Chapter. 17

ADVENT OF MODERN PHYSICS

The year 1900 not only marked the beginning of this century but also the new era of modern physics. It would be rather difficult to mention all the major developments since 1900, but surely there are quite a few events of pivotal importance which could be narrated vividly. With in a span of few years J.J.Thomson experimentally proved the existence of electron which was taken as the fundamental unit of electricity. Rontgen announced the discovery of X-ray. Henry Bequerel discovered the phenomena of radioactivity.

Meanwhile, Max Planck's put forth his famous hypothesis, that in its interaction with matter, radiant energy, behaved as discrete quanta of energy $E = h\upsilon$ Also, during this period, Albert Einstein gave a reconsideration to the fundamental concepts of classical physics which led to his famous theory of special relativity. This was the turning point which made a line of demarkation between the classical and the modern physics. We will discuss briefly some of the theories and experiments which laid down the foundations of modern physics in this chapter.

17.1 FRAMES OF REFERENCE:

We are familiar that the displacement of a point from some fixed origin is specified by two measurements when the point is located on a particular surface, and by three measurements otherwise. For example, the position of a point on earth's surface is completely specified by its latitude and longitude, these measurements give us the distances from North or South of the equator and East or West of the geographic meridian. The location of an aeroplane or helicopter is completely specified when we state that it is 1000 Km East of Karachi, 300 Km North of that city, and at an elevation of 30 Km above sea level.

The most commonly used set of co-ordinates for the above mentioned purpose is the rectangular cartesian system and is often called as the frame of reference. The reference frame is mathematically expressed interms of a set of three mutually perpendicular lines called axes of the frame of reference as shown in fig.(17.1).

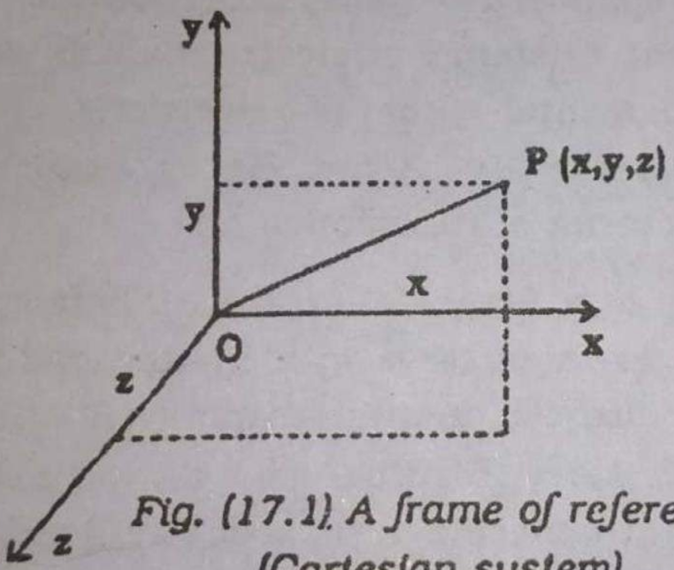


Fig. (17.1) A frame of reference (Cartesian system)

Any point P referred to the frame of reference has its three position coordinates represented by the coordinates (x, y, z). The line joining the origin O to the point P is called the position vector $\underline{r}$ of the point p with respect to the origin O.

Having seen that displacements needs to be specified relative to a given reference frame we must recognize that velocity must also be specified in the same manner. Hence, it will be worthwhile to know how the same motion will appear to be in two reference frames, one of which is moving relative to the other.

To illustrate the above, let us suppose that a rail car is moving towards East at a speed of 60 Km/h and a train is running from the back to the front of the rail car at a speed of 10Km/h. This means that the rali car covers a distance of 1Km, as measured by the measuring rod or chain laid in the west-east direction along the ground, during every minute, as noted by a clock on the ground along the rail track. On the other hand it means that the man covers a distance of $\frac{1}{6}$ Km, as measured by the measuring rod laid along the floor of the rail car, during every minute, as measured by the clock kept on the rail car. If we make the same assumption made by Galileo and Newton that the measuring rod or chains on the rail car are identical with those on the ground and the two clocks show identical time, then if:

u = Velocity of the rail car relative to the ground.

V' = Velocity of the man relative to the train.

We come across with the situation shown in Fig.(17.2).

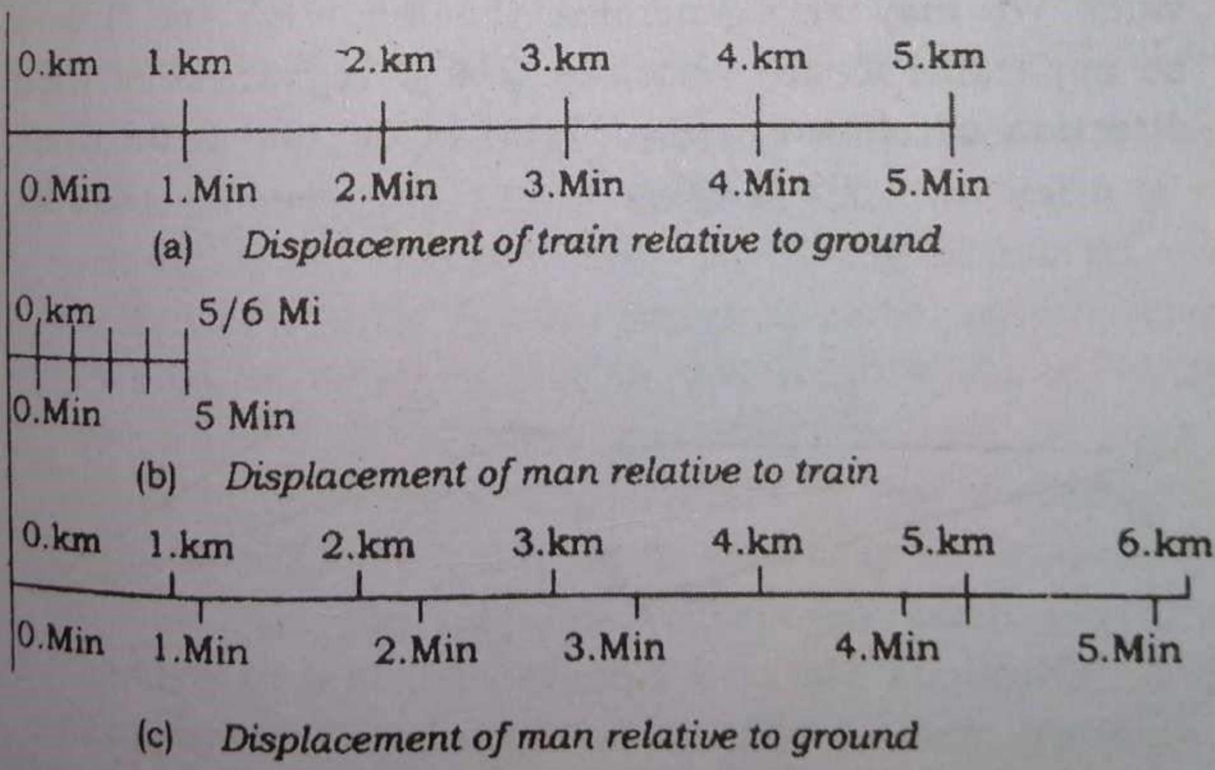


Fig.(17.2)

From the figure of the linear motion as seen in two frames of reference, we see that during an interval of

5 min. the train has covered a distance of 5Km to the East relative to the ground. The man has moved $\frac{5}{6}$ = 0.83 Km East relative to the rail car during the same interval. Hence the displacement of the man relative to the ground during the same interval will be:

(5.0 + 0.83) Km = 5.83 Km towards East.

Similarly, the man's velocity (distance/time) relative to the ground will be

given by:

$$V' = \frac{5.83 \text{ Km}}{5.0 \text{ min}} \text{ Eastward} = 70 \text{ Km/h Eastward.}$$

We, therefore find that the sum of u and v' is equal to v i.e

$$v = U + v' \quad \text{------------------ (17.1)}$$

Considering any other time interval instead of 5 min, we may establish that the above addition remains valid. We may also generalize that equation (17.1) may be applicable to any velocities $\underline{U}$ & $\underline{v}'$ regardless of their direction as shown in Fig.(17.11) below and write that

$$\underline{v} = \underline{U} + \underline{v}' \quad \text{-------------------(17.2)}$$

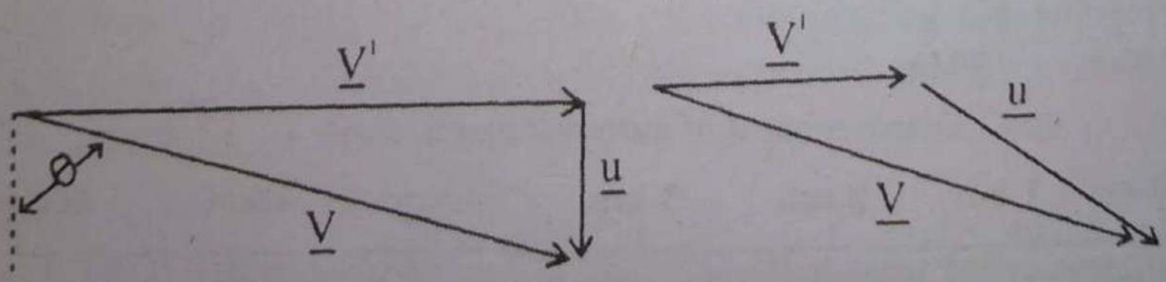


Fig. 17.3 Addition of velocities

Equation 17.2 is known as the Galilean transformation of velocities for two frames one moving relative to the other.

17.2 INERTIAL REFERENCE FRAME:

The word inertial is derived from the Newton's first law of motion which is often called the law of inertia. Inertia is an inherent tendency of inertness possessed by bodies to maintain their state of rest or of uniform motion, unless some external force is applied to change their state of rest or of uniform motion. That is why, we attribute an inertia of either rest or motion to bodies at rest or in motion with a uniform speed in the light of the Newton's first law of motion . In view of this we may define a reference frame moving with constant velocity as an inertial reference frame. With the use of inertial frame we determine whether or not the frame has a non constant velocity i.e whether it is accelerating. Only if the frame is non-accelerating i.e has a constant velocity, will an observer in that frame will experience the validity of the first law: A body at rest will remain at rest in the absence of any unbalanced force acting on it.

For most practical purposes we may consider our earth as an inertial frame. However since the earth is revolving round the sun, and is rotating about its own axis, a coordinate system chosen at earth, require forces to hold an object at rest in this rotating frame. In the strict sense of the word, an object at rest in such a frame of reference will not remain at rest if there is no unbalanced force acting on it. We are however, quite fortunate that the rotational effects on earth are small enough so that the Newton's laws are applicable on the experiments conducted on earth's surface. Thus to a good approximation we may consider that "objects at rest remain at rest" on the surface of the earth even if it is not strictly an inertial frame of reference. Thus, we may also define inertial frame of reference as that in which the Newton's laws are valid.

It can be shown that all inertial frames are equivalent from the point of view of making measurements of physical phenomena. Different observers in different inertial frames may have different values of physical quantities but the basic physical laws.(Relationships between the measured physical quantities) will always remain the same for all observers. For example, let two observers in different inertial frames measure velocity and momentum of two bodies before and after collision. It will be found that they will obtain different values for these quantities for individual bodies. Each observer, will however note that the sum of the velocities and the total momentum is the same before and after the collision in their respective frames. In other wards the law of addition of velocities and the law of conservation will remain the same in both the frames.

17.3. FRAMES OF REFERENCE IN UNIFORM RELATIVE MOTION:

As described in the previous section the Newton's laws of motion remain unchanged in any inertial frame of reference. Hence, if the law of force F= ma is valid in a frame S' at rest at time t = 0, then it will also hold good in another frame S moving at a constant velocity v with respect to the frame S. For simplicity if we assume that the origins of the two frames are coincident at the initial time t =0 the situation will be as shown in Fig 17.4

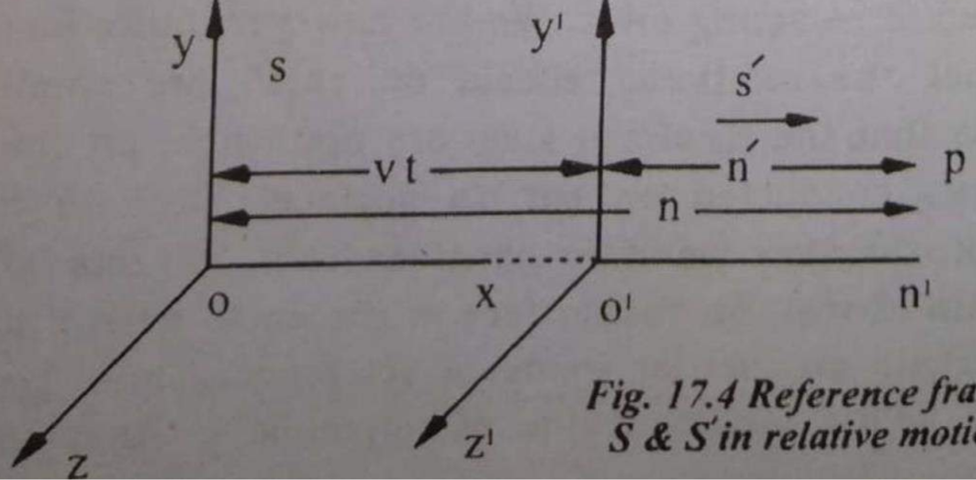


Fig. 17.4 Reference frames S & S' in relative motion

From Fig 17.4 it is seen that the frame S′ moving at a uniform velocity ϑ along the direction of x-axis will be at a distance of ϑt from the origin σ of the frame S at rest. Let us suppose that a measurement is made at the point P designated as (x,y,z) with respect to the frame S, and (x′, y′,z′) from the point of view of the frame S′ to determine the force F = ma on a body of mass 'm' making an acceleration 'a'. From the figure we find that:

$$\left.\begin{aligned} x' &= x - \vartheta t \quad \text{-------------} \\ y' &= y \quad \text{-------------} \\ z' &= z \quad \text{-------------} \end{aligned}\right\} \quad (17.3)$$

and t′= t (i.e.two clocks located in S & S′ are identical in time) from this set of equations, we have transformed the coordinates (x′,y′,z′;t) of the frame S′ in terms of the coordinates (x, y, z, t) of the frame of reference S. This is called the Galilean transformation of space-time coordinates. From this transformation we may infer that:

$$x = x' + \vartheta t \quad \text{-------------------}(17.4)$$

Dividing through out by t we get:

$$\frac{x}{t} = \frac{x'}{t} + v$$

or,
$$v = v' + v \quad \text{------------------} (17.5)$$

where, V is the velocity as measured in S and V′ is the velocity from the point of view of S′ .

Equation (17.5) is the same as the Galilean transformation law of velocities obtained in (17.2 of section 17.1)

Let us now consider the measurement of acceleration 'a' of a body in inertial frame S expressed as:

$$a = \frac{V_2 - V_1}{t_2 - t_1} \quad \text{-------------------}(17.6)$$

where V_1 and V_2 are the velocities in the frame of reference S during a time interval $(t_2 - t_1)$ for the difference of velocity $(V_2 - V_1)$.

Using equation (17.5) we may write:

$$V_2 = V_2' + \upsilon$$

$$V_1 = V_1' + \upsilon$$

subtracting the two equations, we get:

$$V_2' - V_1' = V_2 - V_1$$

Dividing both sides by $t' = t$ we have:

$$\frac{V_2' - V_1'}{t'} = \frac{V_2 - V_1}{t}$$

or $$a' = a \quad \text{------------------ (17.7)}$$

Hence, we have shown that the acceleration a' in the moving frame's is the same as that observed in the stationary frame S.

Since the laws of physics are the same in both the frames of references we assert that:

$$F' = ma'$$

if $$F = ma \quad \text{------------------(17.8)}$$

provided the mass does not depend on the velocity. But from 17.7 we have $a' = a$, hence from 17.8 we get

$$F' = ma' = ma = F$$

Thus the forces F' and F are equal in the two frames and the Newton's second law remains unaltered under Galilean transformations. Since the first law ($F = 0$, $a = 0$) and the third law involving the forces are contained in the second law of Newton, we therefore conclude that the Newton's laws of motion remain the same under Galilean transformations.

Although the Galilean transformations are correct transformations for accelerated frames. These transformations are also not applicable to electromagnetic phenomenon. Since velocities in different frames in relative motion appears to be different, the speed of light c given by the Maxwell's electromagnetic equation should appear different in different inertial frames. This is in contradiction to the experimental observation of the constancy of speed of light demonstrated by Michelson and Morley (1887). These short comings led Albert Einstein to propose his famous special theory of relativity.

17.4. THE PRINCIPLE OF RELATIVITY:

The choice of the frame of reference to describe the motion of an object is of vital importance in view of the fact that the description of the motion of the object may be different in different frames of reference. It may therefore be quite obvious to expect that a correct description of motion may be obtained in a reference frame which is at rest. But it is almost impossible to determine by means of an experiment performed in a reference frame, whether the frame is at rest or in uniform motion. The only possibility is to detect the motion of a frame relative to another frame. For example, assume an observer sitting in a vehicle moving with a uniform speed. Suppose the observer throw an object vertically upwards. He will observe that the object falls along the same vertical path as shown in Fig (17.5).

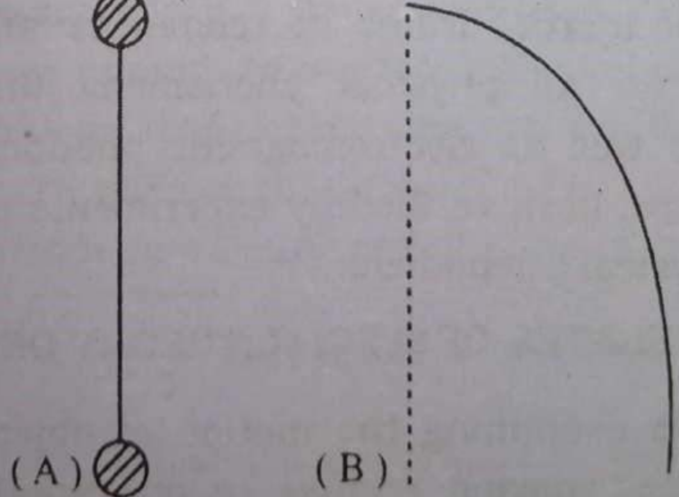


Fig.17.5 Path of an observer as seen in the vehicle (A) and on the ground (B)

For an observer at rest on the ground, the path of the object will appear to be a curve Fig 17.5 (b). The observation will be of opposite nature if the same experiment is conducted on the ground.

As there is no way to find which frame is at rest, there is naturally no means to decide which of the two paths is the correct path. However, we can realize that the motion of the object is much simpler in one frame than that in the other frame of reference. Thus the choice of the frame of reference depends upon the simplicity of the motion of the object. From the above considerations we see that physical phenomena depend, on relative motion. Albert Einstein therefore assumed that all possible reference frames moving at uniform velocity relative one another (i.e inertial frames) are equivalent for the statement and description of physical laws. This simple assumption is called the Principle of relativity.

Based on the constancy of the velocity of light Einstein further generalized this principle to restate it in the form.

All inertial frame of references are completely equivalent for all physical phenomena (including mechanical as well as electromagnetic phenomenon)". This principle have been verified by experiments on a wide variety of physical phenomena.

17.5 POSTULATES OF SPECIAL THEORY OF RELATIVITY;

While examining the motion of objects in frames of references moving relative to one another. Einstein proposed his famous theory of special relativity in the year 1905. The theory is termed as special because it is valid specially for inertial frames and is to be modified into a general theory for accelerated frames of reference. The special theory of relativity is based on two assumptions known as the postulates of special relativity. The two postulates are stated as follows:

(i) There is no preferred or absolute inertial frame of reference i.e all the inertial frames are equivalent for the description of all physical laws.(Newton's laws as well as the Maxwell's Electromagnetic equations).

(ii) The speed of light in vacuum is the same for all observers in uniform translational relative motion and is independent of the motion of the observer and the source.

The free space value of the speed of light is a universal constant c which appears explicitly in the Maxwell's electromagnetic equation.

The value of the speed of light is very nearly equal to $3\times 10^{8}\ ms^{-1}$

17.6 CONSEQUENCES OF SPECIAL THEORY OF RELATIVITY

In developing the special theory of relativity we have seen that frames of reference in relative motion with a constant speed V have been used. If the speed V becomes large enough to approach the velocity of light c, then the Galilean transformations are found to be noticeably wrong. To correct the state of affairs it will be necessary to introduce a factor called Lorentz or Relativistic factor $\sqrt{1-\frac{V^2}{c^2}}$. This factor is infact a measure of departure from Galilean transformation. We see that if $\frac{V}{c}$ where c is the velocity of light is much smaller than as it is in our common situations, then $\frac{V^2}{c^2}$ is so small that the relativistic factor is essentially equal to unity. Under these conditions the classical and the relativity physics predict nearly identical results. However when V approaches c (e.g: $v = \frac{c}{5}$). than the Galilean transformation will be incorrect.

Based on these considerations, if we interpret the results of special theory of relativity we end up in some very interesting consequences. Without going to make actual mathematical calculations, we may summarize the important consequences of the theory of special relativity which are as under:-

1. **Mass Variation**

According to the special theory of relativity, the mass of an object in a frame of reference at rest is called its rest mass m_o. If this mass is measured by an observer moving with a constant speed v relative to the object, then it will not remain constant if the speed v is comparable to c. The mass m in the moving frame will vary according to the mass variation relation given by:

$$m = \frac{m_o}{\sqrt{1 - \frac{v^2}{c^2}}} \quad \text{-------------------(17.9)}$$

This mass variation formula shows that mass changes with the velocity and is not in general a constant nor the same for all observers but is a quantity that:

(a) depend upon the reference frame from which the body is being observed.

(b) is greater than or equal to the rest mass m_o when the body is at rest in the frame of reference from which the body is being observed.

Example 17.1

An electron has a rest mass of 9.1×10^{-31} kg when it is at rest relative to an observer. What will be its mass when it is moving at speed one half the speed of light c..

Solution:-

From the mass variation formula we have:

$$m = \frac{m_o}{\sqrt{1 - \frac{v^2}{c^2}}}.$$

Given, $m_o = 9.1 \times 10^{-31}$ kg

$v = \frac{1}{2}$ c

$$m = \frac{9.1 \times 10^{-31}\ kg}{\sqrt{1-(1/2c)^2/c^2}} = \frac{9.1 \times 10^{-31}\ kg}{\sqrt{1-1/4}} = \frac{9.1 \times 10^{-31}}{1/2\sqrt{3}}$$

or $$m = \frac{18.2 \times 10^{-31}\ kg}{3} = 10.50 \times 10^{-31}\ kg$$

2 LENGTH CONTRACTION

In the theory of special relativity it has been found that the measurement of length of a rod in a stationary reference frame is not the same when the rod is measured by the observer in the moving frame of reference with a velocity relative to the rod, provided the measurement is made along the direction of motion.

Hence, if L_o is the length of the rod in the frame at rest, and L is the length of same rod in the moving frame, them

$$L = L_o \sqrt{1 - \frac{v^2}{c^2}}. \qquad \text{.................(17.10)}$$

Since v/c is less than unity, the length L is less than L_o i.e there is a contraction in length along the direction of motion. This is called the Lorentz-Fitzgerald contraction.

Equation: (17.10) tells us that an observer past whom a system is moving with a speed v measures objects in the moving system to be shortened in length along the direction of motion by a factor $1 - v^2/c^2$. It is important to note that only the dimension along the line of motion is changed and there is no change in the other two perpendicular direction.

With the development of the special theory of relativity it became apparent that there is no physical contraction of the moving objects. There is, however, an apparent contraction of a body for an observer when there is a relative motion of the object and the observer. In the natural sense the observer in the moving frame can not detect the contraction because in his frame it does not exist; where as in the rest frame, it does exist, but the measuring rod in the moving system has shrunk too further we must note that for moderate velocities ($\frac{v}{c}$ <<1) of the objects the contraction in length is negligible as observed in our every day observation.

Example 17.2

The length of a measuring rod is 1m when it is at rest. What will its length be if it is moving with a velocity one third of the speed of light?

Solution:

We have from the length contraction formula:

$$L = L_o \sqrt{1 - V^2/c^2}$$

Given: $L_o = 1\text{m}$ and $v = \frac{c}{3}$

$$L = 1 \times \sqrt{1 - \frac{1}{9}} = \sqrt{\frac{8}{9}} = \frac{2\sqrt{2}}{3}$$

$= 0.943$ m.

3. TIME DILATION:

Time is regarded as an absolute quantity in classical mechanics whereas in the special theory of relativity it is considered to be a relative entity based on the measurement of time in frame of references in relative motion.

The time interval between two events taking place at the same point in space as timed with a clock at rest with respect to that point is called the proper time interval and is denoted by $\Delta t_o = T_o$. The time measured with a clock in motion with respect to the events is known as the relativistic time it is represented by $\Delta t = T$. Both of the time intervals T_o & T refer to the time elapsed between the same pair of events occurring in the two frames moving with a relative speed v. Then, according to special relativity the two times are related by the formula:

$$T = \frac{T_o}{\sqrt{1 - \frac{V^2}{c^2}}} \qquad \text{----------------}(17.11)$$

Equation:- (17.11) represents, what we call as the time dilation phenomena. According to the time dilation formula we mean that from the point of view of an observer at rest, the time of the observer in motion is dilated i.e. the clocks in moving frame run slowly and the Lorentz factor $\sqrt{1 - \frac{V^2}{c^2}}$ gives us the ratio of the rates of clocks for normal speeds, this factor is so close to unity that we are quite unable to detect time-dilation effect, but for speed comparable to the speed of light c the time dilation effect is quite significant.

We can now conclude that for every observer, his own clocks in his frame of reference run faster than do any other clocks which are moving relative to him. We may also note that every observer may consider himself to be at rest and consider all that moves as moving relative to him. This is actually an outcome of the principle of special relativity stated earlier in section(17.4): Every observer is equivalent to every other observer.

Example 17.3

If the average life time of a particle before it decays is 2×10^{-6}s. what will be its average life time if it is moving with respect to an absERVER at speed $\frac{c}{2}$?

Solution

From the time dilation formula we have:

$$T = \frac{T_o}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Given: $T_o = 2 \times 10^{-6}s$

$v = c/2$

$$T = \frac{2 \times 10^{-6}\ s}{\sqrt{1 - 1/4}} = \frac{2 \times 10^{-6}\ s}{\frac{1}{2}\sqrt{3}} = \frac{4 \times 10^{-6}\ s}{\sqrt{3}}$$

i.e $T = 2 - 3 \times 10^{-6}$ s.

4. MASS ENERGY RELATION:

In section (17.5) we have stated the postulates of relativity that the speed; of light is a universal constant. We cannot reach speeds greater than the speed of light by the relativistic addition of velocities. The question is. how to reconcile with this result of special relativity with Newton's second law, F = ma? It would be seen that any constant force, no matter how small, applied for a considerably very long time. should continuously accelerate any mass 'm' at a rate $a = \frac{E}{m}$ until the speed was arbitrarily very large. Einstein, concluded that Energy has inertia i.e the more energy a body possess, the more inertia that body will display. Since, inertia is a property of matter which is associated with mass. Thus from Einstein's argument mass is simply a property attributed to the total energy of the body and only the total energy is required, to know the total mass of the body. thus, in special theory of relativity total energy and mass are related by the famous Einstein's equation.

$$E = mc^2 \qquad \text{-----------------(17.12)}$$

from this relation between mass and energy it has been predicted that any process that changed the mass by a detectable amount would involve huge amounts of energy for example, a mass change of 1 gram is equal to an energy change of 9×10^{13} joules. Such energy transfers will be discussed in more detail in Chapter 19, when we will study the atomic nuclear. We must bear in mind that the relation $E = m\,c^2$ is a direct and logical consequence of the mass variation mentioned earlier in section (17.6) under the results of the special theory of relativity.

Example 17.4

Find the mass associated with the energy of a mass of 10 kg moving with a speed of 100 ms^{-1}

Solution:

The kinetic energy of the mass is given by:

$$E = \frac{1}{2}\, m\, v^2$$

or, $$E = \frac{1}{2} \times 10\ kg \times (100)^2\ \frac{m^2}{s^2}$$

or, $$E = 5\ kg \times 10000\ \frac{m^2}{s^2}$$

i.e $$E = 5 \times 10^4 \, kg - \frac{m^2}{s^2}$$

using, now the mass- energy relation:

$$E = m\,c^2$$

we have:-

$$m = \frac{E}{c^2} = \frac{5 \times 10^4 \, kg - m^2/\, s^2}{(3 \times 10^8)^2 \, kg - m^2/\, s^2}$$

or, $$m = \frac{5 \times 10^4}{9 \times 10^{16}} \, kg = 5.5 \times 10^{-13} \, kg$$

17.7. BLACK BODY RADIATIONS AND QUANTUM THEORY

By a black body we mean an object which can absorb all the radiations that falls on it. For all forms of radiations a hollow sphere of metal with a fine hole in it called cavity is approximately a black body as shown in Fig. (17.6).

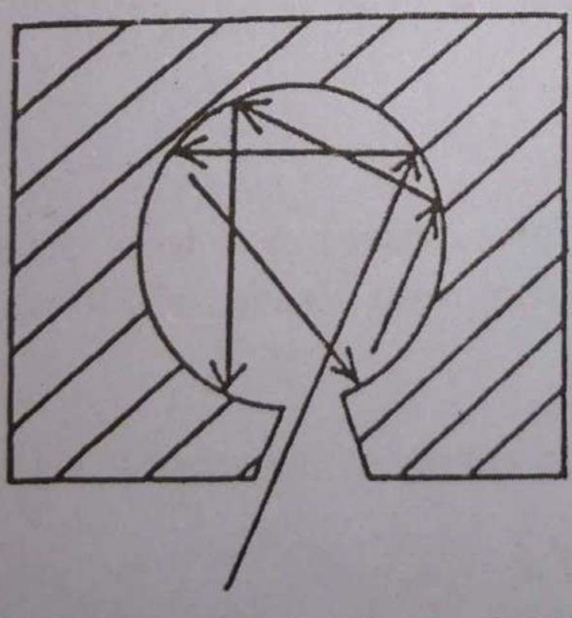


Fig 17.6. A cavity approximately a perfect Black Body

Any radiation entering the hole of the cavity is trapped by multiple reflections inside and very little of it is able to escape. Just as a black body is nearly a perfect absorber, so is the most effective emitter of radiation when heated. Due to this property experiments on black body radiation may be performed by putting such a cavity in a furnace and studying the energy of the emitted radiations from its fine hole as a function of the temperature. The graphs shown in Fig. 17.7.are called Black Body Radiation curves:

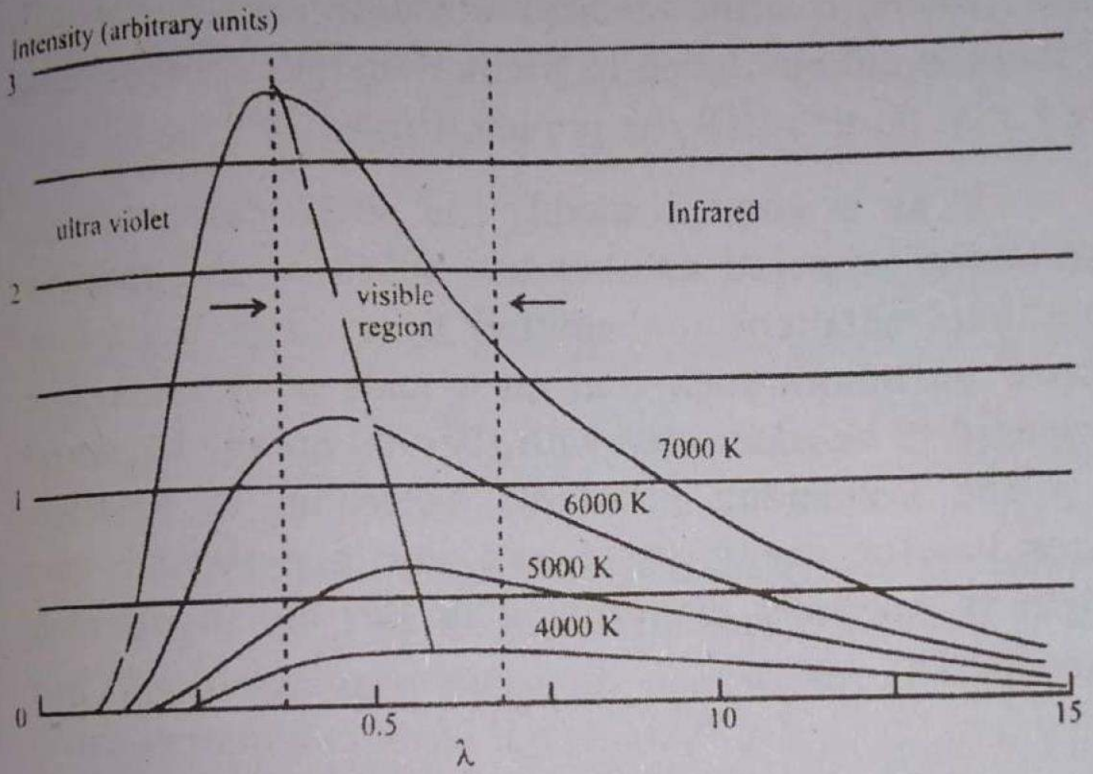


Fig. 17.7

from the blackbody radiation curves we see that the wave length at which maximum energy of radiation occurs is shifted towards the shorter wave length with the rise of temperature. The wave length for maximum radiation λ_{max} is related to the absolute temperature T of the black body by Wien's law given by:

$$\lambda_{max} \times T = \text{constant} \quad \text{----------------} \quad (17.13)$$

The Wien's law is in agreement with the common observation that a white hot furnace is hotter than one which is red hot. This law is valid in regions less than λ_{max} only.

Another relationship known as stefan's law, was also proposed in an attempt to describe the black body radiation curves. This law states that the total energy radiated per second per unit surface area is proportional to the fourth power of absolute temperature i.e

$$E \propto T^4$$

or, $$E = \sigma T^4 \qquad \text{------------------(17.14)}$$

where, σ is the Stefan-Boltzmann constant.However these equations failed to predict energy distribution in the entire curve at all the temperatures.

In an attempt to modify the Wien's law Rayleigh and Jeans proposed another law based on the assumption that radiations are emitted by a large number of atomic oscillators such that each mole of vibration was supposed to be associated with thermal energy kT, where k is the Boltzmann constant. According to Rayleigh-Jeans law the energy associated with a particular wave length is inversely proportional to the fourth power of wave length

i.e. $$E = \frac{(\text{Constant})}{\lambda^4} \qquad \text{------------------(17.15)}$$

This law has been found to give a good agreement with experimental results at large values of l. For wave lengths near and less than λ_{max} . the Rayleigh-Jean law gave values which were found to be much too large i.e total energy tending to acquire infinite value even at short wave lengths.This is called ultra-violet catastrophe, because it is a serious discrepancy from physical point of view, that the energy can not be infinity. By common experience we know that hot bodies actually emits mostly red light and not ultra violet and x-rays.

17.8. PLANCK'S LAW - QUANTUM THEORY:

To over come the difficulties in providing a successful explanation of the Black body curves Planck (1900) proposed a formula that correctly describe the intensity distribution with respect to wave length of a black body. a perfect absorber or emitter of radiation. Fig.(17.7) the previous section. Planck proposed that radiant energy comes out in discrete amounts or quanta of energy. The energy content of each quantum was directly proportional to the frequency υ

Energy of a quantum = a constant x frequency of quantum

or, $$E = h\upsilon \qquad \text{------------------(17.16)}$$

Since , $\upsilon = c/\lambda$, where c is the velocity of light.

we have , $$E = \frac{hc}{\lambda} \qquad \text{------------------(17.17)}$$

The constant, h, is known as Planck's constant, has since proved to be a fundamental constant of nature. By matching theory with observations the value of h was determined to be 6.63×10^{-34} J-S.

The success of Planck's Theory is that it avoided the ultra-violet catastrophe by limiting the energy of a hot body to a finite number of sources the high-frequency quanta require more energy; hence, fever of their would be radiated. On the basis of assumption that energy could only be emitted or absorbed by atomic oscillators in discrete quanta, the Pianck's law would be:

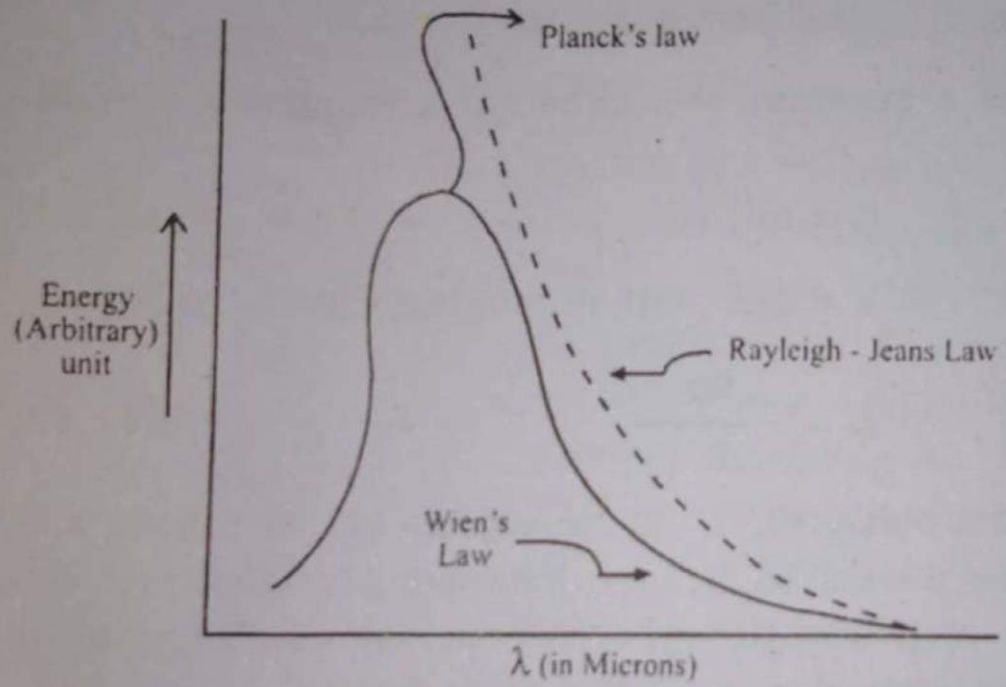
$$E = n\,h\upsilon \qquad \text{------------------(17.18)}$$

where $E = 0.h\upsilon,\ 2h\upsilon$ ---------- Corresponding to

$n = 0,\ 1,\ 2,\ 3,\ 4$ - - - - -

Thus we see that the Planck's law had an important virtue; it worked, that is, it agreed very well with experiment i.e theoretical and experimental curves for radiation matched reasonably. The price for this success was a revolution of concept about electromagnetic radiation not as waves, but as discrete quanta of energy.

A comparison of the different radiation laws is given in Fig.17.8 from which it is evident that the Wien's Law and the Rayleigh-Jean's law are just the special cases of the Planck's law



Fog. 17.8 Comparison of Radiation Laws.

To appreciate how great a departure Planck's law is from classical theory, we may recall that classically, the energy of a wave is related to the amplitude. For example, large waves, such as ocean waves have a large energy. That is no need to relate energy with the frequency. We can have waves of low energy and high frequency and vice versa. According to Planck's theory, however, each electromagnetic wave carries with it a minimum energy that is a function of its frequency. This novel idea took about a quarter of century after it's proposal to be fully welcomed in the modern scientific thought. In this way the planck's law created a revolution in modern Physics.

Example: 17.5.

What will be the energy of an x-ray quantum of wave length 1.0×10^{-10}m?

Solution:

Given:

$$h = 6.63 \times 10^{-34} \text{ J-S}$$

$$\lambda = 1.0 \text{ Å} = 10^{-10} \text{ m}$$

$$c = 3 \times 10^{8} \text{ ms}^{-1} \text{ (velocity of light)}$$

To find, E ?

from Planck's law we have:

$$E = h\upsilon$$

But $\upsilon = \frac{c}{\lambda}$ (wave equation)

$$E = \frac{hc}{\lambda} = \frac{6.63 \times 10^{-34} \text{ J-S} \times 3 \times 10^{8} \text{ m/s}}{10^{-10} \text{ m}}$$

Hence, $E = 19.89 \times 10^{-16}\text{J} = 1.99 \times 10^{-15}\text{J}$

17.9 THE PHOTON.

The photon is, a particle that has no charge and no mass. It can interact with all charged particles as well as with some neutral ones it is electromagnetic radiation and the carrier of electromagnetic forces. Every atom is found to emit and absorb photons of the particular energies and frequencies.

The photon is the lightest particle, being a massless particle its energy is:

$$E = mc^2 = p\,c. \quad \text{------------------(17.19)}$$

where $p = mc$ is the momentum of the photon.

Since $E = h\upsilon$ is the energy of the photon the mass of the photon will be given by

$$mc^2 = h\upsilon$$

or. $$m = \frac{h\upsilon}{c^2} \quad \text{-----------------} (17.20)$$

from, this it is quite obvious that the mass of photon approaches zero as c^2 is a very large quantity occurring in the denominator of equation (17.20)

The photon is a stable particle and, therefore it does not decay spontaneously into any other particle. Its life time is therefore infinite so long it does not undergo interaction with other particles, and is why photons are supposed to be reaching our earth from the farthest distances of the universe. Thus most of our information regarding the universe is carried by photons.

Example: 17 6

Compare the energy of a x-ray photon of wave length 2.0×10^{-10} m with the energy of 2.0μm infrared photon.

Solution:

We have the energy of the photon of x-ray given by:

$$E_1 = h\upsilon_1$$

But. $$\upsilon_1 = \frac{c}{\lambda_1}$$

$$E_1 = \frac{hc}{\lambda_1} = \frac{6.63 \times 10^{-34} \text{ J-S} \times 3 \times 10^8 \text{ m/s}}{2.0 \times 10^{-10} \text{ m}}$$

i.e. $E_1 = 9.94 \times 10^{-16}$J

Now, for the infra-red photon we have:

$$E_2 = \frac{hc}{\lambda_2} = \frac{6.63 \times 10^{-34} \text{ J-S} \times 3 \times 10^8 \text{ m/s.}}{2.0 \times 10^{-10} \text{ m}}$$

i.e. $E_2 = 9.945 \times 10^{-20}J$

$$\frac{E_1}{E_2} = \frac{9.945 \times 10^{-16}J}{9.945 \times 10^{-20}} = 10^4$$

17.10. THE PHOTO ELECTRIC EFFECT:

Hertz in 1887 discovered that when ultraviolet light falls on certain metals, electrons are emitted. This phenomena in which certain metals emitt electrons when exposed to high frequency light is called photoelectric effect. The experimental arrangement to demonstrate the effect is shown in Fig.(17.9).

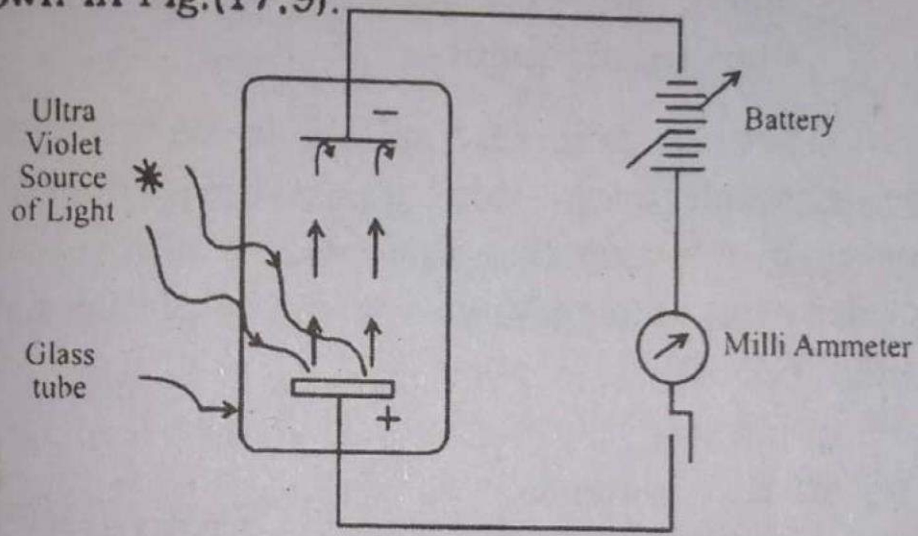


Fig. 17.9 The Photo Electric Effect

The observed photoelectric effect has these features:

(1) Increasing the intensity of the source of light, increases the number of photoelectrons, but not the velocity with which they leave the surface of the metal.

(2) For each substance there is a certain frequency called the threshold frequency below which the effect does not occur.

(3) The higher the frequency of the incident light, the greater the kinetic energy ($\frac{1}{2}mv^2$) of the photoelectrons.

Attempts were made to explain these aspects of the photoelectric effect from the point of view of classical wave theory of light, but no successful explanation was obtained due to the following reasons:

(a) From classical theory, there should be no threshold frequency because at a given time, electrons might absorb enough energy from the incident light to escape from the metal surface at any applied frequency.

(b) The velocity of photoelectrons should depend upon the amplitude of the wave incident on the metal, and therefore upon the intensity rather than the frequency.

Let us first discuss the experimental results of the photoelectric effect. If we draw the photoelectric curves by plotting the photoelectric current I versus the accelerating voltage V we will obtained the curves shown in Fig. (17.10)

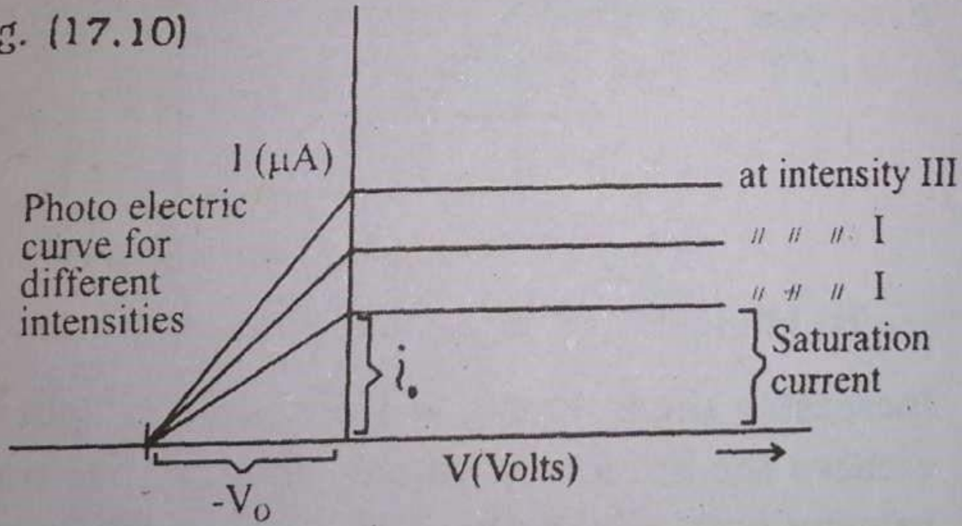


Fig. 17.10

From these curves, it follows that there is a saturation current for different intensities I, II and III etc. and even when the potential V = O, there is some photocurrent I_o and a negative potential $-V_o$ called stopping potential. This behaviour of photoelectric curves indicate that the stopping potential is independent of the intensity of the source and kinetic energy K of the photoelectrons will be maximum for the condition.

$$K_{max} = \frac{1}{2} m\upsilon^2_{max} = eV_o$$

Where, V_{max} = maximum velocity of photoelectrons

e = charge of electron

If photoelectric curves are plotted for different frequencies V_1 and V_2 but the same intensity I of the source, the curves show up the following behaviour Fig. (17.11).

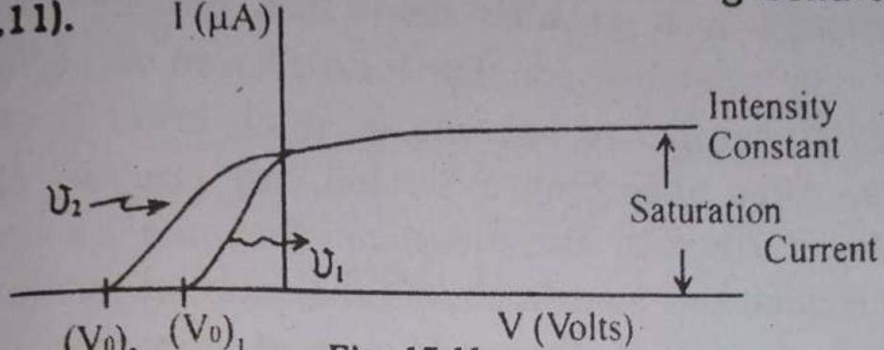


Fig: 17.11

From these curves we infer that the saturation current depends upon intensity and not on the frequency. However, the stopping potential becomes more negative from $(V_o)_1$ to $(V_o)_2$ with the increase in frequency ($V_2 > V_1$). The variation of the maximum energy of photoelectrons K_{max} as a function of frequency υ may be seen from Fig. (17.12) below:

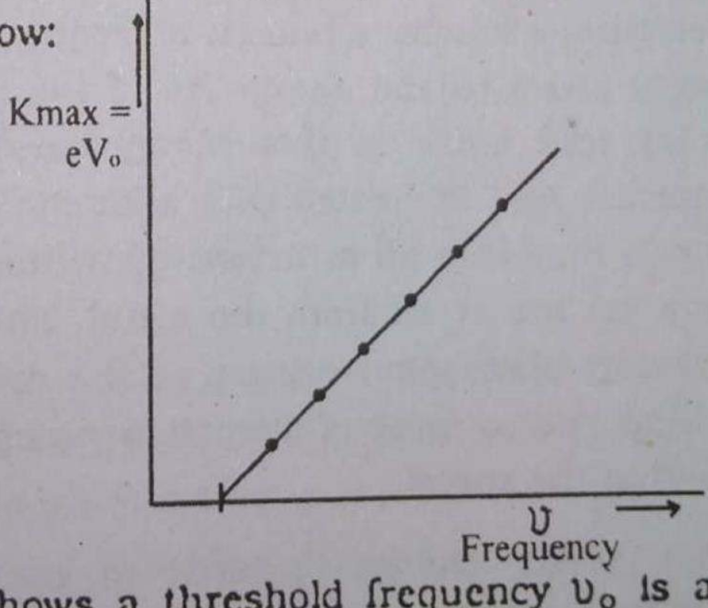


Fig: 17.12

This graph shows a threshold frequency υ_o is a minimum frequency below which no electrons escape the metal surface. The most significant feature of this graph is that the slope of the line gives the ratio h/e. Thus if e is known the measured slope provides a value of Plancks constant h quite independently.

17.11 EINSTEIN'S EXPLANATION OF PHOTOELECTRIC EFFECT ON THE BASIS OF QUANTUM THEORY.

Albert Einstein provided a successful explanation of the photoelectric effect on the basis of quantum theory. He proposed that an electron either absorbs one whole photon or it absorbs none. The chance that an electron may absorb more than one photon is negligible because the number of photons is much lower than the electrons. After absorbing a photon, an electron either leaves the surface of the metal or dissipate its energy within the metal in such a short time interval that it has almost no chance to absorb a second photon. An increase in the intensity of light source simply increases the number of photons and the number of electrons, but the energy for electron remain unchanged. However, the increase of frequency of the light increases the energy of the photons and hence the energy of electrons too.

Thus according to Einstein's quantum theory, an electron absorbs a photon of frequency to acquire an energy equal to the energy $h\upsilon$ of the photon. The electron may lose some of this energy before leaving the metal surface and is ejected with a kinetic energy less than $h\upsilon$. or, it may lose all of its energy within the metal and does not escape at all from the metal. This is true even if the photon absorption occurs at the metal surface because there exist a force of attraction which holds the electrons within the metal.

The energy required to overcome this binding force is called the work function of the particular metal and is denoted by ϕ which is a constant of the metal. Hence the Einstein's equation for photoelectric effect will be written as:

$$K_{max} = \frac{1}{2} m\upsilon^2_{max} = h\upsilon - \phi$$

The graph in Fig. (17.12) shows that the threshold frequency υ_o is a minimum frequency below which no electrons escape out of the metal surface. hence, the condition to find the threshold frequency would be

$$h\upsilon_o = \phi$$

i.e. the Einstein's equation will become:

$$\frac{1}{2} m\upsilon^2_{max} = h\upsilon - h\upsilon_o = h\ (\upsilon - \upsilon_o\ ) \text{ -------(17.22)}$$

The threshold frequency for some metals is shown in table (17.1).

Table 17.1

Photoelectric Threshold Frequency & Work Function of Metals.

Metal	Threshold Freq: υ_o (Hz)	Work Function ϕ in eV
Cesium	4.6×10^{14}	1.9
Beryllium	9.4×10^{14}	3.9
Titanium	9.9×10^{14}	4.1
Mercury	1.09×10^{15}	4.5
Gold	1.16×10^{15}	4.8
Palladium	1.21×10^{15}	5.0

For comparison with these values, it is to be noted that the high frequency end of visible light (violet) has a frequency 8×10^{14} H_z and a photon energy is = 3.3.eV.

Example: 17.7

Sodium light of wave length 5.893×10^{-7} m falls on a photocell. A negative stopping potential of 0.30V is needed to stop the electrons from reaching the collector.

(a) Find the work function of the material of the plate.

(b) **What will be the potential required when light of 4000°A is used?**

Solution:

Data $\lambda = 5.893 \times 10^{-7} m$; $c = 3 \times 10^{8}\ ms^{-1}$

$V_o = 0.3$ Volt i.e. $= 1.6 \times 10^{-19} C$

$h = 6.63 \times 10^{-34}$ J-S

(a) from the Einstein's Photoelectric Equation, we have

$$V_o e = h\nu - \phi$$

$$\phi = h\nu - V_o e$$

or $$\phi = \frac{hc}{\lambda} - V_o e \text{ - - - - - - - - } \therefore c = \upsilon\lambda$$

$$= \frac{6.63 \times 10^{-34} \times 3 \times 10^{8}}{5.893 \times 10^{-7}} - 0.3 \times 1.6 \times 10^{-19}$$

$$= 3.38 \times 10^{-19} - 0.48 \times 10^{-19}$$

$$= 2.90 \times 10^{-19}\ J$$

$$\phi = 2.90 \times 10^{-19}$$

$$= \frac{2.90 \times 10^{-19}}{1.6 \times 10^{-19}}\ eV \quad \therefore \phi = 1.8\ eV$$

(b) $\lambda = 4000°A = 4000 \times 10^{-10}\ m = 4 \times 10^{-7}\ m$:

$\phi = 2.90 \times 10^{-19} J$: $c = 3 \times 10^{8}\ ms^{-1}$

$e = 1.6 \times 10^{-19}$ C ; $h = 6.63 \times 10^{-34}$ J-S

$$\therefore V_o e = h\upsilon - \phi$$

or $$V_o e = \frac{hc}{\lambda} - \phi$$

$$\therefore V \times 1.6 \times 10^{-19} = \frac{6.63 \times 10^{-34} \times 3 \times 10^{8}}{4 \times 10^{-7}} - 2.90 \times 10^{-19}$$

$$V \times 1.6 \times 10^{-19} = 4.97 \times 10^{-19} - 2.90 \times 10^{-19}$$

or

$$V \times 1.6 \times 10^{-19} = 2.07 \times 10^{-19}$$

or

$$V = \frac{2.07}{1.6}$$

$$V = 1.29 \text{ Volts}$$

17.12. PHOTO CELL AND THEIR USES.

A simple photocell based on the photo electric effect is shown in Fig. (17.13)

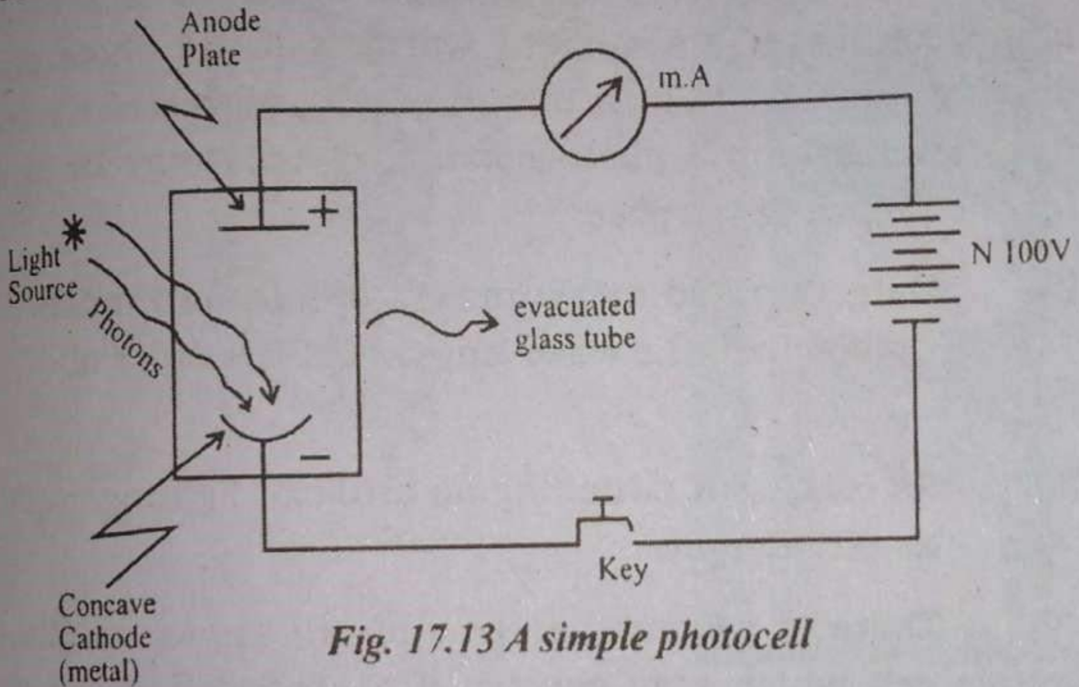


Fig. 17.13 A simple photocell

The photocell or photo tube consist of a evacuated glass tube fitted with an Anode plate and a concave metallic cathode of a appropriate surface. The material of the cathode can be chosen to respond to the frequency range over which the photocell operates. The response can be made proportional to the intensity of the light source. The photocell is connected in a circuit as shown in Fig. (17.13) to operate for a particular use of the cell as a source of photoelectric emission. A photocell can be used in any situation where beam of light falling on a cell is interrupted or broken.

For example:

(a) To count vehicles passing a road or items running on a conveyer belt.

(b) To open doors automatically in a building.

(c) To operate burglar alarms.

Besides Photo-emission cells there are also photo conductive cells in which an internal photoelectric effect may liberate free charge carriers in a material that is otherwise an insulator, and thereby increase its electrical conductivity by as much as 10,000 times when it is illuminated by a light source. Such materials are called photo conductors. A current will flow if the photo conducting material is in a circuit with a source of electromotive force (e.m.f) photo conductive cells may be used for the following purpose:

(i) Detection and measurement of infra-red radiations where the wave length is of the order of 10^{-6} m.

(ii) As relays for switching on artificial lighting, such as street lights.

There is yet another type of cell known as photo voltaic cell which may consist of a sandwich of copper, copper oxide and a thin film (layer) of translucent gold. An e.m.f capable of giving a current of 1mA can be generated when the film is illuminated. Such cells need no source of e.m.f and are frequently used as exposure meters in photography. The exposure meters are required to set the aperture (opening before the camera lens) of the camera compatible to the day light intensity.

Besides the above mentioned uses, the most important use of photocells in of modern everyday life is the production of pictures in television cameras and the sound tracks of motion pictures. In one type of photocell of much usage, the cathode is coated with caesium, a light sensitive material that emit electrons when illuminated. The electrons collect at a second electrode the anode, the current being proportional to the illumination. The sound information is stored on the film in the form of spots of varying widths. When such a film run between the light source the photocell, variations in light intensity reaching the cell cause pulsations in the current that, after being amplified, activates a loudspeaker and reproduces sound. This is diagramatically shown in fig: 17.14

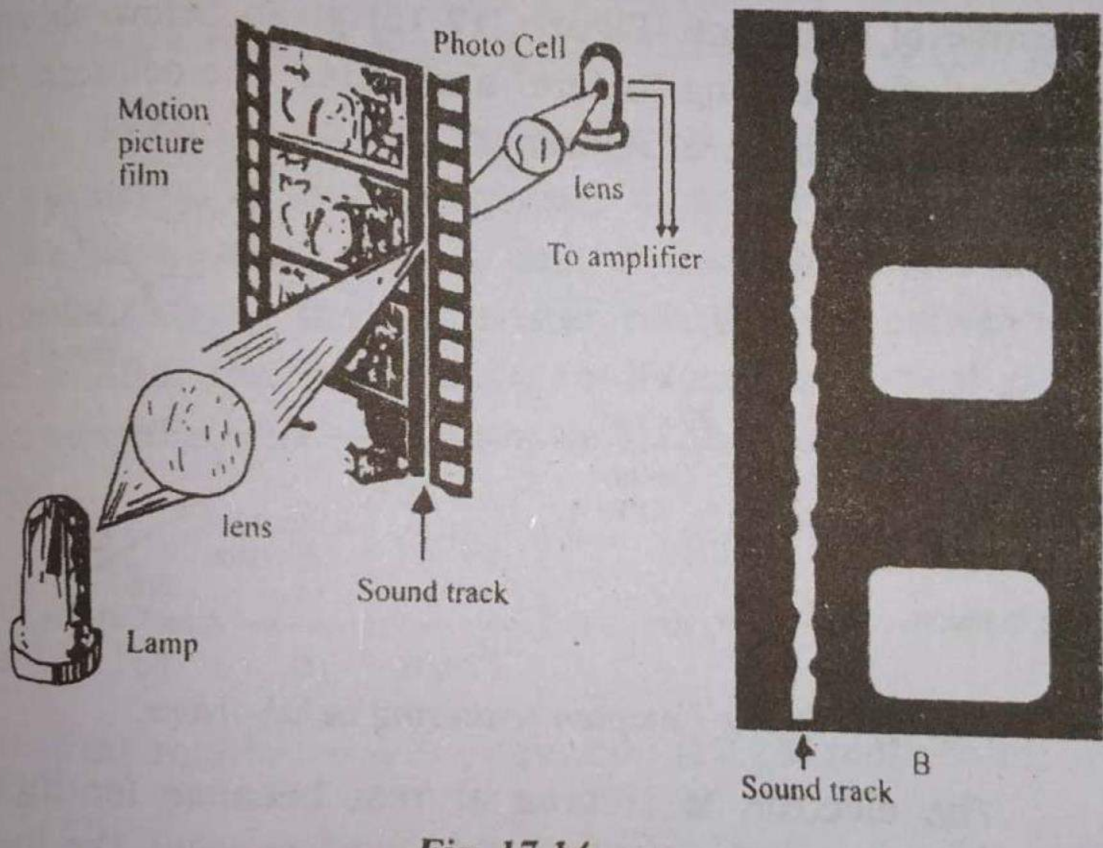


Fig. 17.14

17.13. THE COMPTON EFFECT:

It was reported by many observers that when x-rays are scattered due to interaction with a light body, such as an electron, the scattered rays exhibit lower frequencies (i.e. higher wave lengths) than the incident radiations. Arthur Compton studied this phenomena of change in wavelength in the year 1926. Making accurate measurements, he was the first to propose a theory based on the idea of photon theory of radiation. Since a detailed study of the phenomena was made by Compton, the effect is now called as the compton's effect. Compton effect provided a solid support for photon theory of light since the results obtained were elegantly described by assigning to the photon an energy E =h υ and momentum

$$p = \frac{h\upsilon}{c} \text{ as, } E = h = mc^2 \text{ gives us } \frac{h\upsilon}{c} = mc = p.$$

The scattering is treated to be a two body collision between a photon and an electron.

It is sufficient to analyze the scattering process in a single frame of reference. Figure (17.15) given below shows the event of scattering "before" and "after" the collision in the laboratory frame of reference:

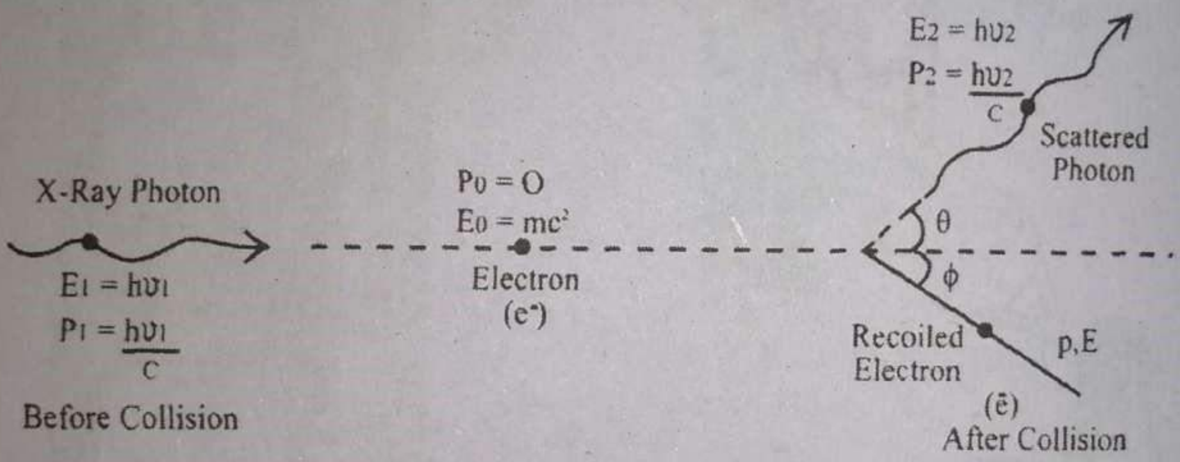


Fig. (17.15) The Compton scattering in lab-frame.

The electron is treated at rest because for high energy photons (and not for visible light quanta), the initial motion of the electrons may be neglected. The photon approaches towards the electron with a frequency υ_1 and is scattered at an angle θ with a lower frequency υ_2. The photon energies before and after collision are $h\upsilon_1$ and $h\upsilon_2$. Whereas the corresponding momentum are $\frac{h\upsilon_1}{c}$ and $\frac{h\upsilon_2}{c}$ The energy and momentum of the recoiled electron are E and P respectively. If the laws of conservation of energy and momentum along and across the direction of approach are applied, we will get the following equations:

Conservation of momntum along the line of impact:

$$\frac{h\upsilon_1}{c} = \frac{h\upsilon_2}{c} \cos\theta + p\cos\phi \quad \text{--------------(17.23)}$$

Conservation of momentum across the line of impact:

$$0 = \frac{h\upsilon_2}{c} \sin\theta - p\sin\phi \quad \text{-----------------(17.24)}$$

Conservation of energy before and after collision

$$h\upsilon_1 + m_0c^2 = h\upsilon_2 + E \quad \text{--------------- (117.25)}$$

To obtain an expression for the final frequency υ_2 as a function of initial frequency υ_1 and scattering angle θ. We have to eliminate p and E from the above three equation, using the relativistic relationship between E and p. After performing some routine mathematical steps and simplification we will finally get the following expression.

$$\frac{1}{\upsilon_2} - \frac{1}{\upsilon_1} = \frac{h}{m_0c^2}(1 - \cos\theta) \quad \text{----------(17.26)}$$

using the relation $\nu = \frac{c}{\lambda}$, equation (17.26) reduces to:

$$\lambda_2 - \lambda_1 = \frac{h}{m_0c}(1 - \cos\theta) \quad \text{-----------------(17.27)}$$

Equation (17.27) is the famous compton formula for the increase in wave length of the scattered photon.

The quantity $\frac{h}{m_0c}$ in the compton's equation is called the compton wavelength and is denoted by

$$\lambda_c = \frac{h}{m_0c} = 2.426 \times 10^{-12}\text{m} \quad \text{---------------(17.28)}$$

17.14 Pair Production and Annihilation of Matter

We have already seen that a low energy Photon on striking with a electron loses its entire energy (Photo electric effect) where as a high energy Photon loses a part of its energy (compton effect). A Photon may also lose its energy in another way, that is a Photon in the vicinity of a nucleus may disappear with the Production of an electron-positron pair.

The positron has been identified to be identical with an electron in mass and carries an equal positive charge and is called the anti-particle of the electron. This phenomenon called pair production is shown schematically in Fig. (17.16) below:

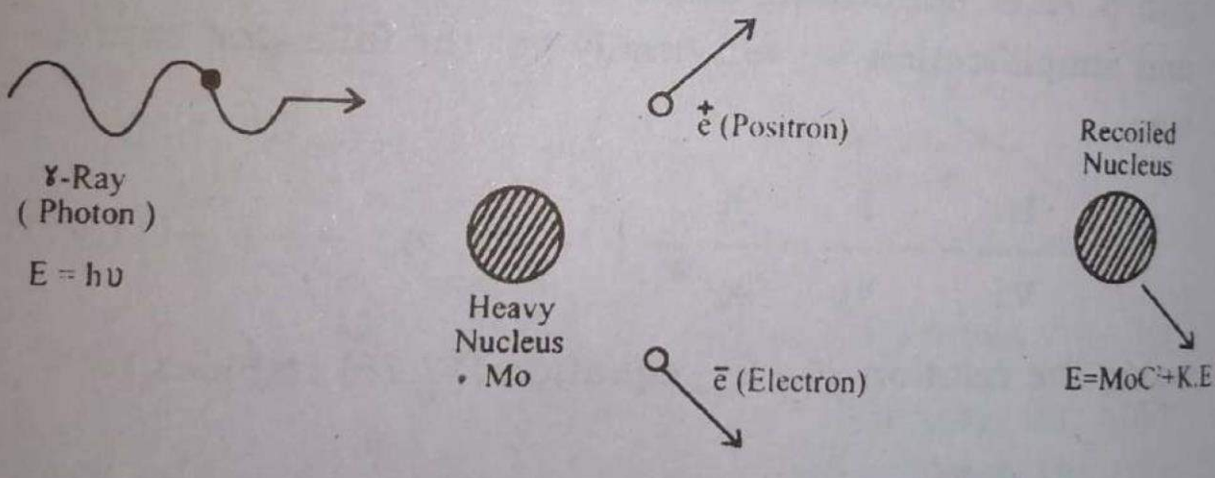


Fig. (17.16) The Pair Production

In order to conserve the charge, the pair production requires that the two particles created by the interaction of photon with matter must have equal and opposite charges and the photon from which the pair was produced must have an energy at least $2m_oc^2$, where m_o is the rest mass of electron and C is the velocity of light. Due to this reason the pair production is not important below 1.02 MeV the energy corresponding to $2m_o$. Note that the role of the heavy nucleus in the vicinity of the incoming photon is just to share some energy and momentum in order to conserve these quantities.

Due to the large mass of the heavy nucleus the recoil kinetic energy of the nucleus $P^2/2m_o$ is negligible as compared with the kinetic energies of the pair. The energy conservation in pair production demands:

$$h\nu = 2m_oc^2 + (K.E)\,\bar{e} + (K.E)\,\overset{+}{e} \quad \text{------------------}(17.29)$$

Since the process of pair production involves the creation of a particle & its anti-particle, it is also some times referred to as the materialization of energy in conformity with the mass-energy equivalence to be discussed further in chapter-19.

It has been observed that a process reverse to the pair production may also occur by the destruction or annihilation of the electron-positron pair with the creation of at least two or more photons. The pair annihilation process is schematically shown below in Fig. (17.17).

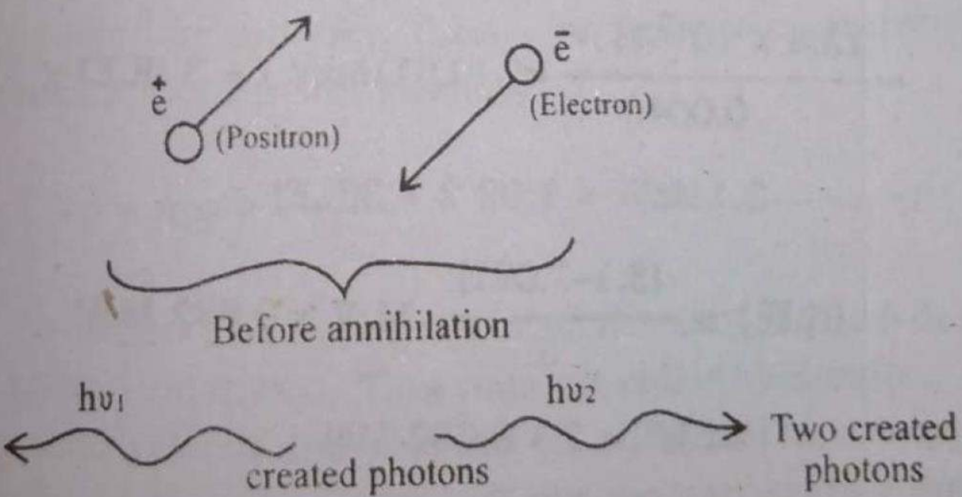


Fig. (17.17) Annihilation of electron positron pair in collision

In pair annihilation a particle and one of its anti-particle come close enough to be converted completely into radiation energy of the two photons moving in opposite direction conserving the total momentum of the creation and annihilation process. Each photon will have an energy equal to rest mass energy m_oc of an electron.

Since the rest mass energy of electron positron pair is 1.02 MeV, each photon created in the annihilation process will have an energy of 0.51 MeV.

The energy conservation equation for the process wil be:

$$(m_o)_{e^+} c^2 + (K.E)_{e^+} + (m_o)_{e^-} c^2 + (K.E)_{e^-} = 2h\upsilon \text{ ------(17.30)}$$

Example: 17.8

A photon of wave length 0.004Å in the vicinity of a heavy nucleus produces an electron positron pair. Find the kinetic energy of each particle if the kinetic energy of positron is twice that of the electron.

Given: $hc = 12.4\times 10^{-3}$ MeV and, $m_oc^2 = 0.511$ MeV

Solution:

From the law of conservation of energy of wave:

Initial Energy = Final Energy

$$= \frac{hc}{\lambda} = 2m_oc^2 + (K.E)_{e^-} + (K.E)_{e^+}$$

or

$$\frac{12.4 \times 10^{-3}\text{ MeV}}{0.0040} = 2\,(0.511\text{MeV}) + 3\,(K.E)\,e^-$$

$$3.1\text{MeV} = 1.02\,2 + 3(K.E)_{e^-}$$

$$\therefore\ (K.E)_{e^-} = \frac{(3.1-1.022)}{3}\text{ MeV} = 0.692\text{ MeV}$$

Hence

$$(K.E)e^+ = 2 \times 0.692\text{ MeV}$$

$$= 1.384\text{ MeV.}$$

Example: 17.9

Pair annihilation occurred due to a head-on collision of an electron and a positron producing 2.5 MeV photon moving in opposite directions. What will be the Kinetic energies of the electron and positron before the collision. Given: $m_oc^2 = 0.511$ MeV.

Solution:

Applying the law of energy conservation we have:

$$2\, m_o c^2 + 2\, K = 2\, (E)$$

When K = K.E of the particle photon before collision.

Thus, we get:

$$2(0.511)\ \text{MeV} + 2K = 2\ (2.5\text{MeV})$$

or, $$1.022\ \text{MeV} + 2K = 5.0\ \text{MeV}.$$

or, $$2K = (5 - 1.022)\ \text{MeV} = 3.918\ \text{MeV}$$

$$\therefore\ K = \frac{3.918}{2}\ \text{MeV} = 1.959\ \text{MeV}$$

17.15 THE WAVE NATURE OF PARTICLES AND DE-BROGLIE HYPOTHESIS.

DeBroglie in 1924 put forth a novel idea called the DeBroglie's hypothesis: If light (electromagnetic radiation) can have particle behaviour, then material particles, such as electrons and protons etc. can also behave in a wave like manner. Thus a particle, like electron can possess a momentum given by:

$$p = m\upsilon = \frac{h}{\lambda} \qquad \text{-----------------(17.31)}$$

Where, m is the mass of the particle (as defined in special relativity). This relation called deBroglie's relation has related the electron (a particle), and the wave character of a frequency . Thus we can write down he wave length associated with the particle i.e

$$\lambda = \frac{h}{m\upsilon} = \frac{h}{\lambda} \qquad \text{-----------------(17.32)}$$

Although, the deBroglie's relation was initially developed for the electron, it is valid for all material objects including particles. However, for massive materials the associated wave length is too small to be measured. For example a mass of 20 kg moving with a velocity of 50 m per second will have its wave length:

$$\lambda = \frac{h}{mv} = \frac{6.63 \times 10^{-34}}{20 \times 50} = \frac{6.63 \times 10^{-34}}{1000}$$

i.e $\lambda = 6.63 \times 10^{-37}$ m

Hence we see that such a small order of 10^{-37} is not measureable.

On the other hand for light particles like an electron moving with a velocity say 10^7 ms^{-1} the wave length will be

$$\lambda = \frac{h}{mv} = \frac{6.63 \times 10^{-34}}{9.1 \times 10^{-31} \times 10^7} \equiv 0.72 \times 10^{-10} \text{ m}$$

this order of magnitude of the wave length falls in the short x-ray wave length and is experimentally measurable.

Example: 17.10

What will be the de Broglie wave length of a mass of 3 kg moving with a velocity of 100 ms^{-1}

Solution:

from deBroglie's relation we have

$$\lambda = \frac{h}{mv} = \frac{6.63 \times 10^{-34}}{3 \times 100} = 2.212 \times 10^{-36} \text{ m}$$

Example: 17.11

What will be the wavelength of a neutron having an energy equal to 0.06 eV.

Given, rest mass energy of neutron $m_0 c^2 = 940 \times 10^6$ eV.

We know that the kinetic energy is given by:

$$K = \frac{1}{2} m_0 V^2$$

$$\upsilon = \sqrt{\frac{2K}{m_0}}, \text{ when given } K = 0.06 \text{ eV}$$

But from, deBroglie's relation we have:

$$\lambda = \frac{h}{m_0 \upsilon} = \frac{h}{m_0 \sqrt{\frac{2K}{m_0}}} = \frac{h}{\sqrt{2m_0 K}}$$

$$\lambda = \frac{12.40 \times 10^{-10}}{10.26} = 1.16 \times 10^{-10} \text{ m}$$

17.16 THE DAVISSON AND GERMER EXPERIMENT:

The theoretical prediction of deBroglie's hypothesis $\lambda = \frac{h}{p}$ was experimentally confirmed by the famous experiment conducted by Davisson and Germer in the year 1927. They were investigating the scattering of an electron beam by the metallic crystal of Nickel. Their experimental set up which was enclosed in a vacuum chamber is schematically shown in Fig. (17.18).

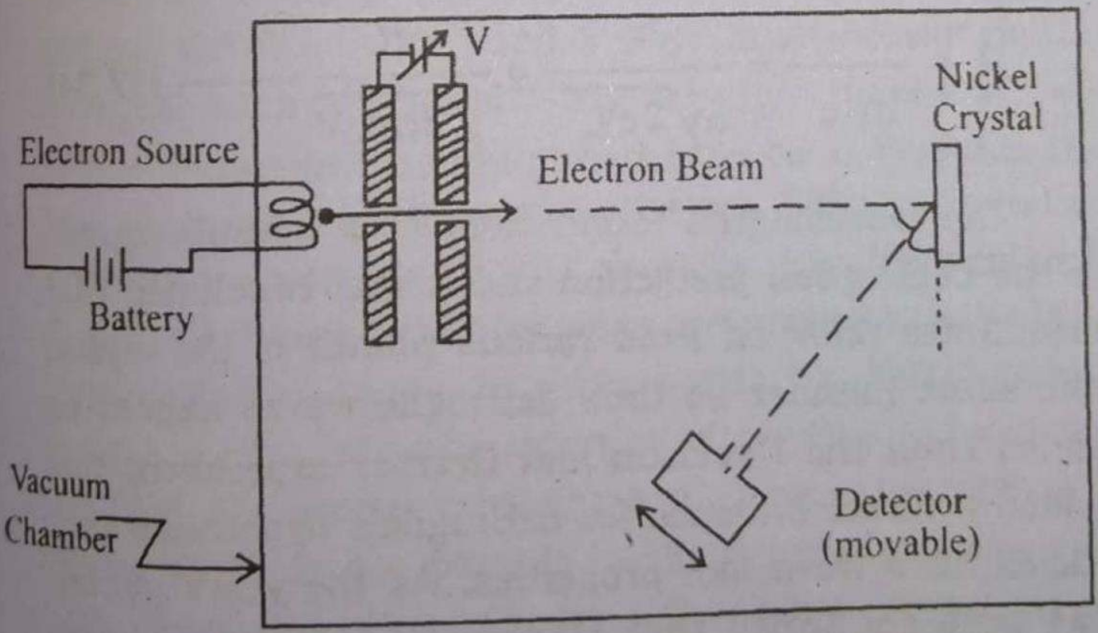


Fig.17.18 Davisson and Germer Experiment

A beam of electron accelerated through the potential V were allowed to strike the nickel crystal. Measurements were made to count the number of electrons scattered by the crystal. Davisson and Germer reported the un-expected result that electrons reflected very strongly at certain angles only and not at other direction. These results remained unexplained for some time until Elasser suggested that perhaps, this was an outcome of deBroglie's relation. Davisson and Germer then further investigated properly oriented crystals to observe if could be possible to interpret that electrons behave as waves of wave length λ as given by the deBroglie's relation (17.32). They calculated the wave length of electron from the known accelerating potential V by applying the relation for kinetic energy of the electron i.e.

$$\frac{1}{2} m_o v^2 = eV$$

so that

$$v = \sqrt{\frac{2eV}{m_o}} \quad \text{----------------- (17.33)}$$

where m_o is the rest mass of electron, e the charge and V its velocity using the deBroglie's relation:

$$\lambda = \frac{h}{m_o v} = \frac{h}{m_o\sqrt{\frac{2eV}{m_o}}} = \frac{h}{\sqrt{2em_oV}} \quad \text{-------- (17.34)}$$

The wavelengths found from this formula agreed with the deBroglie's prediction and it was concluded that electrons are reflected from various planes of the crystal in the same manner as their deBroglie waves should be reflected thus the Davisson and Germer experiment has provided a direct evidence for deBroglie's hypothesis that particles have wave like properties. As the years passed it has been confirmed that other particles neutron, protons, atoms and molecules etc, are associated with the same wave effects as that of electrons.

17.17: WAVE PARTICLE DUALITY:

From the ancient times till the time of Newton.

scientists believed that light propagates in the form of corpuscles (particles). Wave theory of light was then established in 1801 when Young's interference experiments was performed. Finally by the year 1900, Planck postulated his law that electromagnetic radiations are emitted in the form of light quanta (photons). It was again demonstrated that particle behaviour had to be assigned to electromagnetic radiations in order to explain the famous photoelectric effect and compton scattering. Finally de-Broglie (1924) proposed his hypothesis in which he suggested that since light much of the character of particles, that particles, like electrons, could exhibit wave like behaviour. hence, it is said that electromagnetic radiation exhibit a wave-particle duality i.e. in certain situations it shows wave like properties while in other circumstances it acts like a particle. Since these are the only two possible modes of propagation of electromagnetic energy. It is essential to have a clear understanding of the distinction between waves and particles. A particle is identified by its position, momentum, mass, energy and charge. On the other hand a wave is associated with attributes such as amplitude, velocity, intensity, wavelength. frequency. energy and momentum. Besides these attributes one of the most distinet difference between wave and a particle is that particles can be localized at certain positions, whereas wave are spread relatively over large region of space. In other words the deBroglie's formula $\lambda = \frac{h}{p}$, may be interpreted physically by proposing that if material particles are allowed to cross a slit whose width is comparable to the wavelength associated with them, then the particles will exhibit diffraction phenomena in exactly the same manner as photons do in the Young's single slit experiment and wave like character predominate. For photons the frequency and wavelength are given by $\upsilon = E/h$ and $\lambda = h/p$. From these expressions it is evident that the left hand sides of these equations involved the wave aspect of photons through υ and λ, whereas the right hand sides display the particle character through E and P. The linking factor between the two aspects is the Planck's constant h, and hence the particle wavelength will be given by

$$\lambda = h/p = \frac{h}{m\upsilon}.$$

There is also another difference between photons and ordinary objects in the manner through which the wave and particle like properties are related. This is because for a photon $\lambda\upsilon = c$, and hence there is only a single rule to obtain both λ and υ from photon's particle like properties of E and p. On the other hand for ordinary objects, separate rules are required to specify its wave length by the relation $\lambda = \frac{h}{p}$. and frequency by $\upsilon = E/h$.

Inspite of the wave-particle duality of radiation and matter (particles) it has not been possible to witness a single phenomena in which radiation or particle exhibit both wave and particle characters simultaneously. All known physical phenomena clearly fall in two Distinct categories. of exhibiting either the particle or wave like behavior of radiation and matter. Thus, a complete description of either radiation or particles requires in either case that both the wave and particle features be considered, but each in its own proper perspective of the particular phenomena exhibited by either of them.

17.18. THE UNCERTAINTY PRINCIPLE:

In classical physics it is generally assumed that position and momentum of a moving object can be simultaneously measured exactly, that is no uncertainties are inolved in its description. For example if we know the initial position and velocity of an object and the net force acting on it, we can apply Newton's Law (Classical mechanics) to predict exactly (with certainty) its final position and velocity. But, can we make similar measurements simultaneously in microscopic world of atoms and subatomic particles?

It is found that however refined we make our instruments there is a fundamental limitation to the accuracy with which the position and velocity of microscopic particle can be known simultaneously. This limitation was first expressed by Heisenberg (1927) and is known as the uncertainty principle. The Heisenberg uncertainty principle states that it is in principle impossible to measure with accuracy both position and momentum of a particle simultaneously. Thus, if we denote Δx and Δp_x the corresponding uncertainties in position; then according to Heisenberg's principle of uncertainty, the product of these uncertainties must always satisfy the inequality:

$$\Delta x \Delta p_x \geq \frac{h}{2\pi} = 1.05 \times 10^{-34} \text{ J S} \quad \text{-----------(17.35)}$$

Similar relations follow for the other two y and z directions.

Although it is difficult to observe this idea in our normal everyday observations but it is necessary and direct consequences of the deBroglie's hypothesis and the wave particle duality.

Let us now examine whether the uncertainty condition is consistent with experiment. Suppose we want to localize a particle in the y direction as shown in Fig. (17.19).

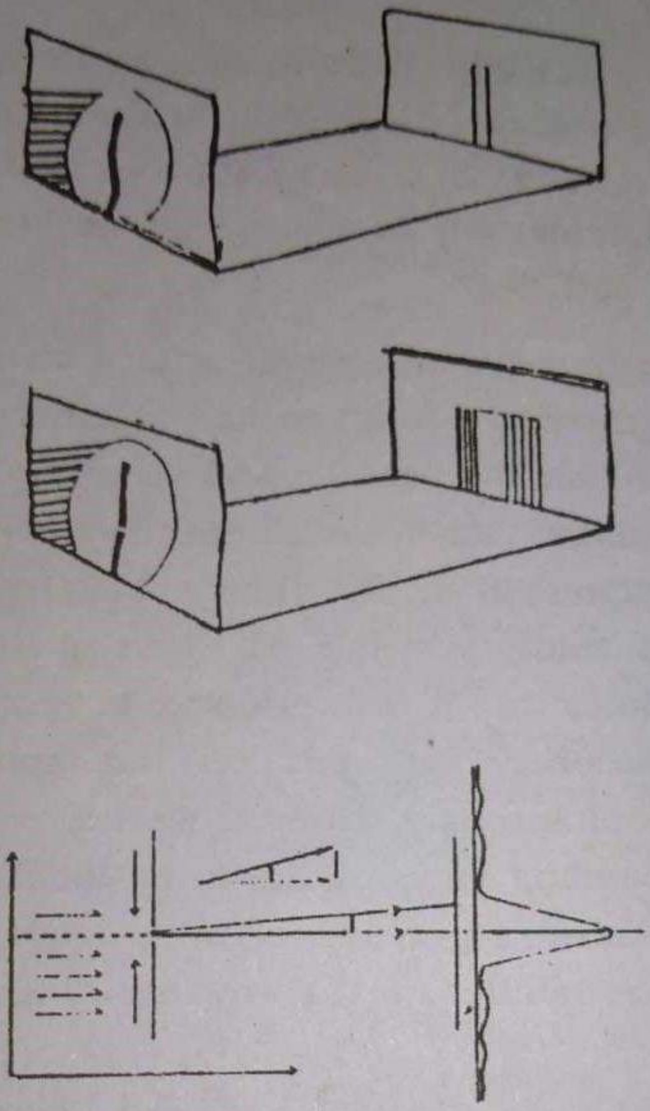


Fig. 17.19

If a particle beam strikes a slit, those passing through the slit must have been localized in a region $\Delta y = d$, where d is the width of the slit. If the particles are ordinary classical particles, they will strike the screen along a thin strip i.e. the projection of the slit on the screen. But if a wave crosses a slit, then there will be a diffraction pattern wider than the width of the slit projected on the screen. The width of the diffraction pattern on the screen will be found to increase as the width of the slit is reduced in size.

A particle arriving at the screen some distance from the centre will have a finite y component of momentum Δp_y along with p_z as shown in figure above.

The relation between the slit width and the distance to 1st minimum i.e. $\theta = \frac{\lambda}{d}$ shows that if we make the slit narrow by reducing d, we increase the diffraction width i.e. increase the uncertainty in p_y hence we may write

$$\theta = \frac{\lambda}{d} = \frac{\lambda}{\Delta y} \quad \text{------------------ (17.36)}$$

The definition of the angle $\theta << 1$ gives:

$$\theta = \frac{\Delta p_y}{P_x} \quad \text{------------------ (17.37)}$$

and, by deBroglie's relation we have

$$\lambda = \frac{h}{P_x} \quad \text{------------------ (17.38)}$$

using the above equations we finally obtain

$$\frac{\Delta p_y}{P_x} = \frac{\lambda}{\Delta y} = \frac{h}{P_x \Delta_y}$$

i.e. $$\Delta p_y \Delta y = h \quad \text{------------------(17.39)}$$

Equation (17.39) is therefore consistent with the Heisenberg, uncertainty principle stated in Equation (17.36).

Similar to the uncertainty relation above there is another principle of uncertainty which limits the accuracy in the measurement of time i.e. if ΔE is the energy uncertainty in time Δt then we have an expression similar to (17.39)

i.e $$\Delta E \Delta t > h$$

The reason why the uncertainty principle is of no importance in our everyday life is that Planck's constant h is so small that the uncertainties in position and momentum of even quite light objects are far too small to be experimentally observed for microscopic phenomena such as atomic processes, the displacements and momentum are such that the uncertainty relation is critically applicable.

Example: 17.12

Find the uncertainty in momentum and the kinetic energy of a electron if it is found to exist in a region equal to the diameter of Hydrogen atom.

Solution:

Given: $\hbar = 1.05 \times 10^{-34}$ J-S and $m = 9.1 \times 10^{-31}$ kg.

diameter of hydrogen atom = 10^{-10} m.

i.e. $V = 1.6 \times 10^{-19} J$

using uncertainty relation

$$\Delta p = \frac{\hbar}{\Delta z} = \frac{1.05 \times 10^{-34} \text{ J-S}}{10^{-10} \text{ m}} = 1.05 \times 10^{-24} \text{ kg ms}^{-1}$$

Now, kinetic energy of electron

$$= \frac{(\Delta P)^2}{2m} = \frac{(1.05 \times 10^{-24} \text{ kg ms}^{-1})^2}{2 \times (9.1 \times 10^{-31} \text{ kg})}$$

$$= 6.1 \times 10^{-19} \text{ J} = 3.8 \text{ eV}.$$

Example: 17.13

Determine the minimum uncertainty in the position of a particle of mass 5×10^{-3} kg moving with a speed of $2ms^{-1}$ The momentum can be determined to a accuracy of one part in a thousand.

Solution:

$(\hbar = 1.05 \times 10^{-34}$ J-S)

Given: $$\frac{\Delta p}{P} = \frac{1}{1000} = 10^{-3}$$

$$\therefore \Delta p = p = 10^{3}p = 10^{-3}mv \; (\therefore p = mv)$$

Now using the uncertainty relation

$$\Delta x \, \Delta p \approx \hbar$$

we get. $$\Delta x \approx \frac{\hbar}{\Delta p} = \frac{1.05 \times 10^{-34}\ \text{J.S}}{10^{-3} \times 5 \times 10^{-3}\ \text{kg} \times 2\text{ms}^{-1}}$$

i.e. $$\Delta x = \frac{1.05}{10} = 10^{-28}\ \text{m} = 1.05 \times 10^{-29}\ \text{m}$$

Example: 17.14

What will be the uncertainty in energy of an electron thrown to a higher state in an atom and falling back to the original state in about 10^{-8} s? Given $\hbar = 1.05 \times 10^{-34}$ J.S.

Solution:

Using the uncertainty relation:

$$\Delta E\, \Delta t \geq \hbar$$

we have, $$\Delta E \approx \frac{\hbar}{\Delta t} = \frac{1.05 \times 10^{-34}\ \text{J.S}}{10^{-8}}$$

$$\therefore \quad \Delta E \approx 1.05 \times 10^{-26}\ \text{J}$$

$$\approx 0.65 \times 10^{-7}\ \text{eV}$$

QUESTIONS

17.1 What do you understand by a frame of reference, and what are inertial frames?

17.2 Give the principle of relativity and explain the postulates of special theory of relativity.

17.3 Discuss the black body radiation and the associated difficulties in explaining the radiation curve.

17.4 Give the essential features of the photoelectric effect. How can these features be explained theoretically.

17.5 Discuss briefly the wave particle duality and the principle of uncertainty.

17.6 How can you demonstrate experimentally that particles have wave-like particles?

17.7 What is a photo-cell? mention some of its uses in the modern life.

17.8 Explain, why is compton effect not observable with visible light?

17.9 What phenomena require a wave description of light? What phenomena require a particle picture of light? How are the two aspects related quantitatively?

17.10 In what way do the particles of light (Photons) differ from particles of matter such as electrons and protons.

17.11 In the photoelectric effect the energy of a photoelectron is less than that of incident photon. Explain.

17.12 How did deBroglie's hypothesis help to explain the stability of the atom?

17.13 With the help of Uncertainty principle show that electrons can not exist inside the nucleus of an atom.

PROBLEMS

17.1 In the inertial frame of a pendulum the time period is measured to be 3 s. What will be the period of the pendulum for an observer moving at a speed of 0.95 c with respect to the pendulum?

Ans: (9.6 s)

17.2 What will be the length of a bar in the stationary frame if its length along the x'-direction is 1m and the motion is with a velocity 0.75c with respect to the observer at rest.

Ans: (0.66m)

17.3 Given $m_o c^2 = 0.511 MeV$. Find the total energy E and the kinetic energy K of an electron moving with a speed $v = 0.85c$.

Ans: (0.970MeV: 0.459Mev)

17.4 The total energy of a proton of mass $1.67 \times 10^{-27} kg$ is three times its rest energy.

Find (a) Protons rest energy

(b) Speed of the proton

(c) Kinetic energy k of proton in eV

Given $c = 3 \times 10^8 ms^{-1}$ and $1eV = 1.60 \times 10^{-19} J$

Ans (939 MeV: $2.82 \times 10^8 ms^{-1}$ and 1878 MeV)

17.5 A particle of rest mass m_o has a speed $\upsilon = 0.8C$. Find its relativistic momentum, its kinetic energy and total energy ?

Ans: $\left[\frac{4}{3} m_o c; \frac{2}{3} m_o c^2; \frac{5}{3} m_o c^2\right]$

17.6 What will be the velocity and momentum of a particle whose rest mass is m_o and whose kinetic energy is equal to its rest mass energy

Ans : $\frac{\sqrt{3}}{2} C$, $\sqrt{3}\ m_o c$

17.7 The sun radiates energy at a rate 3.8×10^{26} w. At what rate the mass of sun diminishes?

Ans: [1.32×10^{17} kg per yr]

(Given $c = 3 \times 10^8 ms^{-1}$ 1 yr = $3.15 \times 10^7 S$)

17.8 What will be the work function of a substance for a threshold frequency of $43.9 \times 10^{13} H_Z$?

Ans: (1.82 eV)

17.9 What will b the value of $\lambda_{min} = \frac{hc}{eV_o}$. If $h = 6.63 \times 10^{-34}$ J.S. $c = 3 \times 10^8 ms^{-1}$.

$e = 1.6 \times 10^{-19}$ C and $V_o = 10^4$ V

Ans: $(1.24 \times 10^{-10}$ m)

17.10 In a compton scattering process, the fractional change in work length of x-ray photons is 1% at an angle $\theta = 120°$, find the wavelength of x-ray used in the experiment.

Ans: $(3.63 \times 10^{-10}$ m)

17.11 Find the wave length of a 2.0g light ball moving with a velocity:

(a) 1.0 mm per century (b) 1.0 ms^{-1}

(Given: $h = 6.63 \times 10^{-34}$ J.S and 1yr $= 3.15 \times 10^7$ S)

Ans: $(1.05 \times 10^{-18}$ m: 3.3×10^{-31} m)

17.12 An electron exist within a region of 10^{-10} m. Find its momentum, uncertainty and the approximate kinetic energy.

Given $\hbar = 1.05 \times 10^{-34}$ J.S and $m = 9.1 \times 10^{-31}$ kg

Ans: $(1.05 \times 10^{-24}$ kg ms^{-1} · 6.01×10^{-19} J)

17.13 Sodium surface is shined with light of wave length 3×10^{-7} m. If the work function of Na = 2.46eV, find the K.E of the photoelectrons and also the cut off wave length $\lambda c = \frac{hc}{\phi}$

(Given, 1eV $= 1.6 \times 10^{-19}$ J)

Ans: (1.68eV, 5.05×10^{-7}m)

17.14 X-rays of wave length λ_o are scattered from a carbon block at an angle of 45° with respect to the incident beam. Find the shift in wave length.

(Given: $h = 6.63 \times 10^{-34}$ J-S, $m = 9.11 \times 10^{-31}$ kg and $c = 3 \times 10^8$ ms^{-1}.)

Ans: $(7.11 \times 10^{-13}$ m)

17.15 If the electron beam in a T.V picture tube is accelerated by 10,000V what will be the deBroglie's wave length?

Ans: $[1.28 \times 10^{-11}$ m]

17.16 What minimum energy photon can be used to observe an object of size 2.5×10^{-10}m.

Ans: $(4.96 \times 10^{3}$ eV)

17.17 What will be the deBroglie's wave length associated with a mass of 0.01 kg moving with a velocity 10 ms^{-1} ?

Ans: $(6.63 \times 10^{-33}$m)

17.18 Certain excited state of hydrogen atom have a life time 2.5×10^{-19} s. What will be the minimum uncertainty in energy?

Ans: $(2.65 \times 10^{-15}$ J)

17.19 X-rays are scattered from a target material. The scattered radiation is viewed at an angle of 90° with respect to the incident beam. Find the compton shift in wave length.

Ans: $(2.42 \times 10^{-12}$m)

17.20 Find the frequency of a photon when an electron of 20 KeV is brought to rest in a collision with a heavy nucleus.

Ans: $(4.84 \times 10^{18}$ Hz).