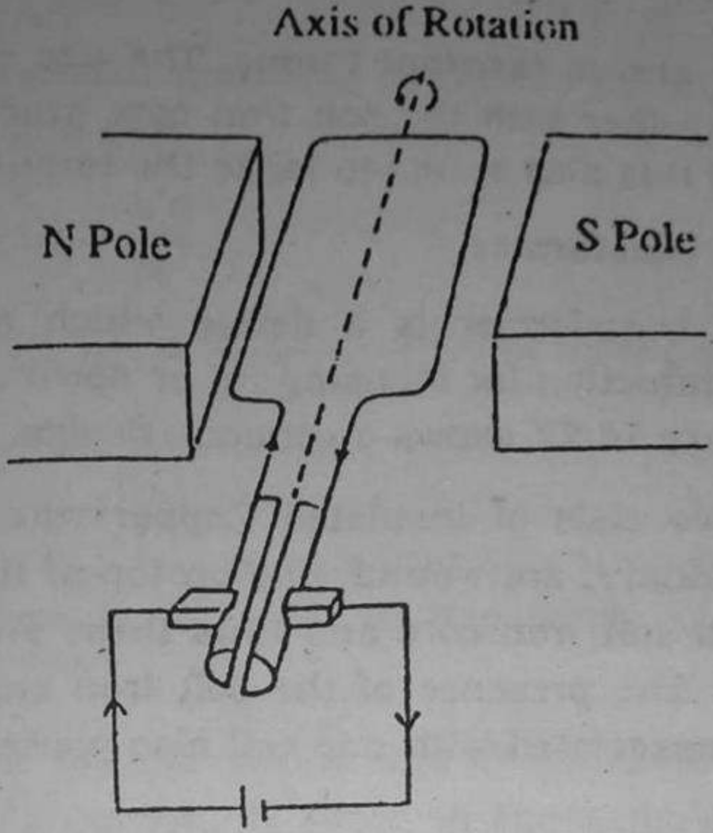


fig 14.26.

beyond this point there will be a torque in the opposite direction to return the coil unless the direction of current is reversed. For the coil to continue to rotate must be a commutator to reverse the direction of current at the proper time. When the plane of the coil is vertical the gaps in the commutator are facing the brushes and momentarily, there is no current in the coil. However, its inertia carries it beyond this position so that side Y comes in contact with brush M and side X comes into contact with brush N.

It follows that the current in the coil always flows in the same direction (clockwise as seen from above) and therefore the coil rotates in an anticlockwise sense no matter what its orientation.

In practice the armature of the motor consists of several equally spaced coils wound on a soft iron core and connected to a commutator which has a corresponding number of sections. The main advantage of using several coils, rather than just one, is that the motor provides an almost constant torque. The use of curved pole pieces together with the soft iron core produces a radial field and this also serves to make the torque constant.

14.13. Transformer.

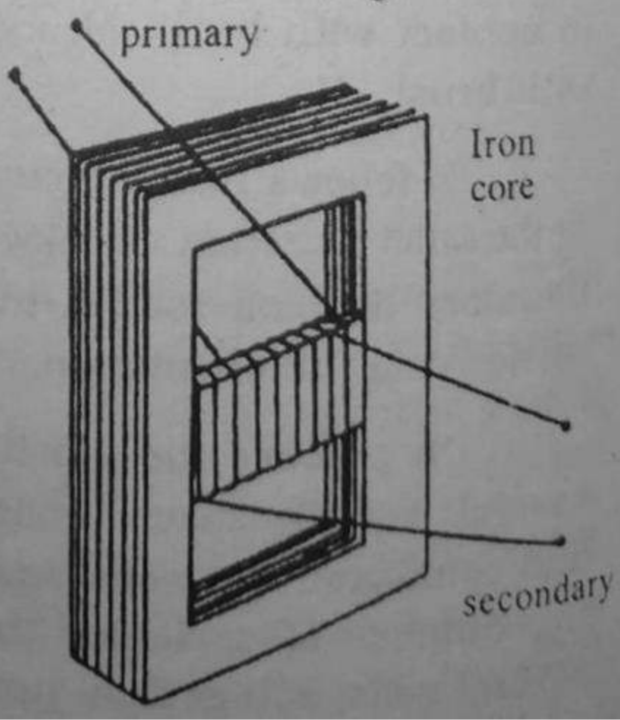
A transformer is a device which makes use of mutual induction for stepping up or down an alternating emf. Figure 14.27 shows a common design.

Two coils of insulated Copper wire, the primary and secondary, are wound, one on top of the other, on a laminated soft iron core and thus these are linked magnetically. The presence of the soft iron ensures that all the flux associated with one coil also passes through the other.

Suppose that an alternating emf E_p is applied to the primary coil. If at some instant the flux in the primary be ϕ then there will be a back emf in it given by the equation.

$$E_p = -\frac{\Delta (N_p \phi)}{\Delta t}$$

The flux through the primary also passes through the secondary and therefore the rate of change of flux in the secondary is also $\frac{\Delta \phi}{\Delta t}$.



It follows that there will be an emf E_s induced in the Secondary and that

$$E_S = -N_S \frac{\Delta \phi}{\Delta t} \quad \text{------------------(14.25)}$$

Where N_S is the number of turns on the Secondary

$$\frac{E_S}{E_P} = \frac{N_S}{N_P} \quad \text{------------------(14.26)}$$

If $N_S > N_p$, the transformer is called a step-up transformer because $E_S > E_p$. A step down transformer has $N_S < N_p$.

When a load (a resistance) is connected across the secondary, a current, I_S flows in the secondary. Suppose that the current in the primary is I_p. If the transformer is 100% efficient.

Power output = Power input

$$I_S E_S = I_p E_p \quad \text{----------------(14.27)}$$

The efficiency of a transformer $\eta = \frac{\text{Power output}}{\text{Power input}}$

The efficiencies of commercial transformers are very high in the range 95 to 99%.

Sources of Power Loss in Transformer.

1. Eddy currents induced on the surface of iron core due to variation of magnetic flux produce heating and therefore reduce the amount of power that can be transferred to the secondary. The core is laminated i.e made up of thin sheets of soft iron each separated from the next by a layer of insulating varnish. This very nearly eliminates eddy current heating.
2. Each time the direction of magnetization of the core is reversed, some energy is wasted in over-

coming internal friction. This is known as hysteresis loss and it produces heating in the core. It is minimized by using special alloys (perm-alloy) for the core material.

3. Some energy is dissipated as heat in the coils (I^2R). This is reduced by using suitably thick wire. The coil which has the smaller number of turns carries the larger current and therefore is wound from thicker wire than the other.

4. Some loss of energy occurs because a small amount of the flux associated with the primary fails to pass through the secondary.

A few uses of transformers are listed below.

1. **Power Transmission.**

Whenever electric energy is to be used at a considerable distance from the generator an alternating current system is used because the energy can then be distributed without excessive loss. On the other hand if a direct current system were used, the losses in transmission would be very great.

Power loss from transmission wires $= I^2 R$ watts.

In A.C. system the terminal voltage at the generator is increased using a step up transformer. Thus the current through transmission wire becomes very small and consequently the power loss I^2R is small. At the other end a step down transformer reduces the voltage to a value 220 — 240 volts that can safely be used.

2. In houses a transformer may be used to step the voltage down from 220 to 4 volt for call bells.

3. Transformers with several secondaries are used where several different voltages are required. For example radio, television circuits etc.

QUESTIONS

14.1. What is flux density and how is it related to the number of lines of Induction expressed in Webers?

14.2. Charged particles fired in vacuum tube hit a fluorescent screen. Will it be possible to know whether they are positive or negative?

14.3. Beams of electrons and Protons are made to move with the same velocity at right angles to a uniform magnetic field of Induction. Which of them will suffer a greater deflection? What will be the effect on the beam of electrons if their velocity is doubled?

14.4. Circular loop of wire hangs by a thread in a vertical plane. An electric current is maintained in the loop anticlockwise on looking at the front face. What direction will the front face of coil turn ?

14.5. Imagine that the room in which you are seated is filled with a uniform magnetic field pointing vertically upwards. A loop of wire which is free to rotate about a horizontal axis in its plane through its centre has its plane horizontal. For what direction of current in the loop as viewed from above will the loop be in a stable equilibrium?

14.6. Two identical loops. One of copper and the other of aluminium, are similarly rotated in a magnetic field of Induction. Explain the reasons for their different behaviour. Is electric generator a generator of electricity? where is the electricity before it is generated? what do such machines generate?

14.7. A loosely wound helix spring of stiff wire is mounted vertically with the lower end just touching mercury in a dish. When a current is started in the spring it executes a vibratory motion with its lower end jumping out and into mercury. Explain the reason for this behaviour.

14.8. Derive the general equation for induced emf beginning with the law of force in a current carrying conductor in a magnetic field.

14.9. What is the mechanism of transfer of energy between the primary and secondary windings of a transformer? A certain amount of power is to be transferred over a long distance. If the voltage is stepped up 10 times, how much is the transmission line loss reduced?

14.10. What is the difference between magneto and A.C generator? what is meant by the frequency of alternating current?

PROBLEMS.

14.1. A horizontal straight wire 5 cm long weighing $1.2\ g.m^{-1}$ is placed perpendicular to a uniform horizontal field of 0.6 $webers\text{-}m^{-2}$. If the resistance of the wire is $3.8\Omega\ m^{-1}$. Calculate the potential difference to be applied between the ends of the wire to make it just self supporting.

(Ans. 3.7×10^{-3} volts)

14.2. A cathode ray tube is set up horizontally with its axis N-s and surrounded by a magnetic sheild. If the voltage across the tube is 900 volts, the distance from electron gun to the screen is 10 cm and vertical component of earth's field is $0\text{–}45 \times 10^{-4}$ webers/m^2. Calculate by how much the spot on the screen will move when the magnetic sheild is removed.

Given that $\frac{e}{m} = 1.8 \times 10^{11}\ Ckg^{-1}$

(Ans: 2.27×10^{-3} m)

14.3. What is the flux density at a distance of 0.1 m in air from a long straight conductor carrying a current of 6.5 amperes? Hence calculate the force per metre on a similar parallel conductor at a distance of 0.1 m from the first and carrying a current of 3 amperes. Will the wires attract or repel, if the directions of currents in the two wires are opposite to each other? Explain how the expression of force between two such conductors is used to define ampere.

(Ans. 13×10^{-6} webers-m^{-2} 39×10^{-6} N)

14.4. A straight metal rod 50 cm long can slide with negligible friction on parallel conducting rails. It moves at right angles to a magnetic field 0.72 webers-m^{-2}. The rails are joined to a battery of emf 3 volts and a fixed series resistance of 1.6 Ω. Find the force required to hold the rod at rest.

(Ans. 0.675 N).

14.5. It is required to produce inside a toroid a field of 2×10^{-3} webers-m^{-2}. The toroid has a radius of 15 cm and 300 turns. Find the current required for this purpose. If toroid is wound on an iron core of permeability 300 times the permeability of free space what increase in B will occur for the same current.

(Ans. 5 Amp. 300 times)

14.6. A proton is accelerated by a potential difference of 6×10^5 volts. It then enters a uniform field B = 0.3 webers-m^{-2} in a direction making an angle of 45° with the magnetic field. what will be the radius of the circular path?

(Ans. 0.26m)

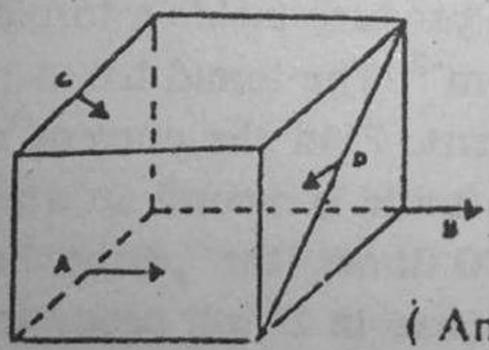
14.7. Two parallel metal plates separated by 5 cm of air have a potential difference of 220 volts. A magnetic field $B = 5 \times 10^{-3}$ webers-m^{-2} is also produced perpendicular to electric field. A beam of electrons travel undeflected through these crossed electric and magnetic fields. Find the speed of electrons.

(Ans. 8.8×10^{5} ms^{-1})

14.8 A coil of 50 turns wound on a rectangular ivory frame 2 cm x 4 cm is pivoted to rotate in a magnetic field of 0.2 webers-m^{-2}. The face of the coil is parallel to the field. How much torque acts over the coil when a current of 0.5 amp passes through it? what will be the torque when the coil is rotated by 60° from its initial position?

(Ans. 0.4×10^{-2} N-m, 0.2×10^{-2} N-m)

14.9. A cube 100 cm on a side is placed in a uniform magnetic field of flux density 0.2 webers-m^{-2}, as shown in the diagram. Wires A, C and D move in the directions indicated, each at a rate of 50 cm.s^{-1}. Determine the induced emf in each wire.



(Ans 0, 0.0707, 0.1 volts)

14.10. What is the mutual Inductance of a pair of coils if a current change of 6 amps in one coil causes the flux in the second coil of 2000 turns to change by 12×10^{-4} webers-m^{-2}.

(Ans. 400 mh)

14.11. An emf of 45 m.volt is induced in a coil of 500 turns. when the current in a neighbouring coil changes from 10 amps to 4 amps in 0.2 seconds.

a) What is the mutual Inductance of the coils?

b) What is the rate of change of flux in the second coil?

(Ans. 1.5 mh, 9×10^{-5} webers-s^{-1})

14.12. An iron core solenoid with 400 turns has a cross section area of 4.0 cm^2. A current of 2 amp passing through it produces B = 0.5 webers$\cdot m^{-2}$. How large an emf is induced in it. If the current is turned off in 0.1 seconds. What is the self Inductance of the solenoid.

(Ans 0.8V L = 40 mh)

14.13. The current in a coil of 325 turns is changed from zero to 6.32 amps, there by producing a flux of 8.46×10^{-4} webers. what is the self Inductance of the coil.

(Ans. 43.5 mh)

14.14. A 100 turns coil in a generator has an area of 500 cm^2 rotates in a field with B = 0.06 webers-m^{-2}. How fast must the coil rotated in order to generate a maximum voltage of 150 volts.

(Ans. 500 rad/s)

14.15. A step down transformer at the end of a transmission line reduces the voltage from 2400 volts to 1200 volts. The power output is 9.0 K.W and over all efficiency of the transformer is 95%. The primary winding has 400 turns. How many turns has the secondary coil? what is the power input. what is the current in each of the coils?

(Ans: N_s = 200, P_p = 9473 watts. I_p = 3.9 amps, Is = 7.5 amp).

14.16. The overall efficiency of a transformer is 90%. The transformer is rated for an output of 12.5 KW.

The primary voltage is 1100 volts and the ratio of primary to secondary turns is 5:1. The iron losses at full load are 700 watts. The primary coil has a resistance of 1.82 ohms.

a) How much power is lost because of the resistance of the primary coils?

b) What is the resistance of the secondary coils?

(Ans: 290 watts. 0.124 ohm).