A TEXTBOOK

OF

PHYSICS

FOR

CLASS XII

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Chapter-11

HEAT

11.1. Heat as a form of energy in Transit.

Many scientists have tried to explain the nature of heat. Up to the beginning of the nineteenth century, it was considered a weightless fluid called caloric which existed in every material body. Hot bodies were said to contain more caloric than the cold bodies. The caloric theory could explain satisfactorily many processes such as heat conduction and mixing of substances in a calorimeter.

This concept of heat fluid was challenged by Count Rumford. He observed while supervising the boring of cannon that as cannon barrels were being bored, a tremendous amount of heat was given off. According to the caloric theory since the metal chips cut off by the drill have lost caloric i.e the heat given off in the drilling process, the chips should not be the same as the original metal, which had not lost caloric. But Rumford was unable to find any difference between the chips and the original metal in respect to their ability to hold or give off heat. In order to further investigate it, Rumford made use of a very dull drill, which was unable to cut the metal. Heat was evolved in apparently unlimited quantity as long as the borer was rotated and the supply of heat was inexhaustible. He concluded that heat was due to the rotation of the borer and not from the metal itself.

As a result of this experiment and experiment performed by Joule, the scientists came to interpret heat not as the flow of substance(caloric) but as transfer of energy when heat flows from a hot body to a cold one. It is energy that is being transferred from the hot to the cold object. Thus heat refers to energy that is transferred from one body to another because of difference in temperature. Heat is not the energy that a body contains in it. It refers to the amount of energy transferred from hot to a cold body. Once the heat energy is transferred to a body, it is converted in to the internal energy of the body. The internal energy is the sum of all the microscopic kinetic and potential energies of the molecules in the body. The S.I unit of heat, as for any form of energy is joule.

11.2. Temperature

If we take two bodies, we can say by the sense of touch whether one is hotter than the other. It is not possible to determine,how much hotter it is at a particular time than the other. We find that it is not possible to determine the degree of hotness by sense of touch. The quantitative determination of the degree of hotness may be termed as temperature. Before describing the measurement of temperature it is appropriate here to define thermal equilibrium, when two bodies at different temperatures are brought in thermal contact with each other. The heat starts flowing from the hot body to the cold body till the temperature of the bodies becomes same, then they are said to be in thermal equilibrium.

11.3. Scales of temperature

It is not possible to determine the temperature of a body accurately by simply sense of touch or by comparing the degree of hotness. So a temperature scale is needed to measure the temperature quantitatively. For this we must have two reference points that are fixed and easily reproduceable. That is, the value of the fixed points must always be the same under similar conditions. The scale depends upon these fixed points.

The melting point of ice and boiling point of water at standard pressure (76 cm of Hg) are taken to be the two fixed points and difference between these two points is divided in different ways called scales of graduation (Celsius and Fahrenheit, etc.).

On the Celsius (Centigrade) scale this interval between these fixed point is divided into hundred equal parts. The lower fixed point is marked 0 and upper fixed point 100. Each part thus represents one degree Celsius (1°c). This scale was suggested by Celsius in 1742.

On the Fahrenheit scale the lower fixed point is marked 32 and the upper fixed point 212; and the interval between them is equally divided into 180 equal parts. Each part represents one degree Fahrenheit (1°F).

There is an other scale called the Kelvin scale.

The lowest temperature on this scale is–273°c. Thus the zero on the Celsius scale will be 273 on the Kelvin scale written as 273K and 100 on Celsius scale will be 373K. The size of the degree on the Kelvin scale is the same as that of Celsius scale.

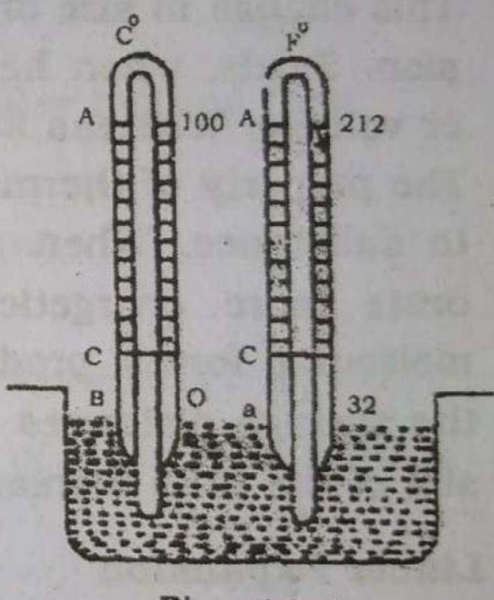


Fig: (11.1)

In order to derive relationship between centigrade and Fahrenheit scales let the two thermometers be placed in a bath and the mercury in each thermometer rises to the same level Fig.11.1 .

We arrive at the relation:

$$\frac{CB}{AB} = \frac{F-32}{212-32} = \frac{C-0}{100}$$

$$= \frac{F-32}{180} = \frac{C}{100}$$

$$= \frac{F-32}{9} = \frac{C}{5} \quad \text{-----} (11.1)$$

11.4. Thermometric Properties

Property of a substance which changes uniformly with the change of temperature is named thermometric property.For example the volume of a liquid in a vessel,the volume of a fixed mass of gas kept at constant pressure,the pressure of a fixed mass of gas maintained at constant volume, electrical resistance of a metal are some of the many measurable physical properties which changes with the change of temperature.

11.5. Thermal Expansion

It is a matter of daily observation that in general bodies undergo change in size when they are heated. This change in size on heating is termed Thermal Expansion. Solids, when heated show increase in length, area or volume. Whereas liquids and gases expand in volume. The property of thermal expansion varies from substance to substance. When a solid is heated,its molecules vibrate more energetically against the action of inter molecular forces producing greater displacement. Since the average distances among the molecules increase, the size of the solid increases.

Linear Expansion

Expansion in length of solids on heating is called linear expansion. The observed expansion in length depends upon the original length and the change in temperature.

It has been found experimentally that the change in length is directly proportional to the original length and the change in temperature of the solid.

Suppose L is the length of a uniform thin metallic rod at some initial temperature. When it is heated through a small temperature ΔT,an increase of length ΔL

takes place.So we can put it as,

$$\Delta L \propto L\,\Delta T$$

therefore $$\Delta L = \alpha\, L\, \Delta T \quad \text{---------- (11.2)}$$

where α is a constant of proportionality, which depends upon the material of rod and called the co-efficient of linear expansion.

Equation 11.2 can be rewritten as:

$$\alpha = \frac{\Delta L}{L\Delta T} \quad \text{------------------- (11.3)}$$

From Eq.11.3, the co-efficient of linear expansion α is defined as the change in length per unit length per Kelvin rise in temperature. Its unit is K^{-1}

If L' is the length of the rod after heating then,

$$\Delta L = L' - L$$

hence equation 11.2 can be written as

$$L' - L = \alpha\, L\Delta T$$

or $$L' = L\,(1+\alpha\,\Delta T) \quad \text{---------------- (11.4)}$$

A list of the average values of common solids is given below in table 11.1.

Table 11.1

S. No.	Substance		(K^{-1})
1	Aluminium	24×10^{-6}	2.4×10^{-5}
2	Brass and Bronze	19×10^{-6}	1.9×10^{-5}
3	Copper	17×10^{-6}	1.7×10^{-5}
4	Glass(ordinary)	9×10^{-6}	0.9×10^{-5}
5	Glass(Pyrex)	$3,2\times10^{-6}$	0.32×10^{-5}
6	Hard Rubber	80×10^{-6}	8.0×10^{-5}
7	Ice	51×10^{-6}	5.1×10^{-5}
8	Invar (Ni-Cr.alloy)	0.9×10^{-6}	0.09×10^{-5}
9	Lead	29×10^{-6}	2.9×10^{-5}
10	Steel	11×10^{-6}	1.1×10^{-5}
11	Concrete	12×10^{-6}	1.2×10^{-5}

Example 11.1.

A steel rod has a length of 10m at a temperature of 25°C. What will be the increase in length if the temperature is raised to 35°C.? Given $\alpha = 1.1 \times 10^{-5} K^{-1}$

Solution

$$L = 10\ m$$

$$\Delta T = 35°C - 25°C = 10°C = 10K$$

from eq:11.2

Since $\Delta L = L\ \alpha\ \Delta T$.

$$= (10m)(1.1 \times 10^{-5} K^{-1} \times 10K)$$

$$= 1.1 \times 10^{-3} m$$

11.6. Volume Expansion

We discussed above expansion in one dimension only but as a matter of fact the solids expand on heating in all the three dimensions i.e. length, breadth and thickness.

Consider a metallic body of volume V at some initial temperature. Its volume changes by ΔV when temperature changes by ΔT. It is found experimentally that

$$\Delta V \propto V \Delta T$$

or $$\Delta V = \beta\ V \Delta T \text{ ----------- } (11.5)$$

where β is a constant of proportionality known as coefficient of volume expansion, its unit is also K^{-1}.

Let there be an object in the shape of a rectangular box of dimensions ℓ, w and h. Its volume V at some initial temperature T is given by

$$V = \ell h.\ w$$

If the temperature changes to $T + \Delta T$, its volume changes to $V + \Delta V$ where each linear dimension changes following the Eq. 11.5. that is $\beta = 3\alpha$

$$V + \Delta V$$
$$= [\ell(1 + \alpha \Delta T)] [h(1+ \alpha \Delta T)] [w(1 + \alpha \Delta T)]$$
$$V + \Delta V = \ell h w (1 + \alpha \Delta T)^3$$
$$= V (1+ \alpha \Delta T)^3$$
$$= V [1+ 3 \alpha \Delta T + 3\alpha^2(\Delta T)^2 + \alpha^3(\Delta T)^3] \text{--------} (11.6)$$

The terms containing $(\Delta T)^2$ and $(\Delta T)^3$ are negligibly small. Therefore the Eq. (11.6) reduces to

$$V + \Delta V = V + (3\alpha) V \Delta T$$

or $$\Delta V = (3\alpha) V \Delta T \text{------------------------} (11.7)$$

Comparing Eq.11.5 and Eq.11.7 we see that

$$\beta = 3\alpha \text{--------------------} (11.8)$$

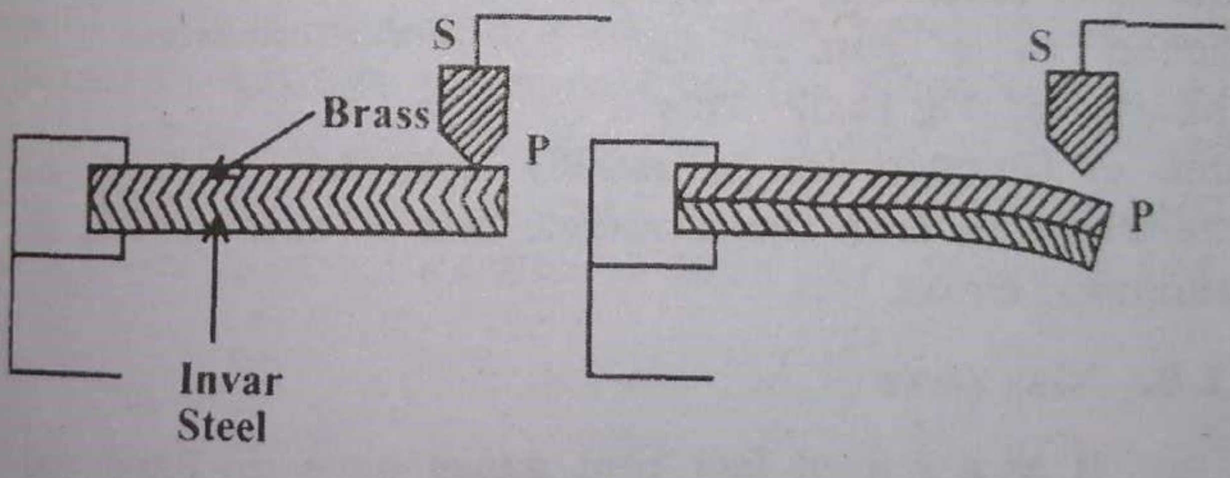


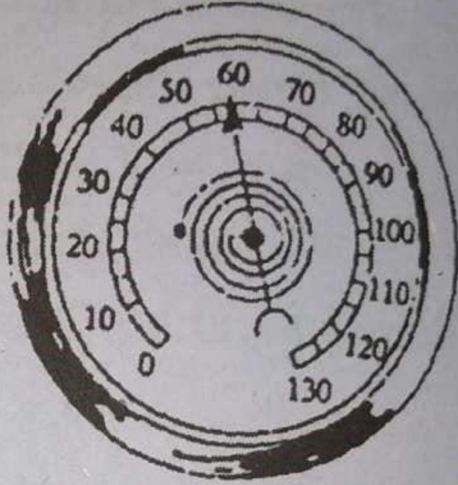
Fig: (11.2)

11.7. Bimetallic Thermostat

One of the examples of thermal expansion is of thermostat devices. Thermostats are well known devices commonly used for maintaining required temperatures. We describe below a common type of thermostat called bimetallic thermostat. The bimetallic strip works as an electric contact breaker in an electrical heating circuit. The circuit is broken when the desired temperature of the bath is reached. Due to the difference in the coefficients of linear expansion of the two metals, the metallic strip bends in the form of a curve and the circuit is broken. In Fig. 11.2 (a) the metallic strip is in contact with the screw S and in the Fig.11.2 (b) the strip curves downwards as it becomes hot and contact at P is broken. Thus the current stops flowing through the heating coil. When the temperature falls, the strip contracts and the contact at P is restored. The two metal strips are very well joined.

Bimetallic Thermometer:

Similarly a bimetallic strip can be used to make a thermometer. In this case the bimetallic strip often is in the form of a coil. Its one end is fixed and the other end is attached to a pointer as shown in Fig.11.3. This kind of thermometer is usually used for ordinary air thermometer, oven thermometer, and in automobiles for automatic choke.



Bimetallic Thermometer
Fig:(11.3)

11.8. Gas Laws

It is a known fact that gases have no fixed volume or shape and their volume can be altered by changing the pressure as well as the temperature. The behaviour of a gas can be described with the help of four variables i.e. pressure, volume, mass and temperature. The relation between any two variables is found experimentally while keeping the other constant.

(1) Boyle's Law

Let us consider the relation between the pressure and volume of a given mass of gas at constant mass and temperature. It was found experimentally by Robert

Boyle in 1660 that for a fixed mass of a gas at constant temperature, the product of pressure (P) Volume (V) is constant. This is known as Boyle's Law. In symbols we can write as:

P V = a constant (temp: remaining constant) 11.9.

(2) Charles's Law

When a given mass of a gas is heated at constant pressure experiments show that the volume V of a given mass of a gas is directly proportional to its absolute temperature T. In symbols, we can put as:

V/T =a constant (pressure remaining constant)----(11.10)

This law was investigated by Charles and is known as Charles's Law

On plotting a graph of volume of the gas against its temperature, a straight line is obtained. It shows that equal changes in temperature lead to equal changes in volume at constant pressure. From the graph it is found that at 0°C the gas still possesses a volume Vo. When the straight line of the graph is, extrapolated to lower and lower temperature axis as shown by the dotted line,

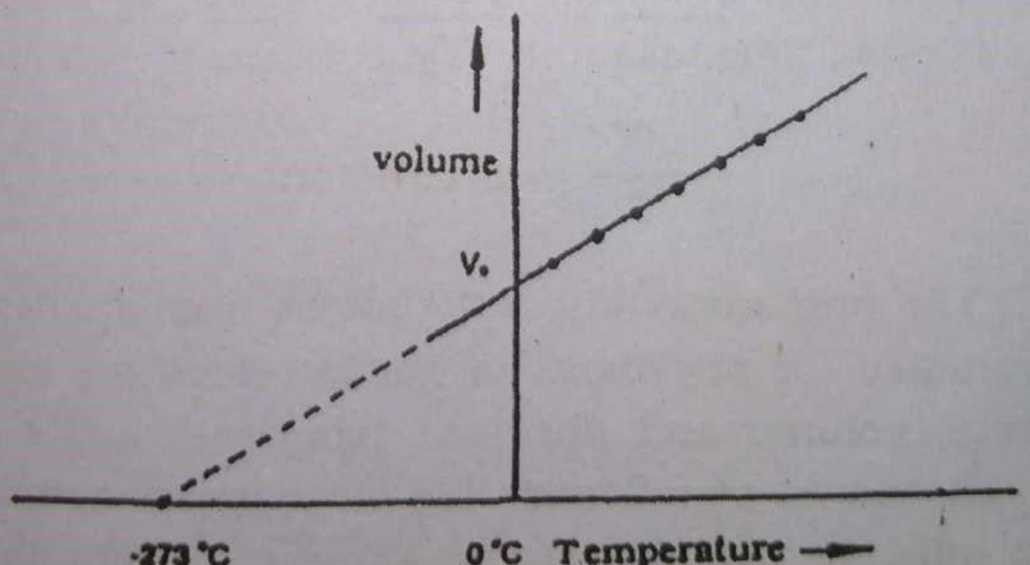


Fig:(11.4)

it intersects the temperature axis at a temperature of -273°C. This implies that if a gas could be cooled to -273°C, it would have no volume. - 273°C is called the absolute zero of the temperature.

11.9. General Gas Law

In order to derive general gas law we make use of both these gas laws. Let P_1, V_1 and T_2 be the pressure volume and absolute temperature of a given sample of a gas of mass m.

According to Boyle's Law, the change in pressure from P_2 to P_1 is accompained by a change in volume from V_1 to V. This can be expressed mathematically

$$P_1 V_1 = P_2 V$$

$$V = \frac{P_1 V_1}{P_2} \quad \text{------------------ (11.11)}$$

Now by changing the temperature from T_1 to T_2 at constant pressure P_2 the volume of the gas changes from V to V_2, then according to Charles's Law we have

$$\frac{V}{T_1} = \frac{V_2}{T_2} \quad \text{------------ (11.12)}$$

Using Equation 11.11 in Equation 11.12, we get

$$\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2} \quad \text{------------ (11.13)}$$

or

$$\frac{PV}{T} = \text{a constant} \quad \text{--------- (11.14)}$$

The proportionality constant in this equation can be evaluated for any mass of gas provided we know its pressure, volume and absolute temperature. Its value, however, has no significance for other given samples because different samples of the same gas differ in mass and hence in number of molecules. It is convenient to express the amount of gas in a given volume in terms of the number of moles, n.

By definition, a mole of any substance is that mass of the substance that contains a specific number of molecules called Avogadro's number, N_A. The value of N_A is approximately 6.022×10^{23} molecules/mole. Avogadro's number is defined to be the number of carbon atoms in 12g of the isotope carbon12. The number of moles of a substance is related to its mass m, through the expression

$$n = m/M$$

where M is a quantity called the molecular weight of the substance, usually expressed in g/mole.

In such cases Eq.11.14 may be written as

$$PV = nRT \qquad \text{----------------11.15.}$$

Where R is called the universal gas constant and does not depends on the quantity of gas in the sample. If P is measured in $N\,m^{-2}$, V in m^3 and T in Kelvin then the value of Universal gas constant is

$$R = 8.314\ J.\ mol^{-1}.\ K^{-1}.$$

Example 11.2.

An air-storage tank whose volume is 112 liters contains 3kg of air at a pressure of 18 atmospheres. How much air would have to be forced into the tank to increase the pressure to 21 atmospheres assuming no change in temperature.

Solution

Here $P_1 = 18$ atm $\quad P_2 = 21$ atm

$V_1 = 112$ liters $V_2 = V_1$

$m_1 = 3$ kg $\qquad m_2 = ?$

$$\frac{P_1 V_1}{m_1} = \frac{P_2 V_2}{m_2}$$

$$\frac{18 \times 112}{3} = \frac{21 \times 112}{m_2}$$

or $\quad m_2 = 3.5$ kg.

The mass of air which must be forced into the tank is (3.50-3.00) = 0.5 kg.

Example 11.3.

Calculate the volume occupied by a gram-mole of a gas at 0°C and a pressure of 1 atmosphere.

Solution

$$P = 1 \text{ atm} = 1.01 \times 10^5 \text{ Nm}^{-2}$$

$$T = 0^\circ C + 273 = 273 \text{ K}$$

$$n = 1$$

$$R = 8.314 \text{ J/mole.K.}$$

$$PV = n R T$$

$$1.01 \times 10^5 V = 1 \times 8.31 \times 273$$

$$V = \frac{1 \times 8.314 \times 273}{1.01 \times 10^5}$$

$$= 0.0224 \text{ m}^3\text{/mole}$$

$$= 22.4 \text{ liters/mole} \quad \text{as } 1\text{m}^3 = 1 \times 10^3 \text{ liters.}$$

11.10. The Properties of Gases; Kinetic Interpretation

The properties of matter in bulk can however be predicted on molecular basis by a theory known as kinetic theory. The first step in the construction of a theory is to set up some sort of model which is simple enough to be treated mathematically. The characteristics of the model are described by a set of fundamental assumptions which for the kinetic theory of gases are:

1. A gas consists of particles called molecules. Depending on the gas each molecule will consist of an atom or a group of atoms. All the molecules of a gas in a stable state are considered identical.

2. Any finite volume of a gas consists of very large number of these molecules. This assumption is

justified by experiments. At standard conditions there are 3×10^{26} molecules in a cubic metre.

3. The molecules are separated by distance large as compared to their own dimensions. The diameter of a molecule, considered as a sphere, is about 3×10^{-10} m.

4. The molecules move in all directions and with various speeds making elastic collisions with one another and with the walls of the container. The walls of a container can be considered perfectly smooth.

5. Molecules exert no forces on one another except during collisions. Therefore in between collisions with other molecules or with the walls of the container, and in the absence of the external forces, they move freely in straight lines.

6. Newtonian mechanics is applicable to the motion of molecules.

11.11. Interpretation of Pressure on Kinetic theory of Gases

In order to calculate the pressure of an ideal gas from kinetic theory. Let us consider (i) N number of molecules contained in a cubical vessel whose walls are perfectly elastic (ii) the faces of the cube which are normal to the x-axis and having edge length L. Figure 11.5 (a). (iii) A molecule which has a velocity v can be resolved into components v_x, v_y, v_z in the direction of the edges.

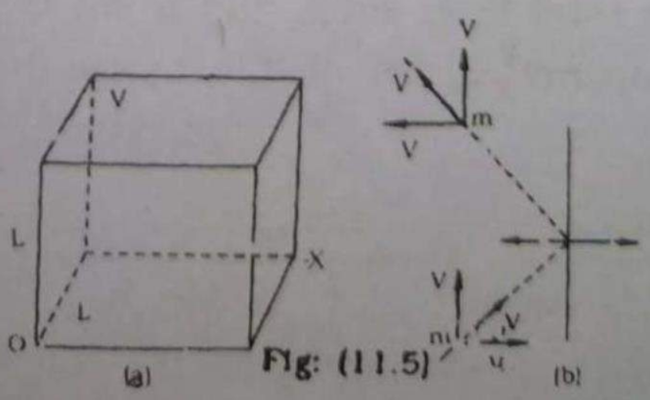


Fig: (11.5)

If we have a molecule which collides with one face A of the cube, it will rebound such that x-component of the velocity v_x is reversed. the v_y and v_z remain unaffected Fig (11.5 (b).) Therefore the momentum before collision is mv_x and after collision $-mv_x$ causing a change of momentúm (momentum after collision -- momentum before collision).

$$mv_x - (-mv_x) = 2mv_x \qquad \text{---------------} (11.16)$$

normal to the face A. Hence the momentum given to the face A will be $2mv_x$, since the total momentum is conserved. The time required by the molecule to reach from one face to the other without collision will be $1/v_x$. The round trip will be covered in time $t = 2\ell/v_x$. The number of collisions per second against the face A would then be

$$\frac{1}{t} = \frac{1}{2\,\ell/v_x} = v_x/2\,\ell$$

Therefore the rate at which the molecule transfers its momentum to face A is found by multiplying the number of collision the molecule suffers at the face A, and the momentum it transferred to the face of each collision.

$$2mv_x \frac{v_x}{2\ell} = \frac{mv^2_x}{\ell} \qquad \text{------------} (11.17)$$

Relation (11.17) is rate of change of momentum of the molecule on striking the face A of the cube i.e. the molecule is exerting a force mv^2_x / ℓ on striking the face A. To obtain the total force on A due to all the gas molecules, we sum up mv^2_x /ℓ for all the molecules and so it will be

$$\frac{mv^2_{1x}}{\ell} + \frac{mv^2_{2x}}{\ell} + \frac{mv^2_{3x}}{\ell} + \text{------} + \frac{mv^2_{nx}}{\ell}$$

where v_{1x} is the x-component of the velocity of 1st particle v_{2x} is that of 2nd particle .... and v_{nx} is that of nth particle. As m is the mass of one molecule then to find the pressure p, divide this force by the area of the face A, namely ℓ^2 ,so

$$P = (m/\ell^3)\,(v_{1x}^2 + v_{2x}^2 + \ldots\ldots + v_{nx}^2) \text{ ----- } (11.18)$$

The number of molecules in unit volume n_v is $\frac{N}{\ell^3}$ where N is the total number of molecules, therefore

$\ell^3 = N/n_v$ and substituting this value in equation (11.18), we have

$$P = mn_v\,(v_{1x}^2 + v_{2x}^2 + \ldots\ldots)/N \text{ ------------ } (11.19)$$

where mn_v is the mass per unit volume which we call density ρ $(v_{1x}^2 + v_{2x}^2 + v_{3x}^2 + \cdots . v_{nx}^2)/N$ is the average value of $\overline{v_x^2}$ for all the particles in the container we call this average square velocity $(\overline{v^2}_x)$. The Square root of $\overline{v_x^2}$ is referred as V_{rms}. Eq. 11.19 can be written as

$$P = \rho\overline{v^2}_x \text{ ---------------- } (11.20)$$

The term $\overline{v_x^2}$ is only one component of the total velocity, since $\overline{v^2} = \overline{v_x^2} + \overline{v_y^2} + \overline{v_z^2}$. On the average. $\overline{v_x^2} = \overline{v_y^2} = \overline{v_z^2}$, due to the randomness of the molecular motions, $\overline{v^2} = 3\,\overline{v_x^2}$ and $\overline{v_x^2} = \frac{1}{3}\overline{v^2}$. Substituting this value into the equation above, we find that

$$P = \frac{1}{3}\rho\,\overline{v^2} \text{ ---------------- } (11.21)$$

Example 11.4.

Calculate the V_{rms} of hydrogen molecules at $0.00^\circ C$ and 1.00 atm pressure, assuming hydrogen to be an ideal gas. Under these conditions hydrogen has a density ρ of 8.99×10^{-2} kg/m^3.

Solution

Since $p = 1.00$ atm $= 1.01\times10^{5}$ Nm^{-2}

so using $v_{rms} = \sqrt{3p/\rho}$. and substituting the values of ρ and P we have

$$v_{rms} = \sqrt{\frac{3 \times 1.01 \times 10^5 \text{ Nm}^{-2}}{8.99 \times 10^{-2} \text{ kg m}^3}}$$

$$= 18\,35.86 \text{ ms}^{-1}$$

This is of the order of nearly two km per second.

11.12. Derivation of the Gas Laws

The gas laws can be derived using Eq 11.21

$$P = \frac{1}{3} \rho \overline{v^2}$$

$$= \frac{1}{3} n_v \, m \, \overline{v^2} \text{ (since } \rho = mn_v) \text{ --------(11.22)}$$

Since n_v represents the number of molecules per unit volume N/V. we can write the Eq. (11.22) as

$$p = \frac{1}{3} \frac{N}{V} m \overline{v^2}$$

$$PV = \frac{1}{3} Nm \overline{v^2} \qquad \text{---------(11.23)}$$

Comparing it with Eq (11.15) we find that

$$\frac{1}{3} Nm \overline{v^2} = n R T. \qquad \text{---------(11.24)}$$

Substituting $n = N/N_A$ and multiplying both sides by 3/2. we obtain the relation

$$1/2 \, m \, \overline{v^2} = (3/2)\,(R/N_A)T$$

Since $R/N_A = k$ (Boltzmann constant) $= 1.38 \times 10^{-23}$ J/ (molecule – K)

Hence $$\frac{1}{2} m \overline{v^2} = (3/2) k T. \quad \text{--------(11.25)}$$

This equation indicates that the average translational kinetic energy per molecule is directly proportional to the absolute temperature and it could serve to define temperature in terms of mechanical quantities.

Thus Eq.11.23 shows that if $\overline{v^2}$ remains constant, the temperature remains constant which is the condition for Boyle's Law.

From Eq 11.23 and 11.25. we have

$$V = \frac{N}{P} kT \quad \text{--------------- (11.26)}$$

If ρ is constant, the factor $\frac{Nk}{P}$ is a constant.
thus $V \propto T$. which is Charles's Law.

11.13. Specific Heat Capacity

It is found experimentally that one kilogram of copper requires considerably less amount of heat to raise its temperature by $1\overset{\circ}{C}$ than one kilogram of water. Likewise one kilogram of paraffin requires about half as much heat to raise its temperature by $1\overset{\circ}{C}$ as does one kilogram of water.These examples demonstrate that equal amounts of different substances absorb different amount of heat when heated to the same temperature.

The examples discussed above introduce the idea of specific heat of a substance which is defined as the amount of heat required to raise the temperature of unit mass of the substance to unit degree rise of temperature. If a body of mass is heated so that its temperature rises from T to $T + \Delta T$ and it absorbs a small amount of heat ΔQ. then its heat capacity C is defined as

$$C = \frac{\Delta Q}{\Delta T} \quad \text{--------------- (11.27)}$$

whereas specific heat capacity $c = \frac{C}{mass} = \frac{\Delta Q}{m\Delta T}$ at temperature T.

The value of specific heat of water is 4200 J $kg^{-1}(K)^{-1}$. Specific Heat is a characteristic of the material of which the substance is made of. The values of specific heat of different materials are given in the table 11.2.

TABLE 11.2.

Specific Heat of some Substances

Sr. No:	Substance	Specific Heat $J\ kg^{-1}\ K^{-1}$
1.	Aluminium	903.00
2.	Carbon	508.200
3.	Copper	386.00
4.	Iron	499.800
5.	Lead	128.100
6.	Mercury	138.600
7.	Silver	230.00
8.	Tungsten	134.800
9.	Water	4180.00
10.	Ice	2100.00

11.14. Determination of the Specific Heat Capacity

The method described below is known as the method of mixtures and is regarded as the most fundamental method of determining the specific heat capacity of solids and liquids. This method is based on the principle; that heat lost by hot bodies is equal to that gained by the cold bodies:

Heat lost = Heat gained.

This is called the law of heat exchange.

The method of mixtures utilises the fact that when a hot substance is mixed with the cold one. the hotter one loses heat and the cooler one absorbs heat until both finally attain the same temperature.

The specific heat of the substance of known mass and temperature is determined by mixing it with liquid of known mass and temperature in a vessel called calorimeter of known mass and temperature. The final temperature of the mixture is measured. The specific heat capacity of the substance can be calculated from the given data. The following example will further illustrate the method of mixtures for calculating the specific heat capacity of a substance.

Example 11.5.

A 50 gram piece of metal is heated to 100°C and then dropped into a copper calorimeter of mass 400 gram containing 400 gram of water initially at 20°C. If the final equilibrium temperature of the system is 22.4°C, find the specific heat of the metal. Specific heat of copper is 386 J kg^{-1} K^{-1}.

Data

m_M = mass of Metal = m_M = 50g = 0.05kg

c = Specific heat of the metal c = ?

T_M =Temperature of the metal piece = T_M = 100°C

T_F = Final temperature = T_F = 22.4°C

m_W = Mass of water = m_W = 400g = 0.4 kg

m_c = Mass of Calorimeter = m_c = 400g = 0.4 kg

C_w = Specific Heat of Water = C_w = 4200 Jkg^{-1} K^{-1}

T_W = Initial temp: of water = T_W = 20°C

c = Specific heat of Copper = c = 386 J kg^{-1} K^{-1}

Solutions From the law of heat exchange.

Heat lost by metal = Heat gained by water + Heat gained by Calorimeter.

$$m_M \times c \times (T_M - T_F) = m_W \times C_W(T_F - T_W) + m_C\, c(T_F - T_W)$$

$$.05kg \times c \times (100 - 22.4)K = o.4kg \times 4200 J/kg^{-1} K^{-1} \times (22.4 - 20)$$

$$+ 0.4\ Kg \times 386\ J/kg^{-1}\ K^{-1}\ (22.4 - 20)$$

$$c = 1134.68\ J/kg^{-1}\ k^{-1}$$

11.15. Molar Specific Heat

As a matter of convenience mole is often used to describe the amount of substance. One mole of any substance is defined as the quantity of matter such that its mass in gram is numerically equal to the molecular weight M. If n is the number of moles and m is the mass in gram of the substance, then

$$n = m/M$$

or $$m = nM \qquad \text{------------------(11.28)}$$

For specific heat we have the relation

$$c = \frac{\Delta Q}{m\Delta T} \qquad \text{------------------(11.29)}$$

Substituting the mass m in 11.29 by nM from 11.28, we get

$$c = \Delta Q/nM\,\Delta T$$

$$M_c = \Delta Q/n\,\Delta T \qquad \text{------------------(11.30)}$$

The product M_c of molecular weight and specific heat is called the molar specific heat and is defined as the quantity of heat required to raise the temperature of one mole of a substance through 1 K. Its units are J-mole^{-1}K^{-1}.

The molar specific heat of a gas depends upon whether or not the gas allowed to expand when it is heated. When the volume of the gas is kept constant throughout heating we call it the molar specific heat at constant volume. It is defined as the amount of heat energy required to raise the temperature of one mole of a gas through 1 K at constant volume. It is designated by the symbol C_V. When the volume of the gas is allowed to increase but its pressure is kept constant throughout heating, we speak of the molar specific heat at constant pressure. It is defined as the amount of heat energy required to raise the temperature of one mole of a gas through 1 K at constant pressure and it is denoted by the symbol C_p.

11.16. Heat and Work in Thermodynamics

Laws governing the conversion of energy to and from heat and the methods employed for such transformations are the subject matter of thermodynamics. Before going in the details of these laws, let us briefly describe the treatment of heat and work in thermodynamics.

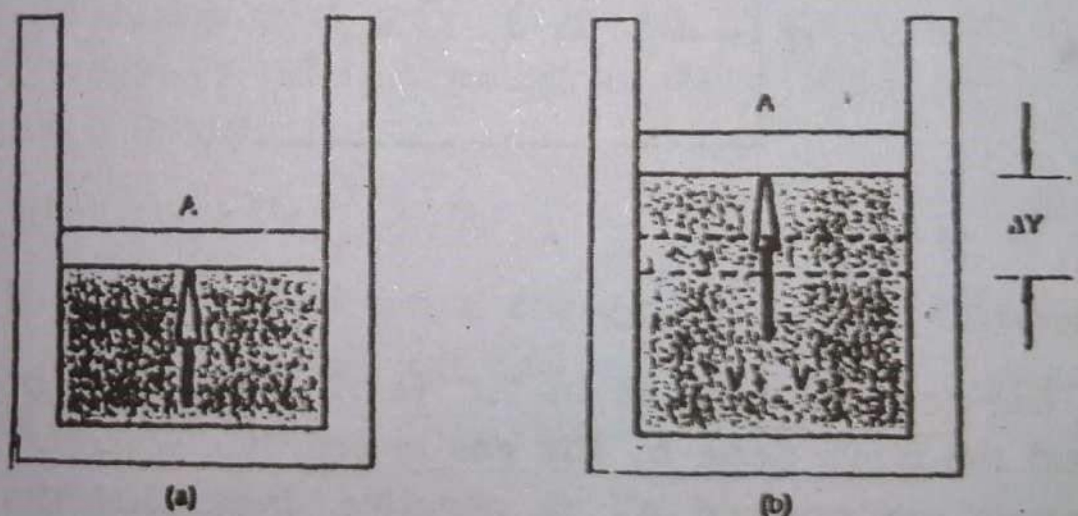


Figure 11.6 Gas contained in a cylinder at a pressure p does work on a moving piston as the system expands from a volume v = Δ v

Fig:11.6

Consider a thermodynamic system such as a gas contained in a cylinder fitted with a movable piston (Fig.11.6). In equilibrium, the gas occupies a volume V and exerts a uniform pressure P on the cylinder walls and piston. If the piston has a cross section area A, the force exerted by the gas on the piston is F= PA. Now let us assume that the gas expands slowly, till it attains the equilibrium state.

As the piston moves up a distance Δy, as shown in fig.11.6(b) the work done by the gas on the system is

$$\Delta W = F\Delta Y = PA\,\Delta Y$$

Since $A\Delta Y$ is the increase in volume of the gas, ΔV, we can express the work done as

$$\Delta W = P\Delta V \qquad \text{------------(11.31.)}$$

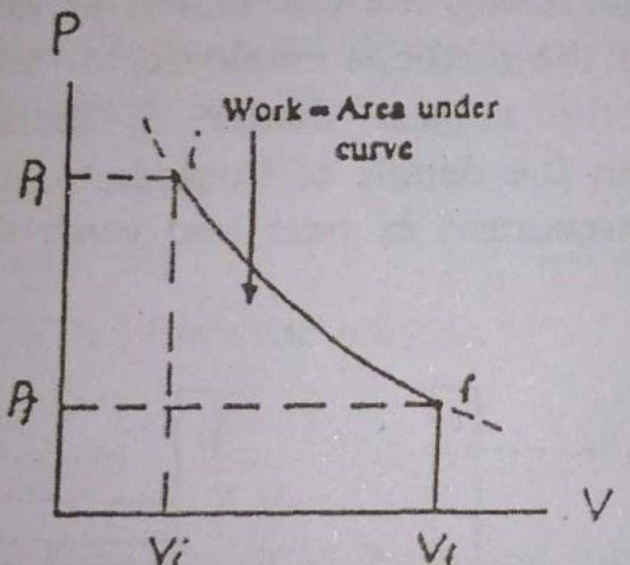


Fig:(11.7.)

If the gas expands, as in Fig. (11.7) then ΔV is positive and the work done by the gas is positive, whereas if the gas is compressed ΔV is negative, indicating that the work done by the gas is negative. (In the latter case, negative work can be interpreted as work being done on the system). Clearly, the work done by the system is zero when the volume remains constant. The total work done by the gas as its volume changes from V_i to V_f can be expressed by the area under the curve in a diagram (Fig.11.7). from Fig. 11.7 the work done in the expansion from the initial state "i", to the final state"f", will depend on the specific path taken between these two states.

11.17. The First Law of Thermodynamics

Let there be a system which absorbs ΔQ amount of heat and as a consequence of this it performs ΔW amount of work. In this process the initial equilibrium state "i" of the system changes to a final equilibrium state "f" in a particular way and $\Delta Q - \Delta W$ is computed. Now this system is changed from the same initial state "i" to the final state "f" but along a different path. This procedure is repeated many times. It is observed that $\Delta Q - \Delta W$ comes out the same in all cases inspite of the fact that ΔQ and ΔW separately depend on the path taken. $\Delta Q - \Delta W$ depends only on the initial and final states.

ΔQ is the energy added to the system and W is equal to the energy that has been extracted, from the system by the performance of work. The difference $\Delta Q - \Delta W$ which is retained within the system is the change in the energy of the system. It follows that the initial energy change of a system is independent of the path and is therefore equal to U_f, internal energy of the system in state "f" minus the internal energy in state "i" or $U_f - U_i$ Therefore, it follows that

$$\Delta Q - \Delta W = U_f - U_i = \Delta U \qquad \text{-----------------(11.32)}$$

The change in internal energy of a system in any process is equals to net heat flow into the system minus the total work W done by the system.

11.18. Special Cases

let us look at some special cases. First consider an isolated system, that is, which does no external work and there is no flow of heat. Then for any process taking place in such a system $\Delta W - \Delta Q = 0$ and $U_2 - U_1 = 0$ or $\Delta U = 0$ which says that the internal energy of a system remains constant. This is the most general statement of the conservation of energy. The internal energy of an isolated system cannot be changed by any process (mechanical, electrical, chemical, or biological) taking place within the system. The energy of a system can be changed only by a flow of heat across its boundary or by the performance of work.

Next consider a process in which the system is taken through a cyclic process, that is, one that starts and ends up at the same state. In this case the change in internal energy is zero.i.e.

$$U_2 = U_1 \text{ and } \Delta Q = \Delta W$$

Thus, although net work W may be done by the system in the process energy has not been created, since an equal amount of energy must have flown into the system as heat ΔQ.

Example 11.6.

A system absorbs 1000 Joules of heat and delivers 600 Joules of work while losing 100 Joules of heat by conduction to the atmosphere. Calculate the change in the internal energy of the system.

Solution

$$\Delta U = \Delta Q - \Delta W$$

$$\Delta Q = +\ 1000\ J - 100\ J = +\ 900\ J$$

$$\Delta W = 600 \text{ Joules}$$

therefore, $\Delta U = 900\ J - 600\ J = 300\ J$

Example 11.7.

A thermodynamic system undergoes a process in which its internal energy decreases by 300 J. If at the same time, 120 J of work is done on the system, find the heat transferred to or from the system.

Solution

$$\Delta U = \Delta Q - \Delta W$$
$$300\ J = \Delta Q - 120\ J$$
$$420\ J = \Delta Q$$

Since this amount of heat leaves the system

So $$\Delta Q = -420 \text{ J.}$$

11.19. Applications of First Law of Thermodynamics

Let us consider the following cases.

1. Isobaric Process

Isobaric process is that process which takes place at constant pressure. In such a process the heat transferred and the work done are both non-zero. When water enters the boiler of a steam engine and is heated to its boiling point, vaporized, and then the steam is superheated, all these processes take place isobarically. Such processes play an important role in mechanical engineering and also in chemistry.

In Fig.11.8. (a,b,c) we show an isobaric process. The system is a gas contained in a cylindrical vessel provided with frictionless air tight piston and is free to move. An infinitesimal heat ΔQ is transferred from the surroundings to the system. The gas expands such that the external pressure p remains unchanged as shown in Fig.11.8. (c). The work done by the gas in moving a piston of area A to a small distance Δx is given by:

$$\Delta W = F \times \Delta x$$

$$= (PA)\ \Delta x$$

$$= P \Delta V$$

Where ΔV is the increase in the volume of the gas from volume V_1 to volume V_2. So we can also write the above equation as:

$$\Delta W = P(V_2 - V_1) \quad \text{-----------------(11.33)}$$

Substituting this value of W in the relation $Q - W = \Delta U$ we get

$$\Delta Q = P(V_2 - V_1) + \Delta U \quad \text{-----------------(11.34)}$$

This is the form of 1st law of thermodynamics in an isobaric process.

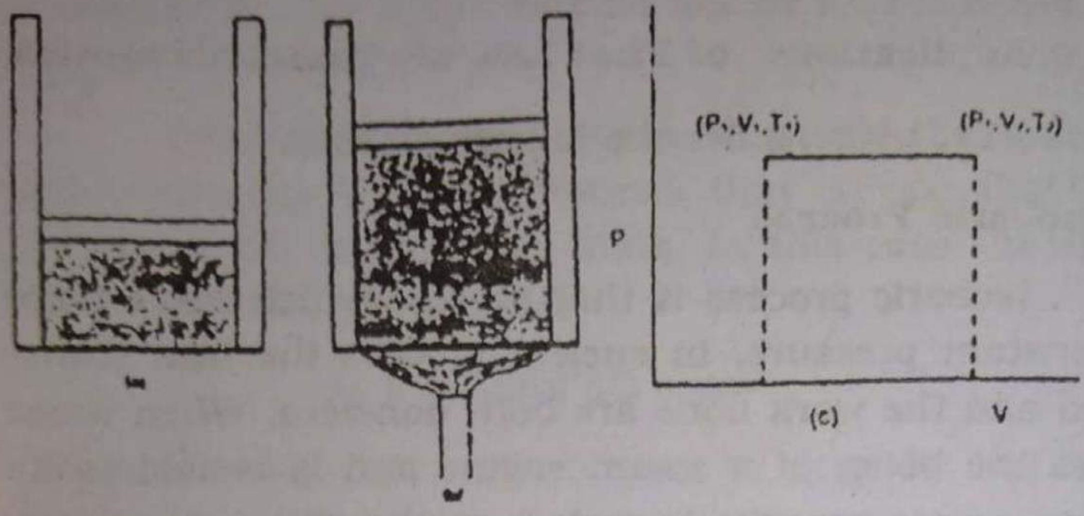


Fig. (11.8) a b c

2. Isochoric Process

Isochoric process is defined as that process in which the volume of the system remains constant.

Consider a cylinder fitted with a piston containing a certain amount of gas. It is heated by supplying an amount of heat ΔQ keeping its volume fixed. During this heating the pressure of the gas increases from p_1 to p_2 (Fig.11.9). Since there is no change in the volume of the gas, so the work done W is zero. From the first law of thermodynamics, putting $W = 0$, we get

$$U_2 - U_1 = \Delta Q \quad \text{(Isochoric process)}$$

$$\Delta U = \Delta Q \qquad \text{------------------(11.35)}$$

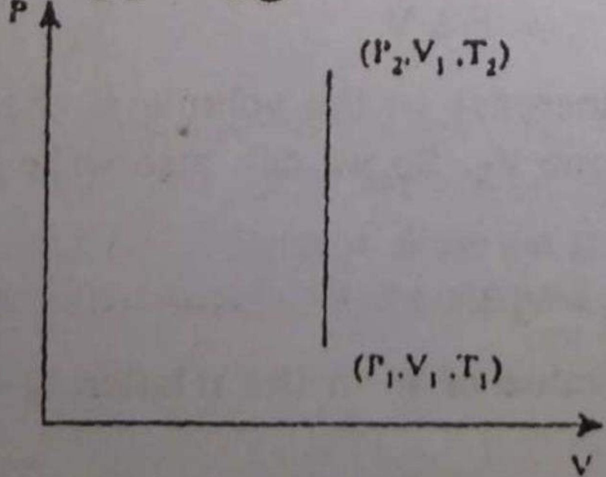


Fig.(11.9.)

3. Isothermal Process

If the temperature of the system remain constant throughout the process, it is called an isothermal process.

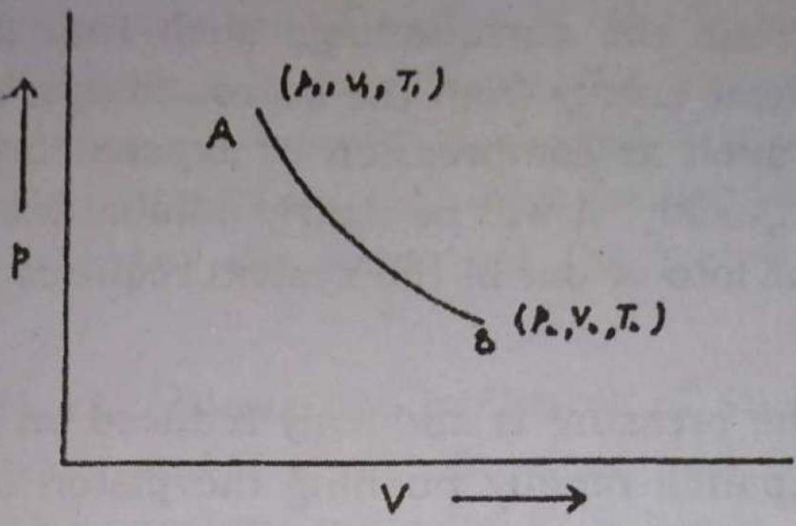


Fig. (11.10)

The pressure, temperature and volume of a gas in a cylinder is shown in (Fig.11.10) by point A. The cylinder has a heat conducting base and non conducting walls and piston and it is placed on a heat reservoir, a body of large heat capacity at a constant temperature T_1. If the pressure of the gas in the cylinder is decreased by decreasing the weights on the piston there should be a fall in temperature. But since the system is perfectly conducting to the reservoir it will absorb heat from the reservoir and maintain a constant temperature from A to B. In order to keep temperature of the gas constant, the changes in its pressure and volume must be carried out very slowly.

The internal energy of the gas does not change during this process i.e. $U_2 - U_1 = 0$. Therefore from the first law of Thermodynamics.

$$\Delta Q = \Delta W \text{ (Isothermal process)} \quad \text{------------(11.36)}$$

Eq.11.36 represents the 1st law of thermodynamics in an isothermal process and the curve in Fig 11.10. is called an isotherm.

4. Adiabatic Process

The process in which no heat flows into or out of the system is called an adiabatic process. During an adiabatic process, the working substance is perfectly insulated from the surroundings such that it does not exchange heat energy from the surroundings. However, if a process such as compression or expansion of a gas is done very quickly, it will be nearly adiabatic because the flow of heat into or out of the system requires finite time.

If the pressure is suddenly reduced on the piston, the gas expands rapidly pushing the piston up and the temperature falls. As no heat exchange can take place, the first law of thermodynamics reduces to

$$\Delta U = -\Delta W \quad \text{------------------(11.37)}$$

A curve between pressure and volume during the adiabatic process is called an adiabatic curve or an adiabatic as shown in Fig. 11.11.

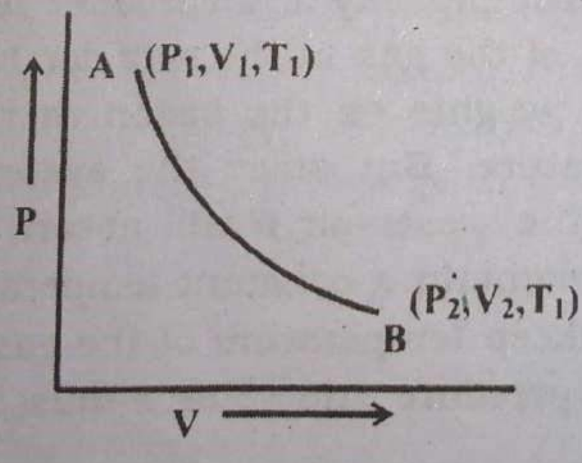


Fig. (11.11)

11.20. Derivation of $C_p - C_v = R$

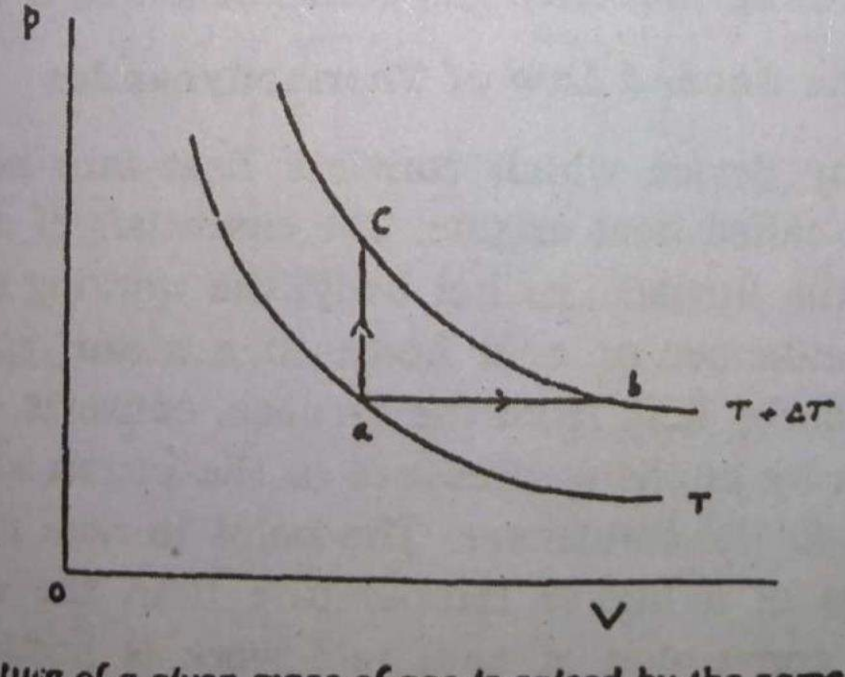
When we heat a substance, its temperature rises under various conditions, for example, the volume may be kept constant or the pressure may be kept constant or both may be allowed to vary in some definite manner. In each of these cases, the amount of heat required per mole per unit rise of temperature (we define this as the molar heat capacity) is different. Only two kinds of molar heat capacities are important for gases, namely, that at constant volume C_V, and that at constant pressure C_P.

Let us consider a piston cylinder arrangement having a certain number of moles of an ideal gas. The cylinder is placed on a heat reservoir, the temperature of which can be raised or lowered as needed. The gas has a pressure p such that its upward force on the frictionless piston just balances the weight of the piston and its load.

Fig. 11.12. shows two isotherms of an ideal gas at temperature T and $T + \Delta T$. Suppose that the gas is taken along the constant pressure path $a \longrightarrow b$ in figure (11.12). The temperature also increases by ΔT along this path. The heat transferred to the gas in this process is given by $\Delta Q = nC_P \Delta T$, where C_P is the molar heat capacity at constant pressure. We note that the volume increases in this process. We call this increase as ΔV. So the work done by the gas is $W = P\Delta V$. Applying the first law to this process, we have

$$\Delta U = \Delta Q - \Delta W.$$

$$= n\,C_P\,\Delta T - P\,\Delta V \quad \text{------------------}(11.38)$$



The temperature of a given mass of gas is raised by the same amount by a constant volume process (a→c).

Fig. (11.12)

Apply the equation of state PV = nRT to the constant pressure process a ⟶ b. For P constant we have, taking $P\Delta V = nR\Delta T$, Eq. (11.38) reduces to

$$\Delta U = n\,C_p\,\Delta T - nR\,\Delta T \qquad \text{------------------(11.39)}$$

Let us consider the constant volume process that carries the system along the path a ⟶ c (Fig. 11.12). By definition $Q = n\,C_v\,\Delta T$, where C_v is the molar heat capacity at constant volume.

Also since ($\Delta V = 0$, so $\Delta W = p\,\Delta V$) is also zero. Therefore from the first law of thermodynamics,

$Q - W = \Delta U$. We have

$$\Delta U = n\,C_v\,\Delta T$$

substituting this into equation (11.39), we get

$$n\,C_p\,\Delta T = nC_v\,\Delta T + nR\Delta T$$

Which gives

$$C_p - C_v = R \qquad \text{-----------------(11.40)}$$

This expression applies to any ideal gas. It shows that the heat capacity of an ideal gas at constant pressure is greater than the heat capacity at constant volume by an amount R, the universal gas constant (8.313 J $mol^{-1}.K^{-1}$)

11.21. The Second Law of Thermodynamics

Any device which converts heat into mechanical energy is called heat engine. The essentials of a heat engine are the furnace, or hot body, the working substance and a condenser or cold body. In a steam engine, the steam absorbs heat from the furnace, converts some of it into work by applying pressure to the piston and rejects the rests to the condenser. The point to note is that the furnace is at a higher temperature than the condenser and this conversion of heat into work is possible when the working substance falls in temperature as shown in Fig 11.13(a). We can generalise it by sayin that the two bodies must be maintained at different temperatures for the working of a heat engine. A continuous supply of work has never yet been obtained from a single supply of heat otherwise we could build a ship which would use far more heat in the ocean water without needing any fuel. This leads to the first way of stating the second law of thermodynamics due to Kelvin.

"It is impossible to derive a continuous supply of work by cooling a body to a temperature lower than that of the coldest of its surrounding".

Let us consider the fact that no one has ever built a refrigerator which will work without a supply of energy. A refrigerator is essentially a machine for conveying heat from one body at a lower temperature to another at a higher temperature as shown in Fig 11.13 (b). In other words, it is only possible to make heat flow from a cold body to a hot body by using up work. The statement of second law of thermodynamics due to Clausius comes from the consideration of this fact of refrigerator and it is stated.

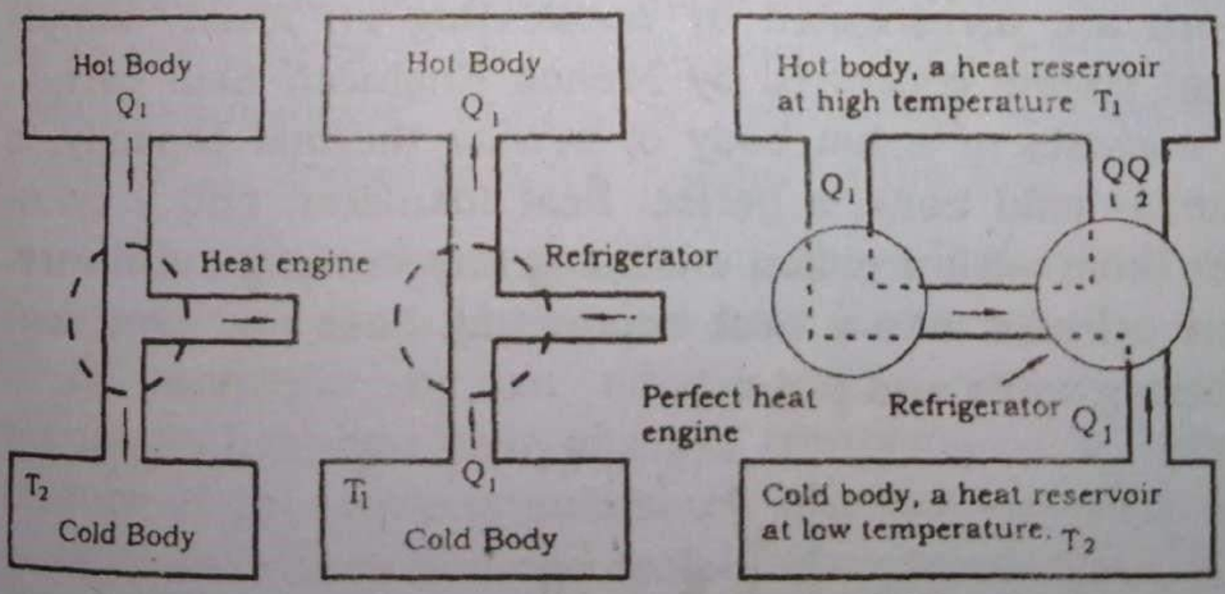


Fig: (11.13) a.b.c.

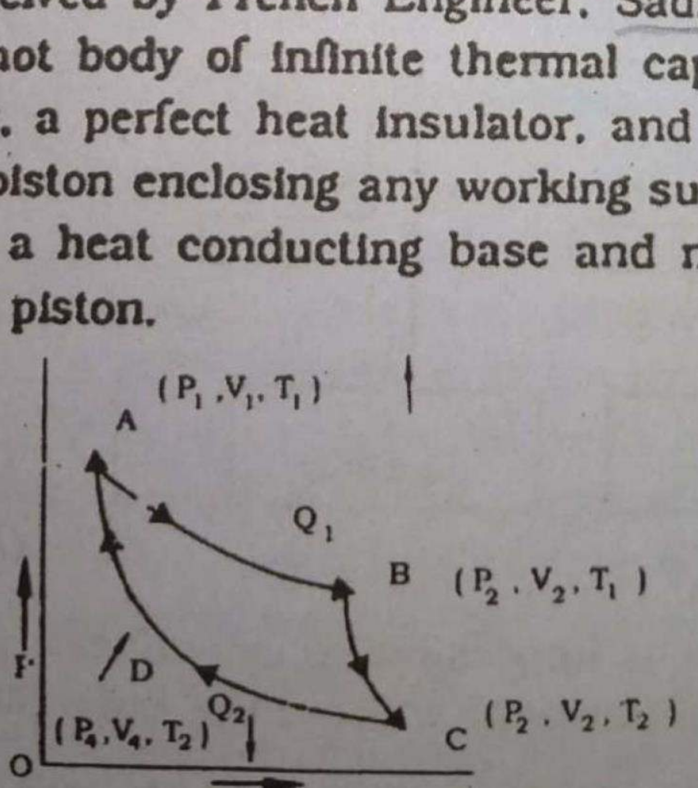
"It is impossible to cause heat to flow from a cold body to a hot body without the expenditure of energy".

We shall now prove that both these statements are equivalent by showing that if either of these statements is supposed to be false, the other must be false also. We suppose that Kelvin's statement is false then we could have a heat engine which takes heat from a source and converts it completely into work. If we connect this perfect heat engine to an ordinary refrigerator we can take heat from the hot body and convert it completely to work. This work can be used to operate the refrigerator which conveys heat from the cold body to the hot body. The net result is a transfer of heat from a cold body to a hot body without the expenditure of work which is contrary to Clausius' statement.

According to Kelvin statement, an engine, when converting heat into mechanical work, can not convert all of it into work. A part of the heat must be rejected to a cooler reservoir, the exhaust. In other words, it is impossible to device a machine that would have an efficiency of 100 percent, even though the first law will be satisfied.

11.22. The Carnot Engine

The law, governing the conversion of heat into work are investigated by considering an ideally simple heat engine conceived by French Engineer, Sadi carnot. It consists of a hot body of infinite thermal capacity, a similar cold body, a perfect heat insulator, and a cylinder fitted with a piston enclosing any working substance. The cylinder has a heat conducting base and non conducting walls and piston.



The working substance is taken through the following cycle of operations known as the carnot cycle (Fig.11.14). Starting at the condition shown by the point A, the cylinder is placed on a hot body (Fig. 11.15)at temperature T_1 and the load on the piston is decreased in order to allow the gas to expand very slowly at temperature T_1. This expansion is strictly isothermal as the bottom of the cylinder is a perfect conductor and the thermal capacity of the hot body is infinite. It is allowed to continue until the volume of the substance reaches the point B, when the cylinder is taken to be perfect heat insulator.

The gas is allowed to expand and this expansion is adiabatic since no heat can enter or leave the system. This expansion is shown along BC and the temperature of the gas falls from T_1 to T_2.

The cylinder is removed from the insulator and is placed on the cold body at a temperature T_2 and the working substance is compressed very slowly by putting the weight on the piston just greater than the force due to the pressure of the working substance. During this compression the heat energy Q_2 is transferred from the gas to the cold body through the conducting base. The working substance undergoes isothermal expansion shown by CD and some work is done on the gas.

When the point D is reached, the cylinder is once again put back on to the perfect heat insulator. The cycle is completed by an adiabatic compression DA because no heat can leave or enter the system. The temperature of the working substance has risen to that of hot body once more and one cycle is completed.

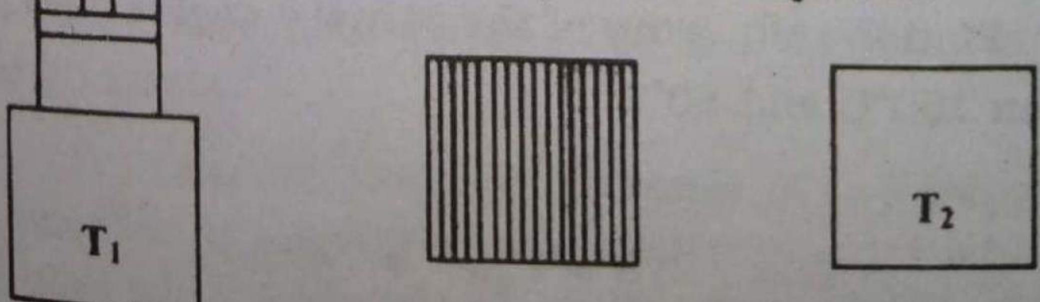


Fig. (11.15)

If Q_1 is the heat absorbed by the working substance from the hot body in the isothermal expansion AB and Q_2 is the heat rejected to the cold body during isothermal compression CD, and W is the external work done by the engine in one cycle, then from the first law of thermodynamics

$$W = Q_1 - Q_2 \qquad \text{------------ Eq. (11.41)}$$

In this cycle, the system comes back to its initial state and hence, there is no change in its internal energy. The efficiency E of a heat engine is defined as

$$E = \frac{\text{output}}{\text{input}} = \frac{W}{Q_1} = \frac{Q_1 - Q_2}{Q_1} = 1 - \frac{Q_2}{Q_1} \qquad \text{------(11.42)}$$

Eq. 11.42. shows that the efficiency of the engine increases as the ratio Q_2/Q_1 decreases. It can also be proved that heat transferred to or from a carnot engine is directly proportional to the temperature of the hot or cold body :

$$Q_2/Q_1 = T_2/T_1 \qquad \text{----------------(11.43)}$$

Thus the efficiency of the carnot engine can be written as

$$\text{efficiency} = 1 - \frac{T_2}{T_1} \qquad \text{----------------(11.44)}$$

It is thus concluded from Eq.11.44 that the smaller the ratio T_2/T_1 the more is the efficiency.

Example 11.8.

Find the efficiency of the carnot's engine working between 150°C and 50°C.

Solution

$$T_1 = 273 + 150 = 423\ K$$
$$T_2 = 273 + 50 = 323\ K$$

$$e = 1 - \frac{T_2}{T_1}$$

$$= 1 - \frac{323}{423} = 100/423 = .236$$

efficiency = 23.6%

11.23. Entropy and the Second Law of Thermodynamics

ENTROPY

We introduce here another new concept known as entropy. We may say that entropy is a measure of molecular disorder and in any process the entropy increases or remains constant, that is the disorder increases or remains constant.

In order to explain this let us consider a large number of molecules of a gas confined in an insulated cylinder fixed with a removable partition as shown in fig 11.16 (a). To start with all the molecules are confined in a volume $V = V_1$ we say that the molecules are localized within a volume V. Suppose now that the partition is removed. The molecules instead of staying confined to volume V, now occupy the whole volume $V_2 = 2V$, as shown in fig 11.16 (b) and are less localized then they were before the partition was removed. Actually, the degree of localization is a measure of disorder. As the system increases in volume, the disorder increases, and we say that the entropy increases. Also, there is almost no chance that all the molecules by themselves will collect in the original volume, V_1. That is why we made the statement "entropy or disorder always increases are remains constant."

We take here another example of the increase of the disorder of a system. Consider equal number of black and white balls contained in a box. The black balls occupy one half of the box while the white balls occupy the other half of the box. We would say that the balls are placed in order. As we shake the box this order of the black and white balls is disturbed. As the box is shaken more, the order is disturbed more. We can say that the disorder increases and the original arrangement cannot be restored no matter how we shake the box.

Scientists describe such a situation by saying that the entropy of the system has increased. These considerations thus lead to define the second law of thermodynamics as follows:

"When an isolated system undergoes a change, the disorder in the system increases".

As said above, entropy is thought of as being synonymous with the 'degree of disorder'. The second law of thermodynamics is then also defined as:

"When an isolated system undergoes a change, the entropy of the system either remains constant or it increases"

It is our experience also that the natural tendency of the things is to go into disordered state.

If ΔQ be the heat transferred to a system at constant kelvin temp T, then

Change in entropy $$\Delta S = \frac{\Delta Q}{T} \qquad \text{---------- (11.45)}$$

If heat is removed from a body, the change in entropy is written as negative. Its SI unit Jk^{-1}

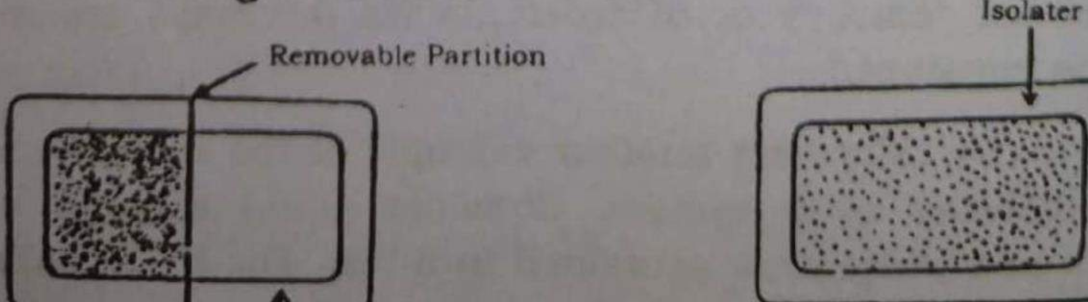


Fig:(11.16)

QUESTIONS

11.1. How do you distinguish between temperature and heat? Give examples.

11.2. Why the earth is not in thermal equilibrium with the sun?

11.3. Is temperature a microscopic or macroscopic concept?

11.4. It is observed that when a mercury in a glass thermometer is put in a flame, the column of mercury first descends and then rises. Explain.

11.5. Is it correct that unit for specific heat capacity is $m^2\ S^{-2}\ (C^{\circ})^{-1}$?

11.6. What is the standard temperature ?

11.7. When a block with a hole in it is heated, why does not the material around the hole expand into the hole and make it small?

11.8. A thermometer is placed in direct sunlight. Will it read the temperature of the air, or of the sun, or of some thing else.

11.9. Will kilogram of hydrogen contain more atoms than kilogram of lead? Explain.

11.10. The pressure in a gas cylinder containing hydrogen will leak more quickly than if it is containing oxygen. Why?

11.11. What are some factors that affect the efficiency of automobile engines?

11.12. What happens to the temperature of a room in which an air conditioner is left running on a table in the middle of the room.

11.13. When a sealed thermos bottle full of hot coffee is shaken, what are the changes, if any in

a) the temperature of the coffee and

b) the internal energy of the coffee.

PROBLEMS

11.1. i) The normal body temperature is 98.4 F . What is this temperature on Celsius scale? (Ans: 36.88°C)

ii) At what temperature do the Fahrenheit and Celsius scales coincide. Ans - 40°.

11.2. A steel rod has a length of exactly 0.2 cm at 30°C. What will be its lenght at 60°C? (Ans: 0.20066 cm).

11.3 Find the change in volume of an aluminium sphere of 0.4m radius when it is heated from 0 to 100°C. (Ans: 0.0019 m^3).

11.4 Calculate the root mean square speed of hydrogen molecule at 800K. (Ans: 3158.6 ms^{-1}).

11.5. a) Determine the average value of the kinetic energy of the partieles of an ideal gas at 0°C and at 50°C.

(Ans: 5.65 x 10^{-21}J), (6.68 x 10^{-21}J)

b) What is the kinetic energy per mole of an ideal gas at these temperatures ?

(3404.5 J/mole), (4026.2 J/mole).

11.6. A 2 kg iron block is taken from a furnace where its temperature was 650°C and placed on a large block of ice at 0°C. Assuming that all the heat given up by the iron is used to melt the ice, how much ice is melted. (Ans: 1.93 kg).

11.7. In a certain process 400 J of heat are supplied to a system and at the sametime 150J of work are done by the system. What is the increase in the internal energy of the system.(250J)

11.8. There is an increase of internal energy by 400 joules when 800 joules of work is done by a system. What is the amount of heat supplied during this process? (Ans: 1200 J).

11.9. A heat engine performs 200J of work in each cycle and has efficiency of 20 percent. For each cycle of operation a) How much heat is absorbed and b) How much heat is expelled?

a) (Ans: 1000 J) (b) (800 J)

11.10. A heat engine operates between two reservoirs at temperatures of 25°C and 300°C. What is the maximum efficiency for this engine? (Ans: 48%)

11.11 The low temperature reservoir of a carnot engine is at 7°C and has an efficiency of 40%. It is desired to increase the efficiency to 50%. By how much degrees the temperature of hot reservoir be increased. (Ans: 93.4 K)

Chapter 12.

ELECTROSTATICS

12.1 Electrons and protons are characterized by a property by virtue of which they exert forces of attraction and repulsion on one another. This property is called charge. On rubbing a body with a suitable material both acquire equal amounts of opposite charges due to transfer of some outer most loosely bound electrons from one body to the other the electric charge is, therefore, conserved. In this chapter we shall investigate quantitatively the force exerted by one charge over another, and the potential at a point in the neighborhood of charged body and charge storing devices etc.

12.2. Coulomb's Law

The first experiment to investigate quantitative law of force between localized charges was carried out by Charles Augustin de Coulomb in 1784. using a torsion balance. The results of the experiment can be stated in the form of a law called coulomb's law. The electric force between two static point charges varies directly to the product of charges with each charge and inversely with the square of the distance between them. If q_1 and q_2 are two point charges distance "r" apart as shown in fig. 12.1.

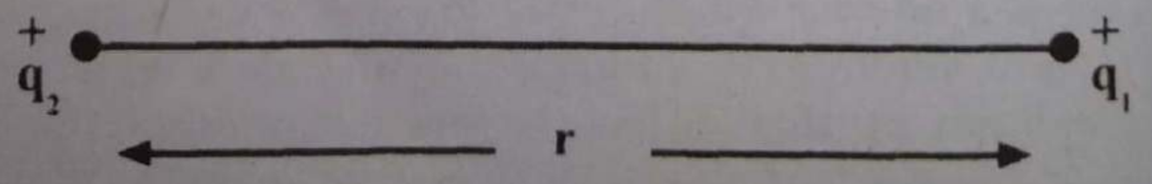


Fig. (12.1)

$$F = K \frac{q_1 q_2}{r^2} \qquad \text{------------------ (12.1)}$$

Where K is the constant of proportionality and its value depends on the medium between the charges. The magnitude as well as the direction of force can be represented by the vector equation

$$\overline{F}_{12} = K \frac{q_1 q_2}{r^2} \hat{r}_{12} \qquad \text{--------------(12.2)}$$

Where $\overline{F}_{12}$ is the force exerted by q_1 on q_2 and $\hat{r}_{12}$ is unit vector along the line joining the two charges from q_1 to q_2.

In case the charges are similar the force is that of repulsion and vice versa.

In SI units the unit of electric charge is coulomb.The unit coulomb which is defined as the amount of charge that flows through a given cross section of a wire in one second if there is a steady current of 1 ampere in the wire.The adoption of unit of current will be explained later.The measured value of K for free space is

$$K = 8.98755 \times 10^9 \ N.m^2.C^{-2}$$

In order to express other equations which will appear in the theory later in a simple form K is expressed in terms of 4π and another constant ϵ_0 called the permitivity of free space

i.e. $K = \frac{1}{4\pi\epsilon_0} = 9 \times 10^9 \ Nm^2C^{-2}$ the value of ϵ_0 is

$8.85 \times 10^{-12} \ C^2/ (Nm^2)$i.e.

$$F = \frac{1}{4\pi\epsilon_0} \times \frac{q_1 q_2}{r^2} \qquad \text{-------------------(12.3)}$$

The value of the constant of proportionality for a medium other than free space between the charges is less and it is written as

$$\frac{1}{4\pi\epsilon_0\epsilon_r} \quad \text{or} \quad \frac{1}{4\pi\epsilon} \quad \text{where } \epsilon = \epsilon_0\epsilon_r \text{ is the}$$

permittivity of the medium

$\epsilon_r = \dfrac{\epsilon}{\epsilon_0}$ is called relative permittivity of the medium.

The force between two given charges, therefore, decreases by introducing a non conducting medium between them and eq. (12.3) can be written as

$$F = \frac{1}{4\pi\epsilon} \frac{q_1 q_2}{r^2} \quad \text{-------------------(14.4)}$$

The magnitude of charge on an electron is

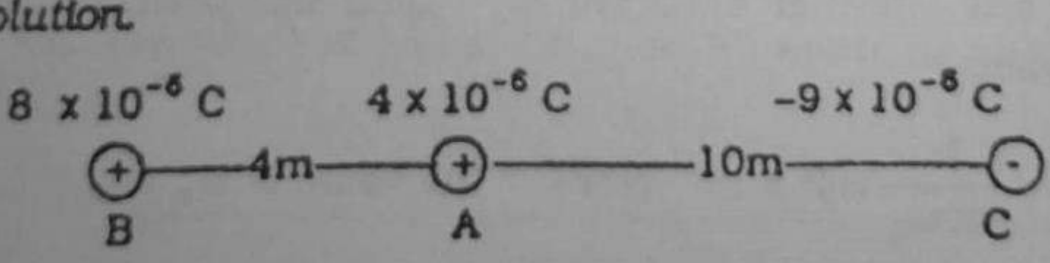
$e = 1.6 \times 10^{-19}$ C.

Thus charge of 1 coulomb is 6.25×10^{18} times the electronic charge.

Example 12.1

A point charge A of $+ 4 \times 10^{-6}$ Coulomb is placed on a line between two charges B of $+ 8 \times 10^{-6}$ C and C of $- 9 \times 10^{-6}$ C. The charge A is 4m from B and 10m from C. What is the force on A?

Solution.



Force on A due to B = $F_1 = K \dfrac{q_1 q_2}{r^2}$

$$= 9 \times 10^9 \frac{8 \times 10^{-6} \times 4 \times 10^{-6}}{(4)^2} = \frac{9 \times 8 \times 4}{16} \times 10^{-3}$$

$= 18 \times 10^{-3}$ N. towards C

Force on A due to C = $F_2 = 9 \times 10^9 \times \frac{4 \times 10^{-6} \times (9) \times 10^{-6}}{(10)^2}$

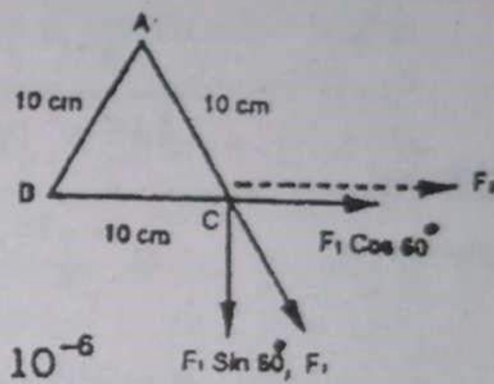
$$= \frac{9 \times 4 \times 9}{100} \times 10^{-3} = 3.24 \times 10^{-3} C, \text{ towards } C.$$

Since these two component forces are in the same direction, the resultant force is their arithmetic sum

$F_1 + F_2 = 21.24 \times 10^{-3}$ N, towards C.

Example 12.2.

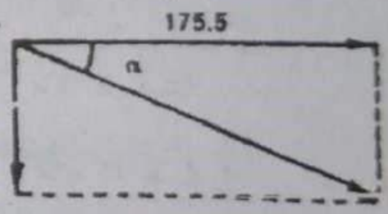
Two charges of magnitudes +10 μ C and + 8 μ C are placed on the corners A and B of an equilateral triangle of sides 10 cm. Find the force on a charge of + 15 μ C placed at the third corner C.



Solution

Force on the charge at C due to charge at A = F_1

$$= 9 \times 10^9 \frac{10 \times 10^{-6} \times 15 \times 10^{-6}}{(.1)^2}$$



= 135 N along AC

Component of F_1 along BC = F_1 Cos 60°

= 135 x .5 = 67.5 N

Component of F_1 in the direction perpendicular to BC

= F_1 Sin 60° = 135 x .866 = 116.9 N.

Force on the charge at C due to charge at B along BC

$$= F_2 = 9 \times 10^9 \frac{8 \times 10^{-6} \times 15 \times 10^{-6}}{(.1)^2} = 108N$$

∴Total force along BC= $F_1 \cos\theta + F_2$ = 67.5+108 = 175.5 N

∴Resultant force on the charge at C

$$= \sqrt{(175.5)^2 + (116.9)^2} = 210.86N$$

The resultant force is in a direction making an angle α with BC such that

$$\alpha = \tan^{-1} \frac{116.9}{175.5} = 33.67°.$$

Example 12.3

The distance between the electron and the proton of Hydrogen atom is about 5.3×10^{-11} m. Compare the electric and the gravitational forces between these two particles.

Solution.

$$\frac{F_{electrical}}{F_{gravitation}} = \frac{\frac{1}{4\pi\epsilon_0} \frac{e^2}{r_2}}{G \frac{m_e m_p}{r^2}}$$

$$= \frac{9 \times 10^9 \times (1.6 \times 10^{-19})^2}{6.7 \times 10^{-11} \times 9.1 \times 10^{-31} \times 1.7 \times 10^{-27}}$$

$$= 2.2 \times 10^{39}$$

i.e. the electrical force is 10^{39} times greater than the gravitation force. That is why gravitational force between electrons and protons of an atom is neglected and only the electrostatic force is taken to provide the centripetal force required to keep the electrons in rotation round the nucleus.

12.3 Electric Field

Before Maxwell and Faraday the forces that act between electric charges were thought of as direct and instantaneous interaction between them and it was called action at a distance.

Faraday conceived that the force between electric charges separated from one another is not a direct action at a distance but it is transmitted through intervening space whether that may be empty or occupied by matter.

An electric charge modifies the space around it such that any other charge brought in this region experiences force of electrical nature.

The modified space around the electric charges is called electric field In other words electric field is a region around a charge body in which another charge experiences an electric force.

(1) **Intensity of electric field**

The intensity of electric field at a particular point is greater if an electric charge experiences a greater force at that point. The difficulty is that the charge that is brought into the field will produce its own field and the originally existed field will be changed. In order to avoid it the field is supposed to be explored by a positive test charge + q_0 of very small magnitude (approaching to zero) so that the original field is not disturbed.

The force experienced by this test charge at a point per unit charge is the measure of intensity of field at that point in the direction of force

$$\vec{E} = \frac{\vec{F}}{q_0} \qquad \text{--------------------(12.5)}$$

The electric intensity is directed from a positive charge and toward the charge in case of a negative charge.

The S.I. unit of electric field intensity is newton/coulomb. (NC^{-1}).

ii) Electric field Intensity near an isolated point charge q

Imagine a very small positive point charge q_0 placed at a distance r from point charge q_0 (fig.12.2)

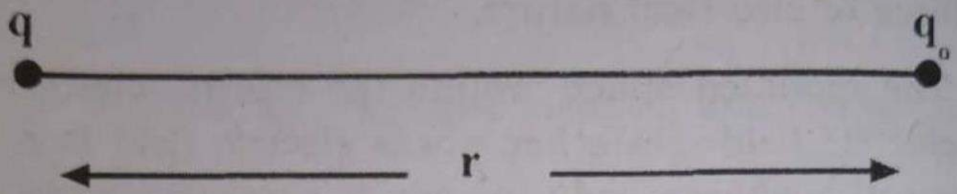


Fig. (12.2)

The magnitude of force on q_0 due to the charge q is

$$F = \frac{1}{4\pi\epsilon_0}\,\frac{q\,q_0}{r_2}$$

∴Electric field Intensity

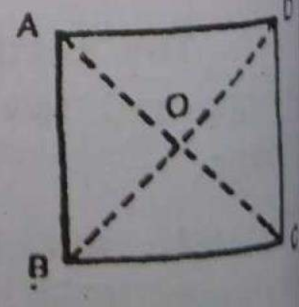
$$E = \frac{F}{q_0} = \frac{1}{4\pi\epsilon_0}\,\frac{q}{r^2} \quad \text{-----------------} \quad (12.6)$$

The direction of the intensity of electric field is that of the electric force.

Example 12.4

Charges each of +3 μ C. are placed at three corners of a square whose diagonal is 6 cm long. Find the field intensity at the point of inter section of diagonals.

Solution



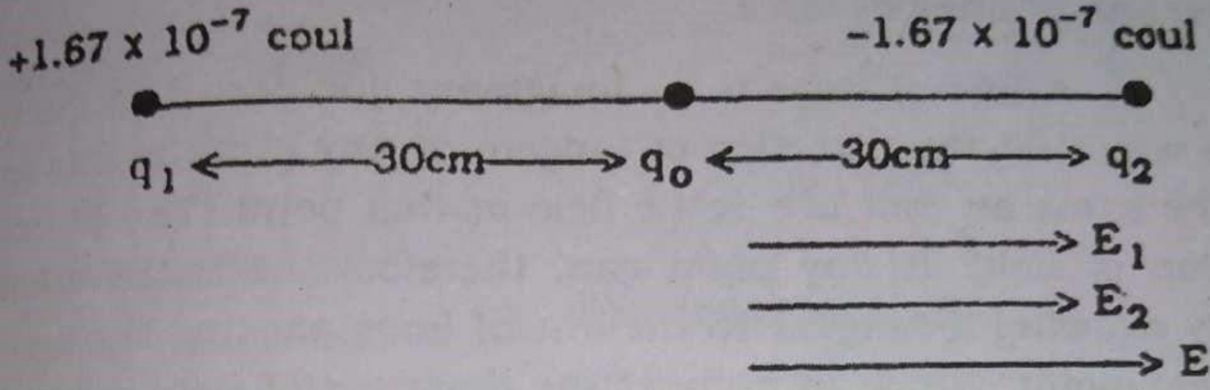
The fields at O due to charges at A and mutually cancel each other being equal and opposite. The net field

$$E = 9 \times 10^9\,\frac{3 \times 10^{-6}}{(3 \times 10^{-2})^2} = 3 \times 10^7 NC^{-1}$$

Example 12.5.

Find the electric intensity midway between the charges $+1.67 \times 10^{-7}$ coul and -1.67×10^{-7} coul separated by a distance of 60 cm

Solution



$$E = E_1 + E_2 = \frac{1}{4\pi\epsilon_0}\;\frac{q_1}{r_1^2} + \frac{1}{4\pi\epsilon_0}\;\frac{q_2}{r_2^2}$$

Since $q_1 = q_2$ and $r_1 = r_2$

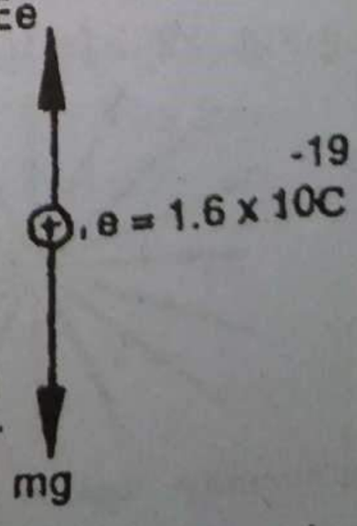
$$E = 2\,\frac{1}{4\pi\epsilon_0}\;\frac{q_1}{r_1^2} = \frac{2 \times 9 \times 10^9 \times 1.67 \times 10^{-7}}{(.3)^2}$$

$$= 3.33 \times 10^4\ NC^{-1}$$

The field is directed along the line from q_1 to q_2.

Example 12.6.

Find the magnitude and direction of the electric field that will counter balance the gravitational force on a proton.



Solution

Let the field be E upwards i.e.

$$E.e = m\,g\,.$$

$$E = \frac{mg}{e} = \frac{1.67 \times 10^{-27} \times 9.8}{1.67 \times 10^{-19}}$$

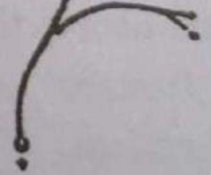
$= 1.02 \times 10^{-7}\ N\ C^{-1}$

12.4 Electric lines of Force

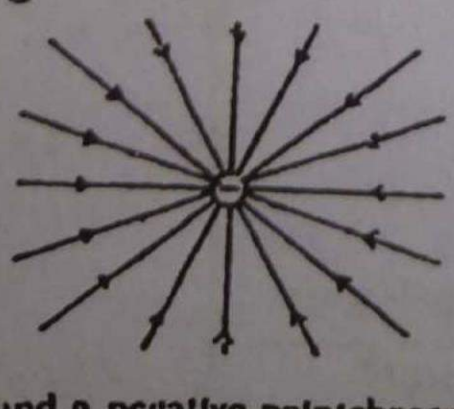
Electric field is a vector quantity and the field in space could be represented by associating a vector to every point of the field.

Faraday did not adopt this procedure and he introduced a novel method of visualizing the field by means of lines of force.

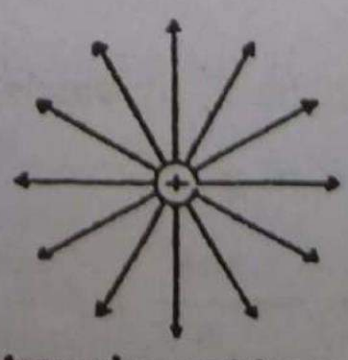
A line of force is an imaginary line drawn in such a way tha' the direction of tangent at any of its points is the same as that of electric field at that point. The direction of field at any point can, therefore, be determined by drawing a tangent to the line of force passing through that point. Since in general the direction of field varries from point to point the lines of force are usually curved. Lines of force do not intersect because the field cannot have two directions at a point. No lines of force originates or terminates in space surrounding the charge. Every line is a continuous line which originates on positive charge and terminates at negative charge. When we speak of an isolated charge it simply means that opposite charges on which the lines terminate or originate have large distances around it.



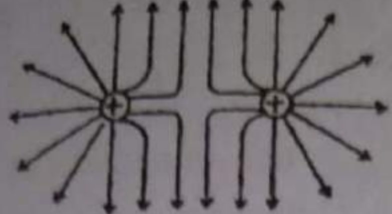
Electric fields due to different charge configurations are visualized by means of field lines in the following Fig. 12.3 (a to f)



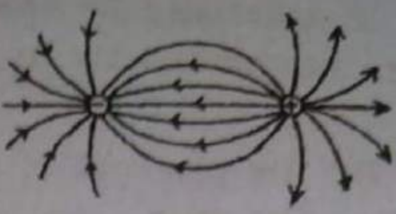
b) Around a negative pointcharge



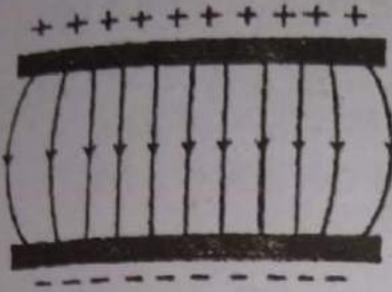
a) Around a positive point charge.



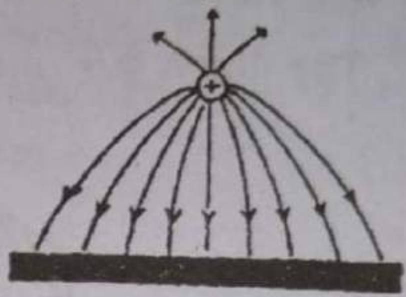
c) Around two like charges field at P is zero. (Neutral Point)



d) Around two unlike point charges



e) Between a pair of parallel oppositely charged conducting plates. Field is uniform in the space where lines of force are parallel metal plate.



f) Between a point charge and an oppositely charged metal plate

Fig.12.3 (a to f)

12.5 Electric Flux and Field Intensity

It is, of course, possible to draw a line of force through every point of an electric field but if this were done the whole space would be filled with lines and no individual line could be distinguished. By suitable limiting the number of lines in space the lines can be made to indicate the magnitude of field at a point. This is accomplished by spacing the lines in such a way that their number per unit area cutting a very small surface held perpendicular to the line of force at a given point gives the electric field at that point.

The total number of lines of force crossing a surface normally is called flux on that surface. The flux per unit area or flux density at any point gives the electric field at that point.

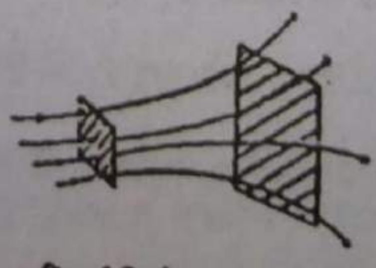


fig 12.4

The flux at a surface is determined by the product of flux density i.e electric field and the projection of it's area perpendicular to the field. or

By the product of area and the component of field normal to the area.

For a very small plane area ΔA such that the field at all of it's points is the same, the flux is given by:

The flux $\Delta \phi = \Delta A (E \cos\theta) = \overline{E} . \Delta \overline{A}$ -------- (12.7)

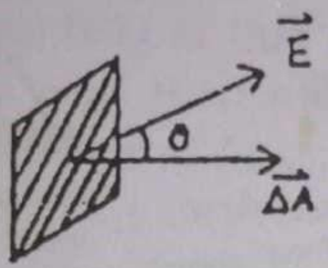
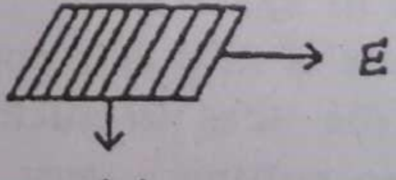
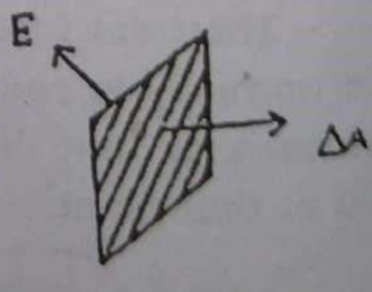


Fig: 12.5 (a)

Thus flux is a scalar product of electric intensity $\overline{E}$ and vector area ΔA. Although area is a scalar quantity, ΔA is taken as a vector quantity of magnitude equal to area and in the direction normal to it



(Fig 12.5(b)



1. The flux is positive when $\theta < 90°$

2. The flux is zero when $\theta = 90°$

3 The flux is negative when $\theta > 90°$

4 The flux is maximum when $\theta = 0°$

$= E\Delta A$.

fig 12.5 (c)

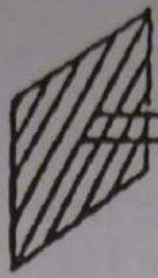


Fig 12.5 (d)

The convention that the electric field is equal to flux density at a point fixes the number of lines originating or terminating on a unit charge

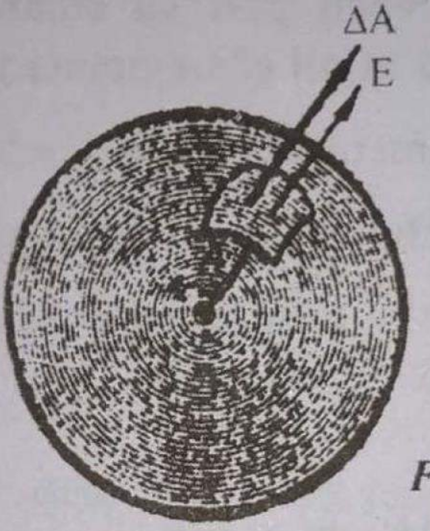


Fig 12.6

Consider an isolated point charge +q (fig .12.6). the lines of force from q will spread uniformly in space around it radially cutting the surface of an imaginary sphere drawn with the position of the charge as centre. normally at all portions. If r is the radius of sph re,

$$\text{Flux density} = \frac{\text{Number of lines Originating from q}}{\text{Surface area of sphere}}$$

The flux over the surface of sphere or the number of lines originating from $q = \Sigma E\Delta A = E\Sigma\Delta A = E\,4\pi r^2$

$$\phi = \frac{1}{4\pi\epsilon_0}\,\frac{q}{r^2}\,4\pi r^2 = q/\epsilon_0 \qquad \text{------------------ (12.8)}$$

We, therefore, see that if a unit charge is supposed to give rise to $\frac{1}{\epsilon_0}$ lines, the flux density at any point will give the intensity of field at that point.

12.6 Gauss's Law

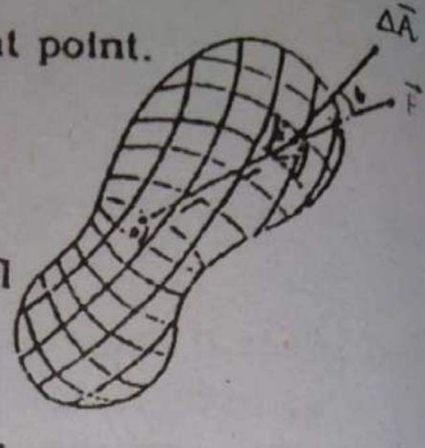


Fig 12.7

Consider a closed surface of any arbitrary size and shape surrounding a point charge q as shown in fig 12.7 electric field vectors at different points of the closed surface will have different magnitudes and directions. In order to determine flux over the closed surface, it is divided into very small parts, each part so small that it is almost plane and the field at all of its points is the same.

$\therefore$ Flux over an element of area $\Delta\phi = \bar{E}.\Delta\bar{A}$

and the flux over the whole surface, $\phi = \Sigma\bar{E}.\Delta\bar{A}$

which is equal to the total number of lines of force originating from q and crossing the closed surface normally i.e.

$$\phi = \Sigma\bar{E}.\Delta\bar{A} = \frac{q}{\epsilon_0}$$

Electric flux being the dot product of two vectors is a scalar quantity and hence if the surface encloses a number of scattered point charges

$q_1, q_2, q_3,$-------------------- q_n, their fluxes can be added algebraically. The total flux ϕ due to all the point charges will be $\phi = \phi_1 + \phi_2 + \phi_3 + \text{-----} + \phi_n$

$$= \frac{q_1}{\epsilon_0} + \frac{q_2}{\epsilon_0} + \frac{q_3}{\epsilon_0} + \cdots\cdots + \frac{q_n}{\epsilon_0}$$

$$\phi = \sum_{r=1}^{n} q_r/\epsilon_0 \qquad \text{------------------(12.9)}$$

This generalization is known as Gauss's law. The total outward flux over any closed hypothetical surface called Gaussian surface is equal to the total charge enclosed divided by ϵ_0 irrespective of the way in which the charge is distributed.

12.7 Applications of Gauss's Law

Gauss's law can be used to calculate the electric field only in those cases of charge distributions which are so symmetrical that by proper choice of a Gaussian surface the flux on it may possibly be evaluated. The following are few examples.

(a) Field of a uniform spherical surface charge at a distance r from its centre.

Let a charge q be uniformly distributed on the surface of a spherical Shell or that of a metal sphere (charge resides on the surface of metal).

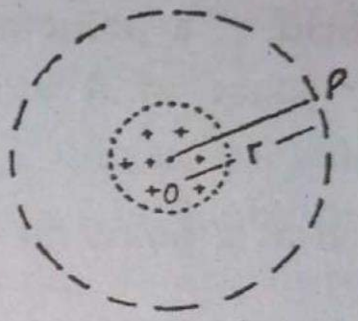


fig.12.8

Our object is to determine that field at a point P outside the shell at a distance r from the centre O.

The field could be determined using coulomb's law by adding vectorially the fields due to individual charge elements. The calculation is tedious because the fields vary both in magnitudes and directions. However, the field can be very easily determined using Gauss's law .

Imagine a Gaussian surface as that of a concentric sphere of radius r that contains the point P (fig.12.8) due to symmetrical charge distribution with respect to every point of Gaussian surface the field has the same magnitude every where on this surface and it is perpendicular to the surface at each point.

Flux over this Gaussian surface is

$$\phi = \Sigma\, E.\, \Delta A$$

$$= E \Sigma \Delta A = E\, 4\pi\, r^2$$

Applying Gauss's law.

$$E 4\pi r^2 = q/\epsilon_o$$

$$E = \frac{1}{4\pi\epsilon_o}\frac{q}{r^2} \qquad \text{------------ (12.10)}$$

i) This shows that the electric field due to a uniform spherical surface charge distribution at an external point is the same as that produced by the same charge when concentrated at its centre.

ii) The field at a point on the surface is considered as the field at a point outside the charged surface but infinitely close to it at r = a (a is the radius of charged shell)

∴ Field at a point on the charged surface will be.

$$E = \frac{1}{4\pi\epsilon_o}\frac{q}{a^2}$$

This can also be expressed in terms of charge density δ (charge per unit area)

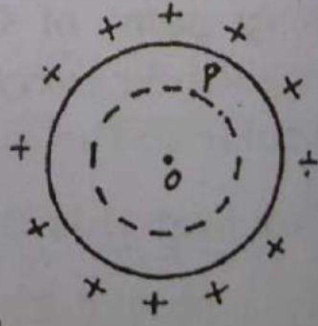
$$q = \sigma\, 4\pi a^2$$

$$\therefore E = \frac{1}{4\pi\epsilon_o}\frac{\sigma\, 4\pi a^2}{a^2} = \frac{\sigma}{\epsilon_o} \qquad \text{---------------- (12.11)}$$

iii) In case if the point P is situated inside the charged surface (fig 12.9) the Gaussian surface passing through P immagined as in case 1 will enclose no charge and hence the flux is zero.

$$E = 4\pi r^2 = 0$$

$$E = 0$$



It means that there exists no field inside a spherical surface charge distribution

.fig 12.9

(b) Electric Intensity due to an infinite sheet of charge

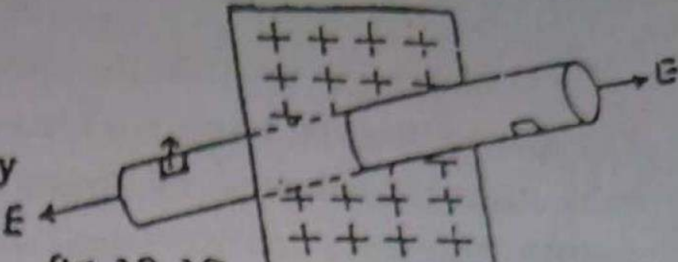


fig 12.10

A single thin sheet of charge can be prepared by depositing charge on one of a sheet of a non conducting material. In case of infinite plane the electric lines of force are perpendicular to the plane at all points.

The figure 12.10 shows a portion of a thin non conducting sheet of infinite size, one side of which is uniformly charged. Let the charge density be σ Cm^{-2}

A convenient Gaussian surface is that of a cylinder of cross section ΔA and length arranged to pierce the sheet, such that flat faces are each parallel to the sheet and are at distances r on either side of the sheet. The charge enclosed within the Gaussian surface is $\sigma \ \Delta A$.

The flux on the curved surface is zero because the angle between the field vector and area Vector of all elements of curved surface is 90°.

Let E be the electric field at a distance r from the sheet. The sum of flux on the two flat faces $\phi = E\Delta A + E\Delta A = 2E\Delta A$.

Applying Gauss's law

$$2E\Delta A = \frac{1}{\epsilon_0} \sigma \, \Delta A$$

$$E = \frac{\sigma}{2\epsilon_0} \qquad \text{------------------(12.12)}$$

The field is independent of r showing that the field is the same at all points on each side of the plane.

If the charge is distributed evenly on both sides of the sheet each gives rise to a field $\frac{\sigma}{2\epsilon_0}$ and the net field at all points on either side will be $\frac{\sigma}{\epsilon_0}$

Although an infinite sheet cannot exist physically the derivation is useful as it yields correct results for real charged sheets of finite size if we consider the points not near to the edges and whose distances from the sheet is small.

(c) Electric Intensity between two oppositely charged plates

Consider two parallel metal plates separated by a small distance as compared to their size as shown in fig 12.11.

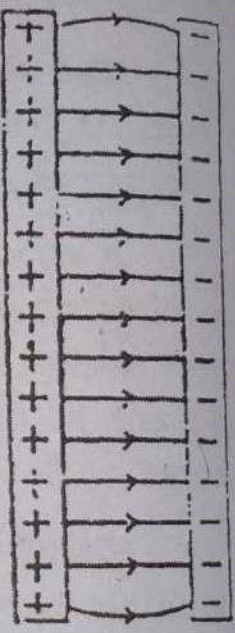


fig. 12.11

The plates carrying equal amounts of opposite charges will each have a charg density σ. Since the lines of force are parallel except near the edges each plate may be regarded to produce the same field as that produced by infinite charged sheets. The magnitude of electric field intensity at any point between the plates due to each plate is $\frac{\sigma}{2\epsilon_0}$ and along the same direction towards the –ve plate the net electric field at any point is

$$E = \frac{\sigma}{2\epsilon_0} + \frac{\sigma}{2\epsilon_0} = \frac{\sigma}{\epsilon_0}$$

$$\overline{E} = \frac{\sigma}{\epsilon_0}\hat{r}$$

$\hat{r}$ is a unit vector from +ve plate to the –ve plate.

12.8 Electric Potential

An electric charge experiences force at all points in an electric field and work has to be done by some external agency to move the charge from one point to the other against the electric force.

The situation is analogous to that of a mass raised to a higher altitude against the gravitational force. Since the work is recovered when the body descends it is considered to be stored as potential energy. We also know that this work is independent of the path along which the body is moved to that altitude.

Similarly the work done in moving a charge against the electric field is stored as electric potential energy. It is independent of the path adopted and hence the potential energy at a given location can be specified.

Work is done on the charge by the electric field when it moves back so that either it is accelerated or work is done on some external agency which prevents it from gaining speed.

Let a very small test charge q move from a point P to a point Q along any arbitrary path in the field as shown in fig.12.12

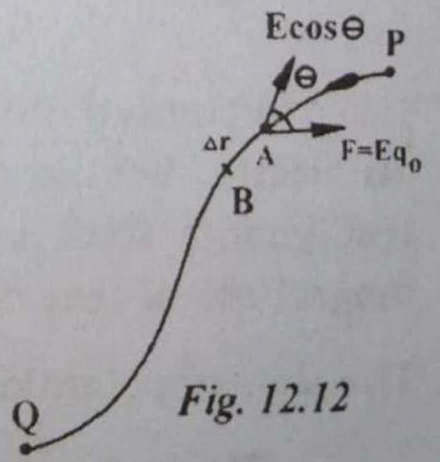


Fig. 12.12

In order to determine the work done the path is divided to small elements, each element being so small that it may be regarded as a straight line and the field at all of its points is the same.

The force on the charge at the point A is $F = Eq_0$

where E is the field at A in the direction shown in the diagram Work done in moving the test charge along an element from A to B $= Eq_o \overline{E} \cos\theta \, \Delta r = q_o E.\overline{\Delta r}$

This process is repeated right from P to Q taking care that the charge is moved very slowly always keeping it in electrostatic equilibrium i.e. when there is no net motion of charge within a conductor or on its surface, the conductor is said to be in electrostatic equilibrium.

$$\therefore \text{Total work} = \sum_{P}^{Q} q_o \vec{E} \cdot \vec{\Delta r}$$

Hence the difference of potential energy of the test charge q_o at point Q and P

$$U_Q - U_P = q_o \sum_{P}^{Q} \vec{E} \cdot \vec{\Delta r} \qquad \text{-----------------} (12.13)$$

Instead of dealing direct with potential energy of a charge it is useful to introduce the more general concept of potential energy per unit charge called potential difference between the points Q and P

$$V_Q - V_P = \frac{W_{P \to Q}}{q_o} = \sum_{P}^{Q} \overline{E} \cdot \overline{\Delta r} \qquad \text{-----------------} (12,14)$$

Potential difference between two given points in an electric field is defined as the work done in moving a test charge from one point to the other divided by the magnitude of test charge.

The unit of potential difference is called Volt

The potential difference between two points in an electric field is 1 Volt if the work done per unit charge in moving the test charge between these points is 1 Joule

12.9 Absolute Potential at a point

Until now we have talked about the difference of potential between the points in the electric field. For defining absolute potential at a point there is a need for an arbitrary choice of zero of electric potential this can be understood by the fact that all heights are measured from sea level which is the conventional choice of the zero level. All temperatures on Celsius scale are measured above the temperatures of ice which is the conventional choice of zero temperature.

The zero reference potential is frequently taken as the potential of earth but for many other purposes the zero of potential is considered as the potential at a point greatly distant from all charges (at infinity)

The absolute potential at a point is the work done per unit charge when a test charge is moved from a point at infinity having zero potential to that point.

12.10. Electric Potential near an Isolated Point Charge

Consider two points A and B in a straight line at distances r_A and r_B respectively from a point charge q as shown in fig.12.13. In order to obtain an expression for the potential difference between the point B and A the work needed to move the test charge from A to B per unit charge is to be determined

Since the electric field is varying from point to point the work is determined in steps, each so small that the field intensity within each step is nearly constant.In the first step the work done per unit charge.

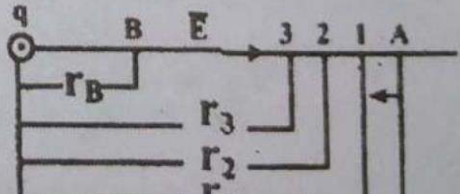


Fig. 12.13

$$\Delta W_1 = \frac{Eq_0(r_A - r_1)}{q_0} = -K\frac{q}{r^2}(r_A - r_1)$$

where r is neither r_A nor r_1 but an average distance between them from q_0. It makes things easier to take r as geometric mean of r_A and r_1

i.e $r = \sqrt{r_A - r_1}$

$$\therefore \Delta W_1 = \frac{Kq}{r_A r_1}(r_A - r_1) = -Kq\left(\frac{1}{r_1} - \frac{1}{r_A}\right)$$

In the next step

$$\Delta W_2 = -Kq\left(\frac{1}{r_2} - \frac{1}{r_1}\right)$$

$$\Delta W_3 = -Kq\left(\frac{1}{r_3} - \frac{1}{r_2}\right) \qquad \text{and so on}$$

$$\Delta W_n = -Kq\left(\frac{1}{r_B} - \frac{1}{r_n}\right)$$

Now

$$\Delta W = \Delta W_1 + \Delta W_2 + \cdots \cdots \Delta W_n = -Kq\left(\frac{1}{r_B} - \frac{1}{r_A}\right)$$

$$\therefore \text{Potential difference } V_B - V_A = Kq\left(\frac{1}{r_B} - \frac{1}{r_A}\right)$$

The absolute potential at the point B is obtained by considering the point A to be situated at infinity.

$$V_B = \frac{Kq}{r_B}$$

Absolute potential due to a point charge + q at a point at a distance r from it

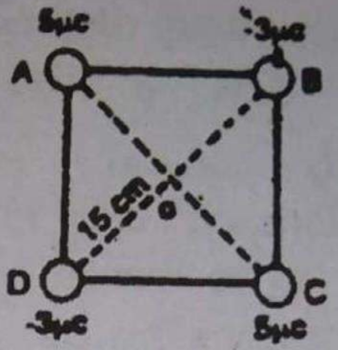
$$V = -\frac{1}{4\pi\epsilon_0}\frac{q}{r} \qquad \text{..................} (12.15)$$

Potential is a scalar quantity and the potentials at a point due to a number of charges are added algebraically.

Example12.7

Find the potential and field due to the charges placed at the ends of the diagonals of a square as shown in the diagram at the point of their intersection. Each diagonal is 30 cm long.

Solution



Potential at O due to charges at A and C

$$= \frac{9 \times 10^9 \times 5 \times 10^{-6}}{.15} + \frac{9 \times 10^9 \times 5 \times 10^{-6}}{.15}$$

$$= \frac{2 \times 9 \times 10^9 \times 5 \times 10^{-6}}{.15}$$

$$= 6 \times 10^5 \text{ Volts}$$

Potential at O due to charge at B and C

$$= \frac{-2 \times 9 \times 10^9 \times 3 \times 10^{-6}}{.15} = -3.6 \times 10^5 \text{ Volts}$$

Total Potential at $O = (6 - 3.6)\, 10^6 = 2.4 \times 10^5$ Volts.

Electric Intensity at O is zero because the field at. A due to the charge at A is cancelled by the field due to charge at C. Similarly the field due to charge at B is cancelled by the field due to charge at D.

Example 12.8.

Find the velocity acquired by an electron in falling through a potential difference of 2000 Volts.

Solution.

$$Ve = \frac{1}{2} mV^2$$

$$V = \sqrt{\frac{2Ve}{m}} = \sqrt{\frac{2 \times 2000 \times 1.6 \times 10^{-19}}{9.1 \times 10^{-31}}}$$

$= 8.4 \times 10^7\ ms^{-1}$

12.11. Relation between electric field and Potential

Consider two points a and b that are separated by a very small distance ΔS on a line of force AB as shown in fig. 12.14. The field is practically constant over the small distance ΔS. If a test charge $+q_0$ is moved from a to b work is done by the electric field on the test charge.

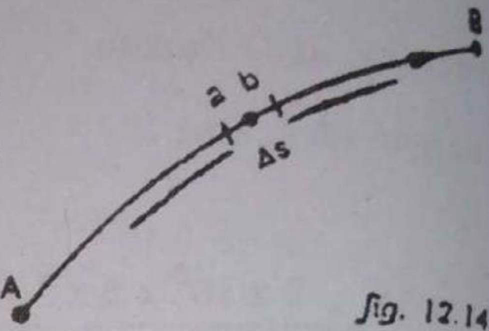


fig. 12.14

The work done by an outside agent in moving a positive charge against the field is conventionally taken + Ve and the work done by the field is taken as −Ve. The reason is that in this way positive work increases the potential energy of test charge.

∴ Work done by the field

$$\Delta W = -F\,\Delta S = q_0 E\,\Delta S$$

Potential difference between the points a and b

$$\Delta V = \frac{\Delta W}{q_0} = -E\,\Delta S$$

$$E = -\frac{\Delta V}{\Delta S} \quad \text{---------------- (12.16)}$$

$\frac{\Delta V}{\Delta S}$ is the rate of change of potential with respect to the distance and it is called potential gradient.

The relation 12.16 states that the electric Intensity at a point in an electric field is equal to the negative space rate of variation of potential at that point.

In case if the test charge is moved in a direction other than that along the line of force then $\Delta V = - E \cos \theta \ \Delta S$.

$$E \cos \theta = - \frac{\Delta V}{\Delta S}$$

In general we can say that the negative space rate of variation of potential at a point in any direction gives the electric field in that particular direction

Component of field along x– direction $E_x = - \dfrac{\Delta V}{\Delta x}$

similarly

$$E_y = - \frac{\Delta V}{\Delta y} \text{ and } E_z = - \frac{\Delta V}{\Delta Z}$$

Example 12.9

An electron is situated midway between two parallel plates 0.5 cm apart. One of the plates is maintained at a potential of 60 Volts above the other. What is the force on the electron.

Solution

$$F = Ee = - \frac{\Delta V}{\Delta S} e = - \frac{60}{5 \times 10^{-3}} \times (-1.6 \times 10^{-19})$$

$$= 1.92 \times 10^{-15} \text{ N}$$

12.12 Electron Volt

Electron Volt is a unit of energy. In atomic Physics, the energies of accelerated fundamental particles are frequently expressed in terms of electron Volts.

Let a charge + q move under the influence of an electric field from a point A to another point B whose potential is lower by ΔV. The electric potential energy of the system is reduced by $q\Delta V$ because this much work has to be done by an external agent to restore the system to its original condition. The decrease of potential energy appear as kinetic energy of the particle.

$$q\Delta v = \frac{1}{2} mv^2 \qquad \text{.................. (12.17)}$$

It suggests that the energy can be expressed as the product of potential difference and charge. If we adopt the quantum of charge e as a unit of charge in place of coulomb we arrive at another unit of energy, the electron volt.

An electron volt is the energy required by an electron in falling through a potential difference of 1 volt.

1 electron- volt = (1 quantum of charge) (1 volt)

$1 \text{ ev} = 1.6 \times 10^{-19}$ (Coulomb) (1 Volt)

$= 1.6 \times 10^{-19}$ Joules.

Example 12.10.

A electron acquires a speed of $10^6 ms^{-1}$ Find its energy in electron Volts

$$\text{Energy} = \frac{1}{2} mv^2 = \frac{1}{2} \times 9.1 \times 10^{-31} \times 10^{12} = 4.55 \times 10^{-19} \text{ J}$$

$$= \frac{4.55 \times 10^{-19}}{1.6 \times 10^{-19}} \text{ eV} = 2.84 \text{ eV}$$

12.13 Equipotential Surfaces

The potential distribution in an electric field may be represented graphically by equipotential surfaces.

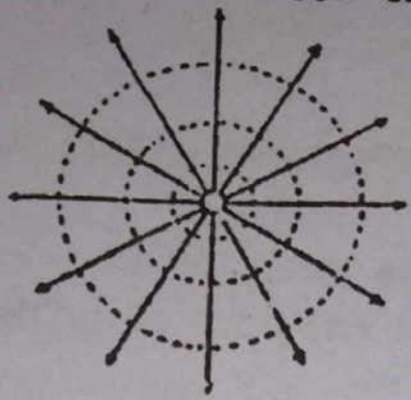
An equipotential surface is that, at all points of which the potential has the same value.Since the potential energy of a charged particle is the same at all points of a given equipotential surface it follows that no work is needed to move the charged particle over such a surface. Hence the surface through any point must be at right angles to the direction of the field at that point. The electric lines of force and equipotential surface are, therefore, mutually perpendicular.

A family of equipotentials can be sketched by drawing surfaces at right angles to the electric lines of force at each point.

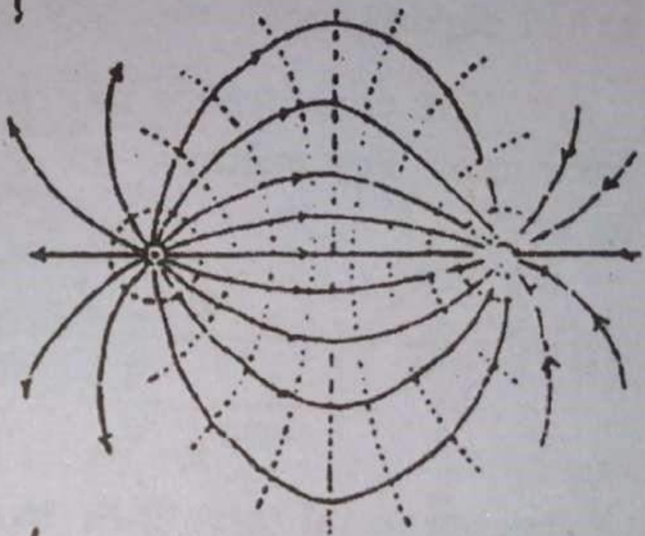
Equipotentials due to a few charge configurations are shown below.

Fig. 12.15

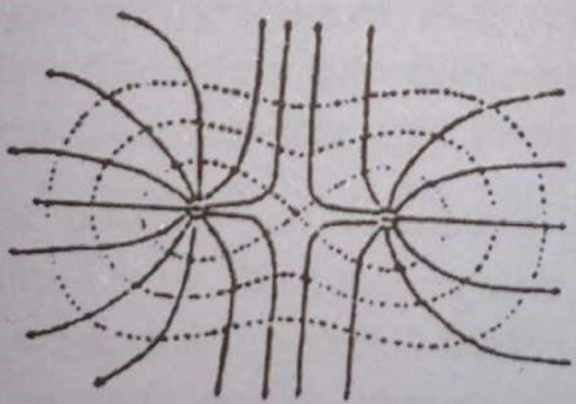
a)



b)



c)



Electric field lines due to different charge configuration.

12.14 Capacitance and Capacitors

The potential of a conductor depends on its own charge as well as the charges on the neighbouring bodies

For an isolated conductor

$q \propto V$

$q = CV$ .... (12.18)

C is a constant for a given conductor.

Electric charges generated by machines cannot be stored on a conductor beyond a certain limit as its potential rises to breaking value and the charge starts leaking to atmosphere.

$C = \frac{q}{V}$ measures how much charge should be placed on conductor to raise its potential to 1 Volt and it is taken as a measure of its capacity of holding electric charge called capacitance.

The capacitance depend, on the size and shape of conductor. For example for a spheric conductor of radius r

$$C = \frac{q}{V} = \frac{q}{\frac{1}{4\pi\epsilon_0}\frac{q}{r}} = 4\pi\epsilon_0 r \qquad \text{---------(12.19)}$$

i.e. proportional to radius. In general we can say for conductors of any shape that the greater the size of a conductor the greater is its capacitance.

The size of a conductor required to store huge amount of electric charge becomes very inconvenient and a device called capacitor is designed to have large Capacity of Storing electric charge without having large dimensions.

The principle of a capacitor is based on the fact that the potential of a conductor is greatly reduced without affecting the charge in it by placing another earth connected conductor or an oppositely charged conductor in its neighbourhood.

A system of two conductors separated by air or any insulating material forms a capacitor.

The conductors have equal and opposite charges and its capacitance is the ratio of the charge on one of the conductor to the potential difference between them.

The unit of capacitance is couls/Volt called farad.

Farad is a large unit and for practical purposes. convenient units are

micro farad $\mu F = 10^{-6}$ farads

pico farad $PF = 10^{-12}$ farads

12.15 Parallel Plate Capacitor

A very common and convenient type of capacitor is a parallel plate capacitor in which the conductors take the form of two plates parallel to each other and separated by a distance very small compared to the dimensions of the plates

Practically the entire field of such a capacitor is located in the region between the plates and it is uniform except at its outer boundary which is negligible when the plates are closer.

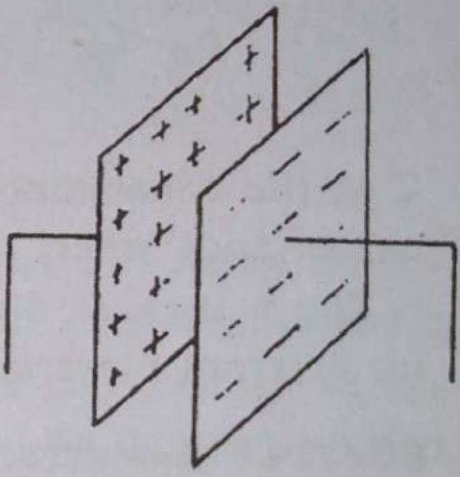


fig. 12 16

The charges on each plate are uniformly distributed on the inner sides of the plates due to attraction between opposite charges

Electric field $E = \dfrac{\sigma}{\epsilon_0}$

Potential difference $V = Ed = \dfrac{\sigma}{\epsilon_0} d$

where d is the distance between plates

Capacitance $C = \dfrac{q}{V} = \dfrac{A\sigma}{\sigma/\epsilon_0 d}$

$$C = \frac{\epsilon_0 A}{d}$$

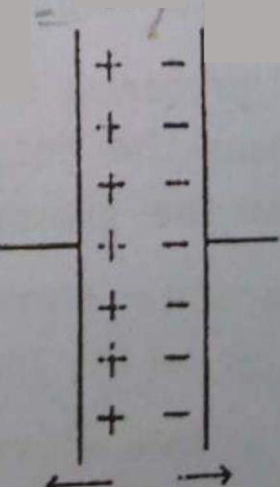


Fig.12.17

where A is the area of the plates.

If an insulating material completely fills the space between the plates

$$C = \frac{\epsilon A}{d} = \frac{\epsilon_r \epsilon_0 A}{d} \qquad \text{---------(12.20)}$$

The equation shows that the capacitance is directly proportional to the area of plates, inversely proportional to the distance between the plates and it is enhanced by ϵ_r if an insulating material called dielectric is introduced between the plates.

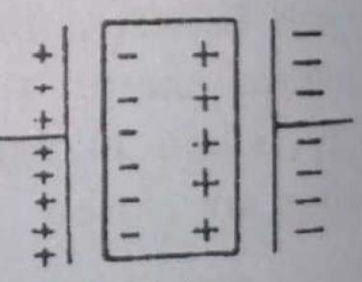
The relative permittivity or dielectric constant

$$\epsilon_r = \frac{\epsilon}{\epsilon_0} = \frac{C}{C_0} \qquad \text{---------(12.21)}$$

C is the capacitance when it has a dielectric. C_0 is the capacitance when the space between the plates is empty. The values of dielectric constants lie between 1—10 for different materials.

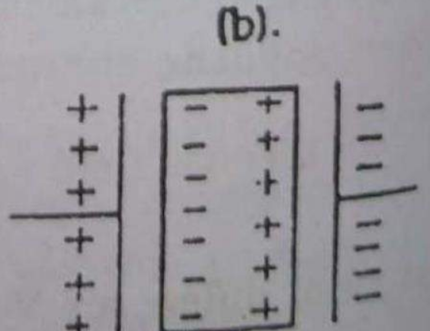
Effect of dielectric in a Capacitor

The electric field between the plates distorts the molecules of the dielectric. The molecules are polarized, one end becoming positive and other negative.



(a)

(b).



The presence of these charges decreases the potential difference between the plates (fig.12.18(a)).

A capacitor connected to a battery will accumulate more charge on the plates as shown in fig.12.18 (b).

In each case the dielectric increases the capaci-

of the capacitor.

Example 12.11.

The parallel plates of an air filled capacitor are 1.0 mm apart. What must the plate area be if the capacitance is 1 farad ?

Solution

$$d = 1.0\ mm = 1 \times 10^{-3}\ m.$$

$$C = 1\ F \quad \epsilon_0 = 8.9 \times 10^{-12}\ C^2 N^{-1} m^{-1}$$

Solution

$$C = \frac{\epsilon_0 A}{d}$$

$$A = \frac{Cd}{\epsilon_0} = \frac{1 \times 10^{-3}}{8.9 \times 10^{-12}} = 1.1 \times 10^{8}\ m^2$$

12.16. Combinations of Capacitors

Capacitors of some fixed values are being manufactured. For a circuit the capacitance of a desired value can however, be obtained by suitable combination of capacitors. Capacitors can be combined in paralleis, series or both.

a) Parallel Combination.

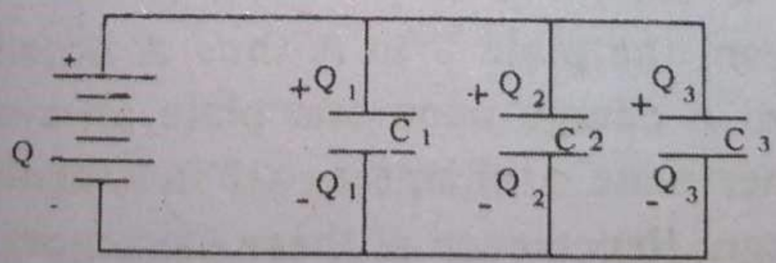


Fig: (12.19)

The figure (12.19) shows three capacitors connected in parallels. The upper plates of each capacitor are connected to a common terminal a and the lower plates to the other common terminal b.

A charge q given to the point a will divide itself to reside on the plates of individual capacitors according to their capacitance's such that

$$q = q_1 + q_2 + q_3$$

The potential difference V across each capacitor is that of the source

If C is the capacitance of the combination and C_1, C_2 and C_3 are the capacitance's of the capacitors joined in parallel

$$CV = C_1V + C_2V + C_3V$$

$$C = C_1 + C_2 + C_3 \qquad \text{------------------- (12.22)}$$

The capacitance of combination is equal to the sum of the capacitance's of individual capacitors. It is greater than the greatest individual one.

(b) Series Combination

The figure shows three capacitors having the right-hand plate of one connected to the left hand plate of the next and so on connected in series. When a cell is connected across the ends of the system, a charge .q is transferred from the plate F to A thus A becomes positively charged. A charge upon one plate always attracts upon the other plate a charge equal in magnitude and opposite in sign. Hence each of these capacitors hold the same quantity of charge.

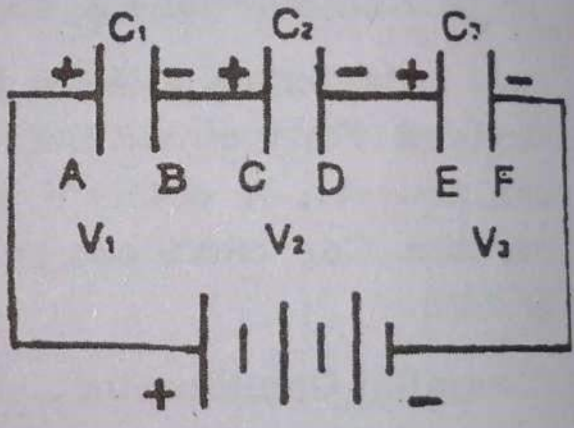


Fig. (12.20)

Let V be the potential difference across the combination. The potential differences across the individual capacitors V_1, V_2 and V_3 are such that their sum is equal to the applied potential difference V.

$$V = V_1 + V_2 + V_3$$

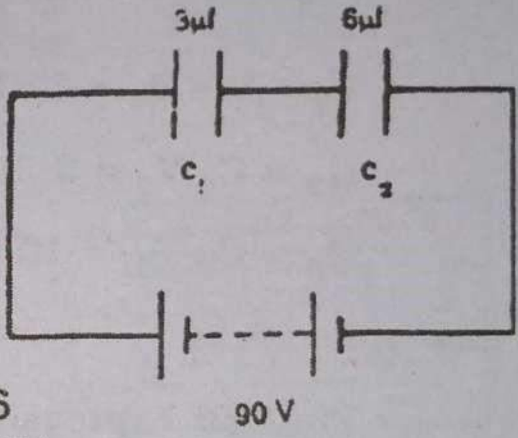
$$\frac{q}{C} = \frac{q}{C_1} + \frac{q}{C_2} + \frac{q}{C_3}$$

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_n} \quad \text{..................(12.23)}$$

Thus to find the resultant capacitance of capacitors in series, we must add the reciprocals of their individual capacitance's. It will give the reciprocal of the resultant capacitance. The resultant is less than the smallest individual capacitance.

Example 12.12

Two capacitors C_1 (3μ f) and C_2 (6μ f) are in series across a 90 Volts d c.supply Calculate the charges on C_1 and C_2 and the p.d.across each.



Solution

Total Capacitance C is given by

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2}$$

$$C = \frac{C_1\, C_2}{C_1 + C_2} = \frac{3 \times 6}{9}$$

$= 2\ \mu$ f.

The charges on C_1 and C_2 are the same and equal to q on C.

$$q = C\,V = 2 \times 10^{-6} \times 90 = 180 \times 10^{-6} \text{ C.}$$

$$V_1 = \frac{q}{C_1} = \frac{180 \times 10^{-6}}{3 \times 10^{-6}} = 60 \text{ volts}$$

$$V_2 = \frac{q}{C_2} = \frac{180 \times 10^{-6}}{6 \times 10^{-6}} = 30 \text{ volts}$$

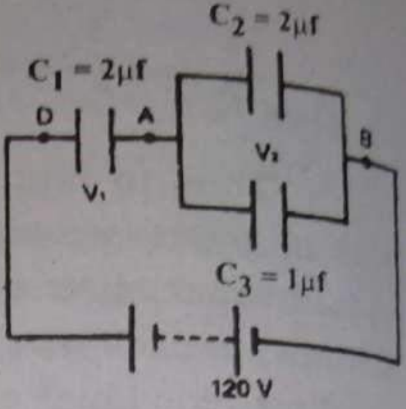
Example 12.13.

Find the charges on the capacitors connected as shown in the diagram across a 120 volts d.c. supply.

Solution.

Capacitance between A and B

$$C' = C_2 + C_3 = 3\ \mu f.$$



Overall Capacitance B to D

$$C = \frac{C_1\ C'}{C_1 + C'} = \frac{2 \times 3}{5} = 1.2\ \mu\ f.$$

Charge stored in this capacitance C

$$Q_1 = Q_2 + Q_3 = CV = 1.2 \times 10^{-6} \times 120 = 144 \times 10^{-6}\ \text{Coul.}$$

$$V_1 = \frac{Q_1}{C_1} = \frac{144 \times 10^{-6}}{2 \times 10^{-6}} = 72\ \text{Volts}$$

$$V_2 = V - V_1 = 120 - 72 = 48\ \text{Volts}$$

$$Q_2 = C_2 V_2 = 2 \times 10^{-6} \times 48 = 96 \times 10^{-6}\ \text{Coul.}$$

$$Q_3 = C_3 V_2 = 10^{-6} \times 48 = 48 \times 10^{-6}\ \text{Coul.}$$

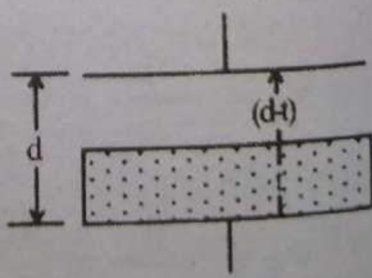
Example 12.14

Find an expression for the capacitance of a compound capacitor, the space between the plates of which is practically filled with a slab of dielectric.

Solution.

The capacitance of a parallel plate capacitor with dielectric completely filling the space between the plate is

$$C_d = \frac{\epsilon_r \epsilon_0 A}{d} = \frac{\epsilon_0 A}{\dfrac{d}{\epsilon_r}}$$



which is equivalent to the capacitance of an air capacitor with a distance $\dfrac{d}{\epsilon_0}$ between the plates.

It means that a thickness d of a dielectric is

equivalent to air thickness $\frac{d}{\epsilon_r}$

Hence the capacitance of a compound parallel plate capacitor with a slab of dielectric of thickness t partially filling the space between the plates.

$$C_d = \frac{\epsilon_0 A}{(d - t) + \frac{t}{\epsilon_r}}$$

(d-t) is the thickness of air space and the thickness t of

dielectric is equivalent of air space $\frac{t}{\epsilon_r}$

12.17 Different types of Capacitors

a) *Multiplate Capacitor:*

A multiplate capacitor consisting of large number of plates each of large area is designed to have large capacitance when N plates are used there are (N-1) individual capacitors in parallel.

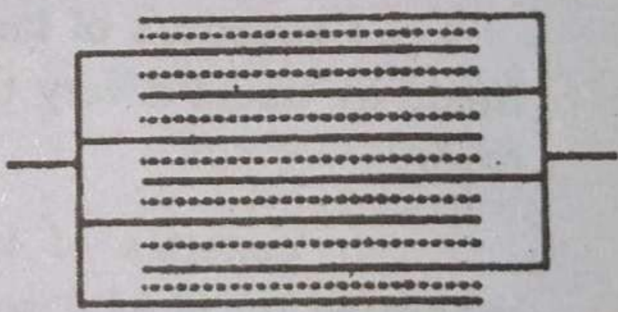
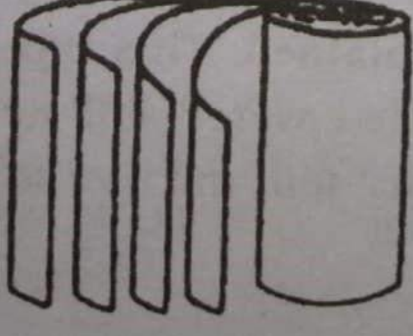


fig.12.21

In high grade capacitors mica is used as dielectric.

Inexpensive capacitors of capacitance upto 10 μF are usually made of alternate layers of tin or aluminium foil and waxed paper. These are frequently wound into rolls under pressure and sealed into moisture-resisting metal container.

fig.12 22



b) Variable Capacitor:

A variable capacitor of the kind used for tuning radio sets is shown in the diagram. It consists of two sets of semi circular aluminium or brass plates separated by air. One set of plates is fixed and the other is rotated by a knob to alter the effective area of the plates.

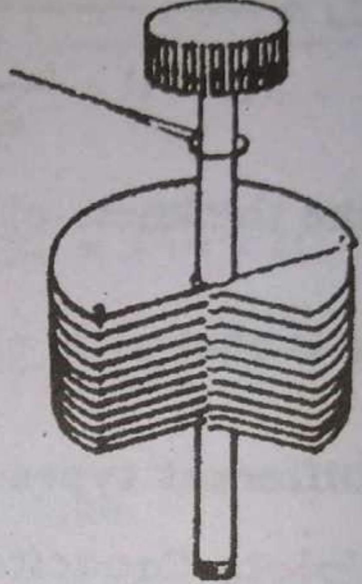


fig12.23

c) Electrolytic Capacitors.

Capacitors of large capacitance upto ·1000 μF are made by using a very thin insulating layer of aluminium oxide.

It consists of two sheets of aluminium equally separated by Muslin soaked in a special solution of ammonium borate. These are rolled up and sealed in an insulating container. Wires attached to the foil strips are then connected to an electric battery and a highly insulating thin film of aluminium oxide forms on the positive foil. A capacitor is thus formed in which the oxide film acts as the dielectric. Owing to the extreme thinness of the film, very large capacitance's which take up very little space may be obtained. This type of capacitor is used such that the Oxide-covered foil never become negative with respect to other foil and +ve and - ve terminals are marked.

12.1 Repulsion is the sure test of electrification. Explain.

12.2 Will a solid metal sphere hold a larger electric charge than a hollow sphere of the same diameter? Where does the charge reside in each case?

12.3 Explain why it is so much easier to remove an electron from an atom of large atomic weight than it is to remove proton?

12.4 Why it is not correct to say that potential difference is the work done in moving unit charge between the points concerned?

12.5 Why is it logical to say that potential of an earth connected object is zero? What can be said about the charge on earth?

12.6 Can an electric potential exist at a point in a region where the electric field is zero? Can the potential be zero at a place where the electric field intensity is not zero? Give examples to illustrate your reasoning.

12.7 An air capacitor is charged to a certain potential difference. It is then immersed in oil.What happens to its (a) capacitance (b) charge (c)Potential.

12.8 Two unlike capacitors of different potentials and charge are joined in parallel. What happens to their potential difference? How are their charges distributed? Is the energy of the system affected?

12.9 Four similar capacitors are connected in series and joined to a 36 Volts battery. The mid-point of group is earthed. What is the potential of the terminals of the group?

12.10 A point charge is placed at the centre of a spherical Gaussian surface. Is the flux changed:-

a) If the spherical Gaussian surface is replace by a cube of the same volume.

b) If the sphere is replaced by a cube of $\frac{1}{10}$ of this volume.

c) If the charge is moved from the centre in the sphere.

d) If the charge is moved outside the sphere.

e) If a second charge is placed inside the sphere.

12.11 Four capacitors each of 2 μf are connected in such a way that total capacitance is also 2 μf. Show what combination gives this value.

12.12 A capacitor is charged by a battery. The battery is disconnected and a slab of some dielectric is slipped between the plates. Describe what happens to the charge, capacitance and potential difference.

12.13 Answer question 12.12 if the battery is not disconnected.

12.14 A capacitor is connected across a battery why does each plate receive a charge of the same magnitude? Will it be true even if the plates are of different sizes?

PROBLEMS.

12.1 Two unequal point charges repel each other with a force of 0.2 newtons when they are 10 cm apart. Find the force which each exerts on the other when they are

(a) 1 cm apart (b) 5 cm apart (Ans. 20N. 0.8N)

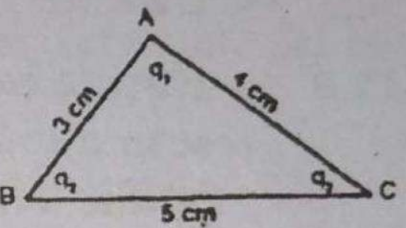
12.2 Two point charges of $+ 1\times 10^{-4}$ and $- 1 \times 10^{-4}$ coul are placed at a distance of 40 cm from each other. A Charge $+ 6 \times 10^{-5}$ coul is placed midway between them. What is the magnitude and direction of force on it.

(Ans. 2700×10^{3} N towards -ve charge.)

12.3 Three point charges each of 4 μ.C. are placed at the three corners of a square of side 20 cm. Find the magnitude of the force on each.

(Ans. 5.03 N. 5.04 N)

12.4 Three charges



$q_1 = + 7\times 10^{-6}$ C

$q_2 = - 4 \times 10^{-6}$ C and

$q_3 = - 5 \times 10^{-6}$ C are placed

at the vertices of a triangle as shown in the diagram.

The sides of the triangle measure 3,4 and 5 cm. Determine the magnitude and direction of the force on the charge q_1.

(Ans. 342.24 N. in the direction making an angle of 35.10° with BA)

12.5 Two small spheres, each having a mass of 0.1 gm, are suspended from the same point by silk threads each 20 cm long. The spheres are given equal charges and they are found to repel each other, coming to rest 24 cm apart. Find the charge on each.

(Ans. 6.86×10^{-8} C).

12.6 Two charges of $+ 2 \times 10^{-7}$ C and -5×10^{-7} C are placed at a distance of 50 cm from each other.

Find a point on the line joining the charges at which the electric field is zero.

(Ans. 86 cm away from 2×10^{-7} C charge)

12.7 What are the electric field and potential at the centre of a square whose diagonals are 60 cm. each when (a) charges each of 2μc are placed at the four corners. (b) charges of + 2μc are placed on the adjacent corners and − 4μc on other corners.

(Ans. (a) O. 2.4×10^5 volts. (b) 8.5×10^5 wc^{-1}. -1.2×10^5 V)

12.8 A particle carrying a charge of 10^{-5} C starts from rest in a uniform electric field of intensity 50 Vm^{-1}. Find the force on the particle and the kinetic energy it acquires when it has moved 1.m.

(Ans. 5×10^{-4} N. 5×10^{-4} J)

12.9 A proton of mass 1.67×10^{-27} Kg and charge 1.6×10^{-19} C is to be held motionless between two horizontal parallel plates 10 cm apart. Find the Voltage required to be applied between the plates.

(Ans. 1.02×10^{-8} Volts)

12.10 A small sphere of weight 5×10^{-3} N is suspended by a silk thread 50 mm long which is attached to a point on a large charged insulating plane. When a charge of 6×10^{-8} C is placed on the ball the thread makes an angle of 30° with the vertical. What is the charge density on the plane.

(Ans. $\sigma = 8.5 \times 10^{-7}$ cm^{-2})

12.11 How many electrons should be removed from each of the two similar spheres each of 10 g so that electrostatic repulsion be balanced by gravitational force.

(Ans. $n = 5.4 \times 10^6$)

12.12 There is a potential difference of 150 volts between two conductors of a power line. A charge of 600 C is carried from one conductor to the other. What work is required? If the time necessary to transport the charge is 1.25 s how much power is used?

(Ans. 9×10^4 .J. 7.2×10^4 watts)

12.13 A metal sphere of 100 mm radius has a charge of 4.25×10^{-9} coul. What is the potential? (a) at its surface (b) at its centre.

What is the potential energy of a charge of 2.5×10^{-6} C at a point 150 mm from the centre of sphere?

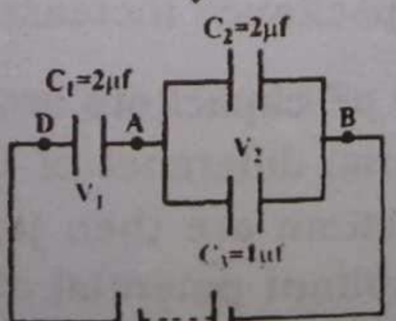
(Ans. 3.825×10^2 volts. 3.825×10^2 volts. 6.375×10^{-4} J)

12.14 An electron having an initial velocity of 10^{+3} cms^{-1} is directed from a distance of 1 mm at another electron whose position is fixed. How close to the stationery electron will the other approach.

(Ans: 0.533 mm)

12.15 Find the equivalent capacitance and charge on each of the capacitor shown in the diagram.

(Ans. 3 μf. q_1 = 10 μc. q_2= 20 μc. q_3= 30 μc)



12.16 Two capacitors of 2 μf and 8 μf are joined in series and a potential difference of 300 volts is applied. Find the charge and potential difference for each capacitor.

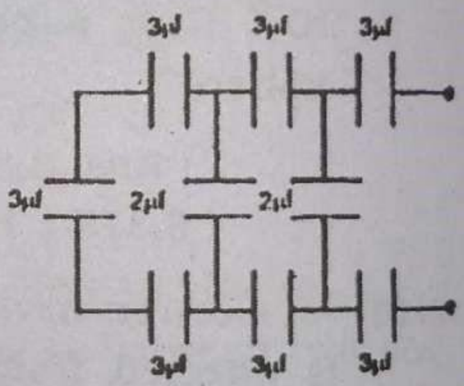
(Ans. $q_1 = q_2 = 4.8 \times 10^{-3}$ C. $V_1 = 240$ Volts, $V_2 = 60$ Volts)

12.17 A capacitor of 100 pF is charged to a potential difference of 50 volts. Its plates are then connected in parallels to another capacitor and it is found that the potential difference between its plates falls to 35 volts. What is the capacitance of the second capacitor.

(Ans. 42.85 pF)

12.18 Find the equivalent capacitance of the combination shown in the diagram.

(Ans:1μf)



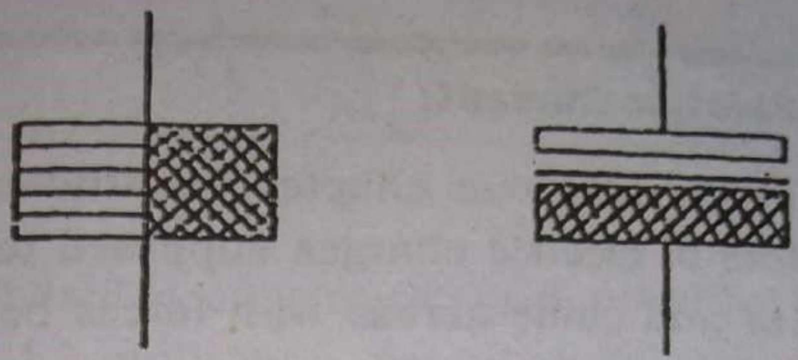
12.19 A parallel plate capacitor has plates 30 cm x 30 cm separated by a distance of 2 cm. By how much the capacitance changes if a dielectric slab of the same area but of thickness 1.5 cm is slipped between the plates. The dielectric constant of the material is 2.

(Ans. Capacitance increase by 2.39×10^{-11} farad)

12.20 Three 1.0 pF capacitors are charged separately to the potential difference of 100, 200 and 300 volts. The capacitors are then joined in parallels. What is the resultant potential difference

(Ans. 200 Volts)

12.21 Compare the capacitance's of two identical capacitors with dielectrics inserted as shown in the diagram. The dielectric constants are K_1 and K_2



$$\left(\text{Ans. } C_b/C_a = \frac{4K_1\, K_2}{(K_1+K_2)^2}\right)$$

12.22 A capacitor of 10 μf and one of 20 μf are connected across batteries of 600 volts and 1000 volts respectively and then disconnected. They are then joined in parallels. What is the charge on each capacitor?

(Ans. 8.66×10^{-3} C. 17.32×10^{-c} C)

12.23 Attempt the problem 12.23 with the difference that the capacitors are joined in series after being charged. as before.

(Ans. $q_1 = q_2 = 1.066 \times 10^{-12}$C)

Chapter. 13

CURRENT ELECTRICITY

13.1. Electric Current:

In the previous chapter we studied the behavior and effects of electric charges supposed to be in stationary states and came across with forces between charges, the electric potential. From this chapter onwards, we shall remain concerned with the study of the properties and effects of electric charges when they are in motion. This field is known as electrodynamics.

The first thing which comes to our mind in the study of electricity is the generation of electric current, which we shall show as the rate of motion of charges in a conductor.

The best conductors are metals, particularly silver, copper, gold, aluminium. In these metals, the electric charges can flow very easily. In metallic conductors, there is always present a large number of free electrons actually detached from their parent atoms which constitute a kind of electron gas. These free electrons are capable of moving freely in random directions in the interatomic space of the metal. In insulators, the electrons are rigidly bound to the atoms and so cannot move and hence no current can flow through them.

In the absence of an electric field across the conductor, the free electrons have a thermal velocity at normal temperature which is of the order of a million metre per second. The velocities of these electrons are randomly directed and in effect the number of electrons moving in one direction is just equal to the number moving in the opposite direction to maintain the neutral state of the conductor.

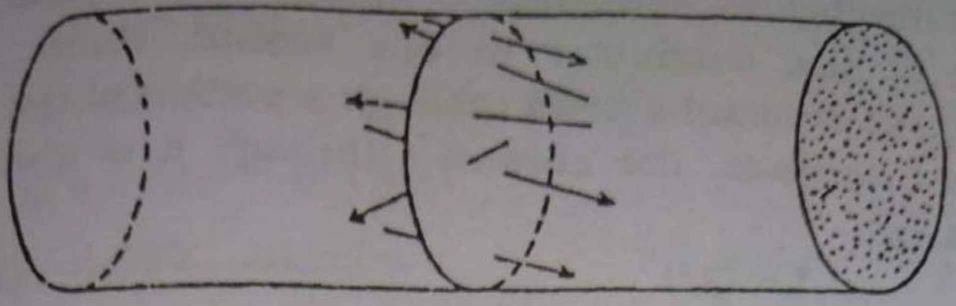


fig. 13.1

On establishing an electric field between the two ends of a wire by applying a potential difference, these free electrons having a negative charge on them, experience a force in the opposite direction to the field and hence acquire an acceleration in that direction. These free electrons which, in the absence of the electric field were moving towards the positive terminal of the battery are now accelerated while those moving away from the said terminal are retarded, the net effect is that an additional component of velocity towards the positive terminal is superimposed on the free electrons over their thermal velocity after they suffered certain number of collisions with the atoms.This additional component of velocity due to the electric field is known as drift velocity. The magnitude of this velocity is of the order of 0.01 metre per second. This drift velocity in the free electrons is really responsible for the generation of electric current in the wire. Thus we see that electric current in a conductor is due to the flow of electrons.

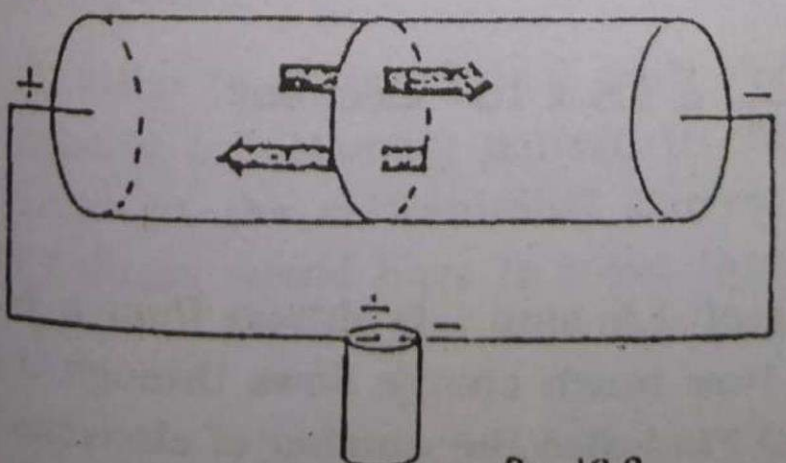


fig. 13.2

We define the strength of current in a conductor as the number of coulombs of charge which pass any section of the conductor in one second. Hence if a charge of Q coulombs flows through a section of the wire in time t seconds, the current I through it is given by the relation.

$$I = Q/t \qquad \text{(13.1)}$$

The unit of current is one coulomb per second. This unit in S.I. is called an "ampere", after the name of the French scientist Andre Marie Ampere. The smaller units of current are milliampere (mA) = 10^{-3} ampere and micro ampere (μ A) = 10^{-6} ampere.

Example 13.1

A current of 2.4 amp. is flowing in a wire. How many electrons pass a given point in the wire in one second if the charge on an electron is 1.6×10^{-19} coulomb.

Reasoning: If we know the total charge passing the point in 1 sec., we can divide that by the charge on an electron and thereby obtain the number of electrons.

Solution:

Since I is the number of coulombs passing a point in the wire in 1 sec. we have that 2.4 coulomb of charge flows that point in 1 second.

But each electron carries a charge 1.6×10^{-19} C. so that number of electrons passing per second is

$$\frac{2.4}{1.6 \times 10^{-19}} = 1.5 \times 10^{19} \text{ electrons}$$

Example 13.2

A current of 1.6 amp is drawn from a battery for 10 minutes. How much charge flows through the circuit in this time? Find also the number of electrons flowing during this time.

Solution:

$$I = 1.6 \text{ amp}$$

$$t = 10 \text{ minutes} = 600 \text{ second}$$

Now $$Q = I \times t = 1.6 \times 600$$

$$= 960 \text{ coulombs.}$$

$$\text{Number of electrons} = \frac{960}{1.6 \times 10^{-19}} = 6 \times 10^{21}$$

13.2. DIRECTION OF A CURRENT:

There are two equivalent ways of describing the same current as a flow of negative charge in one direction or an equal flow of positive charge in the opposite direction. When a current is set up in a wire by connecting its ends respectively to the two terminals of battery, the electrons move along the circuit from the negative terminal to the positive terminal of the battery. This we call as the electronic current.

After the discovery of electron, the electric current was shown due to the flow of electrons and hence the direction of electric current should have been reversed to be that of electronic current but we know that a negative charge moving in one direction is in effect, equivalent to a positive charge moving in the opposite direction in a wire, therefore, the old belief about the direction of current from positive to the negative terminal of the battery has been retained.

In order to differentiate it with electronic current, it has become a customary pattern to consider the electric current as the conventional current in which the positive charges would have to move through the circuit from a point of higher potential to a point at lower potential.

13.3. ELECTRIC RESISTANCE AND OHM'S LAW.

It is a common fact that flow of a fluid in a medium under some applied force experiences some sort of friction or resistance in its path. The electric current we have defined as the flow of electrons in the conductor, it is logical to assume that these electrons would come across some sort of resistance during their flow.

We assigned the strength of the current due to the drift velocity of the electrons which depends upon the strength of the electric field. These free electrons during their course of motion towards the positive terminal of the battery experience collisions with the atoms of the conductors which in turn reduces or some times destroys the velocity gained due to the accelerations.The current I should be proportional to the potential difference V between the two ends of the conducting wire.

or $I \propto V$

$$I = KV \qquad \text{------------------(13.2)}$$

where K is a constant known as the conductance of the material of the wire.

It is customary to speak of the resistance R of the wire instead of the conductance K, which is the reciprocal of the conductance i.e.

$$R = \frac{I}{K}$$

or equation 13.2 becomes

$$I = \frac{V}{R} \qquad \text{------------------(13.3)}$$

This equation is known as Ohm's law, discovered by the German scientist George Simon Ohm who found that a linear relationship existed between current and potential difference, for which the graph is a straight line as shown in fig. 13.3

We define Ohm's law for metallic conductors as "The current through a conductor is directly proportional to the potential difference between the ends of the conductor, provided that physical conditions remain the same". Thus

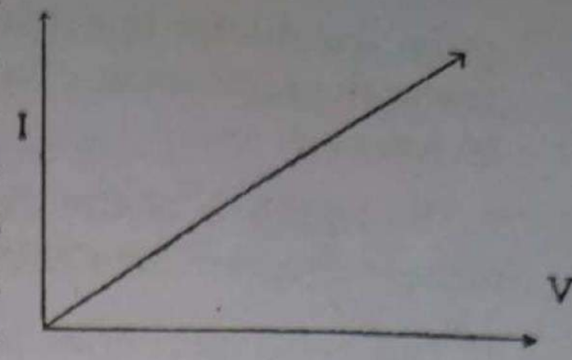


fig 13.3

$$\frac{V}{I} = R, \text{ a Constant}$$

or Resistance = Volts per ampere

The S.I. unit of resistance is the Ohm. Shown as Ω. (The capital Greek letter Omega).

Therefore one ohm is the resistance of a conductor through which a current of one ampere passes when a potential difference of one volt is maintained across the end of the conductor. Large resistances are more conveniently expressed in Kilo Ohm ($K\Omega = 10^3$ Ohm) or Mega-ohm ($M\Omega = 10^6$ Ohm) and small resistances in milliohm ($m\Omega = 10^{-3}$ Ohm) or micro Ohm ($\mu\Omega = 10^{-6}$ Ohm).

It is important to mention that Ohms law is valid only for metallic resistance at a given temperature and for steady currents.

Example:- 13.3

A light bulb has resistance of 150 Ω, find the current in it when it is connected to a 225 volt source.

Solution:

From Ohms law $I = V/R$

$$= \frac{225\ V}{150} = 1.5\ A.$$

13.4 RESISTIVITY

From Ohm's law, we saw that the resistance of a given conductor is constant under certain conditions. Actually the resistance of a conductor depends upon certain factor, such as

i) The length L of the conductor; the longer the conductor, the greater should be its resistance.

ii) The cross sectional area. A of the conductor; thicker wires have less resistance, and

iii) The material of which the conductor is made. Under the same conditions, wire of different metals show different values of resistances.

From the above conditions we see that resistance of a conductor or wire is directly proportional to its length and inversely proportional to its cross sectional area. That is

$$R \propto L/A$$

or $$R = \rho \frac{L}{A} \quad \text{----------(13.4)}$$

The constant of proportionality ρ(Greek letter rho) is known as the resistivity or specific resistance of the material which means that resistivity is a property of the material.

Since $\rho = \frac{RA}{L}$, therefore the S.I. unit of resistivity is Ohm metre (Ω.m).

By definition, the resistivity of a material is the resistance of a wire of the material per unit of its length and per unit area of its cross section, in other words the resistance of a cube of unit length.

Dependence of resistivity upon temperature:

The electrical resistance of most metals increases with the increase in temperature, because atoms sitting on their sites in metal start vibrating more violently about their mean position due to increase in temperature. This increases the probability of collision of free electrons with them which ultimately affects the drift velocity of free electrons for a given applied voltage. As we have seen earlier that the electric current depends upon the drift velocity of the free electrons, so with rising temperature, the decrease in the drift velocity is attributed to the decrease in the electric current or the increase in the electrical resistance for a given applied voltage.

Experimentally it has been observed that the change in the resistance of a metallic conductor with the change in the temperature of the conductor is nearly linear over a wide range of temperature above and below $0^\circ C$. Let us suppose that the resistance of a wire at $0^\circ C$ is R_o and at a higher temperature $t^\circ C$, it is R_t. The change in resistance is $R_t - R_o$ for a change in temperature $(t^\circ - 0^\circ)$ C. If we denote the changes in resistance and temperature respectively by ΔR and Δt, then

$$\Delta R \propto R_o \,.\, \Delta t$$

$$= \alpha R_o \Delta t$$

Where α is the constant of proportionality

or $$R_t - R_o = \alpha R_o . \Delta t$$

or $$\alpha = \frac{R_t - R_o}{R_o - \Delta t} \qquad \text{-------------------(13.5)}$$

Thus the constant α gives the fractional change in resistance per unit resistance per Kelvin change in temperature. It is known as the temperature Coefficient of resistance or of resistivity.

Since resistivity ρ is directly proportional to the resistance of a metal. We can thus derive that

$$\alpha = \frac{\rho_t - \rho_o}{\rho_o \times \Delta t}$$

where ρ_t and ρ_o refer to resistivities at $t^\circ C$ and $0^\circ C$ respectively

or $\rho_t = \rho_o (1 + \alpha . \Delta t)$ ....(13.6)

The following table mentions the values of ρ and α all at $0^\circ C$

Example 13.4

Find the potential difference across two ends of a copper wire to maintain a steady current of one ampere The length of the wire is one metre, radius of cross-section is 0.25 cm.

Solution

The resistance of the copper wire will be given by the formula.

$$R = \rho \frac{L}{A}$$

$$L = 1 \text{ m}$$

$$\rho = 1.54 \times 10^{-8} \,\Omega - \text{m}$$

$$r = 0.25 \text{ cm} = 0.0025 \text{ m};$$

$$A = \pi r^2 = ?$$

$$A = \frac{22}{7} \times (0.0025)^2 \text{ m}^2$$

$$= 19.6 \times 10^{-6} \text{ m}^2$$

Hence $$R = 1.54 \times 10 \times \frac{1}{19.6 \times 10^{-6}} = 0.000785 \,\Omega$$

and the potential difference required will be

$$V = IR$$

$$= 1.0 \text{ A} \times 0.000785 \,\Omega$$

$$V = 0.000785 \text{ Volts}$$

Table 13.1

Resistivities ρ and Temperature Coefficients α. All at $0^{\circ}C$.

Material	ρ (Ω.m)	α $(^{\circ}C)^{-1}$
Silver	1.52×10^{-8}	0.0038
Copper	1.60×10^{-8}	0.0039
Gold	2.27×10^{-8}	0.0034
Aluminium	2.63×10^{-8}	0.0040
Tungsten	5.0×10^{-8}	0.0045
Iron	11.0×10^{-8}	0.0052
Platinum	11.0×10^{-8}	0.00392
Constantan (Cu 60% + Ni 40%)	49×10^{-8}	0.00001
Nichrome (Ni 60% + Fe 24% + Cr 16%)	100×10^{-8}	0.00004
Manganin (Cu 84% + Mn 12% + Ni4%	44×10^{-8}	0.00000
Carbon	35×10^{-6}	-0.0005
Germanium	0.60	-0.048
Silicon	2300	-0.075
Wood	10^{13}	
Glass	$10^{10}-10^{14}$	
Mica	$10^{11}-10^{15}$	

Examples: 13.5.

A rectangular bar of iron is 2 by 2 cm in cross section and is 40 cm long. what is the resistance of the bar when ρ for iron is 1.1×10^{-7} Ω m.

Solution:

We know that $$R = \rho \frac{L}{A}$$

$$= \frac{1.1 \times 10^{-7} \times 0.40}{(0.02 \times 0.02)} = \frac{4.4 \times 10^{-8}}{4.4 \times 10^{-4}}$$

$$= 1.1 \times 10^{-4}\ \Omega$$

Example 13.6.

What would be the resistance of the above bar at 500°C.?

Solution:

We have from the table that $\alpha = 5.2 \times 10^{-3}\ C^{-1}$

$$\frac{R_t - R_o}{R_o} = \alpha\,(t - o)$$

$$\frac{R_t - 1.1 \times 10^{-4}}{1.1 \times 10^{-4}} = 5.2 \times 10^{-3}\ (500 - 0)$$

from which

$$R_t - 1.1 \times 10^{-4} = 1.1 \times 10^{-4} \times 5.2 \times 10^{-3} \times 500$$

$$= 2.86 \times 10^{-4}\ \Omega$$

or $$R_t = (2.86 + 1.1) \times 10^{-4}\ \Omega$$

$$= 3.96 \times 10^{-4}\ \Omega$$

The resistance of the bar is roughly four times higher at this temperature than it was at $0^{o}C$

Example: 13.7.

The resistance of a platinum resistance thermometer is 200.0 Ω at 0°C and 257.6Ω when immersed in hot bath. What is the temperature of the bath when α for platinum is $0.00392°C^{-1}$?

Solution:

$$\frac{R_t - R_o}{R_o \times t} = \alpha$$

$$\text{or} \quad t = \frac{R_t - R_o}{\alpha \times R_o} = \frac{257.6 - 200.0}{0.00392 \times 200} \,^\circ C$$

$$\frac{57.6}{0.784} \,^\circ C = 73.5^\circ C$$

Example: 13.8.

A water heater draws 30 A from a 220 volts power source 15 meters away. What is the minimum cross section of the copper wire that can be used if the voltage is not to be lower than 210 volts at the heater?

Solution:

The voltage drop is (220 − 210) = 10 V and the resistance that corresponds to this drop when the current 30A is

$$R = \frac{V}{I} = \frac{10V}{30A} = 0.33\Omega$$

The total length of the wire involved is twice the distance between the source and the heater, so the length of the wire will be 2× 15m = 30 m.

From the table ρ for copper is $1.6 \times 10^{-8}\ \Omega$ m.

$$\text{Cross-sectional area } A = \frac{\rho.L}{R} = \frac{1.6 \times 10^{-8} \times 30 \times 30}{10} m^2$$

$$= 1.44 \times 10^{-6}\ m^2$$

$$= 1.44\ mm^2$$

13.5. COMBINATION OF RESISTORS.

Quite often we find an electric circuit containing a large number of elements such as resistors, capacitors or batteries inter connected together in a complicated manner. We call such circuits a network.

For the cases of resistors, we consider here few types of networks.

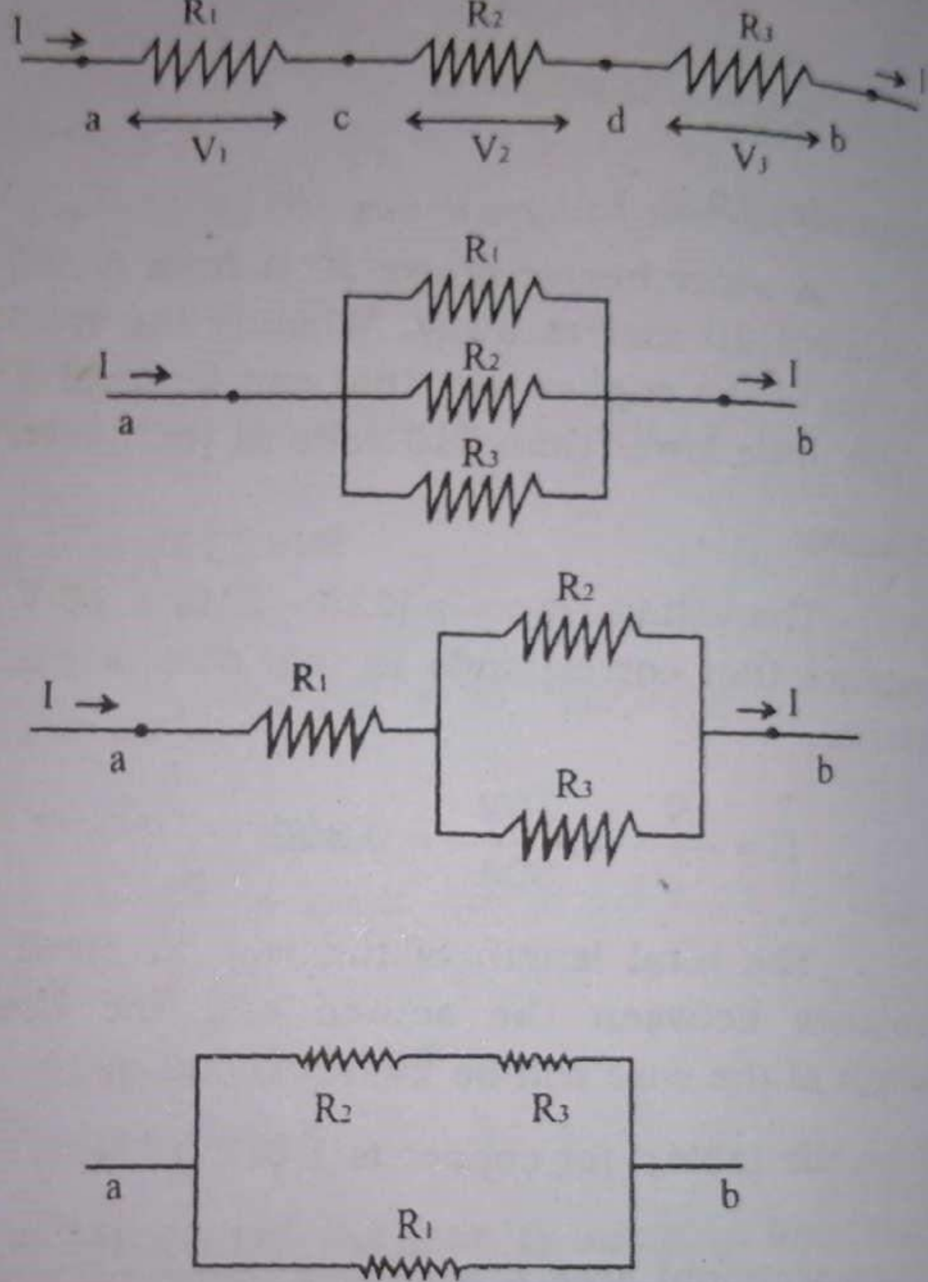


Fig. (13.4) (i to iv)

The above figures 13.4 (i to iv) show four different ways in which three resistors R_1, R_2 and R_3 might be connected to form a network. In fig.13.4 (i) the resistor are joined end to end providing a single path to the current between the points a and b. Such an arrangement of resistors is called series connection. Any number of resistors can be so joined in which the same current will flow through each resistor.

The resistors in fig. 13.4 (ii) are said to be joined in parallel between the points a and b. In this arrangement, each resistor provides an alternate path to the current between the end points a and b which means that current I is divided at the point x into three different paths in the respective resistors and ultimately rejoins at the end point b. In fig 13.4 (iii) resistors R_2 & R_3 are joined in parallel with one another and this combination is in series with resistor R_1 while in fig.13.4 (iv) resistors R_2 and R_3 are in series and this combination is in parallel with R_1.

We can find a single resistor equivalent in value which could replace a combination of resistors in any given network keeping the potential difference and the current unaltered in the circuit. If any one of the network in the given figures were replaced by its equivalent resistance R we could safely write.

$$V_{ab} = R\,I \qquad \text{------------------ (13.7)}$$

where V_{ab} is the potential difference between the points a and b and I is the current in the circuit.

(1). Resistances in Series:

Referring to figure 13.4 (1) . The current I in the series combination is the same in each resistor when the potential difference between the points a and b is V_{ab} :

As $V_1 = IR_1$, $V_2 = IR_2$ and $V_3 = IR_3$

As $V_{ab} = V_1 + V_2 + V_3$

$$IR = IR_1 + IR_2 + IR_3$$

$$R = R_1 + R_2 + R_3 \qquad \text{------------------(13.8)}$$

Hence $R = R_1 + R_2 + R_3$ from eq: 13.8. The equivalent resistance of any number of resistors in series equals the sum of the values of individual resistances.

(2). Resistances in Parallel

In this connection, the potential defferences between the terminals of each resistors must be the same and equal to V_{ab}, while the current I at the point x (in fig.13.4(II)) is divided into three resistors R_1, R_2 and R_3 as I_1, I_2 and I_3 respectively

From Ohm's law

$$I_1 = \frac{V_{ab}}{R_1},\ I_2 = \frac{V_{ab}}{R_2} \text{ and } I_3 = \frac{V_{ab}}{R_3}$$

But $\quad I_1 + I_2 + I_3 = I$

or

$$I = \frac{V_{ab}}{R_1} + \frac{V_{ab}}{R_2} + \frac{V_{ab}}{R_3}$$

$$\frac{I}{V_{ab}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

Since $\quad \frac{I}{V_{ab}} = \frac{1}{R}$ we have

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \quad \text{-----------(13.9)}$$

that is the sum of the reciprocals of individual resistances connected in parallel is equal to the reciprocal of the equivalent resistance. Since we have defined the conductance of a material as the reciprocal of its resistance, hence we can say that in the parallel combination of resistances, the sum of the individual conductance is equal to the equivalent conductance in the network.

The equivalent resistances of network in fig.13.4.(iii) and (iv) can be found by the same method by considering them as combinations of series and parallel arrangements. Thus in fig. 13.4. (iii), the combination of R_2 and R_3 in parallel is first replaced by its equivalent resistance R_4 which then forms a series combination with R_1 giving $R = R_1 + R_4$ and in fig.13.4. (iv) the combination of R_2 and R_3 in series forms a parallel combination with R_1 hence R can be determined.

Example:13.8.

A battery of 6 volts is connected to two resistors of 3Ω and 2Ω joined together in series. Find the current through the circuit and the potential drop across each resistor.(fig.13.5.)

Solution:

Since the resistors are joined in series, the equivalent resistance R will be equal to 3Ω + 2Ω = 5Ω. Therefore the current *I* through the circuit will be

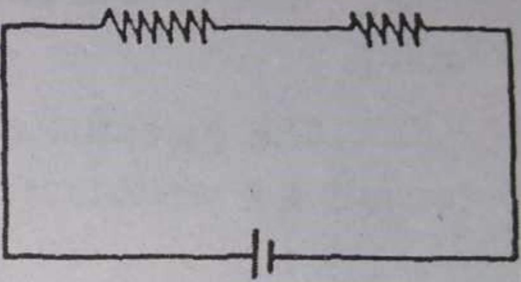


Fig.13.5

$$\frac{6v}{5\Omega} = 1.2\ A$$

The voltage drop across 3Ω resistor

$$V = IR = 1.2A \times 3\Omega = 3.6\ V$$

& the voltage drop across 2Ω resistor

$$V_2 = IR = 1.2A \times 2\Omega = 2.4\ V$$

So that $V_1 + V_2 = 3.6V + 2.4V = 6V$ which is the voltage of the battery.

Example:13.9.

Find the equivalent resistance in the given circuit and also the current I_1, I_2 and I_3

Solution:

For the parallel Combination of R_2, R_3 and R_4 the equivalent

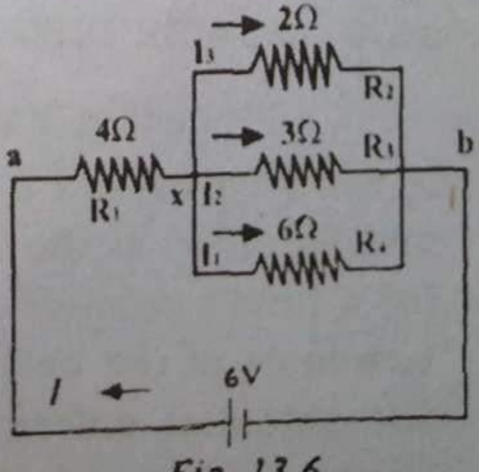


Fig. 13.6

resistance R_5 is given by

$$\frac{1}{R_5} = \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4} = \frac{1}{2} + \frac{1}{3} + \frac{1}{6} = 1$$

or $R_5 = 1\Omega$

This R_5 is now in series with R_1 hence

$$R = R_1 + R_5 = 4\Omega + 1\Omega = 5\Omega$$

The current I in the circuit $= \frac{V}{R} = \frac{6v}{5\Omega} = 1.2A$

Current I_1 can be calculated if we know the potential difference existing between the two end points x and b.

The potential difference between points a and x including a resistance of

$$4\Omega = IR = 1.2\ A \times 4\Omega = 4.8\ v.$$

Therefore the potential difference between points a and b will be

$$6V - 4.8V = 1.2V$$

or the current I_1 across a resistance of $6\Omega = \frac{1.2V}{6\Omega} = 0.2A$

Similarly $I_2 + I_3$ will be found as 0.4 A and 0.6A respectively so that $I_1 + I_2 + I_3 = 0.2A + 0.4A + 0.6A = 1.2A$ which is total current from battery to the circuit.

13.6 POWER DISSIPATION IN RESISTORS

Suppose a battery is connected across a resistor R which produces a potential difference of V across its end fig. 13.7. If the current I flows through this resistor for a time t seconds, the charge transported between the terminals of the battery is given by $Q = I \times t$. Since V is the potential difference causing the transfer of charge, therefore the work done in transfering charge $Q = QV$. This work is done at the expense of the potential energy of the charges as they pass through the resistor. This loss of potential energy is converted into vibrational energy of the atoms to which the electrons collide during their motion and thus the energy lost by the electrons is gained by the atoms of the conductor (resistor) in the form of heat.

Hence the heat developed in the resistor in t second is QV We define power as the rate of doing work i.e the work done or energy spent per unit time, hence the power dissipated as heat due to electric current in the resistor is given by

$$\text{Power} = \frac{\text{Work done}}{\text{time}}$$

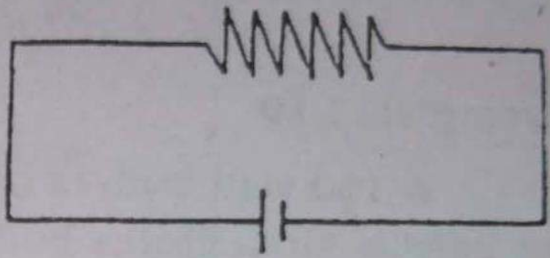


fig.13.7.

$$P = \frac{QV}{t} = VI \text{----------} 13.9.$$

as the current $I = Q/t$

This is the general relation for power delivered from a source of electric current operating at a voltage V. From ohms law equation (13.9) can be converted in terms of resistance R by changing I or V.

Hence we have

$$P = VI = I^2R = V^2/R \qquad \text{------------------} (13.10)$$

Recalling that volt is joule per coulomb and current is coulomb per second the unit of power is

$$\frac{\text{joule}}{\text{coulomb}} \times \frac{\text{coulomb}}{\text{second}} = \frac{\text{joule}}{\text{second}}$$

Which is called watt.

Higher values of power are expressed in kilowatt = 1000 watts or megawatt = 10^6 watts.

If a current I flows steadily through a resistor R for a time t, the total heat energy supplied to the resistor is given by heat energy = Power x time = $V.I.t = I^2Rt = \frac{V^2}{R}t$ joules.

Usually the energy supplied by electric current through its generating stations is measured in terms of a unit known as Kilowatt-hour (KWh) which is the energy delivered by the current in one hour when it supplies energy at the rate of 1000 joule per second i.e.

$$1 \text{ KWh} = 1000 \text{ Joule/Sec} \times 1 \text{ hour}$$

$$= 1000 \text{ J/s} \times 3600\text{s}$$

$$= 36 \times 10^5 \text{ Joules}$$

Example:13.10

A 100 watt bulb is operated by 240 volts. What is the current through the bulb?

Solution:

Since $P = VI$

$100 \text{ watt} = 240 \text{ V} \times I$

$$I = \frac{100 \text{ watts}}{240\text{V}} = 0.416 \text{ A}$$

Example: 13.11.

An electric Kettle of 1000 watts rating boils a certain quantity of water in 8 minutes. How much heat has been generated for boiling this water.

Solution:

$$\text{Heat} = \text{Power} \times \text{time}$$

$$= 1000 \text{ watts} \times 8 \times 60 \text{ second}$$

$$= 48 \times 10^4 \text{ Joules}$$

Example: 13.12.

How much current is drawn by a half horse power electric motor operated from 240 V source of electricity? Assume that the efficiency of motor is 80%.

Solution:

One horse power = 746 watts

$\frac{1}{2}$ horse power = 373 watts

The power input to motor = IV = 240 × 0.8

The out put of the motor = 373 watts.

$$I = \frac{\text{Power out Put}}{\text{Efficiency} \times \text{volt}} = \frac{373\ W}{0.8 \times 240}$$

= 1.95 A

13.7. ELECTROMOTIVE FORCE:

We have seen that an electric field is needed to maintain a current in a conductor,When this electric current passes through a resistor. it dissipate energy which is transformed into heat. Thus to sustain a current in a conductor, some source of energy is needed, so that it could continuously supply power equal to that which is dissipated as heat in the resistor. The strength of such a source is known as electromotive force. Usually the potential difference that exists between the two terminals of a battery or any source of electrical energy when it is not connected to any external circuit is called its electromotive force, simply read as E.M.F and is represented by E.

As charges pass through a source of electrical energy i.e a cell or a battery, work is done on them, the electromotive force may be defined as the work done per coulomb on the charges.

The e.mf of an automobile storage battery is 12 V, which means that 12 joules of work is done on each coulomb of charge that passes through the battery. In the case of a battery, chemical energy is converted into electrical energy by means of the work done on the charges in transit through it; in a generator, mechanical energy is converted into electrical energy and in a thermocouple, heat energy is converted into electrical energy and so on.

The e.m.f of an electrical source bears a relationship to its power output analogous to that of applied force to mechanical power in a machine, which is the reason for its name.

Let us consider a simple circuit in which a resistor R is connected by leads of negligible resistance to the terminals of a battery (fig.13.8.)

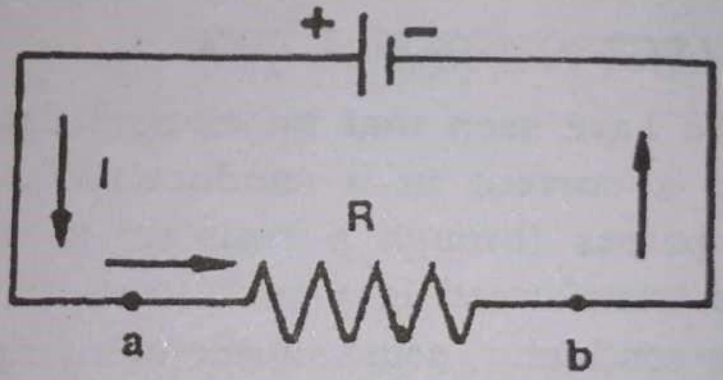


Fig. 13.8.

A current I will flow through the resistor in a direction from a to b, the potential V_a being higher than potential V_b. The same current will flow through the battery from its negative terminal to the positive terminal. The battery is made of some electrolyte and electrodes for the production of e.m.f. and hence when this current flows the battery, it encounters some resistance by the electrolyte present between its two electrodes. This resistance is known as the internal resistance of the battery.

Thus the current in the circuit from Ohm's law is given by the relation

$$I = \frac{E}{R + r}$$, where r is the internal resistance

$$E = IR + Ir \quad \text{------------------(13.11)}$$

Here IR is the voltage to drive the current I through the external resistor R and Ir is" lost voltage" driving current I through the internal resistance of the battery denoting IR. the potential difference between the two terminals of the battery by V we have

$$V = E - Ir \quad \text{------------------ (13.12)}$$

This shows that the potential difference between the terminals of a battery drops when it delivers a current. However when no current is drawn, there is no potential drop across the internal resistance so that the terminal potential difference is equal to its e.m.f. i.e.

$$V = E \quad \text{------------------ (13.13)}$$

In effect, we can say that the internal resistance of a battery governs the maximum current it can supply.

Example:13.13.

A storage battery whose e.m.f is 12 V and whose internal resistance is 0.2Ω is to be charged at rate of 20 A. What applied voltage is required ?

Solution:

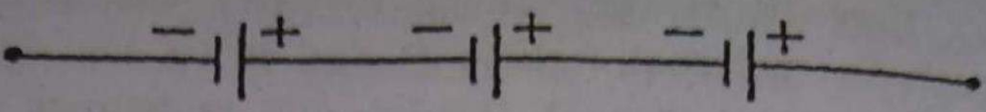
The applied Voltage V must exceed the battery e.m.f E by the amount Ir to provide the charging current I. hence.

$$V = E + Ir = 12\ V + (20A)\ (0.2\Omega)$$

$$= 12\ V + 4\ V$$

$$= 16\ V.$$

When the e.m.f of a single cell is too small for a particular application, two or more can be connected in series (fig.13.9)



(fig.13.9)

The e.m.f of the set is sum of e.m.f of the individual cells and the internal resistance of the set is the sum of individual resistances

$$E - series = E_1 + E_2 + E_3 \text{ ------------ } = E_n$$

$$r - series = r_1 + r_2 + r_3 \text{ ------------- } = r_n$$

A familiar example of such an arrangement is the use of lead acid cells in series to make a 12V battery of a car

When the e.m.f of a battery or cell is sufficient but its capacity is too small, two or more batteries or cells can be connected in parallel to give more current. The total current is the sum of the currrent delivered from the individual battery or cell (fig.13.10).

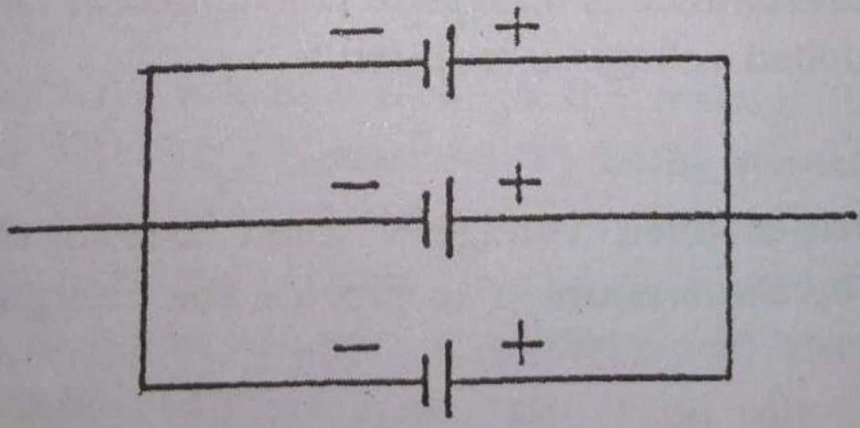


fig.13.10

QUESTIONS:

13.1 Electrons leave a dry cell and flow through a lamp bulb back to the cell. Which terminal, the positive or the negative is the one from which electrons leave the cell? In which direction is the conventional current.

13.2 Both p.d. and e.m.f. are measured in volts. What is the difference between these concepts?

13.3 Can you construct two wires of the same length, one of copper and one of iron, that would have the same resistance at the same temperature.

13.4 Why does the resistance of a conductor rises with the rise in temperature?

13.5 Why is heat produced in a conductor due to flow of electric current?

13.6 When a metal object is heated, both its dimensions and its resistivity increase. Is the increase in resistivity likely to be a consequence of the increase in length?

13.7 It is sometime, said that an electrical appliance, " uses up" electricity. What does such an appliance actually use in its operation?

13.8 Do bends in a wire affect its resistance?

13.9 Resistances of 10Ω . 30Ω and 40Ω are connected in series.If the current in 10Ω resistance is 0.1A, what is the current through the others?

13.10 Ten resistances of different value are connected in parallel. If the p.d.across one of them is 5V, what is the p.d. across the remaining nine resistors?

13.11 For a given potential difference V, how will the heat developed in a resistor depend on its resistance R ? will the heat be developed at a higher rate in a larger or a smaller R?

13.12 Is there any electric field inside a conductor carrying an electric current?

13.13 How does the current flowing in a conductor depend on the number of mobile charge carriers per unit length? On their average velocity? On the charge per carrier?

13.14 (a) What is the equivalent resistance of three 5Ω resistors connected, (i) in series (ii) in parallel.

(b) If a potential difference of 60V is applied across series connection, what is the current in each resistor?

13.15 Can the terminal voltage of a battery be zero?

13.16 Why is the internal resistance of a cell not constant?

PROBLEMS

13.1 A certain battery is rated at 80 ampere hour. How many coulomb of charge can this battery supply

(Ans: 2.88×10^5 C).

13.2 A silver wire 2 m long is to have a resistance of 0.5Ω. What should its diameter be

(Ans: 2.78×10^{-4}m).

13.3 A current of 6A is drawn from a 120 V line. What power is being developed? How much energy in joule and in Kilowatt is expended if the current is drawn steadily for one week.

(Ans: 720 watt. 120.83 kwh, 4.35×10^8 J)

13.4 Currents of 3A and 1.5A flow through two wires, one that has a potential difference of 60V across its ends and another that has a potential difference of 120 V across its ends. Compare the rate at which energy pass through each wire.

(Ans: 1)

13.5 A wire carries a current of 1 A. How many electrons pass a point in the wire an each second? (6.3×10^{18} electrons).

13.6 An electric drill rated at 400W is connected to a 240V power line. How much current does it draw?

(Ans: 1.67A)

13.7 Resistors of 20Ω, 40Ω, 50Ω are connected in parallel across a 50V power source. Find the equivalent resistance of the set and the current in each resistor?

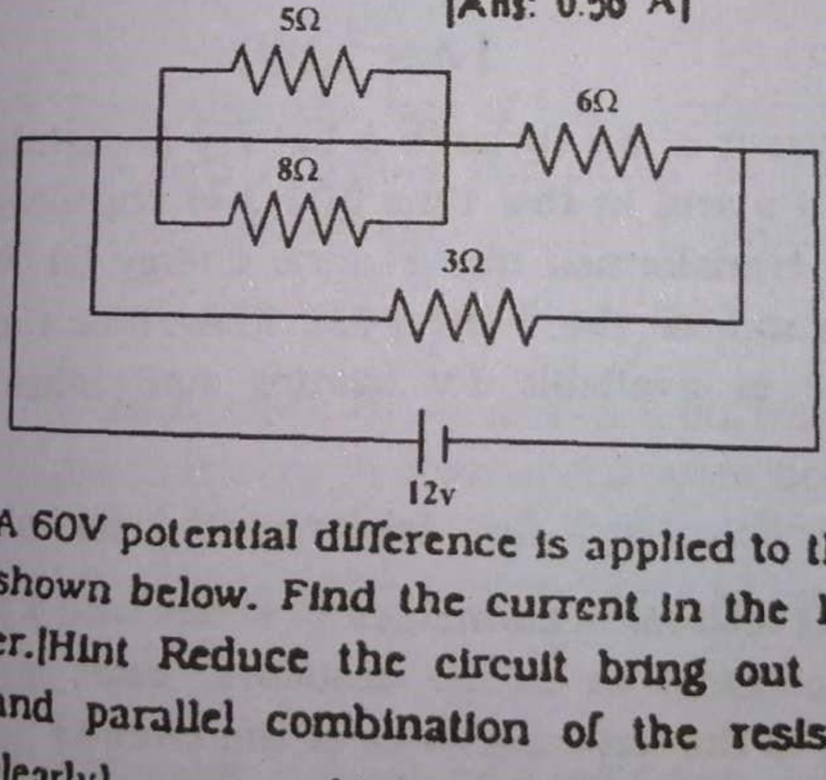
[Ans: 10.5 Ω $I_1 = 2.5A, I_2 = 1.25A, I_3 = 1A$]

13.8 (a) Find the equivalent resistance of the network shown below

|Ans: 2.18 Ω|

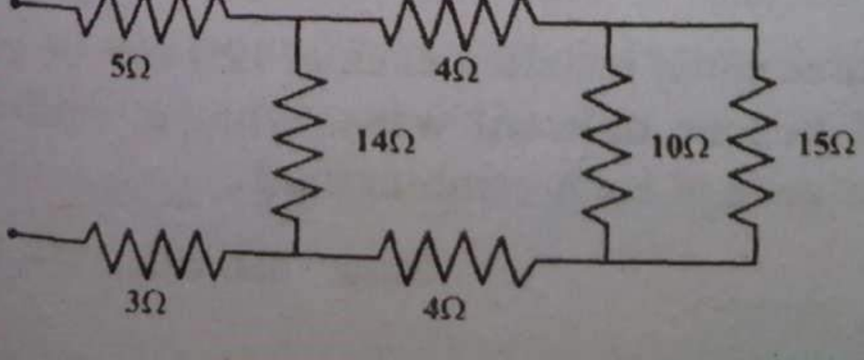
(b) What is the current in 8Ω resister if the potential difference of 12V is applied to the network?

|Ans: 0.56 A|



13.9 A 60V potential difference is applied to the circuit shown below. Find the current in the 10Ω resister.[Hint Reduce the circuit bring out the series and parallel combination of the resistors more clearly].

|Ans: 15 Ω. 1.2 A|



13.10 A source of what potential difference is needed to charge a battery of 20 V e.m.f and internal resistance of 0.1Ω at a rate of 70A.

[Ans: 27V]

13.11 A battery of 24 V is connected to a 10Ω load and a current of 22 A flows. Find the internal resistance of the battery and its terminal voltage.

(Ans: 0.9 Ω, 22 V)

13.12 A 40 Ω resistor is to be wound from platinum wire 0.1 mm in diameter. How much wire is needed?

[Ans: 2.85m]

13.13 The battery of a pocket calculator supplies 0.35A at a p.d. of 6V. What is the power rating of the calculator?

[Ans: 2.1W]

13.14 A current of 5A through a battery is maintained for 30 s and in this time 600 J of chemical energy is transformed into electric energy (a) What is the e.m.f of the battery? (b) How much electric power is available for heating and other uses?

(a) [Ans: 4V], (b) 20W)

13.15 A 12Ω resistor is connected in series with a parallel combination of 10 resistors, each of 200Ω. What is the net resistance of the circuit?

(Ans: 32Ω]

13.16 Three equal resistors each of 12Ω can be connected in four different ways. What is equivalent resistance of each combination?

[Ans: 4Ω, 8Ω, 18Ω, 36Ω]

13.17 Find the resistance at 50 °C of a copper wire 2 mm in diameter and 3 m long.

[Ans: 0.0184 Ω]

13.18 The resistance of a tungsten wire used in the filament of a 60w bulb is 240Ω when the bulb is hot at a temperature of 2020 °C what would you estimate its resistance at 20 °C

[Ans: 25.4 Ω]

13.19 A water heater that will deliver 1 kg of water per minute is required. The water is supplied at 20 °C and an output temperature of 80 °C is desired. What should be the resistance of the heating element in the water if the line voltage is 220V?

[Ans: 11.5Ω]

13.20 Prove that the rate of heat production in each of the two resistors connected in parallel are inversely proportional to the resistances.

[Ans: $P = \frac{v^2}{R}$; $P \propto \frac{1}{R}$]

13.21 A 240V cloth dryer draws a current of 15A. How much energy in Kwh and Joules does it use in 45 minutes operation and how much will be the cost at the rate of Rs.1.45 per unit of electric energy?

[Ans: 2.7 kwh, 9.72×10^6 J, Rs: 39]

13.22 A resistor is made by winding on a spool a 40 m length of constantan wire of diameter 0.8 mm. Calculate the resistance of the wire at a) 0 °C. b) 50 °C. Assume ρ at 0 °C .

(Ans: 49×10^{-8} Ω — m. (a) 38.99 Ω (b) 39.01 Ω)

Chapter 14.

MAGNETISM AND ELECTRO-MAGNETISM

It has been known from centuries that a magnet exerts force on another magnet from a distance. The phenomenon is similar to that of force between electric charges or gravitational force between material particles.

14.1. Magnetic field due to Current

In 1819 Christian Oersted, Professor of Physics at Copenhagon discovered that a pivoted magnetic needle is deflected from its normal north - south direction when even a current bearing wire is held parallel to it as if the current behaves as a magnet.

The force that acts on the magnetic needle can be described by visualizing the needle as being situated in the field created by electric currents through conductors or magnets. This is called magnetic field of Induction (Magnetic field would be a more suitable name but it has been acquired for historical reason by another magnetic quantity to be discussed in higher classes).

In 1820 Ampere observed that two current bearing conductors exert forces on each other and suggested that the magnetic condition of magnets is caused by currents within the body of magnets. These currents may now be identified with the motion of electrons in the atoms of the magnetic material.

When electric charges are at rest they exert electrostatic forces of attraction or repulsion on each other. When the charges are in motion they still exert these electrostatic forces but, in addition, magnetic forces appear because of motion. Isolated moving positive or negative charges create both electric and magnetic fields but an electric current through a conductor produces only a magnetic field because the electric field of moving electron is neutralized by the field of fixed protons in the conductors.

A magnetic field is a region in which a force is experienced on a moving charge or a magnet.

This force depends upon.

1. The magnitude of charge q.

2. The speed of the moving charge ϑ.

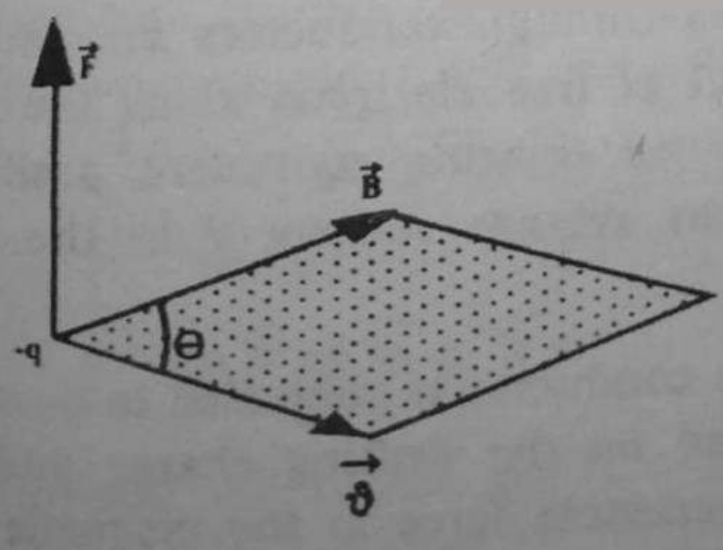
3. The magnetic field of Induction B.

The magnetic Induction B is a Vector quantity defined from the relation.

$$\vec{F} = q(\bar{\vartheta} \times \bar{B}) \qquad \text{-------------------(14.1)}$$

The direction of force is perpendicular to both the direction of motion of charge and the direction of B given by cross product rule.

It also follows that the force on the charge in the field is zero either when the charge is stationary or moving along the direction of B.



The magnitude of B is given by

$$B = \frac{F}{q\vartheta \, \text{Sin}\,\theta} \quad \text{------------------(14.2)}$$

A unit magnetic field of Induction is said to exist at a point where the force per unit charge experienced by a positive test charge, moving with a velocity of $1 ms^{-1}$ in the direction perpendicular to the field is 1 newton.

$$\text{Unit of B} = \frac{\text{newton}}{\text{coul} \times \text{metre/second}} = \frac{\text{newton}}{\text{ampere} \times \text{metre}}$$

It is called tesla (T).

Example 14.1.

A proton enters a uniform magnetic field of Induction B = 0.300 Tesla in a direction making an angle of 45° with the direction of field. What will be the magnitude of force if the velocity of proton is $10^4 ms^{-1}$?

Solution:

$$F = q\,V\,B\,\text{Sin}\,\theta = 1.6 \times 10^{-19} \times 10^4 \times 0.3 \times \text{Sin}\,45°$$

$$= 3.4 \times 10^{-16}\,N$$

14.2. Force on a Current Carrying Conductor in a Uniform Magnetic Field.

Currents through conductors are caused by the directional drift of free electrons along the conductors. Conventionally we imagine equivalent positive charges drifting with an average velocity V in the direction of current.

When a conductor is subjected to a magnetic field force is exerted on the drifting charge and hence the conductor experiences force in the magnetic field of Induction.

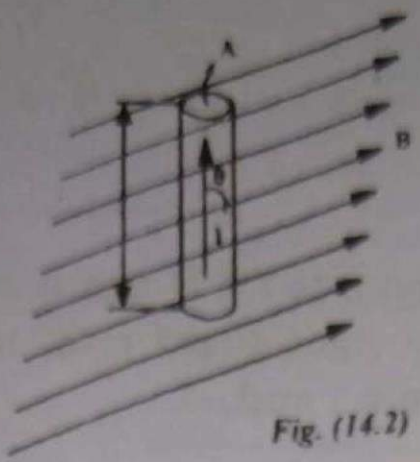


Fig. (14.2)

Consider a linear conductor, of length L and carrying a current I, be subjected to a uniform magnetic field of Induction B which makes an angle θ with direction of current as shown in Fig.14.2.

If there are n number of free electrons per unit olume the total moving charge

$$q = n A \ell e$$

where A is the area f cross section of conductor.

The force on the conductor

$$\vec{F} = q(\vec{\vartheta} \times \vec{B}) = n A \ell e\ (\vec{\vartheta} \times \vec{B})$$

This expression can be changed in terms of the current by a little mathematical manipulation. Consider the length of conductor, a Vector in the direction of $\vec{\vartheta}$ which can be written $\hat{a}\,\ell$ where $\hat{a}$ is a unit Vector in the direction of $\vec{\vartheta}$. Similarly $\vec{\vartheta}$ can be written as $\hat{a}\,\vartheta$ where $\vec{\vartheta}$ is the magnitude of drift velocity.

$$\vec{F} = n A \ell e\,(\hat{a}\vec{\vartheta} \times \vec{B})$$

$$\vec{F} = n A ev\,(\hat{a}\,\ell \times \vec{B})$$

$$\vec{F} = n A e\vartheta\,(\vec{\ell} \times \vec{B})$$

The average drift velocity $\vartheta = \dfrac{\ell}{t}$ where t is the time taken by q to cross the length of the conductor.

$$\vec{F} = \frac{nAe\ell}{t}(\vec{\ell} \times \vec{B}) = \frac{q}{t}(\vec{\ell} \times \vec{B})$$

$$\vec{F} = I(\vec{\ell} \times \vec{B}) \quad \text{..................} \quad (14.3)$$

The magnitude of force is $I \ell B \sin\theta$ and the direction of force is given by right hand rule. It is in the direction perpendicular to both I and B.

$$B = \frac{F}{I \ell \sin\theta}$$

Example 14.2.

A steady current of 25 A is passing through a horizontal power line 50 m in length held between two poles in the north south direction. The earth's magnetic field is 10^{-4} tesla at that place and angle of dip is 60°, find the force on the wire.

Solution:

Angle of dip is the angle between the direction of earth's magnetic field and the horizontal in the magnetic meridian.

$$\vec{F} = I(\vec{\ell} \times \vec{B}) = I \ell B \sin\theta$$

$$= 25 \times 50 \times 10^{-4} \times \sin 60^\circ$$

$$= 0.108 \text{ N}$$

14.3. Torque on a Current Carrying Rectangular Coil placed in a Magnetic Field

In Fig. 14.3 a rectangular coil is suspended in a uniform magnetic field and the plane of the coil is parallel to the field. A current I is flowing round the coil in the direction shown.

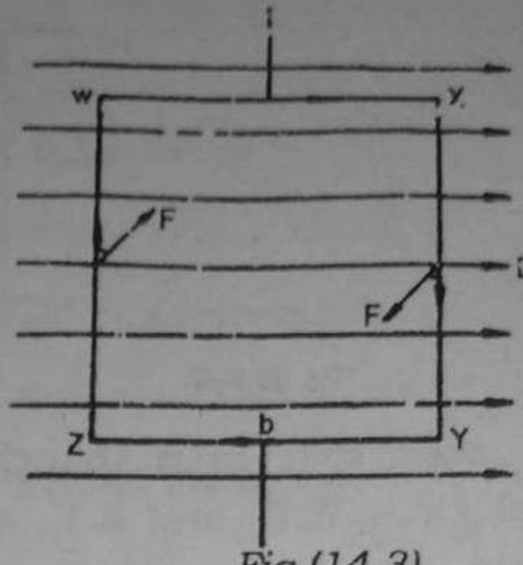


Fig (14.3)

The vertical side WZ of the coil experiences a force F which is directed perpendicularly into the paper. There is an equal and opposite force on xy. Since the plane of the Coil is parallel to the field there is no force on either WX or ZY. the forces on WZ and XY constitute a couple whose torque τ is given by

$$\tau = F b$$

where b is the width of the coil. The directions of the current in WZ and XY are each at 90° to the magnetic field, and therefore from equation 14.3

$$F = B I \ell \sin 90^\circ . = B I \ell$$

where ℓ is the length of each vertical side of the coil.

$$\therefore \tau = B I \ell b = B I A. \qquad \text{Where } A = \text{Area of coil.}$$

For a coil of N turns.

$$\tau = B I A N$$

As soon as the coil turns under the influence of the torque it ceases to be parallel to the field. Figure 14.4 shows the situation of the coil when it is at some angle α to the field.

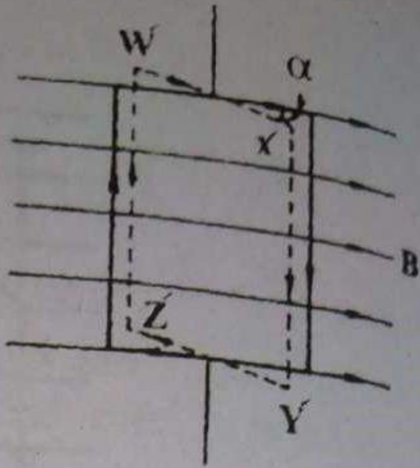


Fig. (14.4)

It can be seen that even though the Coil has turned its vertical sides $W'Z'$ and $X'Y'$ are still perpendicular to the field. The forces acting on vertical sides therefore have the same magnitude and the same direction as they had before the coil turned. However the separation of the forces alter so that the torque τ has a reduced value given by $\tau = F\, b \cos\alpha$

Therefore in general

$$\tau = B\,I\,A\,n \cos\alpha \quad \text{--------------------(14.4)}$$

For values of α other then zero the field exerts forces on $W'X'$ and $Z'Y'$ but these forces are always parallel to the axis about which the Coil is turning in opposite directions and therefore make no contribution to the torque. The formula is also valid for circular Coil.

Example 14.3.

A 50 turn rectangular Coil of wire 4 cm by 5 cm is suspended in a vertical plane.

A horizontal uniform magnetic field of 2.0 $\frac{N}{Am}$ is maintained in the plane of Coil. Compute the torque experienced by the coil when a current of 0.3 amp flows through it

Solution:

$N = 50$

$A = 4 \times 5 = 20\ \text{cm} = 0.002\ \text{m}^2$

$\alpha = 0 \qquad\qquad B = 2$

Torque

$$\tau = B\,I\,A\,N\,\text{Cos}\,\alpha$$

$$= 2 \times .3 \times .002 \times 50 \times 1$$

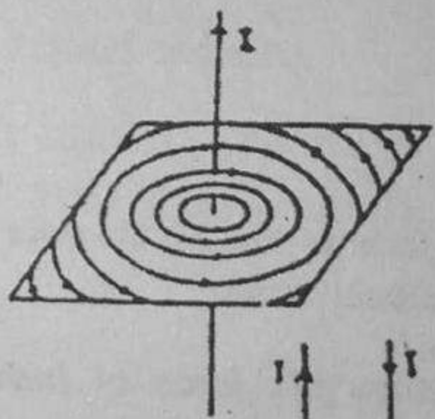
$$= 0.06 \text{ newton-metre}$$

14.4. Magnetic flux and flux density

Magnetic field of Induction in a region can be visualized by magnetic lines of induction just as electric field was represented by electric lines of force.

Lines of induction are defined in the same way as electric lines are defined. Unlike electric lines of force the magnetic lines of induction are endless and continuous lines in the field region and can be traced using a small compass needle. Figure 14.5. Shows the lines of Induction in a few cases.

(a) Magnetic lines of Induction around a long Straight Current carrying wire in a plane perpendicular to the wire.



(a)

The lines are concentric circles with their centres at the wire. If the wire is grasped by right hand with the thumb in the direction of current, the fingers encircle the conductor in the direction of field. This is called right hand grip rule.

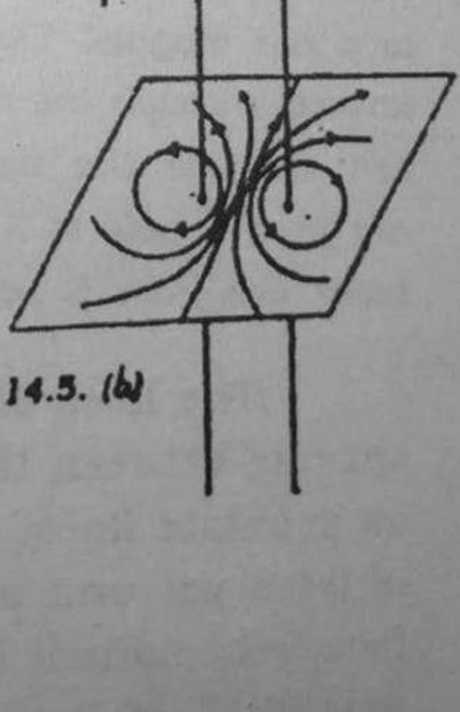
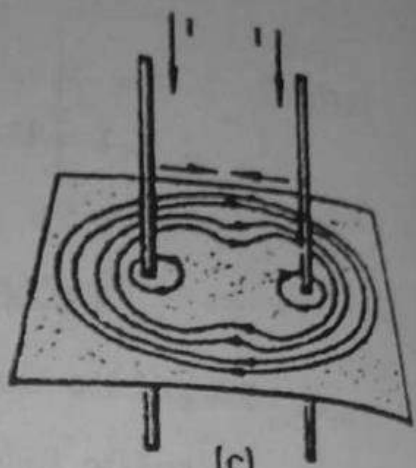


Fig 14.5. (b)

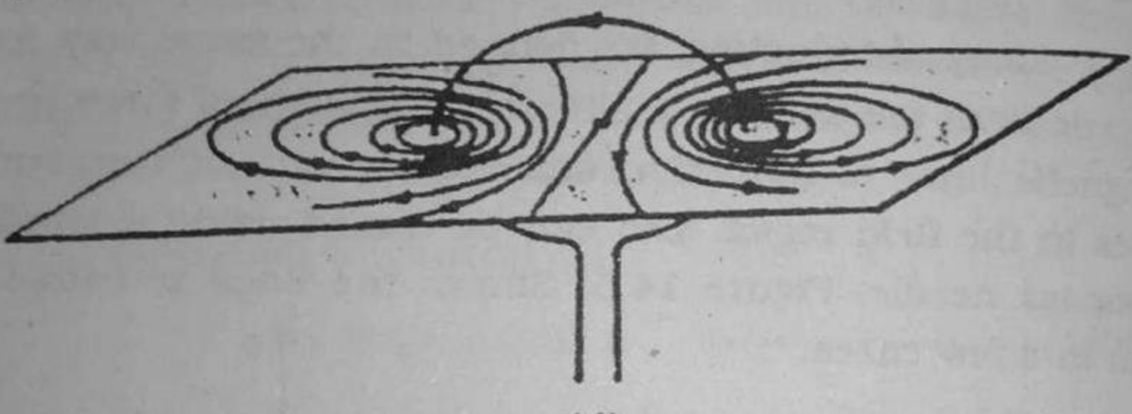
(b) Lines of Induction due to two long parallel wires carrying current in opposite directions.

(c) Lines of Induction due to two long parallel wires carrying currents in the same direction.



(c)

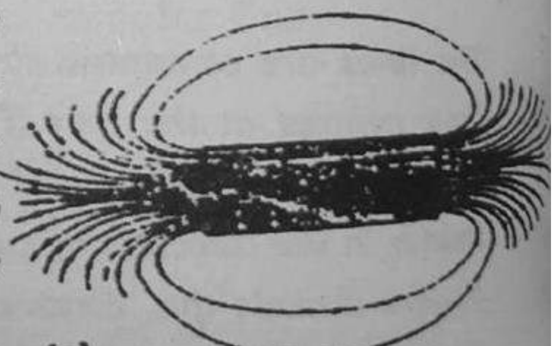
(d) Lines of force of Induction due to a circular loop. The lines enter one face which is the South pole emerge from other face which is North pole. On looking at a face if the current appears clock wise the face is South pole.



(d)

The face is the North pole if on looking on it . the current is anticlock wise.

(e) Lines of force of Induction due to a bar magnet. The lines are continuous through the material of magnet. Outside the magnet the lines are from N pole to S pole but inside there are from S pole to N pole.



(e)

The lines of Induction are to be drawn with some spacing between them so that they may be distinguished as separate lines. It is so manipulated that the number of lines per unit area passing through a very small surface held normal to the lines at a point is equal to the magnitude of B at that point.

The magnetic flux over a surface is defined in the same way as electric flux. It is the number of magnetic lines of Induction crossing the surface normally.

The magnetic flux over a small surface at every point of which the field is the same is

$$\Delta \phi_m = \vec{B} \quad \Delta\vec{A} \qquad \text{-------------(14.5)}$$

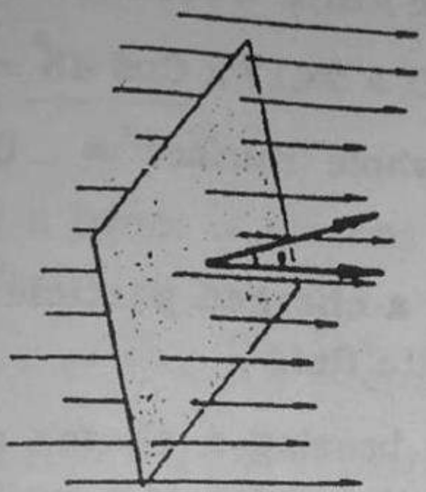


Fig. (14.6)

Magnetic flux is expressed in the unit called weber.

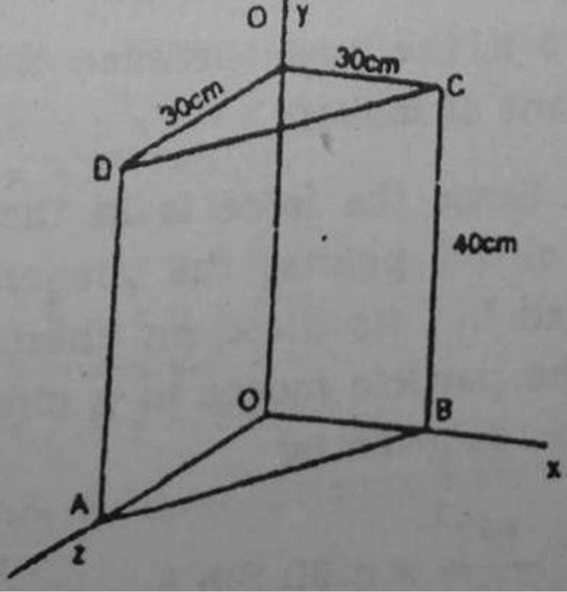
Since the magnetic field of Induction is the number of lines passing normally per unit area it is the flux density and its units are Weber m^{-2}.

$$B = \frac{\Delta \phi_m}{\Delta A_n} \qquad \text{------------------ (14.6)}$$

Example 14.4.

Find the magnetic flux across the Surface of triangular prism shown in the diagram if the flux density is 2 tesla along X-axis

$\phi_m = BA \cos \theta$



a) **Flux across the face**

$$OADO' = B\,A \cos\theta = 2 \times (.3 \times .4) = 0.24$$

$$DC = \sqrt{30^2 + 30^2} = \sqrt{1800} = 42.42 \text{ cm} = .42 \text{ m}$$

$\therefore$ area of the face ABCD $= .42 \times .4 = 0.17 \text{ m}^2$

Flux across ABCD $= 2 \times .17 \times \cos 45^\circ = 0.24$

Flux over the whole Surface $= 0.24 + 0.24$
$= 0.48$ Webers.

14.5 (i) Force on a charged particle moving on a magnetic field .

When a particle bearing a charge q and moving with a velocity v enters the region of a uniform magnetic field of Induction. B it is acted upon by a force

$$F = q(\vec{\vartheta} \times \vec{B}) = q\,VB \sin\theta$$

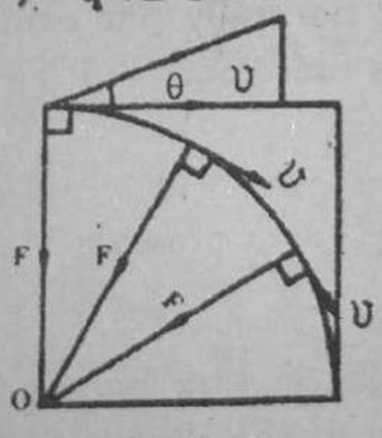


Fig. 14.7

when θ is the angle between the plane of the field and the plane of motion.

Since the force is in the direction perpendicular to ϑ at all points, the magnitude of ϑ remains unchanged but its direction changes from point to point and the particle moves in a circular path. The centripetal force is given by

$$\frac{m\vartheta^2}{r} = q\,\vartheta B \sin\theta$$

$$r = \frac{m\vartheta}{q\,B\,\text{Sin}\,\theta} \qquad \text{------------(14.7)}$$

where r is the radius of the circular arc and m is the mass of the particle.

In case if the magnetic field is perpendicular to the plane of motion.

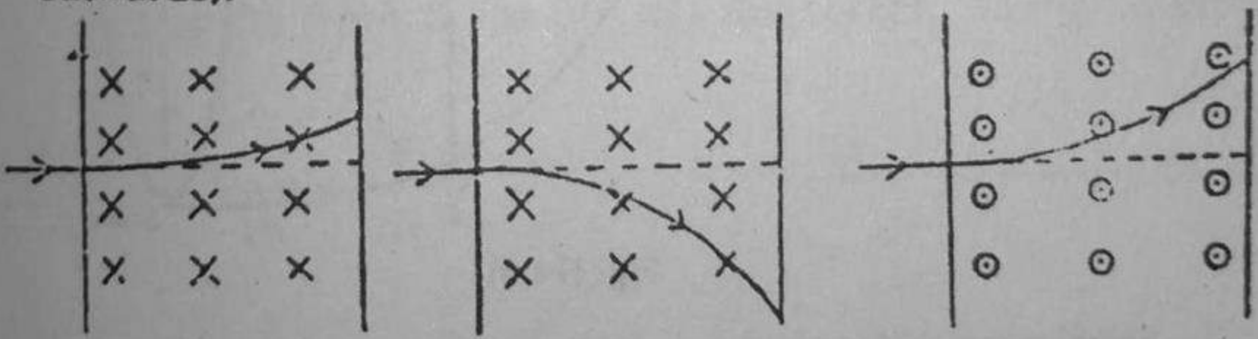
$$r = \frac{m\vartheta}{qB}$$

For a beam of protons $F = e\ (\vec{\vartheta}\ \times\ \vec{B})$ and for a beam of electrons

$$F = -\,e(\vec{\vartheta}\ \times\ \vec{B}) \ = e(\vec{\vartheta}\ \times \vec{B})$$

The figure 14.8 illustrates the direction of deflection in a few cases when the plane of field is perpendicular to the plane of motion.

(It is a convention that a cross (x) indicates a field directed inwards of the page and a dot (.) means a direction outwards).



a) Trajectory of a Beam of Protons.

b) Trajectory of a Beam of Electrons.

c) Trajectory of a Beam of Electrons.

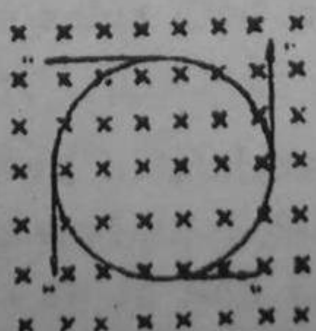


Figure 14.8 (d). Trajectory of a Beam of electron when the field exists in an extended region.

(ii) Determination of Charge to Mass ratio of an electron.

The above knowledge was utilized by Sir J.J. Thomson to determine $\frac{e}{m}$ of an electron.

The apparatus consists of a highly evacuated pear shaped glass bulb into which several metal electrodes are sealed. (fig.14.9).

Electrons are produced by heating a tungsten filament by passing a current through it.

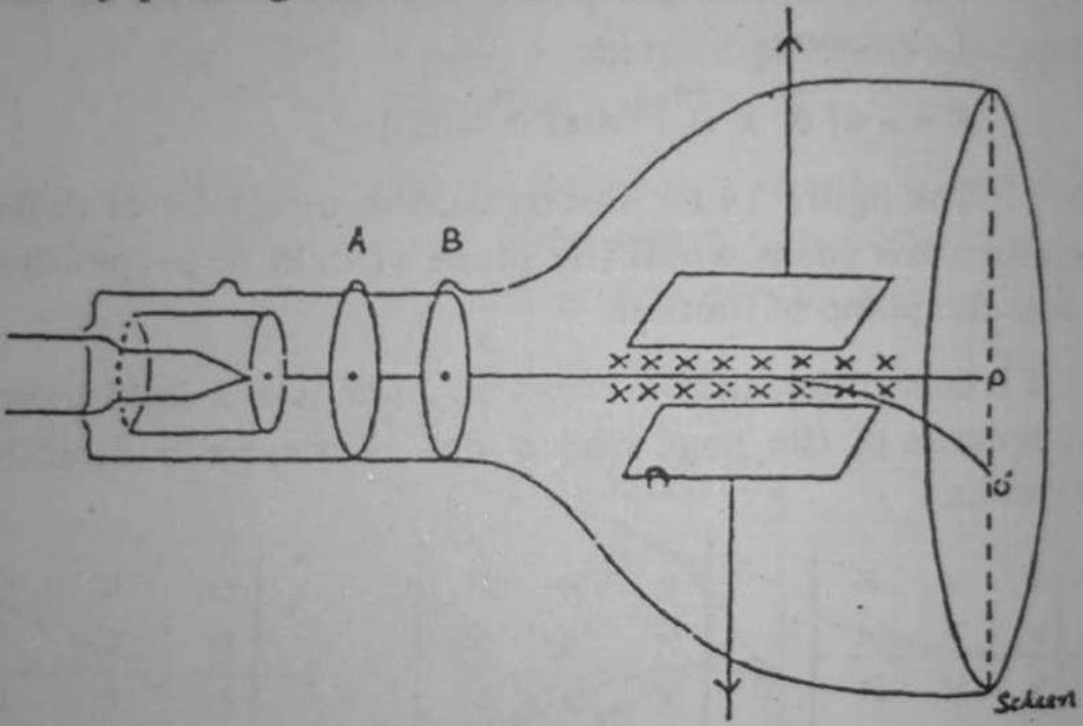


Fig. (14.9)

The electrons moving side ways are also directed towards the screen by applying a negative potential on a hollow cylinder open on both sides surrounding the filament.

Electrons are accelerated by applying a potential difference of above 1000 Volts between the filament and the metal disc A with a hole at its centre. A further potential difference of 500 Volts is applied between the discs A and B. This arrangement focuses the electron beam to the hole of the disc B from where it further proceeds in a straight line. If the total potential difference between the filament and the disc B is V, the velocity acquired by the electrons is given by the energy equation.

$$\frac{1}{2} m v^2 = V_e$$

$$v = \sqrt{\frac{2V_e}{m}} \quad \text{------------------(14.9)}$$

The beam strikes the screen coated with Zinc Sulphide after passing through the middle of two horizontal metal plates and a spot of light is produced at "O" on the screen where the beam strikes and its position is noted.

A magnetic field of induction B is produced along y-axis by two identical current carrying circular coils placed on either side of the tube at the position of plates. The force due to magnetic field on moving electrons makes them move in a circular path and the light spot shifts from 0 to 0′, on the screen. Using equation 14.7

$$\frac{e}{m} = \frac{v}{rB} \quad \text{------------------ (14.10)}$$

$\frac{e}{m}$ can be computed from this expression if

the radius of the circular arc in which the beam moves in the field region is determined. The radius is calculated from the shift of light spot.

A better method of determining. V is as under:

An electric field E is produced between the plates by applying a suitable potential difference to exert a force Ee on the electrons opposite to that due to magnetic field. The potential difference V_1 is so adjusted that the two fields neutralize each others effects and the spot comes back to its original position O

$$E = \frac{V_1}{d}, \text{ where } d = \text{distance between plates.}$$

$$Ee = e\vartheta B$$

$$\vartheta = \frac{E}{B} \quad \text{------------------(14.11)}$$

Substituting in eq 14.10

$$\frac{e}{m} = \frac{E}{rB^2} \quad \text{------------------(14.12)}$$

$$e/m = 1.75888 \times 10^{11}\ Ckg^{-1}$$

As $e = 1.60207 \times 10^{-19}$ C

$$m = \frac{1.60207 \times 10^{-19}}{1.75888 \times 10^{11}} = 9.1084 \times 10^{-31}\ kg$$

14.6. Ampere's Law

Ampere's Law is some what analogous to Gauss's Law of electrostatics and it helps to determine the magnetic field of induction in a few cases of current configurations.

Consider first a long straight wire carrying a current, I in the direction shown in the fig. 14.10 the lines of force are concentric circle with their common centre on the wire. Hence the magnetic fields at all points on a curve taken in form a circle round the wire is tangential and of the same magnitude. Biot and Savart experimentally found that the magnitude of the field depends directly on twice the current I and inversely on the distance r from the conductor.

$$B \propto \frac{2I}{r}$$

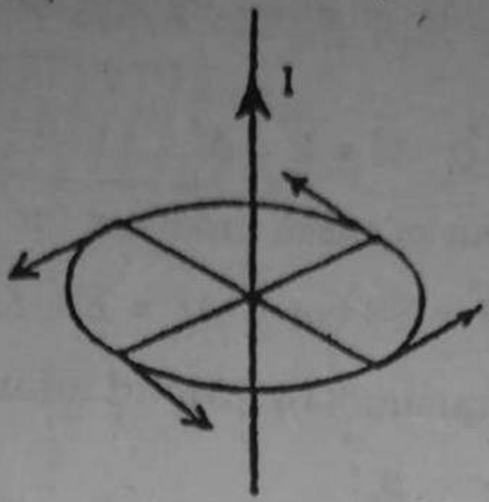


Fig.(14.10)

The constant of proportionality is written as $\frac{\mu_o}{4\pi}$

and its value is 10^{-7}. μ_o is called permeability of free space.

$$B = \frac{\mu_o}{4\pi} \times \frac{2I}{r}$$

$$B = \frac{\mu_o I}{2\pi r} \quad \text{-----------------(14.13)}$$

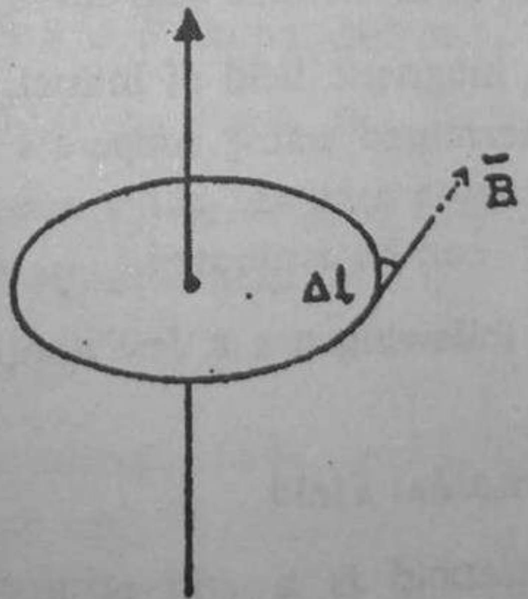


Fig.(14.11)

Again let the circle be divided into small elements each of length $\Delta \ell$. Multiplying the length of each element by the tangential component of field which is in the direction of $\Delta\ell$ for all elements and for this special case we get

$$B\cos 0^\circ \,\Delta\ell = \vec{B}\cdot\Delta\ell$$

The sum of these products for all the elements is

$$\Sigma\vec{B}\cdot\Delta\vec{\ell} = \Sigma B\cos 0^\circ\,\Delta\ell = \Sigma B\,\Delta\ell = B\,\Sigma\,\Delta\ell$$

using equation(14.13)and taking $\Delta\ell = 2\pi r$ we get

$$\Sigma\vec{B}\cdot\Delta\vec{\ell} = \frac{\mu_o I}{2\pi r}\times 2\pi r = \mu_o I \qquad \text{----------(14.14)}$$

This relation is called Ampere's Law and it is true for a closed curve of any shape taken in the magnetic field because the distance of the element from the conductor is not involved in this expression. The law states that the sum of the products of the tangential component of magnetic field of induction and the length of an element of a closed curve taken in the magnetic field is μ_o times the current which passes through the area bounded by this curve. If the closed curve is taken in the magnetic field such that it encloses no current then $\Sigma\vec{B}\cdot\Delta\vec{\ell} = 0$. as is the case with curve C_2 in the Fig.14.11.

The magnetic field of Induction due to a current can be determined using Ampere's Law only if we can possibly imagine a closed curve around which the quantity $\Sigma\vec{B}\cdot\Delta\vec{\ell}$ can be evaluated.

The following are a few applications of Ampere's Law.

a) Solenoidal Field

A Solenoid is a coil of insulated Copper Wire wound on a long cylinder with close turns.

Except at the ends the lines of magnetic Induction are fairly parallel and closely packed inside the solenoid indicating that the field is strong and uniform in the middle portion of the solenoid. Outside the solenoid the lines are widely separated and the field is weak.

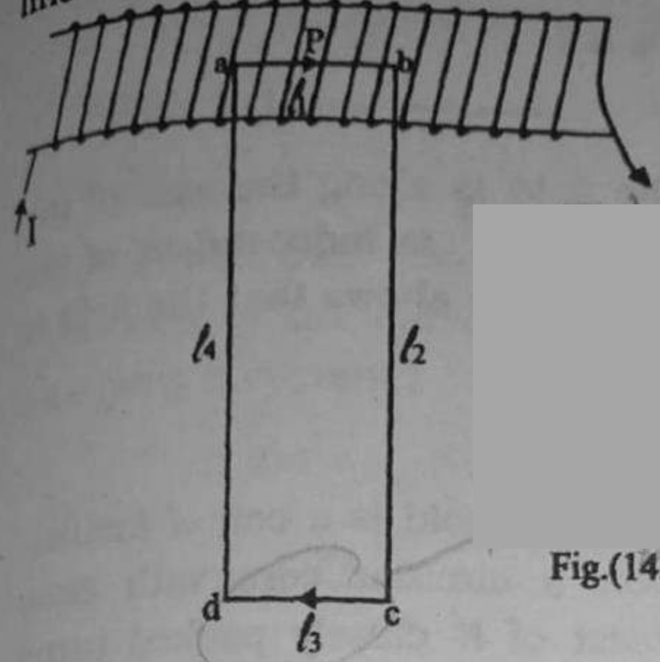


Fig.(14.12)

In order to determine the magnetic Induction at a point P on the axis of Solenoid well inside it, imagine a rectangular loop abcd with the side ab on the axis and the side cd far away where the field is zero as shown in fig. 14.12. It is divided into four elements ℓ_1, ℓ_2, ℓ_3 and ℓ_4. By Ampere circuital law

$$\sum_{\ell=1}^{4} (\vec{\Delta\ell} . \vec{B})_\ell = \mu_o \times \text{current enclosed}$$

Inside the solenoid $\vec{B}$ is parallel to ℓ_1 so

$$(\Delta\vec{\ell}_1 . \vec{B})_1 = \ell_1 B \cos 0^\circ = \ell_1 B$$

The field outside the solenoid is very small, it can be neglected and put equal to zero.

$$\therefore (\Delta\vec{\ell}_3 . \vec{B})_3 = 0$$

As $\vec{B}$ is perpendicular to ℓ_2 and ℓ_4 inside the solenoid, the field is zero i.e.

$$(\Delta\vec{\ell}_2 . \vec{B})_2 = (\Delta\vec{\ell}_4 . \vec{B})_4 = 0$$

$$\therefore \sum_{\ell=1}^{4} (\Delta\vec{\ell} . \vec{B})_\ell = \ell_1 B = \mu_o \times \text{current enclosed.}$$

If there are n turns per unit length of the solenoid, and each turn carries a current I, the current enclosed by the loop abcda will be $n \ell_1 I$.

$$\therefore \ell_1 B = \mu_0 n \ell_1 I$$

i.e $\quad B = \mu_0 nI \quad$ ------------------- (14.15)

The direction of the field is along the axis of the solenoid. Eq.(14.15) shows that $\vec{B}$ is independent of the position within the solenoid which shows that the field is uniform within a long solenoid.

b) **Toroidal Field**

A toroid or a circular solenoid is a coil of insulated Copper wire wound on a circular core with close turns. Let the toroid consist of N closely packed turns and carry a current I.

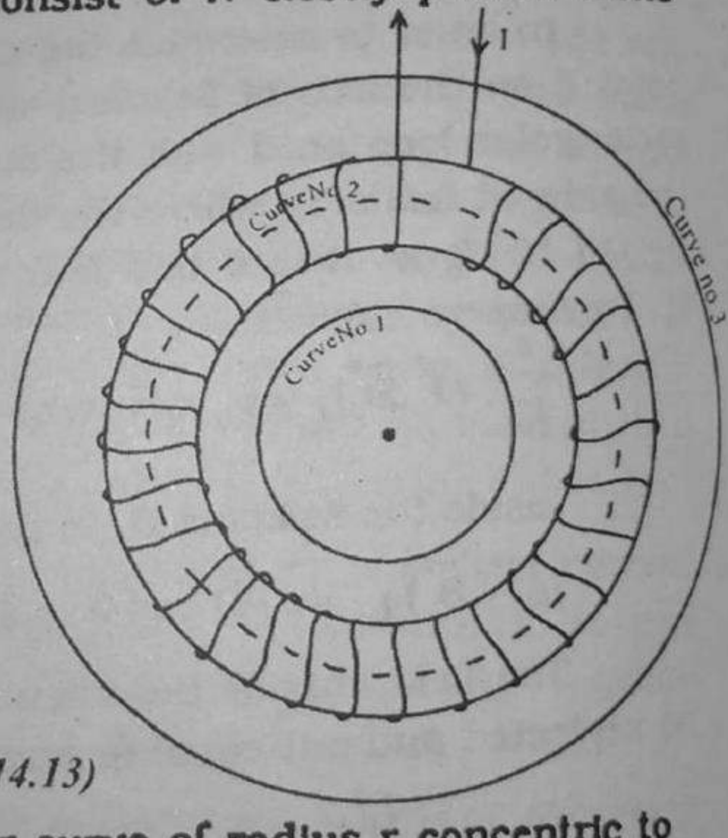


Fig. (14.13)

Imagine a circular curve of radius r concentric to the core as shown in Fig 14.13.

It is evident from symmetry that the field at all points of the curve must have the same magnitude and it should be tangential to the curve at all points.

$$\therefore \Sigma \vec{B} \cdot \vec{\Delta \ell} = \Sigma B \Delta \ell \cos 0^\circ = B \Sigma \Delta \ell = B 2\pi r$$

Let us consider the following cases:-

1. If the circular path (marked 1) is outside the core on the inner side of the toroid it encloses no current.

$$B\,2\pi r = 0$$

$$\therefore B = 0$$

2. If the circular path (marked 2) is within the core the area bounded by the curve will be threaded by N turns each carrying a current I

$$B\,2\pi r = \mu_o NI$$

$$\therefore B = \frac{\mu_o NI}{2\pi r} \qquad \text{------------------(14.16)}$$

which is the same as that at the centre of the solenoid.

3. If the circular path (marked 3) is out side the Core on the outer side of toroid the area bounded by the curve will be threaded by each turn twice but in opposite directions and the algebraic sum of all the currents is zero

$$B = 0.$$

A toroid, therefore, produced a uniform magnetic field of induction which is confined in the space occupied by the core.

Example 14.5.

There is a current of 25 A in a long straight wire. What is the flux density at a point 3 cm from the wire.

Solution:

$$B = \frac{\mu_o I}{2\pi r} = \frac{4\pi \times 10^{-7} \times 25}{2\pi \times .03} = 1.67 \times 10^{-4}\ T$$

Example 14.6.

A toroid has 3000 turns. The inner and outer diameter are 22 cm and 26 cm. Calculate flux density inside the core when there is a current of 5.0 amp.

Solution:

Mean Radius = 12 cm

$$B = \frac{\mu_o NI}{2\pi r} = \frac{4\pi \times 10^{-7} \times 3000 \times 5}{2\pi \times .12} = 0.025\ T$$

14.7. Electromagnetic Induction

In the year following the discovery by Oersted that an electric current produces magnetic field of Induction, the question before the scientists was whether a magnetic field can, in some way, induce an electric current. Continuous attempts were made for about ten years to detect currents in the Coils by subjecting them to strong magnetic fields but the answer was that such an effect apparently does not exist.

In 1830 Joseph Henry in the United States and a year later Faraday in England independently observed that an emf is set up in a coil placed in a magnetic field when ever the flux through the coils changes. The effect is called electro-magnetic Induction and if the coil forms a part of a closed circuit, the induced emf causes a current to flow in the circuit.

Experiments show that the magnitude of emf depends on the rate at which the flux through the coil changes. It also depends on the number of turns N on the coil and it is useful to define a quantity called flux linkage as being the product of the number of turns and the flux through the coil.

Flux linkage = N ϕ ------------------(14.17)

It is not necessary that a conductor is in the form of a coil in order for it to be able to acquire an emf. Emf can be induced in a straight conductor when ever it is caused to cut across magnetic Induction lines.

The magnetic flux through a circuit can be changed in a number of different ways:-

1). By changing the relative position of the coil with respect to a magnet or current bearing solenoid.

2). By changing current in the neighbouring coil or by changing current in the coil itself.

It should be noted that early attempts were bound to fail because the coil has resistance and energy is necessary to force a current through it. No energy is expended when the coil is lying motionless in the field.

Laws of Electromagnetic Induction.

A detailed investigation of magnetic field of induction leads to two laws known as Faraday's laws.

1. An emf is induced in a coil through which the magnetic flux is changing. The emf lasts so long as the change of flux is in progress and becomes zero as soon as the flux through the coil becomes constant.

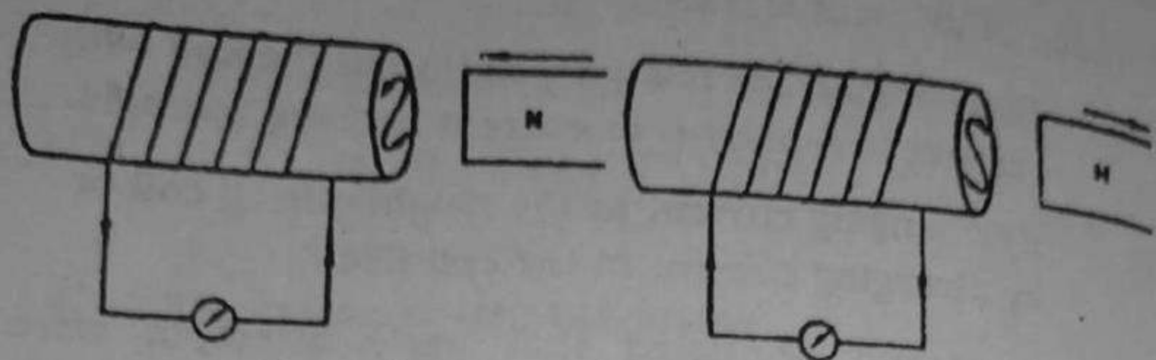
2. The magnitude of induced emf depends only upon the number of turns and the time rate of change of flux linked with the circuit. It can be expressed as

$$\xi = -\frac{d}{dt}(N\phi) \qquad \text{----------------(14.18)}$$

The negative sign is introduced for the reason to be explained in the next article.

14.8. Lenz's Law

The directions of induced current as shown in fig.(14.14) was carefully studied by Lenz and the results were generalized most elegantly into a rule in 1835 called Lenz's Law.



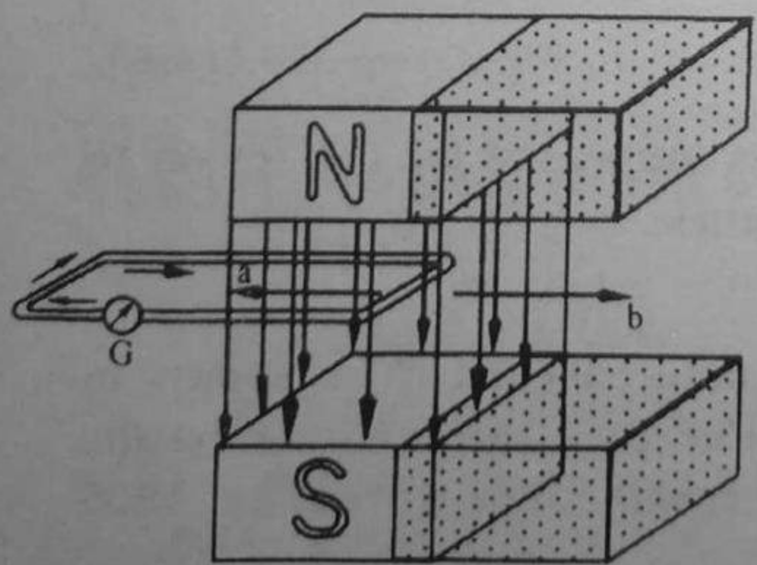
(a) fig. (14.14) (b)

The law states that the induced current always flows in such a direction as to oppose the change which is giving rise to it. This is why a negative sign is introduced in equation (14.18).

It can be explained with the help of following examples.

a). When the N pole of a bar magnet is approaching the face of the coil it becomes a North face by the induction of current in anticlockwise direction to oppose the forward motion of the magnet. Fig. 14.15.

b). When the N pole of the magnet is receding the face of the coil becomes a south pole due to a clockwise induced current to oppose the backward motion.



The direction of the induced current is such that the magnetic force on the wires is always opposite to the applied force.

Fig. (14.15)

c). Let a wire ab of length ℓ which makes a part of a circuit be pulled towards left. According to Lenz's Law this motion should be opposed by the current induced. The current must produce a force towards right.

Example.14.7.

A coil of 600 turns is threaded by a flux of 8×10^{-5} webers. If this flux is reduced to 3×10^{-5} webers in 0.015s, what is the average induced emf?

Solution:

$$\xi = -N \cdot \frac{\Delta\phi}{\Delta t} = \frac{-600\,(8-3) \times 10^{-5}}{.015} = -2.0 \text{ Volts}$$

14.9. (a) Self - Induction.

A coil through which a current is flowing has an associated magnetic field. If, for any reason, the current changes, then so too does the magnetic flux and an emf is induced in the coil. Since this emf has been induced in the coil by a change in the current through the same coil, the process is known as Self Induction.

In accordance with Lenz's Law, the emf opposes the change that has induced it and it is therefore known as a back emf.

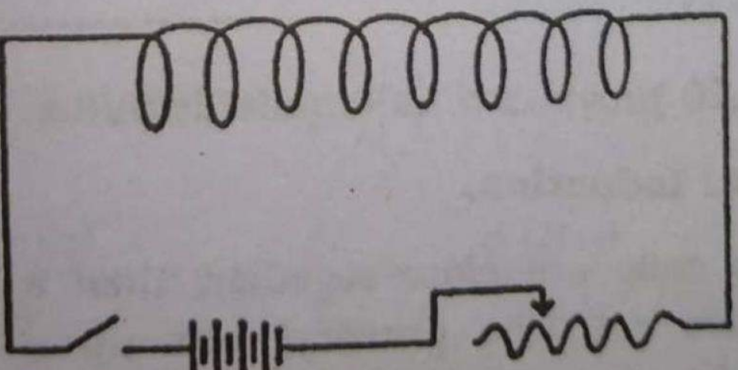


Fig. 14.16.

If the current is increasing, the back emf opposes the increase. If the current is decreasing, it opposes the decrease. The measure of the ability of a coil to give rise to a back emf is known as the Self-Inductance of the coil. It is defined by

$$\xi = -L\frac{\Delta I}{\Delta t} \quad \text{-----------------} \quad (14.19)$$

where

ξ = the back emf induced in the coil in Volts

L = the Self-Inductance of the coil. The unit of self Inductance is henry.

$\frac{\Delta I}{\Delta t}$ = the rate of change of current in the coil amp. s^{-1}

The Self Inductance of a coil is 1 henry if the current varying through it at the rate of 1 ampere per second induces a back emf of 1 Volt.

The value of L depends on the dimensions of the coil, the number of turns and the permeability of the core material.

If ϕ is the flux through a coil of N turns when it is carrying a current I.

$$\xi = -\frac{\Delta (N\phi)}{\Delta t} = -L\frac{\Delta I}{\Delta t} = -L\frac{\Delta (LI)}{\Delta t}$$

or $\quad N\phi = LI \quad$ -----------------(14.20)

Equation 14.20 provides an alternate definition of L.

(b) **Mutual Induction.**

If two coils are close together, then a changing current in one coil (the primary) sets up a changing magnetic field in the other (the Secondary) and so induces an emf in it. The effect is known as mutual Induction.

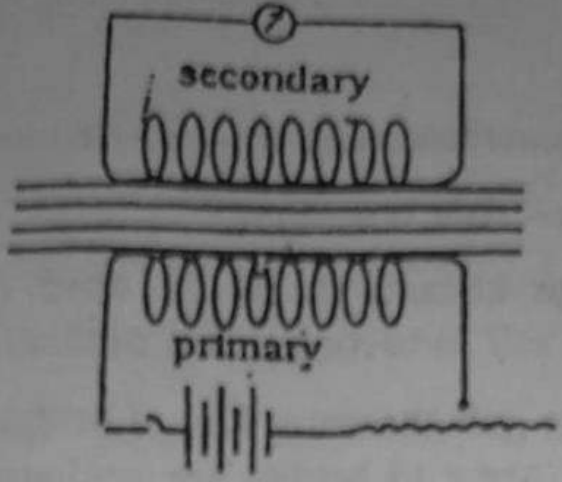


Fig. (14.17)

The mutual Inductance, M, of the pair of coils is defined by

$$\xi_2 = -M \frac{\Delta I_1}{\Delta t} \quad \text{------------------(14.21)}$$

Where ξ_2 is the back emf induced in the Secondary coil.

$\frac{\Delta I_1}{\Delta t}$ = the rate of change of current in the primary,

M = the mutual Inductance of the pair of coils M has the same value no matter which of a given pair of coil is taken to be the primary. Its unit is also henry.

If the rate of change of flux - linkage in the Secondary is

$\frac{\Delta(N_2 \phi_2)}{\Delta t}$ as a result of the current in the

primary changing at the rate $\left(\frac{\Delta I_1}{\Delta t}\right)$ then

$$\xi_2 = -\frac{\Delta(N_2\phi_2)}{\Delta t} = -M\frac{\Delta I}{\Delta t} = -\frac{\Delta(MI_1)}{\Delta t}$$

where N_2 is the number of turns in the secondary coil

It follows that

$$N_2 \phi_2 = M I_1. \quad \text{------------------(14.22)}$$

This equation provides an alternative definition of M.

(c) **Non Inductive Winding.**

In bridge circuits such as used for resistance measurements, self Inductance is a nuisance.

When the galvanometer key of bridge is closed the currents in the arms of bridge are re-distributed unless the bridge happens to be balanced. While the currents are being re-distributed these are changing and self Induction delays the reading of a new equilibrium. Thus the galvanometer deflection at the instant of closing the key does not correspond to steady state which the bridge will eventually reach. It may, therefore, be misleading.

To minimize their self Inductance, coils of the bridge and resistance boxes are so wound as to set up extremely small magnetic fields. The wire is doubled back on itself before being coiled up as shown in the figure. Such a coil is said to be non-Inductive.

In this type of winding, current flows in opposite directions in the double-wires and consequently,the magnetic flux set up by one wire is neutralized by that due to the outer wire. Hence Self Induced emf's will not be produced when the current through the circuit changes

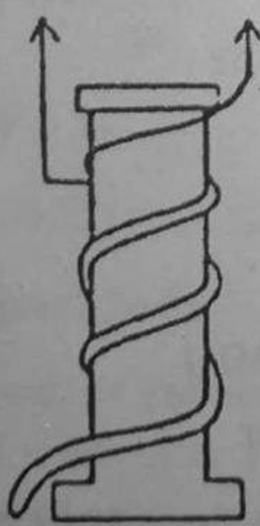


fig. (14.18)

Non Inductive coil

(d) **Motional EMF.**

When, a conductor is moved across a magnetic field, a potential difference appears across its ends. This potential difference is known as motional emf.

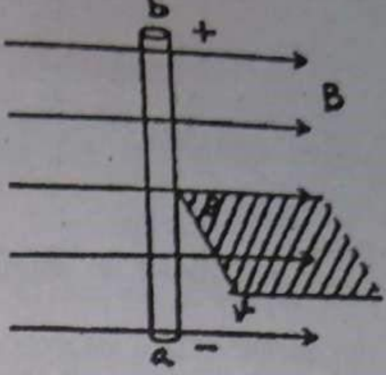


fig.(14.19.)

Consider a wire of length of ℓ moving across the magnetic field of Induction $\vec{B}$ with a velocity $\vec{v}$ as shown in the figure 14.19. Each free electron of the conductor is moving with the conductor and thus experiences a force

$$\vec{F} = -e(\vec{v} \times \vec{B}) = e(\vec{B} \times \vec{v}) \quad \text{from b to a}$$

The electrons go on accumulating at the end a leaving the end b with a positive charge till the force of electric field balances the force due to the motion of conductor. Thus a potential difference is set up from b to a.

Let the total charge that flows be q

$\therefore$ Potential difference = work done by unit charge

$$V = \frac{F\ell}{q} = \frac{q\,v\,B\,\mathrm{Sin}\,\theta\ell}{q} = vB\ell\sin\theta .$$

If the conductor is moving at right angles to the field $\theta = 90°$.

$$V = vB\ell \qquad \text{------------------(14.23)}$$

Example14.8.

A pair of adjacent coils has a mutual Inductance of 1.5 henrys. If the current in the primary changes from 0 to 20 ampere in 0.050 s what is the average induced emf in the Secondary? If the Secondary has 800 turns what is the change of flux in it?

Solution:

$$\xi_s = M \frac{\Delta I_1}{\Delta t} = 1.5 \times \frac{20}{.050} = 600 \text{ Volts}$$

$$\xi_s = N_s \frac{\Delta \phi}{\Delta t}$$

$$600 = 800 \times \frac{\Delta \phi}{0.050}$$

$$\Delta \phi = 0.038 \quad \text{Webers.}$$

Example 14.9.

A circuit in which there is a current of 5 amp is changed so that the current falls to zero in 0.1 s. If an average emf of 200 volts is induced, what is the self inductance of the circuit.

Solution:

$$\xi = L \frac{\Delta I}{\Delta t}$$

$$200 = L \frac{5}{0.1}$$

$$L = 4 \text{ henrys.}$$

Example: 14.10

The flux density B in a region is 0.5 weber/ m^2 directed vertically up wards. Find the emf induced in a straight wire 5 cm long perpendicular to B when it is moved across the field in a direction at an angle of 60° with the horizontal with a speed of 100 cm. s^{-1}

Solution:

Since the angle between the direction of motion and horizontal is 60°, the angle between the field and motion is 30°.

Induced emf = $B v \ell \sin\theta$

$= 0.5 \times 1 \times .05 \times .5$

$= 0.0125$ Volts

14.10. Alternating Current Generator (Dynamo).

An electric generator is a device to convert mechanical energy into electrical energy .

The principle of the generator is that an emf is induced in the coil due to changing magnetic flux linkage when it is rotated between the poles of a magnet. The essential parts of an alternating current generator are:-

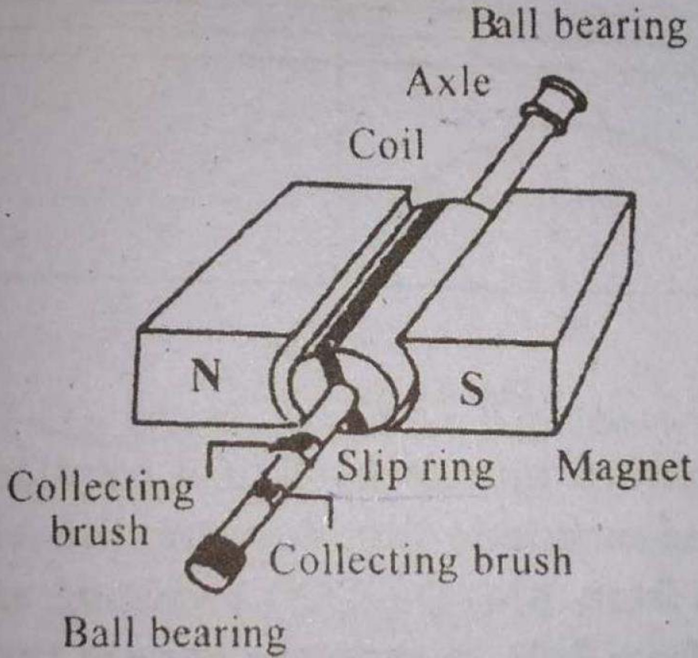


Fig. (14.20)

1. **Field Magnet.** It is strong permanent horse-shoe magnet, which produces a strong and uniform magnetic field of Induction $\vec{B}$ between its poles.

2. **Armature.** It is a soft iron cylinder mounted on an axle which rotates on ball bearings thus rotating the cylinder between the poles of the magnet. A coil of Insulated copper wire of large number of turns is wound on the cylinder in the grove cut length wise as shown in the figure.

3. **Slip Rings and Collecting Brushes.** The ends of

the coil are joined to two separate copper rings fixed on the axle. Two carbon brushes remain pressed against each of the rings which form the terminals of the external circuit.

Let the coil of an area A with number of turns N start rotating clockwise from the position marked .1. in the figure 14.21

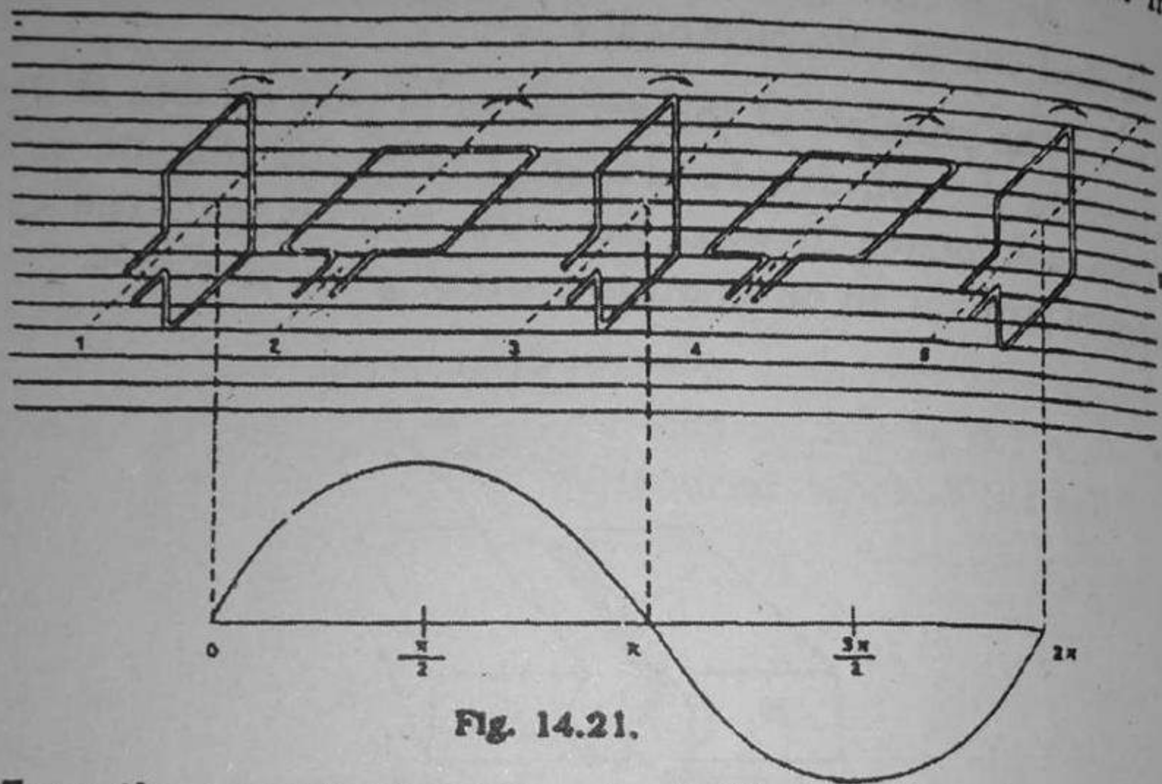


Fig. 14.21.

When the armature rotates by half a rotation from position 1 to 3. The magnetic flux entering the left hand side face decreases from BAN-O-(-BAN) inducing an emf clockwise (as seen from left) in order to oppose the cause.

At positions 1 and 3 the coil is rotating for sometime almost parallel to the lines of magnetic flux and the rate of change of flux and hence the induced emf is zero. The emf is maximum at positions 2 and 4 of the coil where the rate of change of flux is maximum.

During the second half rotation from position 3 to 5 the emf is in the reverse direction because the flux through the face under consideration increases from -(BAN)-O-BAN. Thus an alternating emf is generated. At thermal power house the armature of the generator is rotated by steam turbines and at hydroelectric power house it is set into rotation by the turbines driven by water fall.

A Motional emf $(\vec{v} \times \vec{B})$ $N\ell$ is setup in each of the sides ab and cd in opposite directions when the coil is rotated because these sides are moving in opposite sense with respect to the magnetic field.

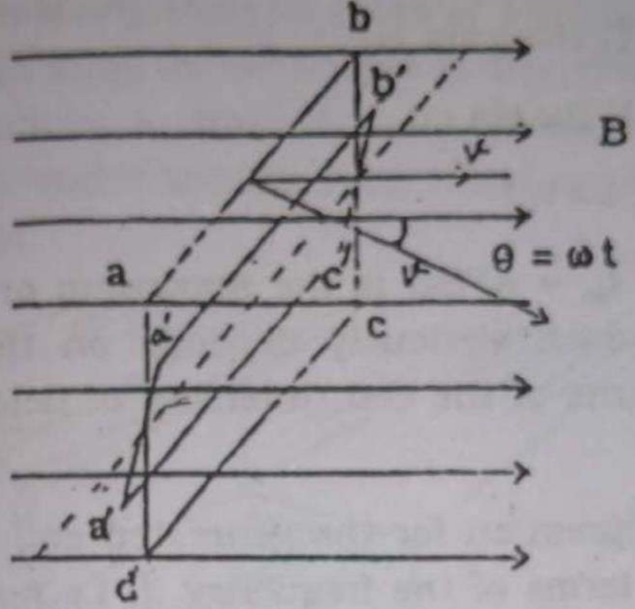


Fig. (14.22)

Since the other two sides are moving in the same sense with respect to field. The emfs are induced in the same direction and cancel each other. Total emf in the coil when it is at the position a'.b'.c'.d'

$\xi = 2\ vBN\ell\ \text{Sin}\ \omega t$

v = Linear Velocity

B = magnetic field of Induction

N = number of turns

ℓ = Length of the coil

ω = angular velocity

t = time in which the coil moves from the position abcd to a' b' c' d'

Since each particle of the sides ab and cd rotates in a circle of radius equal to half the width of the coil b

$$v = \frac{b}{2}\omega$$

$$\xi = 2\frac{b}{2}\omega B N \ell \sin \omega t$$

$$\xi = (b\ell) NB\omega \sin \omega t$$

$$\xi = A N B\omega \sin \omega t$$

$$\xi = \xi_o \sin \omega t$$

where ξ_o = ANBω is the maximum or peak value of the emf which obviously depends on the area and number of turns of the coil. Intensity of field and speed of rotation.

The expression for the generated emf can also be expressed in terms of the frequency .f. i.e number of rotation per second

$$\xi = \xi_o \sin 2\pi ft \qquad \text{------------------ (14.24)}$$

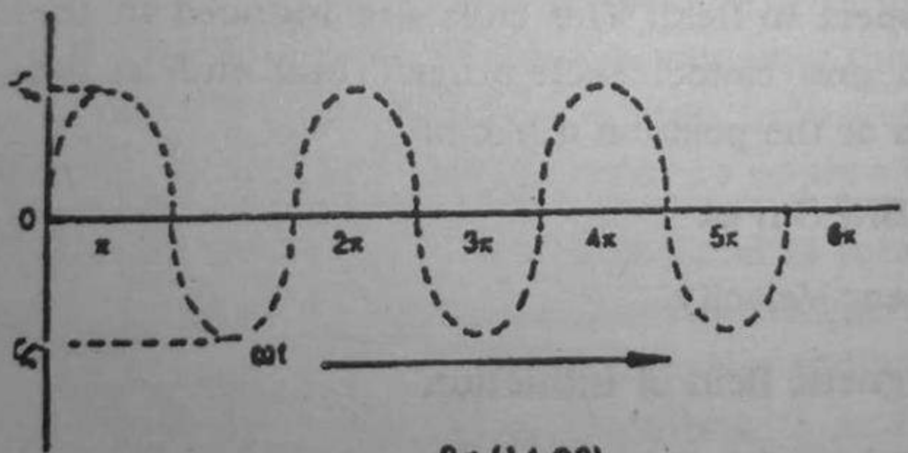


fig (14.23)

Any small generator employing a permanent magnet is commonly called a magneto and it is used in ignition system of petrol engines. motor bikes and motor boats. etc.

The field magnets of large generators are electromagnets and these generators are called alternators. The performance of A.C generator is more satisfactory when the armature is stationary and the field magnet rotates around the armature. Stationary armature is called stator and rotating magnet rotor.

14.11. D.C. Generator.

By replacing the slip rings of an A.C. generator by a simple split ring, or commutator, the generator can be made to produce a direct current through the external circuit. A generator modified for this function is called d.c. generator.

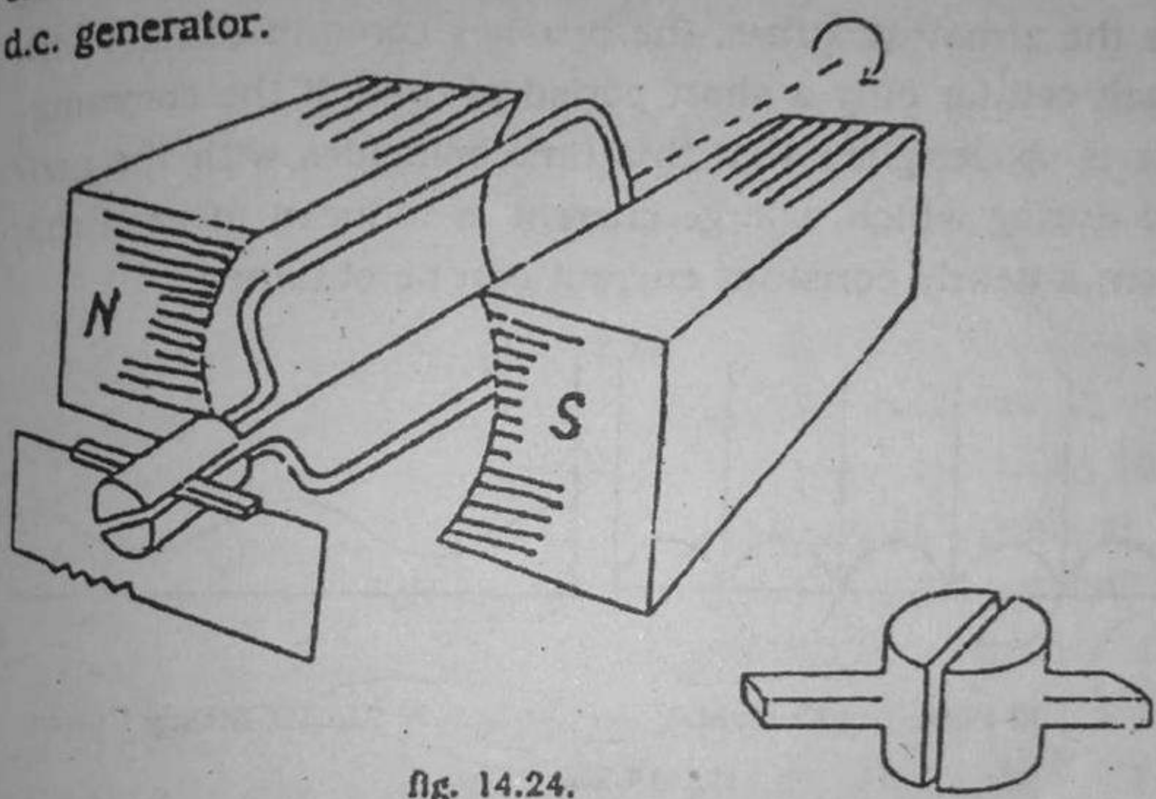
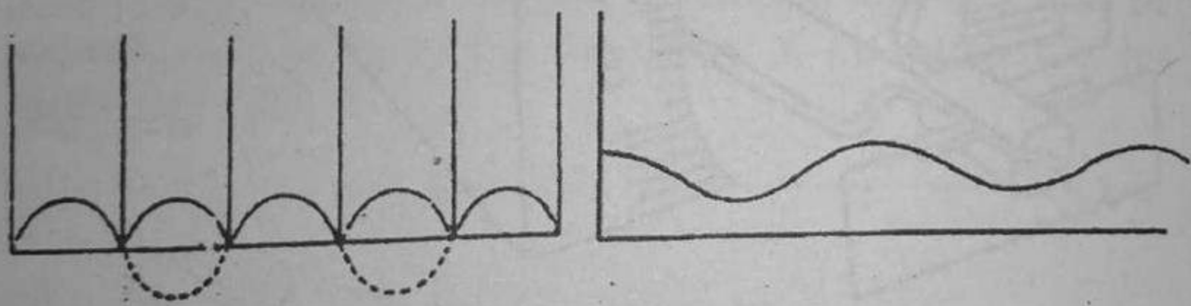


fig. 14.24.

Fig. 14.24 shows the armature coil of this generator after some time as it rotates in a clockwise direction from its vertical position. The induced emf increases gradually from zero to a maximum during the first quarter of the revolution and decreases to zero during the second quarter of the revolution.

As the coil rotates past its next vertical position, an emf is induced in the coil in the reverse direction just in the case of an A.C generator. But at this moment, the two halves of the split ring exchange contact with the brushes so that the direction of induced emf in the external circuit remains unaltered. This emf again increases from zero to a maximum and then falls to zero when the coil is back in the initial vertical position.

The variation of the emf with the rotation of the coil is shown in fig.14.25. Although the current is unidirectional, it fluctuates from zero to maximum. Such a current is called pulsating current. To obtain a steady current, a number of coils are mounted at different angles round the armature and the commutator is divided into a corresponding number of segments with each coil connected to two diametically opposite segments. Thus, as the armature turns, the brushes come in contact with each coil for only a short period of time. If the commutator is so designed that this time coincides with the period during which a large current is induced in each coil, then a nearly constant current can be obtained.



a) Pulsating Current. b) Almost Steady Current.

Fig. 14.25.

14.12. Electric Motor.

An electric motor is a device which converts electrical energy into mechanical energy.

A simple D.C. motor is shown in fig 14.26. It follows from article 14.3 that when a current is passed through a coil capable of rotation in a magnetic field of induction, it experiences a couple.

$\tau = BIAN \cos\alpha$. To rotate the coil in anticlockwise direction.The couple becomes zero when the face of the coil becomes perpendicular to the field. If the coil turns

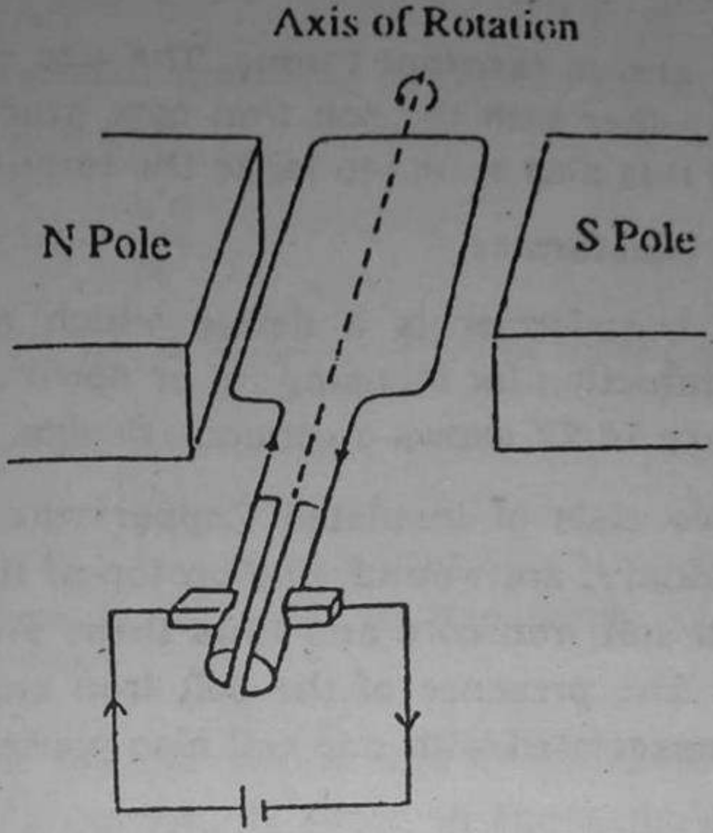


fig 14.26.

beyond this point there will be a torque in the opposite direction to return the coil unless the direction of current is reversed. For the coil to continue to rotate must be a commutator to reverse the direction of current at the proper time. When the plane of the coil is vertical the gaps in the commutator are facing the brushes and momentarily, there is no current in the coil. However, its inertia carries it beyond this position so that side Y comes in contact with brush M and side X comes into contact with brush N.

It follows that the current in the coil always flows in the same direction (clockwise as seen from above) and therefore the coil rotates in an anticlockwise sense no matter what its orientation.

In practice the armature of the motor consists of several equally spaced coils wound on a soft iron core and connected to a commutator which has a corresponding number of sections. The main advantage of using several coils, rather than just one, is that the motor provides an almost constant torque. The use of curved pole pieces together with the soft iron core produces a radial field and this also serves to make the torque constant.

14.13. Transformer.

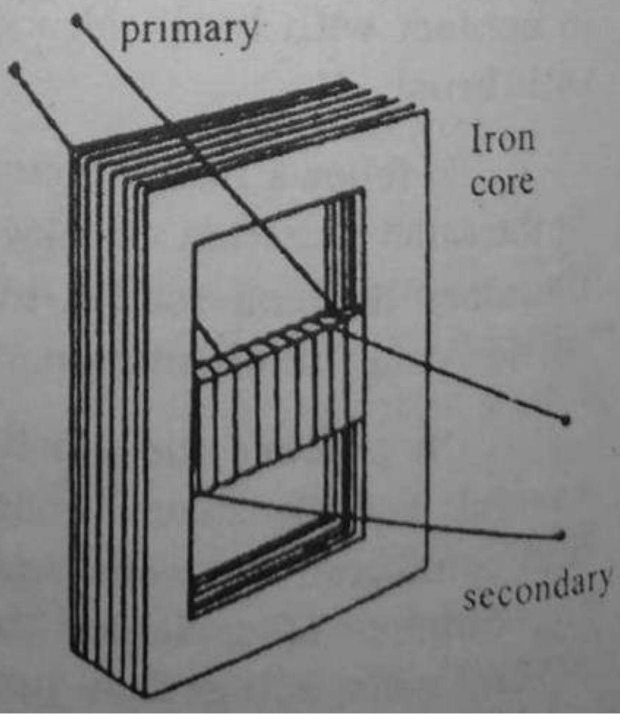
A transformer is a device which makes use of mutual induction for stepping up or down an alternating emf. Figure 14.27 shows a common design.

Two coils of insulated Copper wire, the primary and secondary, are wound, one on top of the other, on a laminated soft iron core and thus these are linked magnetically. The presence of the soft iron ensures that all the flux associated with one coil also passes through the other.

Suppose that an alternating emf E_p is applied to the primary coil. If at some instant the flux in the primary be ø then there will be a back emf in it given by the equation.

$$E_p = -\frac{\Delta (N_p \phi)}{\Delta t}$$

The flux through the primary also passes through the secondary and therefore the rate of change of flux in the secondary is also $\frac{\Delta\phi}{\Delta t}$.



It follows that there will be an emf E_s induced in the Secondary and that

$$E_S = -N_S \frac{\Delta \phi}{\Delta t} \qquad \text{------------------(14.25)}$$

Where N_S is the number of turns on the Secondary

$$\frac{E_S}{E_P} = \frac{N_S}{N_P} \qquad \text{------------------(14.26)}$$

If $N_S > N_P$, the transformer is called a step-up transformer because $E_S > E_P$. A step down transformer has $N_S < N_P$.

When a load (a resistance) is connected across the secondary, a current, I_S flows in the secondary. Suppose that the current in the primary is I_P. If the transformer is 100% efficient.

Power output = Power input

$$I_S E_S = I_P E_P \qquad \text{----------------(14.27)}$$

The efficiency of a transformer $\eta = \dfrac{\text{Power output}}{\text{Power input}}$

The efficiencies of commercial transformers are very high in the range 95 to 99%.

Sources of Power Loss in Transformer.

1. Eddy currents induced on the surface of iron core due to variation of magnetic flux produce heating and therefore reduce the amount of power that can be transferred to the secondary. The core is laminated i.e made up of thin sheets of soft iron each separated from the next by a layer of insulating varnish. This very nearly eliminates eddy current heating.

2. Each time the direction of magnetization of the core is reversed, some energy is wasted in over-

coming internal friction. This is known as hysteresis loss and it produces heating in the core. It is minimized by using special alloys (perm-alloy) for the core material.

3. Some energy is dissipated as heat in the coils (I^2R). This is reduced by using suitably thick wire. The coil which has the smaller number of turns carries the larger current and therefore is wound from thicker wire than the other.

4. Some loss of energy occurs because a small amount of the flux associated with the primary fails to pass through the secondary.

A few uses of transformers are listed below.

1. **Power Transmission.**

Whenever electric energy is to be used at a considerable distance from the generator an alternating current system is used because the energy can then be distributed without excessive loss. On the other hand if a direct current system were used, the losses in transmission would be very great.

Power loss from transmission wires $= I^2R$ watts.

In A.C. system the terminal voltage at the generator is increased using a step up transformer. Thus the current through transmission wire becomes very small and consequently the power loss I^2R is small. At the other end a step down transformer reduces the voltage to a value 220 — 240 volts that can safely be used.

2. In houses a transformer may be used to step the voltage down from 220 to 4 volt for call bells.

3. Transformers with several secondaries are used where several different voltages are required. For example radio, television circuits etc.

QUESTIONS

14.1. What is flux density and how is it related to the number of lines of Induction expressed in Webers?

14.2. Charged particles fired in vacuum tube hit a fluorescent screen. Will it be possible to know whether they are positive or negative?

14.3. Beams of electrons and Protons are made to move with the same velocity at right angles to a uniform magnetic field of Induction. Which of them will suffer a greater deflection? What will be the effect on the beam of electrons if their velocity is doubled?

14.4. Circular loop of wire hangs by a thread in a vertical plane. An electric current is maintained in the loop anticlockwise on looking at the front face. What direction will the front face of coil turn ?

14.5. Imagine that the room in which you are seated is filled with a uniform magnetic field pointing vertically upwards. A loop of wire which is free to rotate about a horizontal axis in its plane through its centre has its plane horizontal. For what direction of current in the loop as viewed from above will the loop be in a stable equilibrium?

14.6. Two identical loops. One of copper and the other of aluminium, are similarly rotated in a magnetic field of Induction. Explain the reasons for their different behaviour. Is electric generator a generator of electricity? where is the electricity before it is generated? what do such machines generate?

14.7. A loosely wound helix spring of stiff wire is mounted vertically with the lower end just touching mercury in a dish. When a current is started in the spring it executes a vibratory motion with its lower end jumping out and into mercury. Explain the reason for this behaviour.

14.8. Derive the general equation for induced emf begining with the law of force in a current carrying conductor in a magnetic field.

14.9. What is the mechanism of transfer of energy between the primary and secondary windings of a transformer? A certain amount of power is to be transferred over a long distance. If the voltage is stepped up 10 times, how much is the transmission line loss reduced?

14.10. What is the difference between magneto and A.C generator? what is meant by the frequency of alternating current?

PROBLEMS.

14.1. A horizontal straight wire 5 cm long weighing 1.2 $g.m^{-1}$ is placed perpendicular to a uniform horizontal field of 0.6 webers-m^{-2}. If the resistance of the wire is 3.8Ω m^{-1}. Calculate the potential difference to be applied between the ends of the wire to make it just self supporting.

(Ans. 3.7×10^{-3} volts)

14.2. A cathode ray tube is set up horizontally with its axis N-s and surrounded by a magnetic sheild. If the voltage across the tube is 900 volts, the distance from electron gun to the screen is 10 cm and vertical component of earth's field is 0–45 × 10^{-4} webers/m^2. Calculate by how much the spot on the screen will move when the magnetic sheild is removed.

Given that $\frac{e}{m} = 1.8 \times 10^{11}$ Ckg^{-1}

(Ans: 2.27×10^{-3} m)

14.3. What is the flux density at a distance of 0.1 m in air from a long straight conductor carrying a current of 6.5 amperes? Hence calculate the force per metre on a similar parallel conductor at a distance of 0.1 m from the first and carrying a current of 3 amperes. Will the wires attract or repel, if the directions of currents in the two wires are opposite to each other? Explain how the expression of force between two such conductors is used to define ampere.

(Ans. 13×10^{-6} webers-m^{-2} 39×10^{-6} N)

14.4. A straight metal rod 50 cm long can slide with negligible friction on parallel conducting rails. It moves at right angles to a magnetic field 0.72 webers-m^{-2}. The rails are joined to a battery of emf 3 volts and a fixed series resistance of 1.6 Ω. Find the force required to hold the rod at rest.

(Ans. 0.675 N).

14.5. It is required to produce inside a toroid a field of 2×10^{-3} webers-m^{-2}. The toroid has a radius of 15 cm and 300 turns. Find the current required for this purpose. If toroid is wound on an iron core of permeability 300 times the permeability of free space what increase in B will occur for the same current.

(Ans. 5 Amp. 300 times)

14.6. A proton is accelerated by a potential difference of 6×10^5 volts. It then enters a uniform field B = 0.3 webers-m^{-2} in a direction making an angle of 45° with the magnetic field. what will be the radius of the circular path?

(Ans. 0.26m)

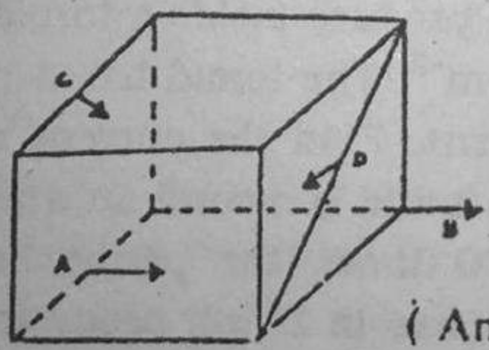
14.7. Two parallel metal plates separated by 5 cm of air have a potential difference of 220 volts. A magnetic field $B = 5 \times 10^{-3}$ webers-m^{-2} is also produced perpendicular to electric field. A beam of electrons travel undeflected through these crossed electric and magnetic fields. Find the speed of electrons.

(Ans. 8.8×10^5 ms^{-1})

14.8 A coil of 50 turns wound on a rectangular ivory frame 2 cm x 4 cm is pivoted to rotate in a magnetic field of 0.2 webers-m^{-2}. The face of the coil is parallel to the field. How much torque acts over the coil when a current of 0.5 amp passes through it? what will be the torque when the coil is rotated by 60° from its initial position?

(Ans. 0.4×10^{-2} N-m, 0.2×10^{-2} N-m)

14.9. A cube 100 cm on a side is placed in a uniform magnetic field of flux density 0.2 webers-m^{-2}, as shown in the diagram. Wires A,C and D move in the directions indicated, each at a rate of 50 cm.s^{-1}. Determine the induced emf in each wire.



(Ans 0,0.0707,0.1 volts)

14.10. What is the mutual Inductance of a pair of coils if a current change of 6 amps in one coil causes the flux in the second coil of 2000 turns to change by 12×10^{-4} webers-m^{-2}.

(Ans. 400 mh)

14.11. An emf of 45 m.volt is induced in a coil of 500 turns. when the current in a neighbouring coil changes from 10 amps to 4 amps in 0.2 seconds.

a) What is the mutual Inductance of the coils?

b) What is the rate of change of flux in the second coil?

(Ans. 1.5 mh, 9×10^{-5} webers-s^{-1})

14.12. An iron core solenoid with 400 turns has a cross section area of 4.0 cm^2. A current of 2 amp passing through it produces B = 0.5 webers·m^{-2}. How large an emf is induced in it. If the current is turned off in 0.1 seconds. What is the self Inductance of the solenoid.

(Ans 0.8V L = 40 mh)

14.13. The current in a coil of 325 turns is changed from zero to 6.32 amps, there by producing a flux of 8.46×10^{-4} webers. what is the self Inductance of the coil.

(Ans. 43.5 mh)

14.14. A 100 turns coil in a generator has an area of 500 cm^2 rotates in a field with B = 0.06 webers-m^{-2}. How fast must the coil rotated in order to generate a maximum voltage of 150 volts.

(Ans. 500 rad/s)

14.15. A step down transformer at the end of a transmission line reduces the voltage from 2400 volts to 1200 volts. The power output is 9.0 K.W and over all efficiency of the transformer is 95%. The primary winding has 400 turns. How many turns has the secondary coil? what is the power input. what is the current in each of the coils?

(Ans: N_s = 200, P_p = 9473 watts. I_p = 3.9 amps, Is = 7.5 amp).

14.16. The overall efficiency of a transformer is 90%. The transformer is rated for an output of 12.5 KW.

The primary voltage is 1100 volts and the ratio of primary to secondary turns is 5:1. The iron losses at full load are 700 watts. The primary coil has a resistance of 1.82 ohms.

a) How much power is lost because of the resistance of the primary coils?

b) What is the resistance of the secondary coils?

(Ans: 290 watts. 0.124 ohm).

Chapter 15.

ELECTRICAL MEASURING INSTRUMENTS

15.1 Introduction

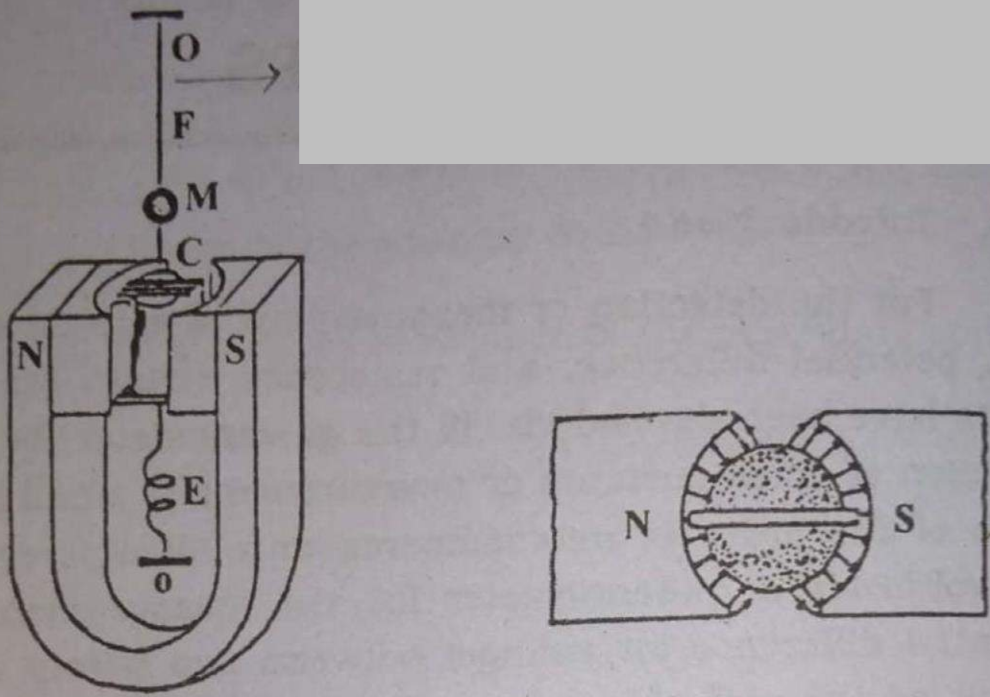
For the detection or measurement of electric current, potential difference, and resistance certain instruments have been devised viz. (i) the galvanometer for the detection of small currents or measurement of small currents of the order of microamperes or milliamperes (ii) the voltmeter or potentiometer for the measurement of potential difference (or voltage) between two points of a circuit or the emf of a source (iii) the ammeter for the measurement of large currents (iv) the wheatstone bridge, the meter bridge, the post office box and the Ohmmeter for the measurement of resistance.

15.2 The moving coil galvanometer

The moving coil galvanometer is a basic electrical instrument. It is used for the detection (or measurement) of small currents.

Its underlying principle is the fact that when a current flows in a rectangular coil placed in a magnetic field it experiences a magnetic torque. If it is free to rotate under a controlling torque, it rotates through an angle proportional to the current flowing through it. The rotation or deflection thus indicates a current through it. Although the galvanometer deflects full-scale by a small current (a few microamperes or milliamperes), nevertheless it can measure such small currents if the deflection is properly calibrated.

The primitive form of the moving coil galvanometer was developed by the French scientist D'Arsonval. Edward Weston improved upon D'Arsonvals original version and gave us the modern moving-coil galvanometer



(a) Moving coil galvanometer.
Fig: 15.1

(b) Concave pole piece and soft iron cylinder makes the field radial and stronger.

As shown in Fig. 15.1, the essential parts of a moving coil galvanometer are:

i) A U-shaped permanent magnet with cylindrical concave pole-pieces

ii) A flat coil of thin enamel insulated wire (usually rectangular)

iii) A soft iron cylinder.

iv) A pointer

v) A scale

The flat rectangular coil of thin enamel insulated wire of suitable number of turns wound on a light non-metallic (or aluminium) frame is suspended between the cylindrically concave pole pieces of the permanent U-shaped magnet by a thin phosphor bronze strip. One end of the wire of the coil is soldered to the strip. The other end of the strip is fixed to the frame of the galvanometer and connected to an external terminal. It serves as one current lead through which the current enters or leaves the coil. The other end of the wire of the coil is soldered to a loose and soft spiral of wire connected to another external terminal. The soft spiral of wire serves as the other current lead. A soft-iron cylinder, coaxial with the pole pieces, is placed within the frame of the coil and is fixed to the body of the galvanometer. In the space between it and the pole pieces, where the coil moves freely, the soft iron cylinder makes the magnetic field stronger and radial such that into whatever position the coil rotates, the magnetic field is always parallel to its plane.

When a current passes through the galvanometer coil it experiences a magnetic deflecting torque which tends to rotate it from its rest position. As the coil rotates it produces a twist in the suspension strip. The twist in the strip produces ah elastic restoring torque. The coil rotates until the elastic restoring torque due to the strip does not equal and cancel the deflecting magnetic torque and then it attains equilibrium and stops rotating any further.

In the previous chapter the, deflecting magnetic torque was derived as:

Deflecting magnetic torque = BIAN cos α

Where B = strength of the magnetic field.

I = current in the coil

A = Area of the coil

N = Number of turns in the coil

and α = the angle of deflection of the coil

The restoring elastic torque is proportional to the angle of twist of the suspension strip provided it obeys

Hooke's Law. Thus restoring elastic torque = c θ, where θ is the angle of twist of the suspension strip (θ is different from but proportional to α), and c is the torque per unit twist of the suspension strip for equilibrium.

Deflecting magnetic torque = Restoring elastic torque

or $$BIAN \cos \alpha = c\,\theta$$

or $$I = \frac{c}{BAN \cos \alpha}\,\theta \quad \text{------------(15.1)}$$

If the magnetic field were uniform (as with flat pole-pieces) α would continuously increase with θ and cos α factor would not be constant. Then the current I would not be proportional to θ and the scale of the galvanometer not linear. However, due to the radial magnetic field the plane of the coil is always parallel to the field irrespective of the position the coil rotates. So, α, the angle between the plane of the coil and the direction of the field is always zero, hence cos α = 1 i.e. constant as are B,A and N.

The equation 15.1 therefore, reduces to

$$I = \frac{c}{BAN}\,\theta \quad \text{------------------(15.2)}$$

$$\therefore I \propto \theta$$

Thus the current through the coil is directly proportional to the angle of twist of the suspension (or deflection), θ, giving a linear scale

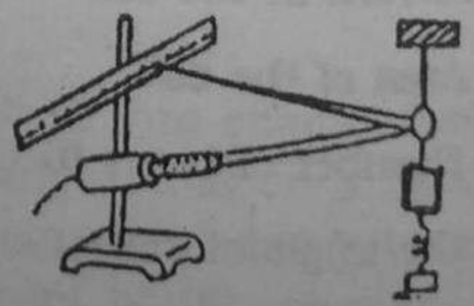


Fig. 15.2 Lamp and Scale Arrangement

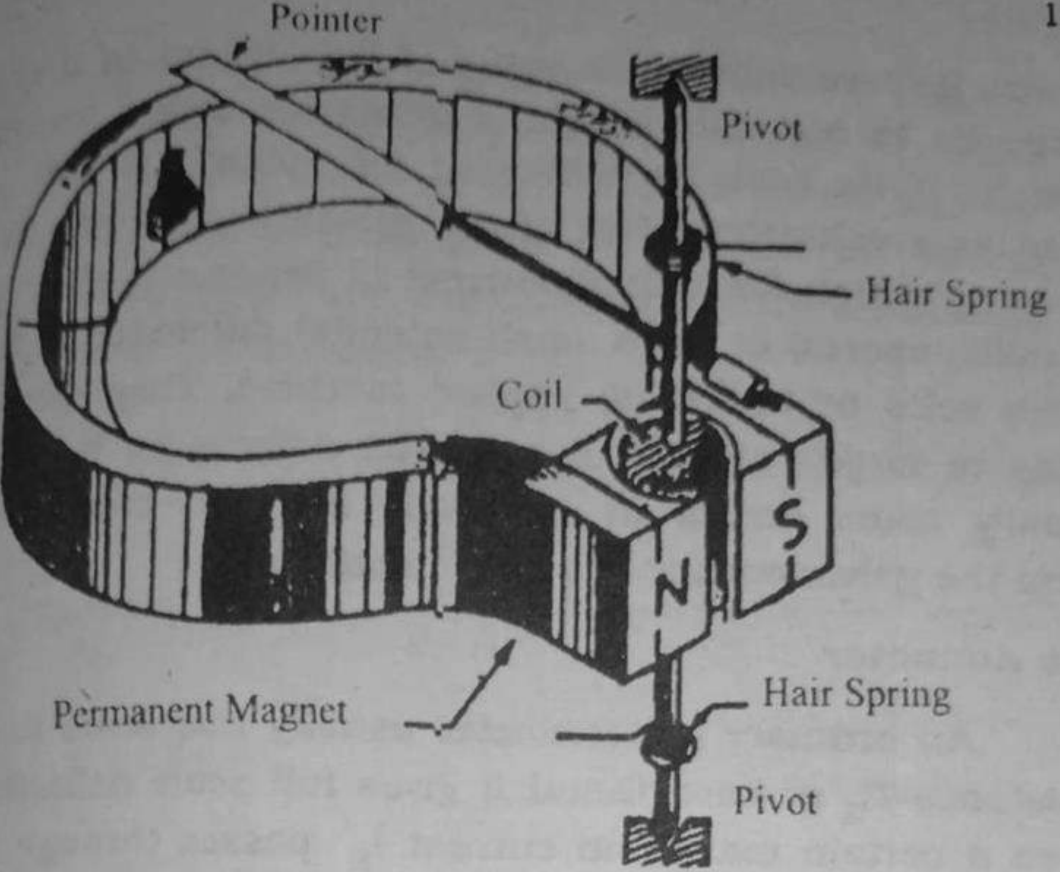


Fig. 15.3 Pivoted moving coil galvanometer

The coil instead of being suspended by a strip is pivoted between two jewelled bearings. The controlling (restoring) torque is provided by two hair-springs one on either side of the coil and curling in the opposite sense. The two ends of the coil are connected one to each spring. The hair springs thus also serve as current leads to the coil. A light aluminium pointer is fixed to the coil which moves over a calibrated circular scale with equal divisions which measures the deflection (in divisions) or current (in micro amperes) directly.

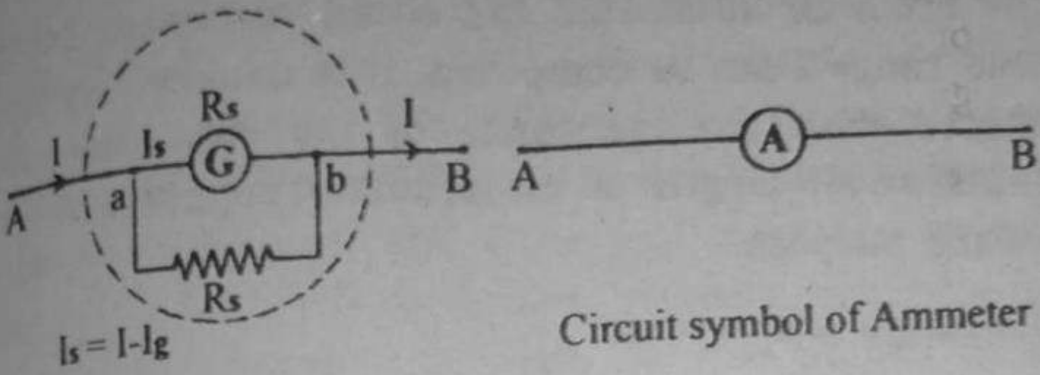
15.3 Ammeter and Voltmeter.

Ammeters and voltmeters are simple pivoted type moving coil galvanometers with suitable modifications. We have already seen that when a current flows in the galvanometer its coil is deflected. The current is proportional to the deflection. If the scale is calibrated for the current, it can be used as an ammeter. Again since the galvanometer coil has a fixed resistance, the current in it is proportional to the potential difference (or voltage) across its terminals. Each value of the current in it corresponds to a certain potential difference across its terminals. If its scale is calibrated for voltage, it can be used as a voltmeter. Most of the galvanometers give full scale deflection for a small current (a few micro amperes or milliamperes) or for a small potential difference (a few micro volts or millivolts) applied to them. They cannot measure large currents or potential differences that we usually come across in our daily life. For measuring them the galvanometer has to be modified.

The Ammeter

An ordinary galvanometer usually has some fixed resistance R_g of its coil and it gives full scale deflection when a certain maximum current I_g passes through it. This maximum current is called the range of the unaided galvanometer. A current that lies within this range can be measured directly with the galvanometer, thus it can serve as an ammeter. For measuring a current this ammeter must be connected in series with the circuit to allow through it the full current which is to be measured. If R_g is large, the insertion of this ammeter will increase the resistance in the circuit and decrease the current it is intended to measure an undesirable situation. So as not to affect the current to be measured an ammeter essentially must have very small resistance.

On the other hand if it is desired to measure a current much larger than I_g the galvanometer cannot be used directly as an ammeter. If tried it will certainly be changed. To measure any current upto I larger than I_g, the galvanometer has got to be modified such that while the current in the main circuit is I, the current in the galvanometer coil never exceeds I_g. This objective is achieved by connecting a by-pass resistance, called a shunt, of appropriate small value across in parallel with the galvanometer coil which allows the large excess current through itself while a known fraction of I within the value I_g passes through the galvanometer coil. The main current in the galvanometer coil and the shunt together is always a simple multiple of the current in the coil which can be found and the scale calibrated accordingly to read the main current directly.



(a)

Circuit symbol of Ammeter

(b)

Figure 15.4

Consider a galvanometer G whose resistance is R_g and which gives full scale deflection when current I_g flows through it. Fig. 15.4. Suppose we want to modify it into an ammeter to measure a maximum current I (or to increase its range to I). A shunt R_s of appropriate small resistance sheuld be connected in parallel with the galvanometer such that while the current in the main circuit is I the current in the galvanometer coil is I_g producing full scale deflection and that in the shunt is $I_s = I - I_g$.

As it is clear, the galvanometer coil of resistance R_g and the shunt resistance R_s are connected in parallel between junctions a and b. the potential difference, V_g, across the galvanometer and that across the shunt, V_s are the same, so $V_g = V_s = V$, say

From Ohm's Law (V = IR), we have :

$$V_g = I_g R_g$$

and $$V_s = I_s R_s = (I - I_g) R_s$$

Hence $$(I - I_g) R_s = I_g R_g$$

$$\therefore R_s = \left(\frac{I_g}{I - I_g} \right) R_g \qquad \text{-----------(15.3)}$$

From equation 15.3 the shunt resistance for any desired range I can be computed. It is usually very small compared with the resistance of the galvanometer. A suitable small length of an ordinary copper wire may serve the purpose.

Multi-range Ammeters.

Sometimes an ammeter has more than one range which means it has as many different shunts as the ranges. The desired range is selected by inserting the proper shunt in position. In one type, one end of each shunt is permanently connected to a common terminal while the other end of each is connected through a range switch to a second common terminal (Fig. 15.5(c)).

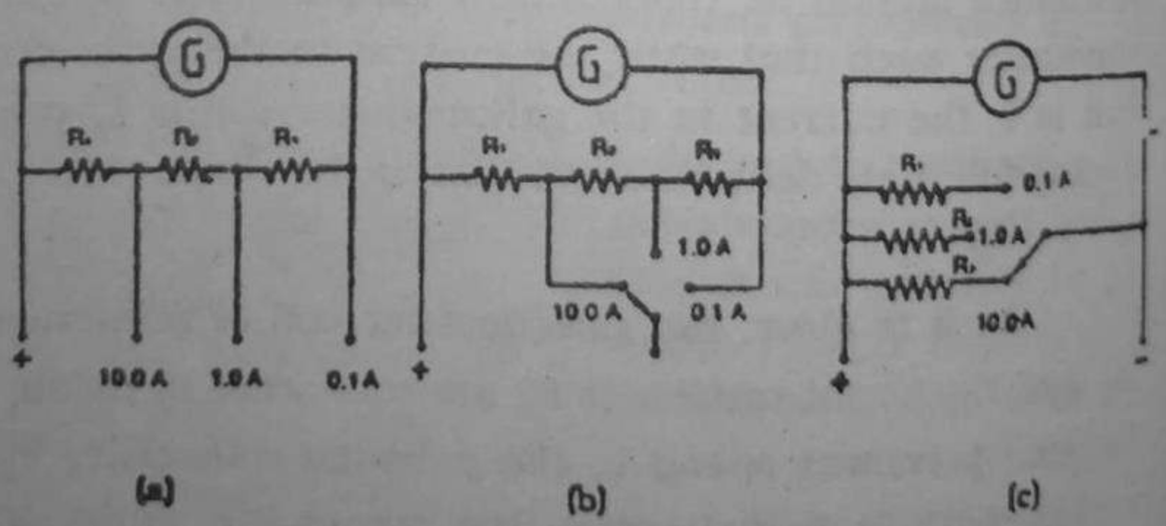
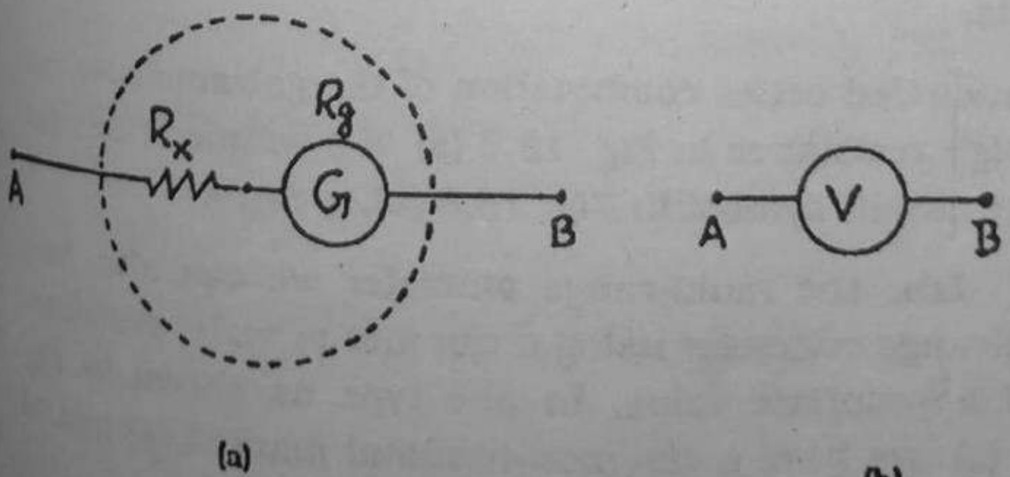


Fig. 15.5

In the other type the shunts are arranged as shown in Fig. 15.5 (a) and (b). In (a) separate range terminals are provided together with the common terminal marked (+) and in (b) the proper range terminal can be connected by a range switch to the second common terminal.

The Voltmeter

A voltmeter measures the potential difference between any two points of a current carrying circuit (or between the two terminals of a source). For doing so its terminals must be connected to these points. Evidently, used as a voltmeter, the galvanometer cannot directly measure the potential difference between two points of a circuit with accuracy even if the p.d. is within its small range V_g for full scale deflection, because when the galvanometer is connected between the two points its coil provides a conducting path of no large resistance. A current flows through the galvanometer and the original current between the points decreases and the p.d. between the points decreases too, since it is proportional to the current. Thus this voltmeter changes the p.d. it is called upon to measure. Moreover for measuring a p.d. much higher than its small range V_g the galvanometer will be unsuitable. Thus for measuring the p.d. accurately without affecting it in any way the voltmeter must not draw any current i.e. it must essentially have very large resistance



(a) (b)

Fig. 15.6

For converting a galvanometer in to a voltmeter of a desired range an appropriate high resistance of the order of kilo ohms is connected in series with it. This resistance is commonly known as multiplier resistance. Consider a galvanometer G whose resistance is R_g and which deflects full scale for the current I_g. Suppose we want to convert it into a voltmeter measuring a p.d up to V volts. An appropriate high resistance R_x must be connected in series with it such that for the p.d.V applied between the ends of the above combination the current in the galvanometer is Ig which produces full scale deflection. Now the total resistance between the terminals is $R_x + R_g$. Thus by ohms law,

$$(R_x + R_g) I_g = V$$

$$R_x + R_g = \frac{V}{I_g}$$

$$\therefore R_x = \frac{V}{I_g} - R_g \quad \text{---------------} (15.4)$$

The equation (15.4) helps to calculate the value of the series high resistance for the conversion of the galvanometer into a voltmeter of any desired range V volts. When the proper high resistance is connected in series with the galvanometer it is converted into a voltmeter of range V volts. The scale can then be calibrated from O to V volts.

The encircled series combination of the galvanometer and the high resistance in Fig. 15.6 (a) is a voltmeter denoted by the circuit symbol in Fig. 15.6 (b).

Like the multi-range ammeter we can also have multi-range voltmeter using a number of series resistances of appropriate value. In one type as shown in Fig. 15.7 (a), we have a common terminal marked (+) and as many other terminals as the ranges.

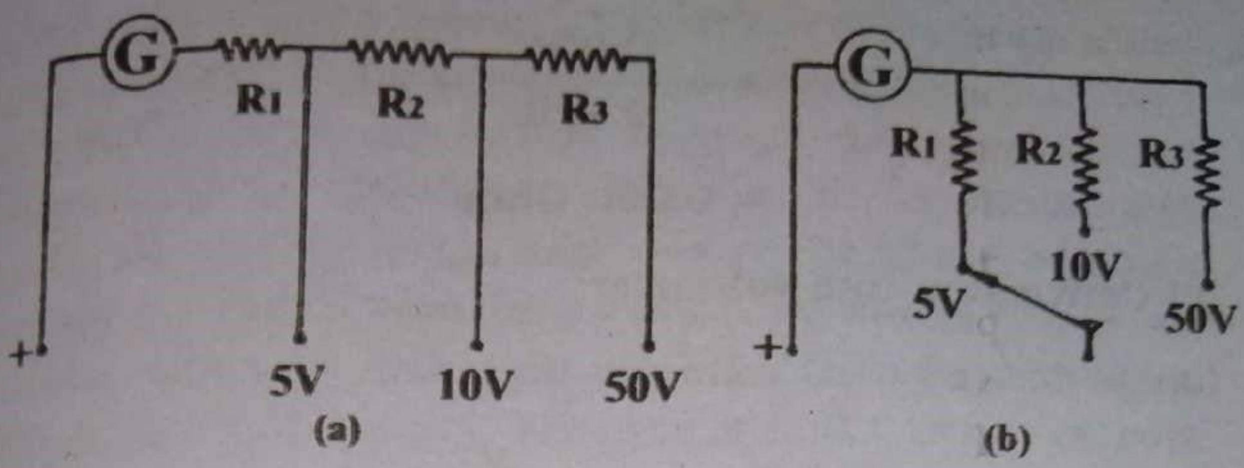


Fig. (15.7)

In the other type one terminal is common marked (+) while different range terminals can be connected by a range switch to the other common terminal (Fig.15.7[b])

Example: 15.1

A galvanometer has a resistance of 20 ohms and gives full-scale deflection when a current of 0.001 ampere flows in it. Find out (i) the value of the shunt resistance to convert it into an ammeter of range 10 amperes. (ii) the value of the series resistance to convert it into an voltmeter of range 10 volts.

Solution:

Resistance of the galvanometer, $R_g = 20$ ohms

Current for full scale deflection, $I_g = 0.001$ A.

(i) conversion into ammeter :

Range desired = 10 amperes

shunt resistance $= R_s = \left(\frac{I_g}{I - I_g}\right) R_g$

$$= \left(\frac{0.001}{10 - 0.001}\right) \times 20$$

$$= \frac{0.001}{9.999} \times 20$$

$$= 0.002 \text{ Ohm}$$

(ii) Conversion into voltmeter:

Range desired = 10 volts

Series high resistance, $R_x = \frac{V}{I_g} - R_g$

$$= \frac{10}{0.001} - 20$$

$$= 10000 - 20$$

$$= 9980 \text{ ohms}$$

(i) shunt resistance = 0.002 ohms
(ii) series high resistance = 9980 ohms Ans.

15.4. Wheatstone Bridge

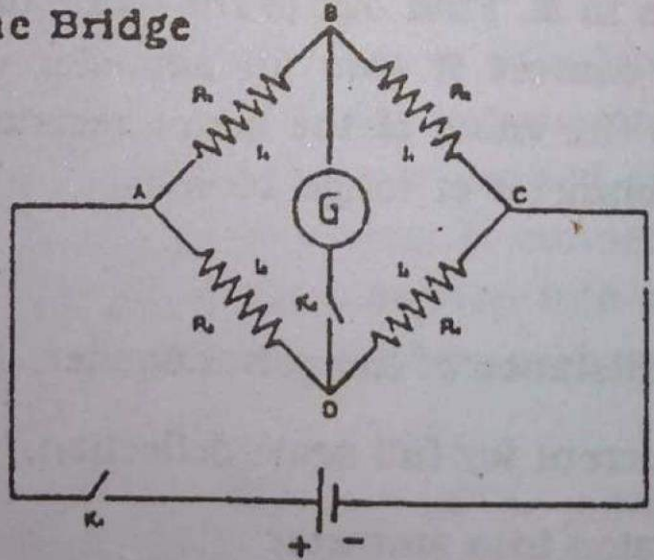


Fig. 15.8

If four resistances R_1, R_2, R_4 and R_3 are connected end-to-end in order to form a closed mesh ABCDA and between one pair of opposite junctions, A and C, a cell is connected through a key while between the other pair of opposite junctions, B and D, a sensitive galvanometer G is connected through another key, the circuit so formed is called a wheatstone bridge.

In the above bridge if the key K_1 is closed first some current flows through the cell and the resistances R_1, R_2, R_3 and R_4. If the key K_2 is also closed a current will usually be found to flow through the galvanometer indicated by its deflection. However, if the resistances R_1, R_2, R_3 and R_4 (or at least one of them) are adjusted a condition can always be attained in which the galvanometer shows no deflection at all i.e. no current passes through it. Then the p.d. between B and D must be zero i.e. B and D must be at the same potential. This implies that the p.d. between B and A (i.e. across R_1) must equal that between D and A (i.e. across R_3). Also the p.d. between B and C (i.e. across R_2) must equal that between D and C (i.e. across R_4). Since no current flows through the galvanometer the current in R_1 equals that in R_2, say I_1 and the current in R_3 equals that in R_4, say I_2, since

$V_{BA} = V_{DA}$ it follows from Ohm's Law that

$$I_1R_1 = I_2 R_3 \qquad \text{----------------- (15.5)}$$

also since $\quad V_{BC} = V_{DC}$,

$$I_1R_2 = I_2R_4 \qquad \text{------------------ (15.6)}$$

From equations (15.5) and (15.6) we have

$$\frac{R_1}{R_2} = \frac{R_3}{R_4} \qquad \text{-----------(15.7)}$$

This is an important relation true only for a balanced wheatstone bridge i.e. when no current flows in the galvanometer with both keys K_1 and K_2 closed. Under balance condition if any three resistances are known the fourth (unknown) can easily be computed. A number of resistance measuring instruments have been devised which make use of this important principle. Examples are the meter bridge, the post office box Carey Foster's bridge, callendar and Griffiths bridge, etc. Here we will discuss only the first two.

15.5 Meter Bridge

The meter bridge, also called slide-wire bridge, is an instrument based on Wheatstone Principle. It consists of a long, thick copper strip bent twice at right angles. Two small portions are cut off from it near the bends to provide the gaps across which two resistances a known one and an unknown may be connected. Each of the three pieces of the strip is provided with binding screws. A uniform wire (of manganin or constantan) one meter long and of fairly high resistance is stretched alongside a meter scale and connected to the ends of the strip (Fig. 15.9). It is this one meter long wire to which the instrument owes its name.

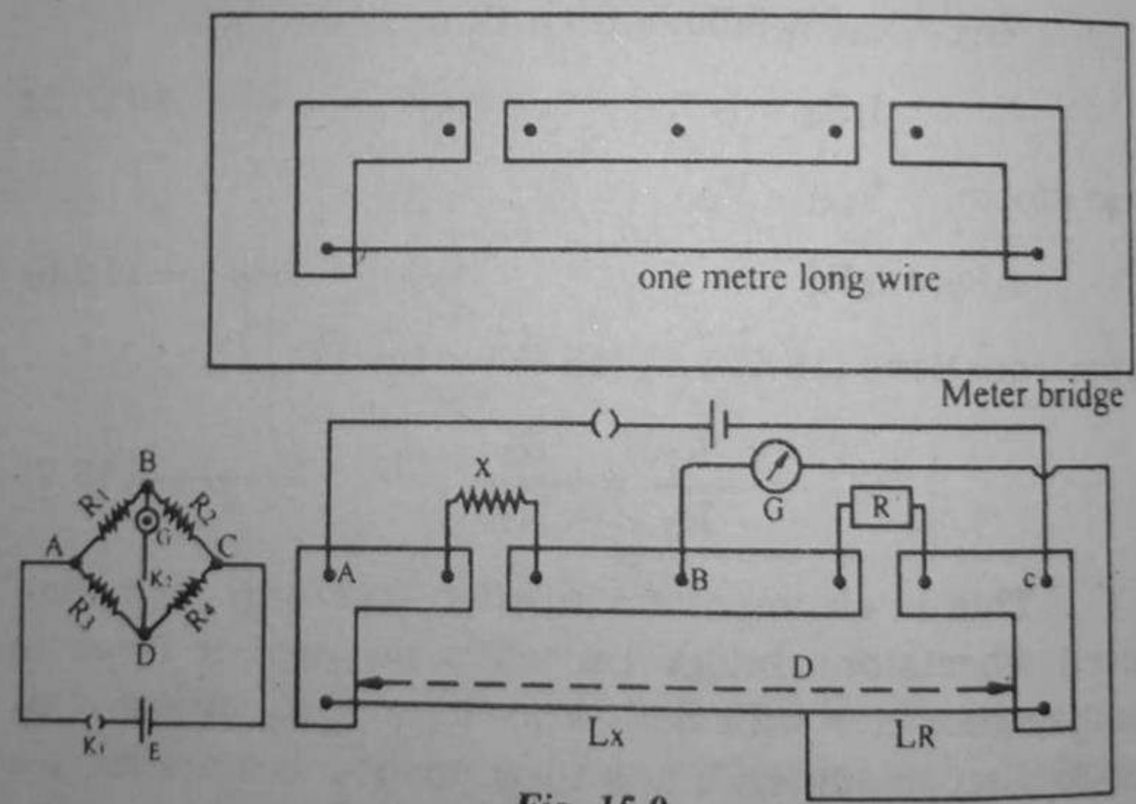


Fig. 15.9

For measuring an unknown resistance X, it is connected in one gap of the meter bridge and a standard resistance box, R, in the other. A cell and a galvanometer are connected as shown. The jockey is moved along the wire to obtain the balance point D. Under balance condition if the length of the wire segment AD towards X is L_x and the length of the wire segment BD toward R is L_R, then their resistances are ρL_x and ρL_R respectively, where ρ is the resistance per unit length of the wire. Then according to Wheatstone Principle:

$$\frac{X}{R} = \frac{\rho L_x}{\rho L_R} \quad \text{or} \quad \frac{X}{R} = \frac{L_x}{L_R}$$

Using this formula, X can be calculated if R is a known resistance.

15.6 Post Office Box (P.O.Box)

Post Office Box is another instrument based on Wheatstone Principle. It is so named because it was first introduced for finding the resistance of telegraph wires and for fault-finding work in the post and telegraph office. It is more compact and easier to use.

In a P.O Box the arms P and Q called the ratio arms, usually consist of three resistances each, viz. 10, 100, and 1000 ohms, so that any decimal ratio from 1:100 to 100:1 may be used. The third arm, R, is an ordinary set of resistances. The unknown resistance, X, to be measured, forms the fourth arm.

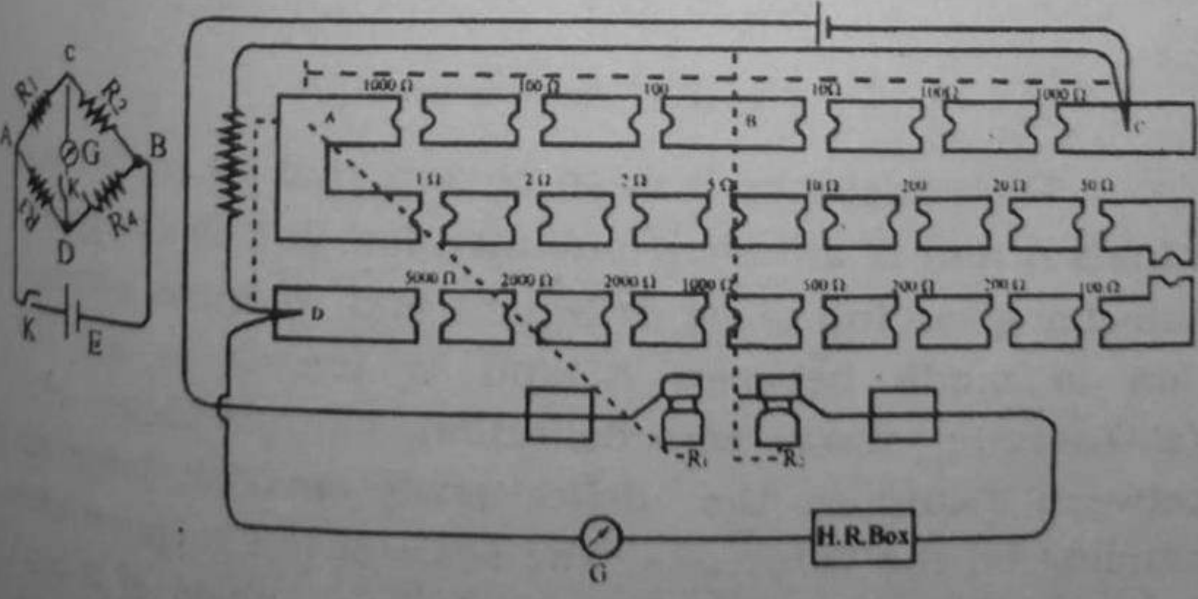


Fig. 15.10

Introducing the ratios 1:1, 10:1, 100:1 in turn the balance or null position is traced by adjusting R. Balance is usually obtained at the ratio 100:1 for some value of R. With this value of R, the value of X can easily be calculated using the relation of Wheatstone Bridge:

$$\frac{P}{Q} = \frac{R}{X} \quad \text{or } X = R\left(\frac{Q}{P}\right)$$

15.7. The Ohmmeter

Although not a very accurate instrument, the Ohmmeter is a useful device for quick measurement of resistance. It includes a sensitive galvanometer G, adjustable resistor R and a torch cell E connected in series between two terminals A and B as in Fig. 15.11.

The unknown resistance X to be measured is connected between the terminals A and B.

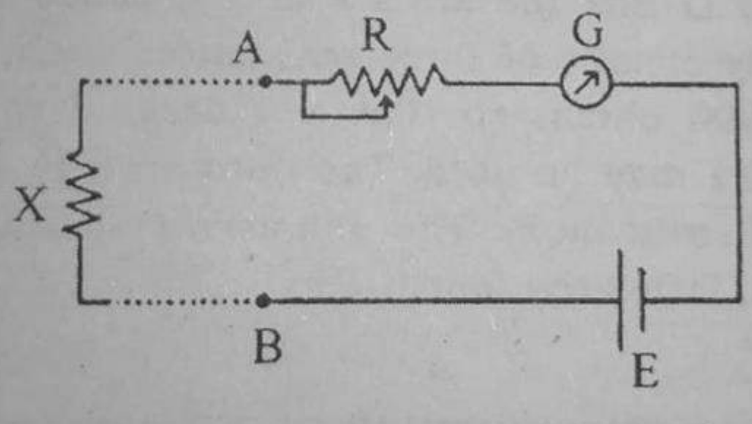


Fig. 15.11 An Ohmmeter

The resistance R is so chosen that when the terminals A and B are short circuited (i.e. X = 0) the galvanometer gives full scale deflection and when no connection is made between A and B (i.e. X = ∞) the galvanometer shows zero deflection. For the values of x between o and ∞ the deflection is small or large depending on the value of x. The scale of the galvanometer is first calibrated with different known values of x and then the circuit serves as an Ohmmeter to measure any unknown resistance approximately.

The scale of the Ohmmeter, however, is not linear.

Using different combinations of R in series and different shunts across the galvanometer worked by a range in switches. The Ohmmeter can be adopted for different accuracies eg: 1Ω accuracy, accuracy in tens of ohms in hundreds of ohms in thousands of ohms (kilo ohms) in mega ohms, etc.

15.8. Potentiometer

A potentiometer is a device for measuring the potential difference (or voltage) between two points of a circuit or the E.M.F. of a current source.

Consider a uniform resistance wire AB of length L and resistance R, across which is connected a source of constant E.M.F. (e.g. an accumulator) through a key and a rheostat to adjust and maintain a constant current I through it. (Fig. 15.12)

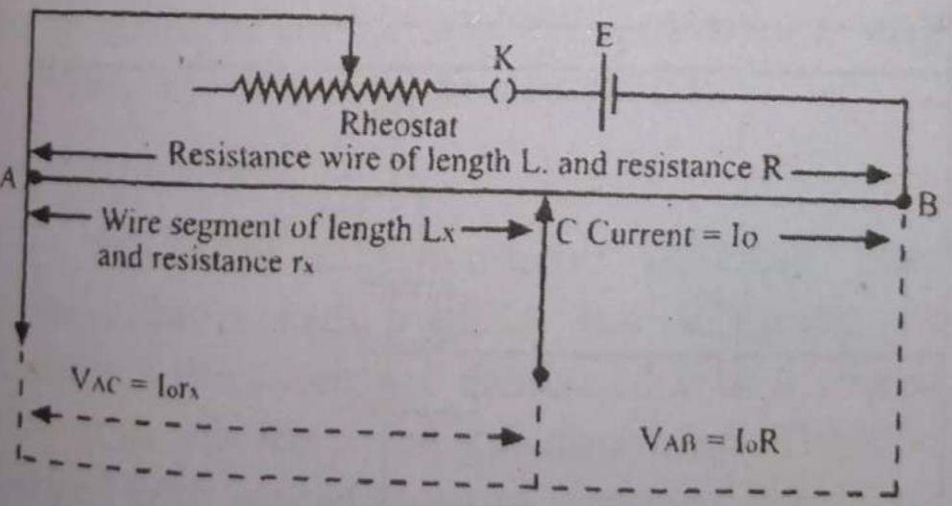


Fig. 15.12 A Potential divider

As the current flows, the potential difference between A and B = V_{AB} = IR.

A running point C can be considered on the wire such that the length of the wire AC = l_x. and its resistance = r_x. The potential difference between A and C =

$V_{AC} = I.r_x$. As C is a running point, the resistance r_x and the potential difference V_{AC} change continuously from zero value to the maximum values R and V_{AB} as C moves from A to B. V_{AC} is zero when C coincides with A. It continuously increases as C moves away from A. It is maximum (= V_{AC}) when C coincides with B. The point C thus divides the p.d., V_{AB}, across the wire AB into two parts — V_{AC} which can be applied usefully to some other circuit, and V_{CB} which may be kept idle or may be applied elsewhere. Therefore, this circuit is called potential divider. (Instead of one point C more than one point: C_1, C_2, C_3, etc. can also be chosen to divide V_{AB} into as many parts as the number of wire segments AC_1, C_1C_2,--- ----------etc.).

The potential divider of Fig. 15.12 can be used to measure an unknown E.M.F (of a cell) or some other potential difference, or the ratio of the E.M.Fs of two cells. When used in this way, it is called a Potentiometer.

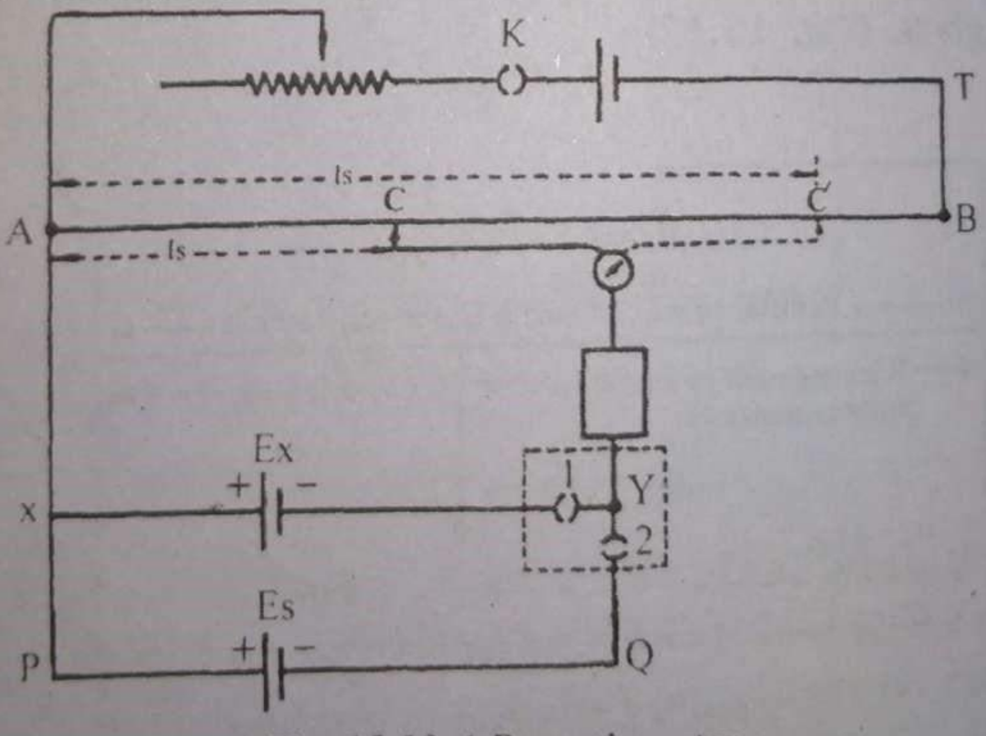


Fig. 15.13 A Potentiometer

In a potentiometer the length of the wire AB may be 1.0 meter or 5.0 or 10.0 meters. The larger the length, the greater is the accuracy of measurement.

The circuit is set up as shown in Fig. 15.13. The positive terminals of a cell of unknown E.M.F. E_x. and a standard cell of E.M.F.Es are connected to the terminal A to which the positive terminal of the current driving battery of E.M.F.E is connected The negative terminals of both the cells are joined to the jockey through a two-way key and a sensitive galvanometer.

Using the two-way key first cell E_x only is introduced into the galvanometer branch and the balance point C and length 1x are found for it. At the balance.

$$E_x = V_{AC} = Ir_x = 1\rho \ell_x \quad \text{------------------(15.8)}$$

where ρ is the resistance per unit length of the wire.

Then putting E_x out of circuit, cell E_s only is introduced and the balance point 'C' and length ℓ_s are determined.

Now $$E_s = V_{AC} = Ir_s = I\rho \ \ell_s \quad \text{---------- (15.9)}$$

Therefore, $$\frac{E_x}{E_s} = \frac{\ell_x}{\ell_s} \quad \text{---------- (15.10)}$$

The equation (15.10) gives the ratio of the two emf in terms of the ratio of their balancing lengths. If E_s is known, E_x can easily be computed.

The AVOmeter

The multi-range ammeter, voltmeter and Ohmmeter have been explained in the foregoing discussion. Sometimes the three are combined into a compact single metre with one common galvanometer. The circuit is so arranged with a selector-cum-range switch that it can be used to measure the currents, the voltages or the resistances of different ranges (or orders). A rectifier circuit is also included in the instrument to convert A.C currents into D.C currents before they pass through the galvanometer. Thus r.m.s. values of A.C currents and voltages can also be measured with the instrument. This compact meter is usually called universal meter or an Ampere-Volt-Ohm-meter abbreviated as AVOmeter.

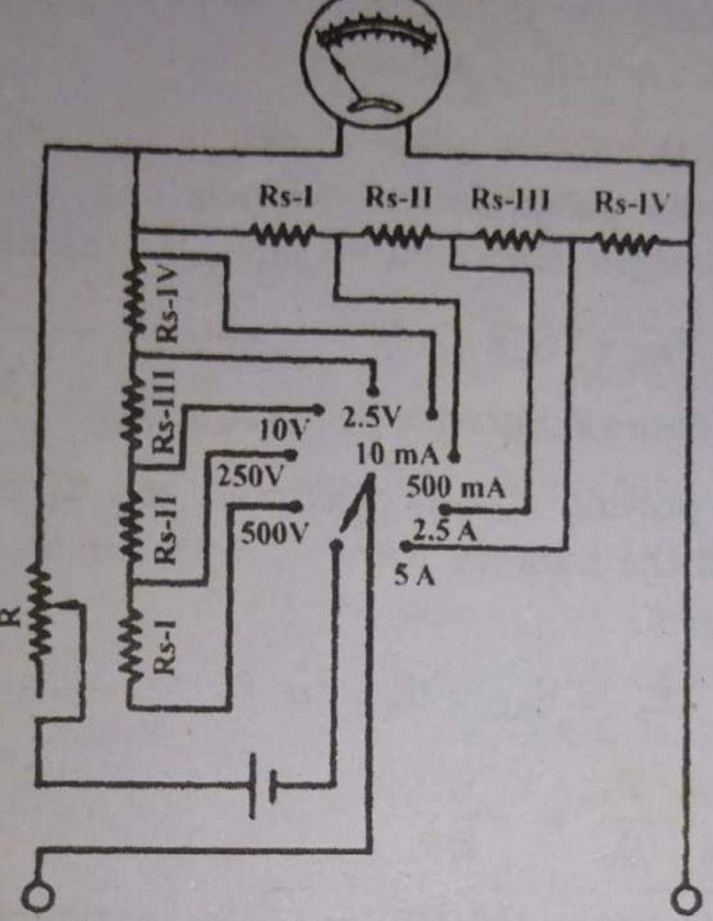


Fig. 15.14

QUESTIONS.

15.1 What is the function of the concave pole pieces and the coaxial soft-iron cylinder in the moving coil galvanometer?

15.2 Why is it necessary to have some form of controlling couple in the moving coil galvanometer?

15.3 What is meant by the sensitivity of a galvanometer? On what factors does it depend? How can we have large sensitivity of a moving coil galvanometer?

15.4 Which galvanometer usually has greater sensitivity, Aluminium pointer and scale type or lamp and scale type? Why?

15.5 We want to convert a galvanometer into (a) an ammeter (b) a voltmeter. What do we need to do in each case?

15.6 Why is it necessary for an ammeter to have zero or negligibly small resistance?

15.7 What necessary condition must a voltage-measuring device satisfy?

15.8 Why must an ammeter be connected to a circuit in series and a voltmeter in parallel?

15.9 An ammeter and a voltmeter of just suitable ranges are to be used in a circuit. What might happen if by mistake their positions are interchanged?

15.10 The terminals of ammeters are usually made of thick and bare wire while those of voltmeters are quite thin and well insulated. Explain why?

15.11 Why is a potentiometer considered one of the most accurate voltage measuring device?

15.12 Describe a circuit that gives a continuously varying potential difference between zero and a certain maximum value.

15.13 What is a wheatstone bridge? How is it used for measuring an unknown resistance?

15.14 In a balanced wheatstone bridge, will the balance be affected if the positions of the cell and the galvanometer are interchanged?

15.15 In a slide-wire bridge, is it absolutely necessary to have the bridge wire one metre long?

15.16 A Post Office Box is a compact wheatstone bridge. Then why is it so named?

15.17 Which is the more accurate instrument a meter bridge or a P.O.Box?

PROBLEMS

15.1 A galvanometer has a resistance of 50 Ohms and it deflects full scale when a current of 10 milliamperes flows in it. How can it be converted into an ammeter of range 10 amperes?

Ans. (By connecting a shunt of 0.05 Ω)

15.2 A galvanometer whose resistance is 40 Ohms deflects full-scale for a potential difference of 100 millivolts across its terminals. How can it be converted into an ammeter of 5 ampere range?

Ans. (By connecting a shunt of 0.02 Ω)

15.3 The coil of a galvanometer which has a resistance of 50 ohms and a current of 500 micro amperes produces full-scale deflection in it. Show by a diagram how it can be converted to (a) an ammeter of 5 ampere range and compute the shunt resistance. (b) a volt meter of 300 volt range and compute the series resistance.

Ans. (a) R_s = 0.005 Ω (b) R_x = 599950 Ω

15.4 A galvanometer of resistance 2.5 ohms deflects full-scale for a current of 0.05 amperes. It is desired to convert this galvanometer into an ammeter reading 25 amperes full-scale. The only shunt available is of 0.03 ohm. What resistance R must be included in series with the galvanometer coil as shown in Fig. 15.15 for using this shunt?

Ans. (12.47Ω)

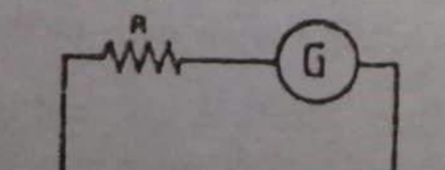


Fig. 15.15

15.5 An ammeter deflects full-scale with a current of 5 amperes and has a total resistance of 0.5 ohms what sunt resistance must be connected to it to measure 25 amperes full-scale?

Ans.(0.125 Ω)

15.6 A moving coil galvanometer G has a resistance of 50 ohms and deflects full-scale with a current of 0.005 ampere. What resistance R_1, R_2 and R_3 must be connected to it as shown in Fig. 15.16

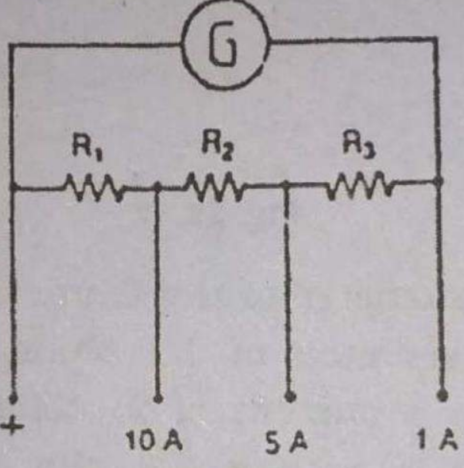


Fig. 15.16

to convert into a multi-range ammeter having ranges of 1A, 5A and 10A.

Ans.(R_1 = 0.0251 Ω, R_2 = 0.05 Ω R_3 = 0.201 Ω)

15.7 A 300-volt voltmeter has a total resistance of 20,000 ohms. What additional series resistance must be connected to it to increase its range to 500 volts?

Ans: (R_x = 13333 Ω)

15.8 The resistance of a moving-coil galvanometer is 25 ohms and current of 1 milliampere causes full-scale deflection in it. It is to be converted into a multi-range voltmeter. Find the series resistances R_1, R_2 and R_3 to give the range of 5 volts, 50 volts and 500 volts at the range terminals as shown in Fig. 15.17.

(Ans. $R_1 = 4975\ \Omega$, $R_2 = 45000\ \Omega$, $R_3 = 450000\ \Omega$)

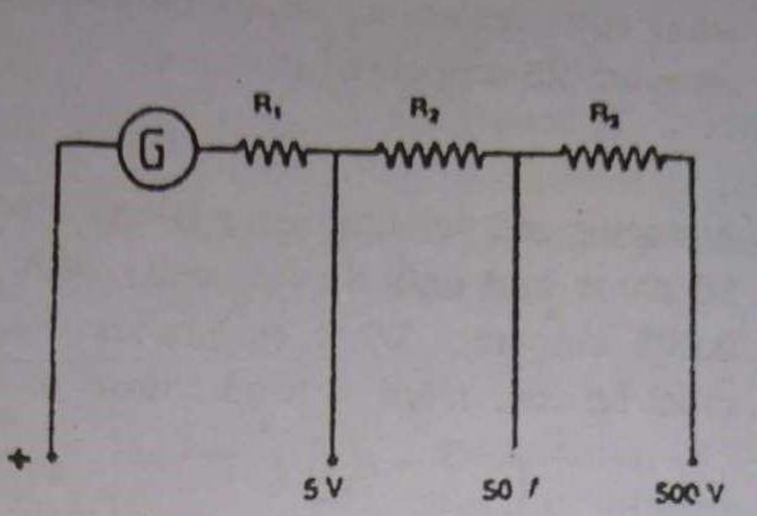


Fig. 15.17

15.9 The galvanometer of the Ohmmeter in Fig. 15.11 has a resistance of 25 ohms and deflects full scale with a current of 2 milliamperes in it. The e.m.f. E of its cells is 1.5 volts.

(i) What is the value of the series resistance R?

(ii) To what values of x connected to its terminals do the deflection of $\frac{1}{5}$, $\frac{1}{2}$ and $\frac{4}{5}$ full-scale correspond?

(iii) Is the scale of the Ohmmeter linear?

Ans. (i) $R = 725\ \Omega$, (ii) $X_1 = 3000\ \Omega$, $X_2 = 750\ \Omega$, $X_3 = 187.5\ \Omega$, (iii) No.)

15.10 A constant potential difference of 25 volts is applied across a uniform resistance wire AB, 100 cm long. Terminals are soldered to three points C,D,F on the wire respectively 16,32 and 64 cm from A, Fig. (15.18). Find the potential differences (between each pair of points given in the subscripts) (i) V_{AC} (ii) V_{AD} (iii) V_{AF} (iv) V_{CD} (v) V_{CF} (vi) V_{DF} (vii) V_{CB} (viii) V_{DB} and (ix) V_{FB}.

Ans. (i) 4V, (ii) 8V, (iii) 16V (iv) 4V ,(v) 12V, (vi)

8V, (vii) 21V, (viii) 17V , (ix) 9V

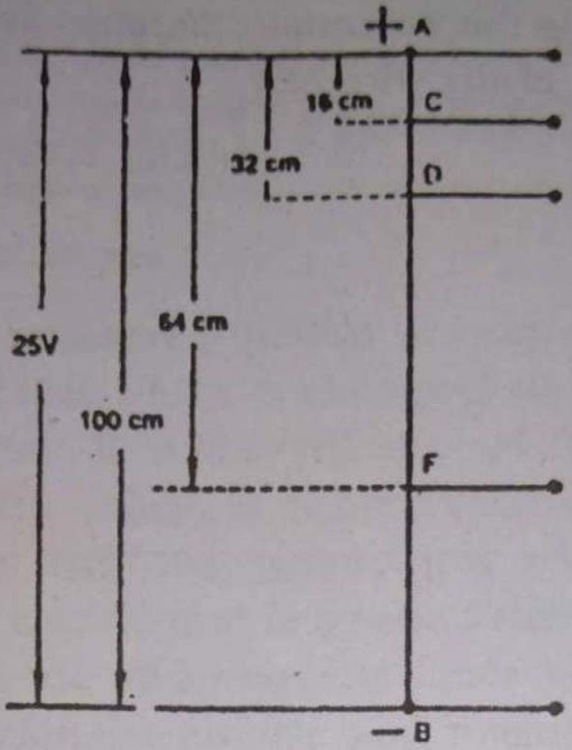


Fig. 15.18

15.11 A potentiometer is set up to measure the emf, E_x of cell (Fig. 15.19). The potentiometer wire is 120 cm long. E_s is the emf of a standard

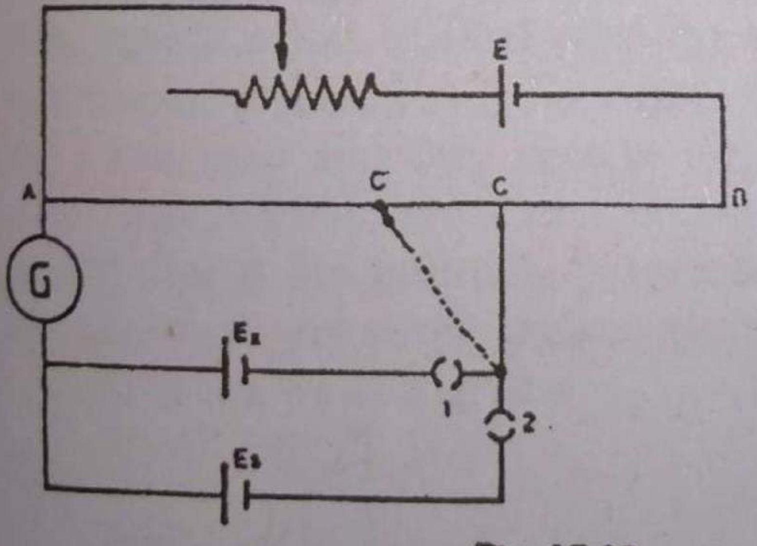


Fig. 15.19

cadmium cell equal to 1.018 volts. When the key 1 only is closed to include the emf E_x in the galvanometer circuit, the galvanometer gives no deflection with the sliding contact at C, 56.4 cm from A. When the key 2 only is closed to include the emf E_s in the galvanometer circuit, the balance is obtained at C', 43.2 cm from A.

(a) What is the emf E_x of the cell?

(b) What is the potential difference across the entir length of the wire AB?

(Ans.(a) 1.329V (b) 2.828

Chapter - 16
ELECTROMAGNETIC WAVES AND ELECTRONICS

Electromagnetic waves :

Faraday discovered the law of magnetic induction that a magnetic field,which is changing with time,causes an induced electric field.Maxwell showed that the opposite is also true:A changing electric field causes an induced magnetic field.This symmetrical relationship between changing electric and magnetic fields shows that if a change of electric and magnetic fields is taking place through any region,the electric and magnetic fields will propagate out of this region in the surrounding space.Such moving electric and magnetic fields are known as electromagnetic waves.When an electromagnetic wave is passing through some point in space,both the electric and magnetic fields at that point are changing with time.Maxwell showed that the electric field E and the magnetic induction B fluctuate. E and B are zero at the same time and they reverse direction together with each cycle.Another prediction of the Maxwell theory is that E and B are mutually perpendicular to each other and that both are perpendicular to the direction of propagation of the wave: Fig:16.1.

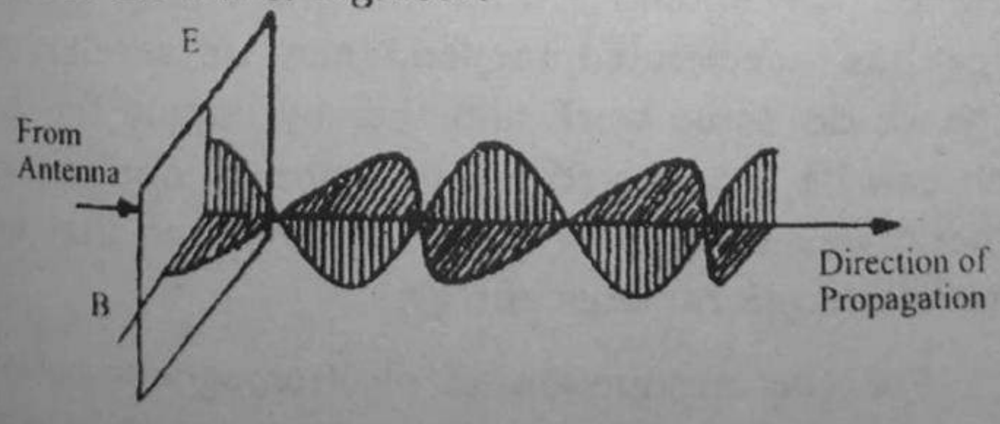


Fig: 16.1 (The E and B vectors in a plane electromagnetic waves travelling along the positive X-axis. The field vector are mutually perpendicular and are in phase.)

It can be shown that the speed (c) of the e.m waves depends upon the permeability and the permitivity of the medium through which it propagates

$$C = \sqrt{\frac{1}{\mu_0 \epsilon_0}}$$

$$= \frac{1}{\sqrt{4\pi \times 10^{-7} Tm/A \times 8.854 \times 10^{-12}\ C^2/Nm^2}}$$

$$\cong 2.998 \times 10^8 ms^{-1}$$

Thus the speed of electromagnetic wave is the same as the speed of a light wave in free space.Maxwell therefore proposed that the light is an electromagnetic radiation.

16.1 Production of electromagnetic waves (Radio waves)

All electric and magnetic fields arise due to moving charges.Our problem is to know the conditions where these fields would radiate away from the source from which they originate.

Now electric charge at rest gives rise to coulomb field around the charge which is stationary in space:a charge moving with constant speed is equivalent to a steady electric current which generates a magnetic field in the surrounding space.These fields also do not radiate.Hence we should expect radiation only when the charge has accelerated motion.When we accelerate the charge,we do some work and thereby supply the energy which can be propagated out in space in form of electromagnetic waves.Thus we know the condition that an accelerated charge radiates energy.

For the production of electromagnetic waves,we have a simple generator of electromagnetic waves as shown in Fig.16.2 where an antenna is formed by two metal rods connected to an alternating source of potential of frequency υ.This source is known as oscillator having wave shape of the potential as shown in Fig 16.2

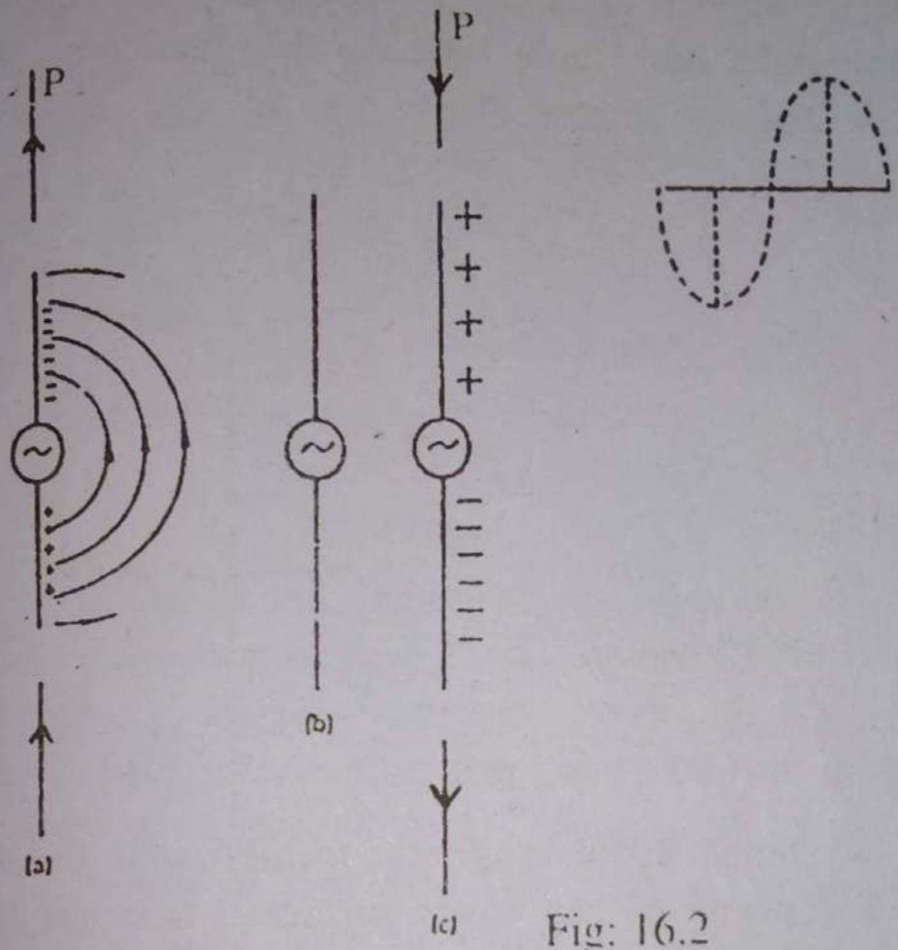


Fig: 16.2

The oscillator causes to flow back along the antenna.The system is shown at several times. In (a) $\frac{1}{4}$ cycle after start,negative charges,have been pushed to the end of the upper rod and an excess of positive charges remains on the bottom rod.At a time that is $\frac{1}{4}$ of a cycle later (b) the charges have come back together again and the rods are momentarily uncharged.Since later,at(c),the oscillator has reversed polarity and the charges are reversed,with the top rod positively charged.At any one instant,say (a),we see that the electric field surrounding the antenna is similar to that due to a pair of equal and opposite charges.At point P in (a), for instance,the electric field is upwards,but half a cycle later,at (c) it is downward.during the time between (a) and (c), positive charge is flowing up the antenna from bottom end to the top end.This upward current produces a magnetic field and the right hand rule tells us that the lines of induction of the magnetic field are of the form of circles.Fig:16.3

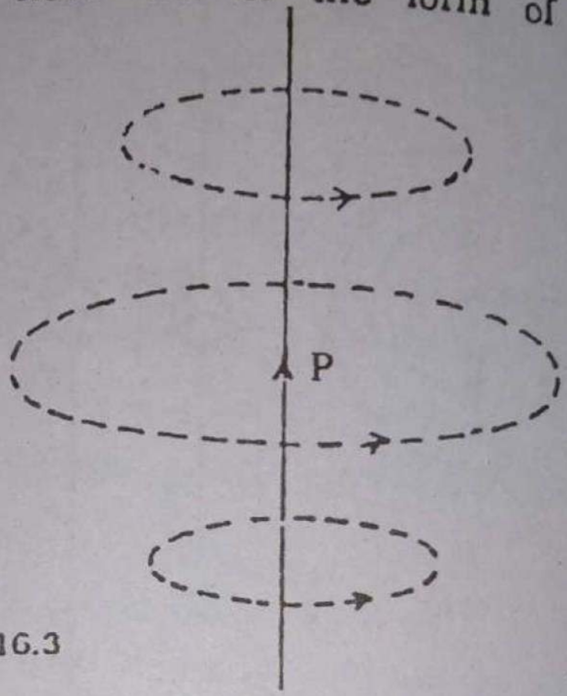


Fig.16.3

At point P,the magnetic induction is perpendicular to the plane of the paper. directed horizontally away from us.Later while charges are flowing back down the antenna,the magnetic induction B reverses direction but is still horizontal.

Thus we see that at P two fields coexist and that the E-field and the B-field are perpendicular to each other.At P,the wave is propagated horizontally,broad sides on to rods,at a point such as Q,the magnetic induction is zero (head on view of the current) and hence no wave is propagated along a direction parallel to the length of the antenna.

At any instant,the energy is stored in the space surrounding the antenna.Fig:16.4 shows the electric field (the accompanying magnetic field is not shown for simplicity) for a given instant.

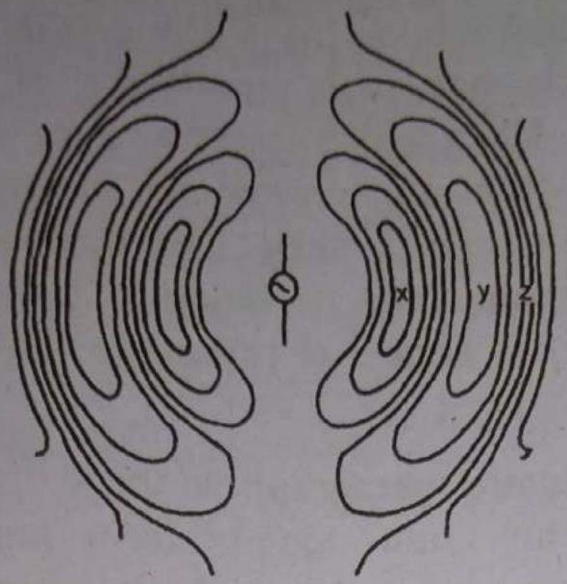


Fig. 16.4

The field in the region X is upwards, at Y it is zero and in the region Z, it is downward.Electric potential energy is stored wherever there is an electric field;the diagram shows that the energy at this instant is localized in the region X and Z separated by region Y where no energy is stored.As the wave spreads out,the regions of the energy concentration spreads out.

This is how the wave model explains the propagation of energy by radiation.As electric and magnetic fields can exists in vacuum and hence no medium is needed to transmit energy by an electromagnetic wave.

16.2 Information Carried by Electromagnetic waves and their Reception.

It is a common experience that sound waves(20 Hz to 20,000 H_Z) in air can be heard only over short distances. When long distance communication is needed and the free space is the communication channel, antennas radiate and they receive the signal. From the theory of the antenna,it turns out that antennas operate effectively only when their dimensions are of the order of magnitude of the wavelength of the signal being transmitted. For example,an audio tone of $4kH_z$ when converted into its electrical analog (i.e when passed through a transducer) corresponds to a wavelength of $(3 \times 10^8 (ms^{-1})/3 \times 10^3 H_z)$. 10^5m which is certainly an impracticable length of an antenna required for the transmission and reception of a tone of $3kH_z$. The required practical lenght of the antenna may be obtained by translating/shifting the electrical analog of the audio tone to a higher frequency.

In the foregoing paragraph we have discussed the transmission of an audio tone,problem regarding the practicability of the antenna and the possible solution.We now discuss another problem whose solution once again needs frequency translation/shifting.

During conversation we generate an audio signal which has a range. Suppose the audio range extends from $20H_z$ to $10kH_z$. The ratio of the highest audio frequency to the lowest($10kH_z$) is 500.and therefore the antenna designed for one end of the range would be entirely too short or too long for the other end. Once again we can overcome this problem by translating/shifting the audio signal to a higher frequency. For example,the audio signal is translated to a frequency of 10^6H_z,then the range occupied is from $(20H_z + 10^6H_z)$ to $(10^4H_z + 10^6H_z)$,yeilding a ratio of 1.00. Therefore translation of an audio signal allows the use of same antenna for both the lowest frequency and the highest frequency of the radio signal. Thus transmission of intelligence (i.e voice,music,code) is accomplished by translating the electrical analog corresponding to the intelligence signal with a high frequency (radio wave) having constant amplitude and fixed frequency generated locally by the transmitting equipments. The process of combining audio frequency (a–f) and radio frequency (r–f) waves to accomplish translation is called modulation.

The higher frequency wave having constant amplitude and a fixed frequency is called carrier wave,whereas the audio signal is referred to as a modulating signal. Therefore modulation is a technique by which some characteristic of the carrier wave is varied with time in accordance with the modulating signal (intelligence) Mathametically, a carrier wave is represented by a sinusoidal waveform

$$V_c\,(t) = A \sin(2\pi f_c t + \phi) \qquad \text{------------(16.1)}$$

where

A - represents the amplitude of the carrier wave

f_c - represents the frequency of the carrier wave

ϕ - represents the phase angle.

The waveform represented by Eq. 16.1 is sketched in Fig: 16.5 (a).

A single tone (i.e a tone having one frequency) modulating signal can be represented by

$$V_m(t) = B \sin 2\pi f_m t$$

where B and f_m represent the amplitude and frequency of the modulating signal respectivly. The wave form is sketched in Fig 16.5 (b).

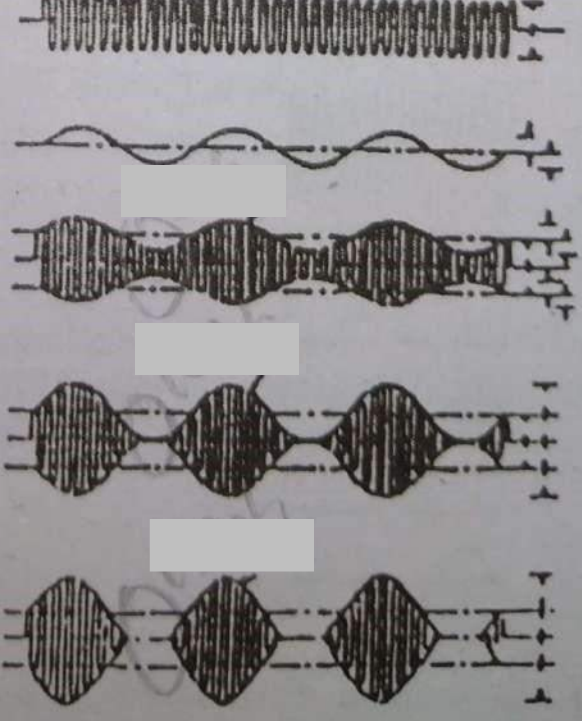


Fig. 16.5 a.b.c.

The sinusoidal wave form expressed in Eq:16.1 can be varried by using one of the three parameter namely. (i) the amplitude of the carrier wave A. (ii) the frequency of the carrier wave f_c and (iii) The phase angle θ. therefore three types of modulations can be devised namely (i) Amplitude Modulation (ii) Frequency Modulation (iii) Phase Modulation.

(a) AMPLITUDE MODULATION

In amplitude modulation (abbreviated A-M) the carrier wave amplitude, A, is varied in accordance with the modulating signal voltage.

Mathematically an amplitude modulated carrier for zero phase angle($\phi = 0$) from which the orignal signal is easily recoverable is generated by adding to the product of the modulation signal and carrier wave, the carrier signal itself.

$$V_{AM}(t) = A \cos 2\pi f_c t + A \cos 2\pi f_c t \times B \cos 2\pi f_m t \quad \ldots (16.2)$$

The modulated wave form is sketched in Fig 16.5(c) shows that the out line of the modulated carrier wave is similar in form to the modulating signal, accordingly this outline is commonly called the modulation envelope. A schematic diagram of an amplitude modulator is shown in Fig. 16.6

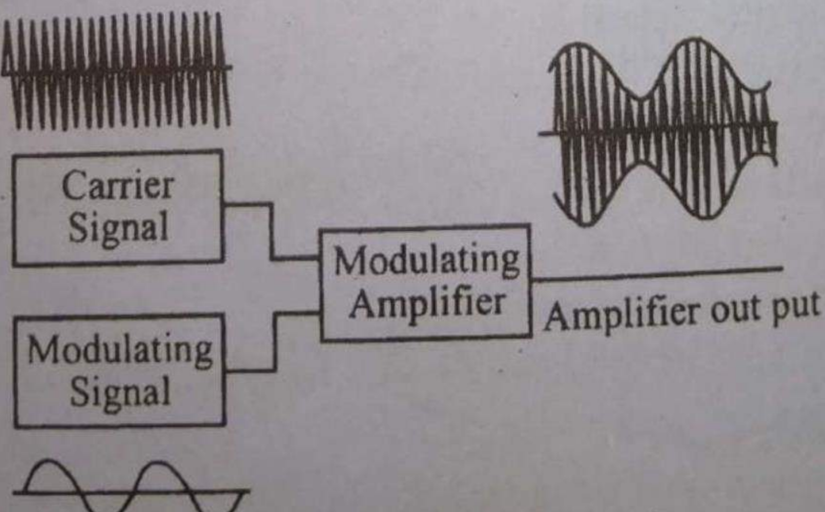


Fig 16.6 Amplitude modulation performed in an amplifier circuit

(b) PERCENTAGE OF MODULATION

In amplitude modulation it is common practice to refer to the percentage of modulation, and designated as M. The percentage of modulation shows the extent to which the modulating signal modulates the carrier wave (i.e. the extent to which a carrier has been amplitude modulated). Then if the modulation is symmetrical, the percentage of modulation is defined as

$$M = \frac{\text{Amplitude of modulating signal(volts)}}{\text{Amplitude of the carrier wave(volts)}} \times 100\% \quad ...(16.3)$$

$$M = \frac{B}{A} \times 100\,\% \quad \text{------------------(16.4)}$$

Let
$$M = \frac{B}{A} \times m_a \quad \text{------------------(16.4)}$$

Where m_a is termed as modulation index and both m_a and the modulation function, V_m (t), are constrained such that

$$V_m\,(t)_{max} < 1.0 < m_a < 1$$

then
$$M = m_a \times 100\%$$

The Fig: 16.5(c), Fig: 16.5(d), Fig: 16.5(e) shows the effect of different amount of modulations (i) the Fig 16.5(c) shows 50% modulation(m_a= 0.5) the carrier wave is under modulated and the power output is reduced.(ii) Fig 16.5(d) shows100% modulation (m_a = 1), the carrier wave is fully modulated with maximum undistorted power output. (iii) Fig 16.5(e) shows the modulation exceeds 100% (m_a > 1), the carrier wave is over modulated and the output of the transmitter will be distorted version of the original modulating signal.

(c) SIDEBANDS

The second part of Eq.16.2 contains product of the carrier wave and the modulating signal. By virtue of this multiplication two new frequencies are generated and are equal to the sum and the difference of the carrier wave frequency and the modulating signal frequency, namely, (i) $(f_c + f_m)$, (ii) $(f_c - f_m)$.

In broadcasting a radio program, the modulating signal frequency (which corresponds to a single tone) varies continually over the frequency range (f_{min} to f_{max}) of the modulating signal being transmitted. Accordingly, the single value of $(f_c + f_m)$ is replaced by a band of frequencies extending from $(f_c + f_{min})$ to $(f_c + f_{max})$ called upper sideband as show in Fig: 16.7. Similarly, the single value of $(f_c - f_m)$ is replaced by a band of frequencies extending from $(f_c - f_{max})$ to $(f_c - f_{min})$ called lower sideband as shown in Fig 16.7. The width of each band is $(f_{max} - f_{min})$ and the system needs a channel bandwidth of twice the maximum frequency of modulating system.

$$\text{i.e} \quad (f_c + f_{max}) - (f_c - f_{max}) = 2f_{max}$$

It is important to note that the information (i.e, the intelligence signal) is being carried by the sidebands which are symmetrically located around the carrier wave frequency.

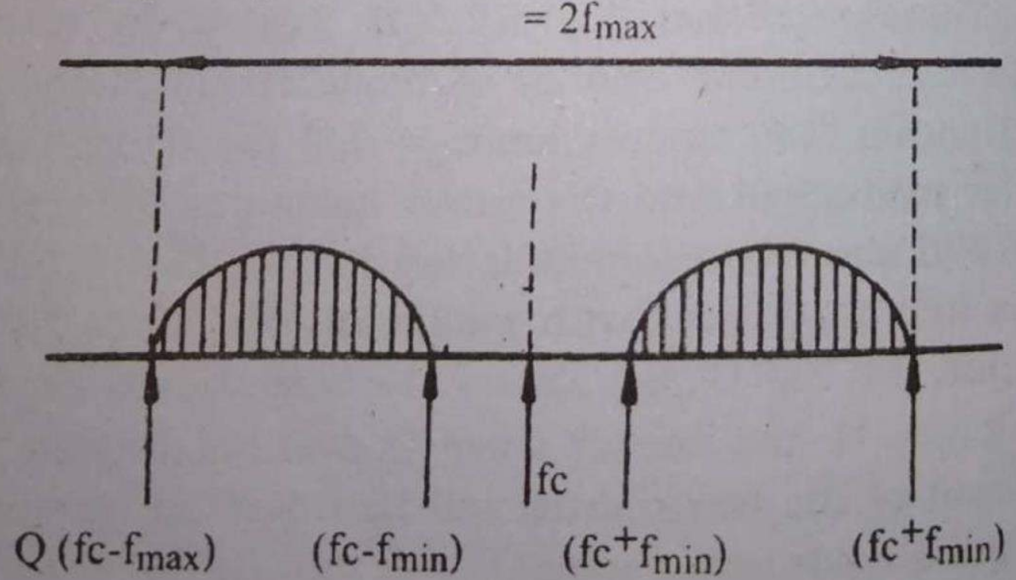


Fig. 16.7 Sidebands and Channel Bandwidth

(d) FREQUENCY MODULATION

In frequency modulation, the amplitude of the modulated carrier wave is maintained at its original strength. The frequency of the modulated carrier wave varies in proportion to the amplitude of the modulating signal, and at a rate determined by the frequency of the modulating signal. Fig:16.8 (a) shows unmodulated carrier. Fig:16.8 (a and b) shows that the frequency of the modulated carrier wave increases as the signal voltage incraeses and that it decreases as the signal voltage decreases. Comparison of Fig:16.8 (b) and Fig:16.8(c) shows that the variation in frequency is determined only by the amplitude of the signal, and the rate of variation in frequency of the carrier wave is determined by the frequency of the modulating signal.

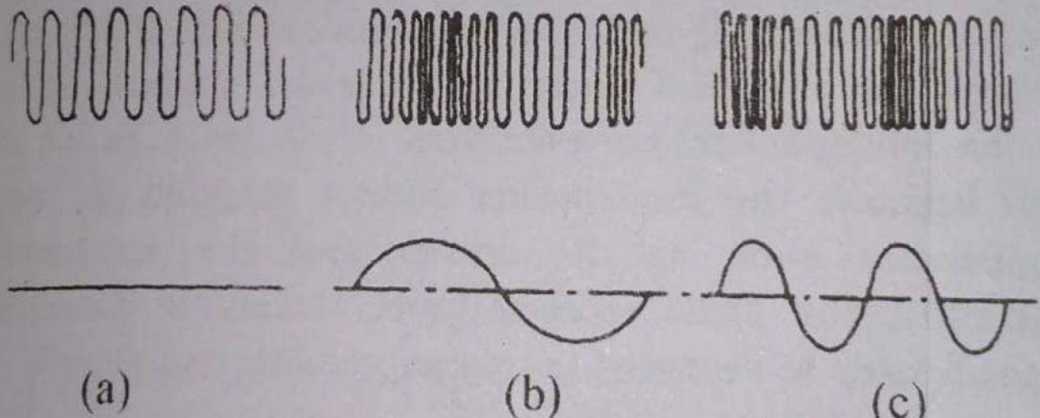


Fig: 16.8

The frequency of FM transmitter without any modulating signal input is called the centre frequency (or resting frequency) and corresponds to the original frequency of the FM transmitter. FM broadcast transmitters operate at frequencies of 88MHz to 108MH_z. When modulating signal is applied,the variation in frequency either above or below the resting frequency is known as frequency deviation, and the total deviation is called the carrier swing.

(e) RECEPTION OF ELECTROMAGNETIC WAVES

The process by which the original modulating signal,or intelligence,is recovered in the radio receiver is referred to as detection or demodulation.It was previously explained that an amplitude modulated carrier which has been modulated by intelligence (i.e voice,music) consists of the carrier wave,upper sideband and lower sideband frequencies and certainly does not contain the modulating frequencies.Therefore,the modulating signal,or frequencies must be reproduced in the receiver to complete the transmission and reception of intelligence.

The carrier with its upper and lower sidebands are radiated from the transmitting antenna in the form of electromagnetic waves.These electromagnetic waves,in turn,induce small voltages into the receiving antenna. These voltages are fed to a tuned radio frequency amplifier with sufficient bandwidth to include the upper and lower sidebands. If the receiver included only linear amplifiers, the amplified carrier and sidebands would be fed to the loudspeaker, however, this would produce no result because the loudspeaker cannot respond to radio frequencies such as the carrier and the sidebands. Therefore, the radio receiver must include a detector/ demodulator to recover the original modulating signal.

Since each modulating frequency is the difference between a sideband frequency and the carrier frequency, it seems that a non linear device is needed to recover the modulating frequencies from the modulated carrier wave.

The most common technique of AM modulation,known as envelop detection is illustrated in Fig.16.9. The received AM wave is passed through a diode which acts a nonlinear device and eliminates the negative portions of the waves,converting it into a positive function with a nonzero average value. Since the message is contained in the low frequency variations(i.e the envelop) of this average about some nominal value. the resistor R and the capacitor C combination as shown in Fig16.9 recovers the modulation by separating out the envelope from the high frequency components as shown in Fig16.9.

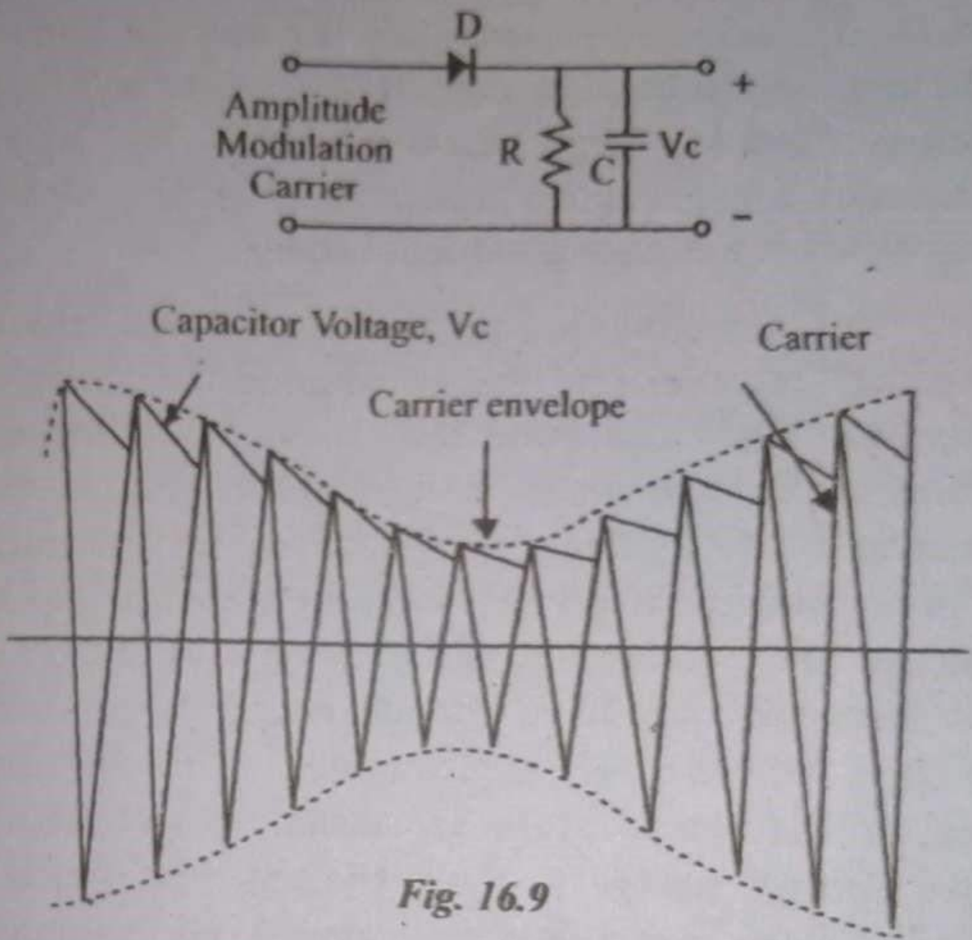


Fig. 16.9

16.3 The Band Theory of Solids.

One of the active fields of research today is called Solid State Physics. It deals with the structure and electrical properties of solids. According to Bohr atomic model electrons in a single atom can occupy only certain energy-levels. The detailed discussion of which is given in chapter 18. The lowest possible energy level is called the ground state and higher ones are called excited states. When atoms or molecules are in solid state, their outer electrons overlap. Hence their energy levels are changed some what and because of their interaction, the energy-levels are spread out into energy bands. The outer electrons can be considered to be in either of two bands. The lower valence band, which corresponds to the ground state;or the upper conduction band. No electron can have an energy in the "forbidden" energy gap between the two bands. Normally the electrons reside in the valence band where they are held more tightly to individual atom. In an insulator, the valence band is full, the conduction band is essentially empty, and the forbidden gap is fairly large as whown in Fig.16.10 (a). Hence normally there are no free states for the electrons to move into the higher conduction band.

In a conductor, on the other hand, there is no gap (fig. 16.10(b) and the two bands usually overlap or there is simply one band that is not filled and the electrons are free to move easily to other states. They can thus move about freely and carry on electric current. In a pure semiconductor the forbidden energy gap between valence and conduction bands is narrow Fig. 16.10(c). A few electrons may have enough energy to jump the gap, so there will be a very slight amount of electrical conduction. If the temperature is raised, more electrons will have enough energy to jump the gap and which further decreases the resistivity. In a doped semiconductor, the impurity provides additional energy states between the bands thus increasing the electrical conduction.

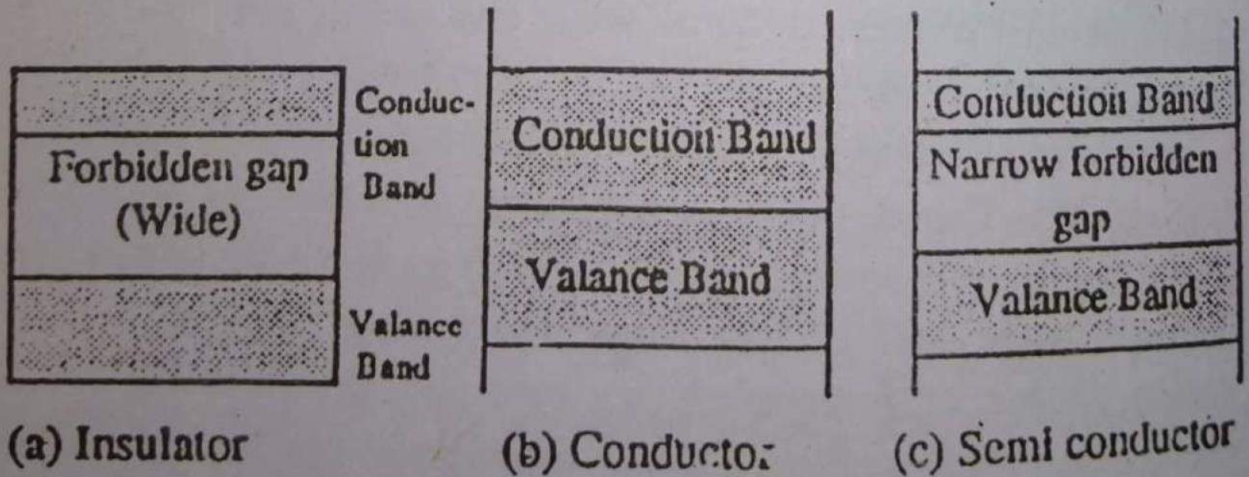


Fig. 16.10 (a) (b) (c)

16.4 Semi Conductors

The elements in the group IV i.e. silicon and germanium are semiconductors. They have narrow forbidden gap. When electrons are transferred from the valence band to the conduction band through some excitation, the valence band no longer remains filled and there are then energy levels whithin the valence band avajlable for valence electrons in it, which can move successively just like cars in a traffic jam. When an opening appears in front of the first car, it moves froward to fill it, leaving an opening behind for the following car. This creation of openings and fillings by the car is repeated so that the net effect is that physical motion of the car is forward while opening behind then moves backward.In the electrical case, the vacancy created by the removal of an electron is a small positively charged region (absence of negative charge) is called a hole which is supposed to behave like a positively charged electron. When a nearby electron moves into a positive hole, it leaves behind another positive hole and so on the process continues till the end. Thus there is a net transport of positive charge which is called hole migration. Collectively electrons and holes are called carriers. Group IV elements Ge,Si are intrinsic semiconductors which form covalent bonds in which two atoms share one or more pairs of electrons. The following diagram fig. 16.11 of covalent bond for Ge shows the hole migration. Suppose that the electrons at A acquire some energy due to some excitation (say thermal) and become free so that covalent bond at A is broken. The electrons move through the crystal thus leaving behind a hole at A in the bond. Finding this vacancy, the electron at B may jump in there to occupy the vacant hole. This creates a hole at B, which may be filled by the electron at C and the process continues through successive electron movements till ultimately a hole appears at G. This may be said that a negative charge has moved from G to A or it will be convenient to regard a postive charge has moved from A to G.

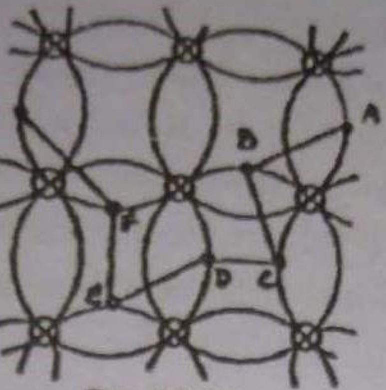
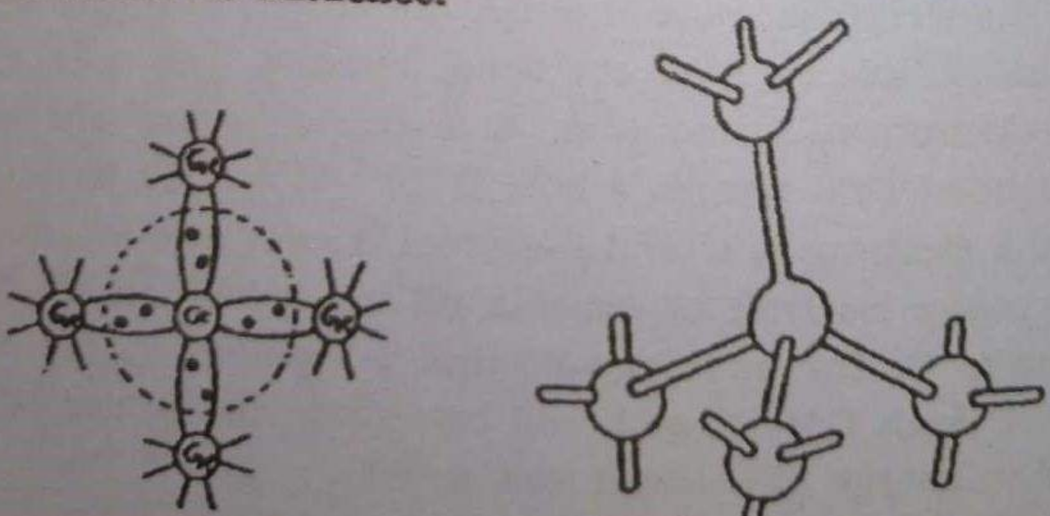


Fig. 16.11

In intrinsic semiconductors of group IV elements, the number of conduction electrons is equal to the number of holes.

16. 5 Atomic binding in semiconductors :

Semiconductors like germanium and silicon,have crystalline structure.Their atoms are arranged in an ordered array known as crystal structure.Both these materials are tetravalent i.e.each has four valence electrons in its outer most shell.The neighbouring atoms forms covalent bonds by sharing four electrons with each other so as to achieve inert gas structure (i.e. 8 electrons in outermost orbit).A two dimentional view of the germanium crystal lattice is shown in Fig;16.12 (b). Each pair of lines represent a covalent bond.The dots represent the valence electrons.It is seen that each atom has 8 electrons under its influence.



(b) Fig:16.12 (a)

A three dimentional view of germanium crystal lattice is shown in Fig:16.12 (a),where each atom is surrounded symmetrically by four other atoms forming a tetrahedral crystal.Each atom shares a valence electron with each of its four neighbours,thereby forming a stable structure.In case of pure (intrinsic) germanium, the covalent bonds have to be broken to provides electrons for conduction.There are many ways of breaking the covalent bond and hence setting the electrons free.

16.6 Preparation of semiconductors :

Intrinsic semiconductors Ge and Si of group IV have quite low values of conductivity due to small number of electrons and holes produced by thermal excitation.

To improve the conductivity and hence to change the electrical properties of semiconductors, elements from group III and V are added as an impurity in Ge and Si, and the semiconductor materials are said to be doped.

Thus doping is the addition of an impurity in a pure semiconducting material. Doped semiconductor materials are termed as extrinsic semiconductor, in which the concentration of impurity atoms is about one part per million.

There are two processes of doping.

(i) Donor doping

(ii) Acceptor doping.

(i) Donor doping:

Consider the intrinsic semiconductor Ge, doped with material of group V elements (arsenic, antimony or phosphorus) which have five electrons in their outermost orbit. Each Ge atom has four valence electrons, therefore each atom has four neighbourring atoms bound to it.

Four of the five electrons of antimony, the doping material, will form bonds with four electrons of Ge, while the fifth electron will remain as free charge carrier. When an electric field is applied, this free electron of antimony will be easily excited to jump to the conduction band from the valence band. Thus every antimony atom introduced into Ge contributes a conduction electron without creating a hole. Hence in addition to the electrons and holes available in Ge, the addition of antimony greatly increases the conductivity of the material. In this case, antimony is called a donor impurity and it makes Ge, an n-type (n for negative) semiconductor.

(ii) Acceptor doping:

If group III elements (boron, aluminium or gallium), which have three electrons in their outermost orbit, are added as an impurity in the intrinsic semiconductor Ge or Si, a deficiency of electrons in the crystal structure is introduced since three electrons of gallium are involved in the covalent bonding with the three atoms of Ge and the covalent bond with the fourth atom remains incomplete, thus creating a hole. Since after doping with group III elements the crystal structure becomes capable of accepting extra electrons, it is called acceptor doping and the material is called p-type (p for positive) semiconductor, due to the fact that there is a vacancy for negative charge or there is an existence of a hole. After application of an electric field their holes migrate to add to the conductivity. Thus we can prepare n-type or p-type extrinsic semiconductors by the addition of impurity in the intrinsic semiconductors from group V or III elements. In n-type semiconductor, current carried is mainly through electrons while in p-type semiconductor, the current is carried through holes.

16.7 Crystallography:

A crystal is a collection of atoms or molecules in which each atom is placed precisely in a definite pattern with respect to its neighbour in the solid. This pattern is repeated over and over again throughout the crystal. The study of the geometric form of crystalline solids by using X-rays, electron or neutron beams constitute the science of crystallography. Crystalline solids are those in which the atoms or molecules are arranged in a very regular and orderly fashion in a three dimentional pattern. Each atom or molecule is fixed at a definite point in space at a definite distance from and in a definite angular orientation to all others surrounding it. This internal spatial symmetry of atomic or molecular orientation is an essential feature of crystalline state. The angular arrangement of the space positions of the atoms in a crystal is called space lattice or lattice array.

16.8 Crystal Lattice and Unit Cell:

The entire lattice structure of a crystal consists of identical blocks of unit cells. The unit cell is the smallest block or geometrical figure from which the crystal is built up by repetition in three dimensions. The unit cell is a parallelopiped, which by moving in the direction of each of the coordinate axes X, Y and Z, arranged parallel to edges of the figure, the crystal lattice can be constructed. The length of the side of a unit cell is the distance between atoms of the same kind nd is known as lattice constant, as shown in fig. 16.13.

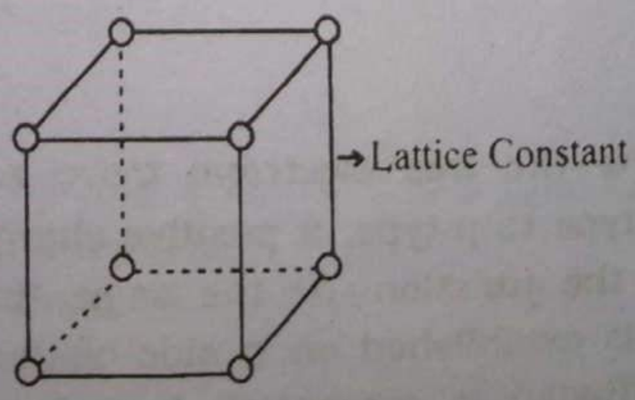


Fig. (16.13)

16.9 The p.n. Junction or Semi Conductor Diode

When a block of p-type material is put adjacent to a block of n-type material, the common plane is termed as p-n junction and the device is called semiconductor diode. Fig. 16.14(a) show a p-type and a n-type semiconductor. In a p-n junction, we know that n-type material has high concentration of free electrons while p-type material has those of holes. Therefore at the junction there is a tendency for the free electrons to diffuse over to the p-side and holes to the n-side. This process is called <u>diffusion</u>.

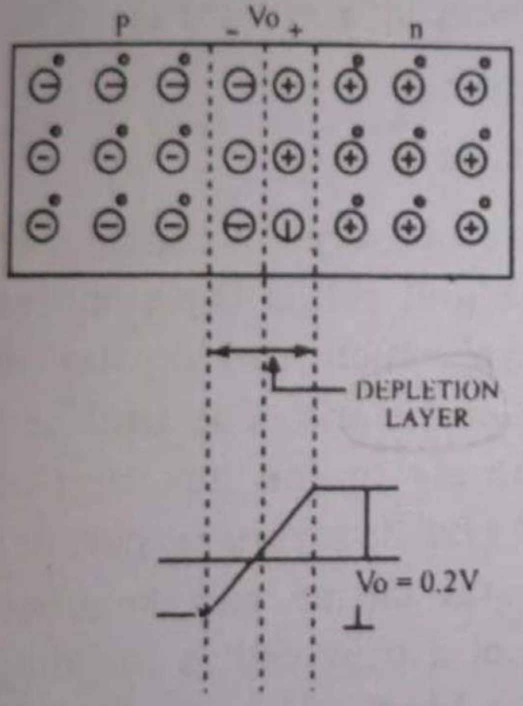


Fig. 16.14 (b)

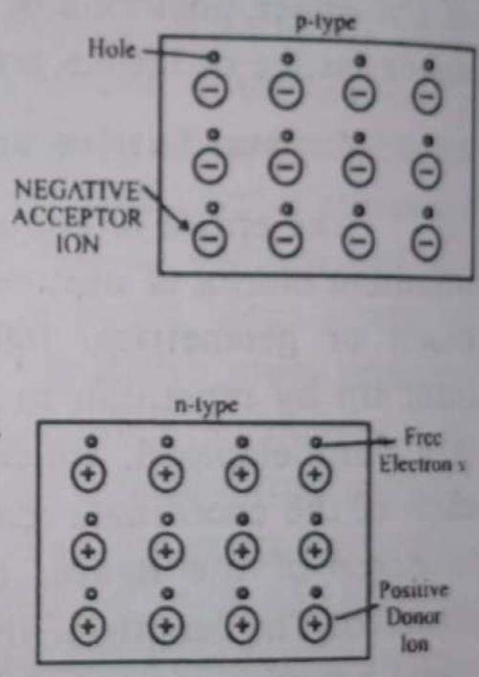


Fig. 16.14 (a)

As the free electrons move across the junction from n-type to p-type, a positive charge is built on the n side of the junction. At the same time, a net negative charge is established on p side of the junction and further diffusion is prevented. It is because now positive charge on n side repels holes to cross from the p side to n-type and negative charge on p side repels free electrons to enter from n-type to p-type. Thus, a barrier is set up against further movement of electrons and holes. This is called Potential barrier or Junction barrier, V_o, which is of the order of 0.1 to 0.7 volts. The potential distribution diagram is shown in fig. 16.14(b). This potential barrier gives rise to electric field which prevents the respective majority carriers from crossing the barrier region. It is to be noted that only inside the barrier, there is a positive charge on n side and a negative charge on the p side. This region is called depletion region. Outside the barrier on each side of the junction, the material is still neutral. When a p-n junction diode is connected across a battery, it permits the flow of current in one direction, similar to that of vacuum tube diode. It is a two terminal device and is symbolically represented in the following figure 16.14 (c).

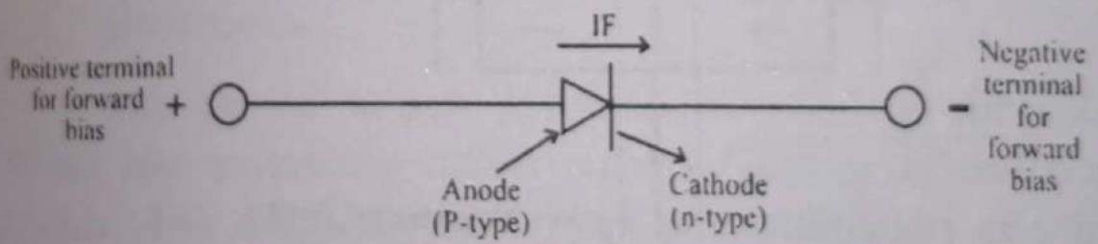


Fig. 16.14 (c)

The application of some electric potential across the diode is known as biasing.

16.10 Biasing

The potential difference across a p-n junction can be applied in two ways, namely

(i) forward biasing and

(ii) reverse biasing.

(i) **Forward biasing:**

When external voltage applied to the junction is in such a direction that it cancels the potential barrier, thus permitting current flow. It is called forward biasing. To apply forward bias, connect positive terminal of the battery to p-type and negative terminal to n-type as shown in fig.16.15 The applied forward potential establishes an electric field which acts against the field due to potential barrier. Therefore the resultant field is weekend and the barrier height is reduced at the junction as shown in fig. 16.15. As potential barrier voltage is very small (0.1 to 0.3 V), therefore, a small forward voltage is sufficient to completely eliminate the barrier. Once the potential barrier is eliminated by the forward voltage, junction resistance becomes almost zero and a low resistance path is established for the entire circuit. Therefore current flows in the circuit.

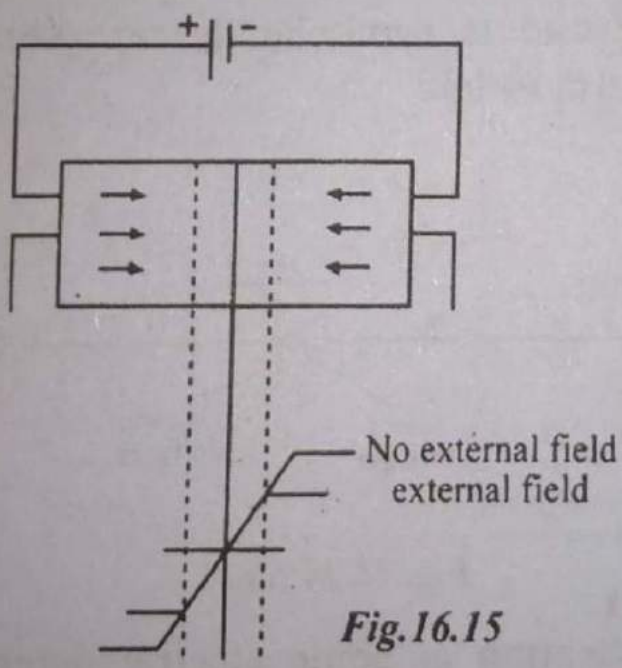


Fig.16.15

(ii) Reverse biasing :

When the external voltage applied to the junction is in such a direction that potential barrier is increased, it is called reverse biasing.To apply reverse bias, connect negative terminal of the battery to P-type and positive terminal to n-type as shown in fig.16.16. It is clear that applied reverse voltage establishes an electric field which acts in the same direction as the field due .to potential barrier.Therefore, the resultant field at the junction is strengthened and the barrier height is increased as shown in fig. This increased potential barrier prevents the flow of charge carriers across the junction.Thus, a high resistance path is established for the entire circuit and hence the current does not flow.

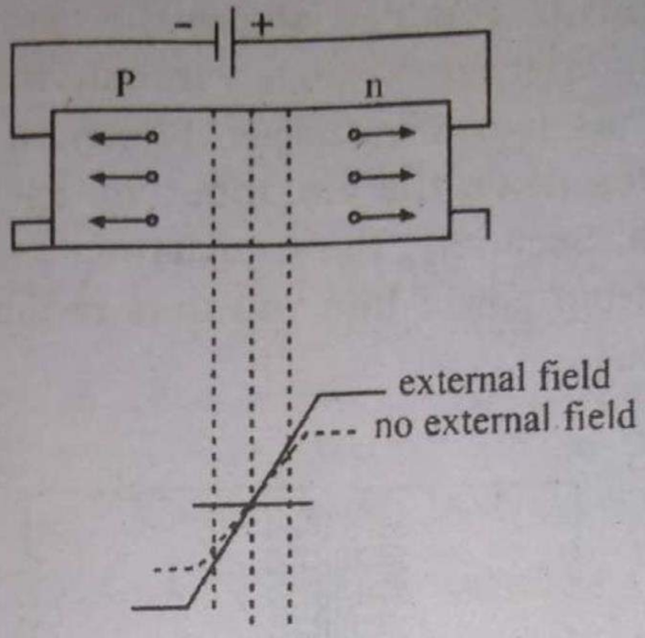


Fig. 16.16

16.11 Semi Conductor Diode (Crystal Diode) Rectifiers.

The device which converts alternating current/ voltage into pulsating direct current/voltage is called a rectifier. The following two types of rectification can be obtained using a diode (i) halfwave rectifier and (ii) full wave rectifier.

A semiconductor diodes, also known as crystal diodes, can be used for rectification purposes. The details of each is given as follows.

(i) **Halfwave rectifier :**

In half wave rectification, the rectifier conducts current only during the positive half cycles of input a.c. supply. The negative half cycles of a.c. supply are suppressed i.e. during negative half cycles, no current is conducted and hence no voltage appears across the load in the external circuit. Therefore, current always flows in one direction through the load, after every half cycle.

Circuit details :

Figure 16.17 shows the circuit where a single diode acts as a halfwave rectifier. The a.c. signal to be rectified is applied in series with the diode and load resistance R_L. The d.c. output is obtained across the load R_L. Generally, a.c. supply is given through a transformer. The use of transformer has two advantages. Firstly, it allows us to step up or step down the a.c. input voltage as the situation demands. Secondly, the transformer isolates the rectifier circuit from power line and thus reduces the risk of electric shock.

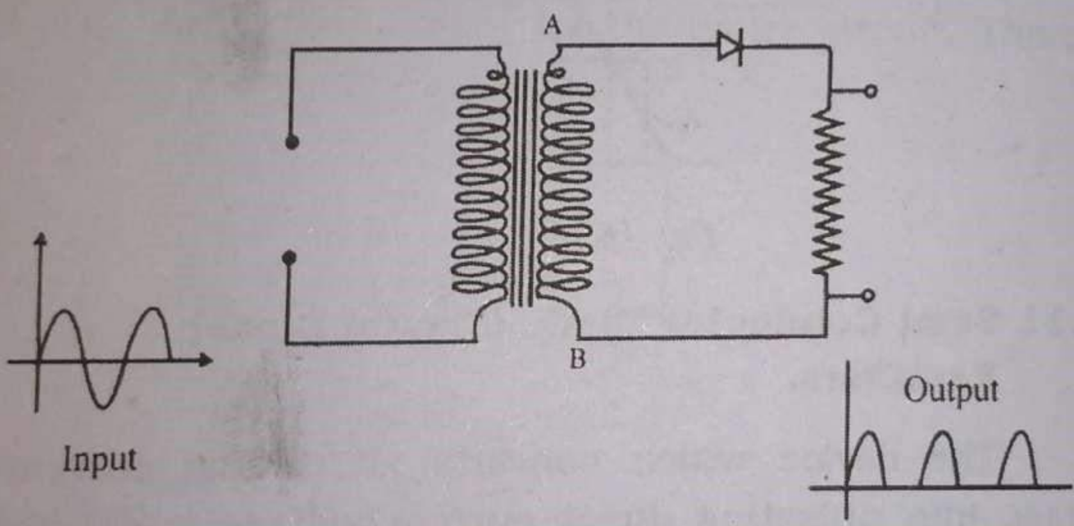


Fig. 16.17

Operation :

The a.c. voltage across the secondary winding AB changes polarities after every half cycle. During the positive half cycle of input a.c. voltage, end A becomes positive with respect to end B. This makes the diode forward biased and hence it conducts current. During the negative half cycle, end A is negative with respect to end B. Under this condition the diode is reverse biased and it conducts no current, therefore the current flows through the diode during positive half cycles of input a.c. voltage and it is blocked during the negative half cycles. In this way current flows through load R_L always in the same direction. Hence d.c. output is obtained across R_L. It may be noted that output across load is pulsating d.c. These pulsations in the output are further smoothened with the help of filter circuits.

(ii) Fullwave rectifier :

In fullwave rectification, current flows through the load in the same direction for both half cycles of input a.c. voltage. This can be achieved with two diodes working alternately. For the positive half cycle of input voltage, one diode supplies current to the load and for the negative half cycle, the other diode does so; current being always in the same direction through the load.Therefore, a fullwave rectifier utilizes both half cycles of input a.c. voltage to produce the d.c. output.

(iii) Construction of Fullwave rectifier :

The circuit employees two diodes D_1 and D_2 as shown in fig.16.18. A center tapped secondary winding AB is used with two diodes connected so that each uses one half cycle of input a.c. voltage. In other words D_1 utilizes the a.c. voltage appearing across the upper half OA of secondary winding for rectification while diode D_2 uses the lower half winding OB.

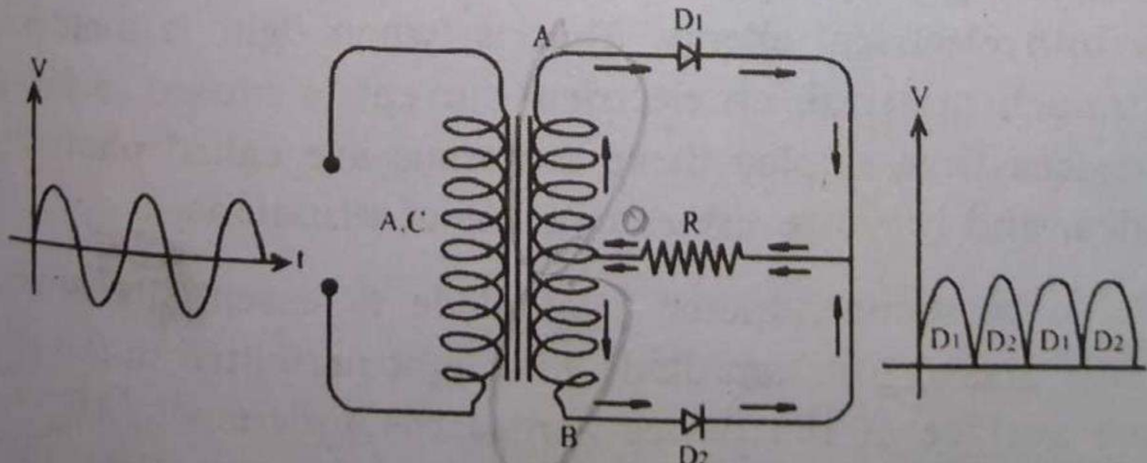


Fig. 16.18

Operation :

During the positive half cycle of secondary voltage, the end A of secondary winding becomes positive and end B negative. This makes the diode D_1 forward biased and diode D_2 reverse biased. Therefore, diode D_1 conducts while diode D_2 does not. The conventional current flow is through diode D_1, load resistor R_L and the upper half of the secondary winding, as shown by the dotted arrow. During the negative half cycle, end A of the secondary winding becomes negative and end B positive. Therefore, diode D_2 conducts while diode D_1 does not. The conventional current flows is through diode D_2, load R_L and lower half winding as shown by solid arrow. Referring to fig. 16.11 it may be seen that current in the load R_L is in the same direction for both half cycles of input voltage. Therefore, d.c. output is obtained across the load R_L for both cycles of the input a.c.

The full wave rectifier output is more efficient source f power and is easier to filter than a half wave rectifier output. Also, its average value of d.c. voltage is much higher than fo the half wave rectifier.

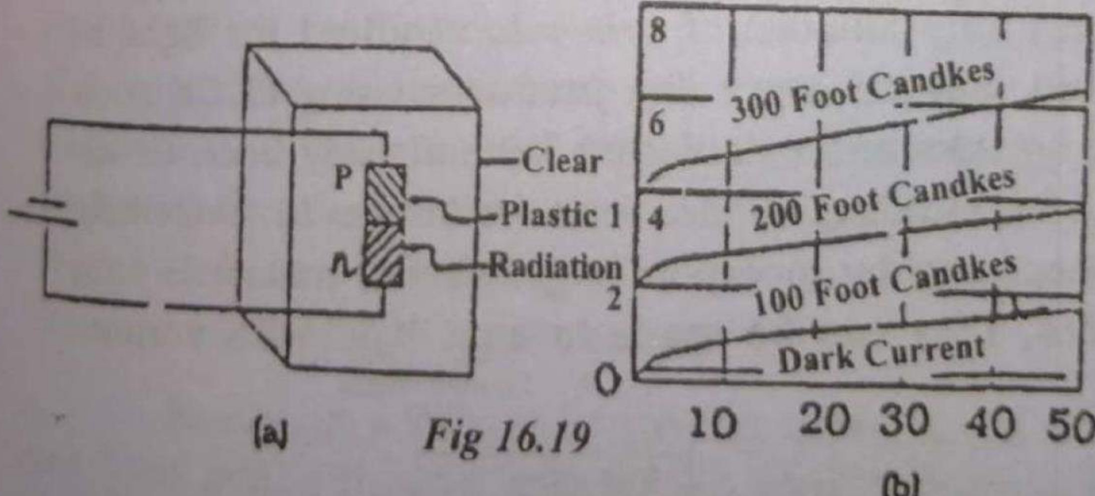
16.12 Photodiodes:

Certain type of semiconductor materials have the interesting property that they convert light energy directly into electrical energy. That is, when light is incident on such material, an electrical current is caused to flow. Devices that employ these materials are called photodiodes, and they are used in variety of situations.

A semiconductor photodiode is essentially a reverse biased junction diode with light permitted to fall on cne surface of the device across the junction fig.16.19. The remaining sides are kept unilluminated. Then the diode current varies almost linearly with the light flux. Fig. 16.19 shows the structure. The p-n junction is embedded in a clear plastic capsule. All the sides of the plastic capsule except the illuminated one are painted black or enclosed in a metallic case. Fig. 16.19 shows the voltage ampere characteristic curves for a typical germanium photodiode. With no illumination, there exists only the small reverse saturation current (I_o). On illuminating the reverse biased p-n junction, new electron-hole pairs are formed, the concentration of which is proportional to the incident light flux. Under large reverse bias condition, the total reverse current I is given by :

$$I = I_s + I_o$$

where I_s is the short circuit current and is proportional to the light intensity. Curves for three different values of illumination are shown in fig. 16.19(b). As with the barrier layer photoelectric cell, the spectrum sensitivity of photodiode is determined by photoelectric properties of semiconductor material. Photodiode compare favourably with photoelectric cells in that they have smaller size and weight, higher integral sensitivity and a lower working voltage. Photodiodes are used in high speed reading of computer punched cards and tapes, light detection system, light operated switches, production line counting of objects etc.



(a) *Fig 16.19* (b)

16.13 Light Emitting Diode (LED).

The Light Emitting Diode (LED) is, as the name implies, a diode that will give off visible light when it is energized. In any forward biased p-n junction there is, within the structure and primarily close to the junction, a recombination of holes and electrons. The recombination requires that the energy possessed by the unbound free electron be transferred to another state. In all semiconductor p-n junctions some of this energy will be given off as heat and some in the form of photons. In silicon and germanium the greater percentage is given up in the form of heat and the emitted light is in significant. In other materials, such as gallium arsenide phosphide (Ga As P) or gallium phosphide (Ga P), the number of photons of light energy emitted is sufficient to create a very visible light source. The process of giving off light by applying an electrical source of energy is called electroluminescence. As shown in fig.16.20, the conducting surface, connected to the p-material, is much smaller to permit the emergence of the maximum number of photons of light energy. Note in the figure that the recombination of the injected carriers due to the forward biased junction is resulting in emitted light at the site of recombination. There may, of course, be some absorption of photon energy in the structure itself but a very large percentage are able to leave, as shown in the figure.

LEDs can be used in battery operated devices because only tiny amount of power is required for light to be emitted. Several every day products using LEDs such as digital clocks and calculators have already been developed. Light emitting diodes can be made in extremely small sizes, and by incorporating different materials into the diodes, they can be made to emit light with various colours.

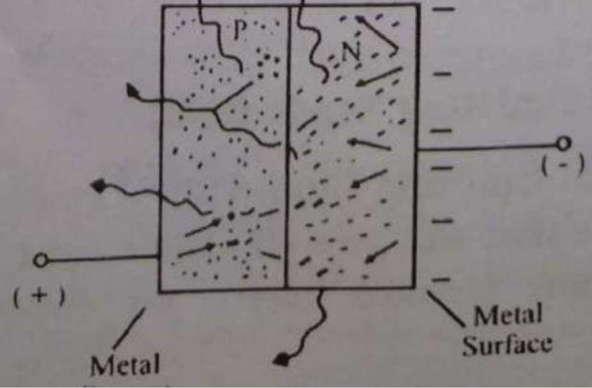


Fig (16.20)

16.14 Solar cell

A solar cell is a photodiode that is used to extract energy from sun light. The basic construction of a silicon p-n junction is shown in Fig. 16.21. As shown in the top view, every effect is made to ensure that the surface area perpendicular to the sun is a maximum. Also note that the metallic conductor connected to the p-type material are such that they ensure that a maximum number of photons of light energy will reach the junction.

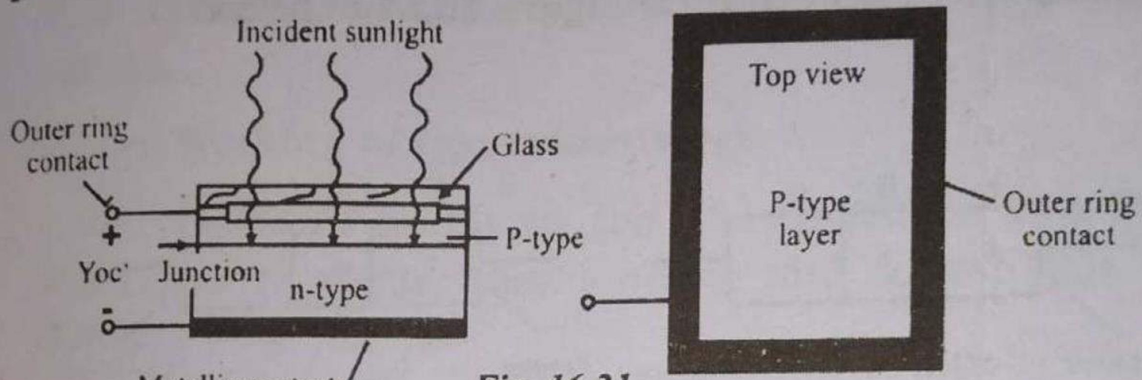


Fig. 16.21

A photon of light energy in this region may collide with a valence electron and impart to it sufficient energy to leave the parent atom. The result is a generation of free electron, and holes. This phenomena will occur on each side of the junction. In the p-type material the newly generated electrons are minority carriers and will move rather freely across the junction as explained for the basic p-n junction with no applied bias. A similar discussion is true for the holes generated in the n-type material. The result is an increase in the minority carrier flow which is opposite in direction to the conventional forward current of a p-n junction.

Selenium and silicon are the widely used material for solar cell, although gallium arsenide, indium arsenide and cadmium sulphide, among others are also used. Commercial silicon solar cell have high stability and conversion efficiency approximately 14%. Solar cells are used for converting solar light energy in to electrical energy in space vehicles.

16.15 Transistor:

The word transistor is the combination of two words, first transfer and second resistor. It is an active semiconductor device that has three electrodes. The first device that was invented was the "bipolar junction transistor" (BJT).

The transistor may consist of an p-type material sandwiched between two n-type materials called npn transistor, as shown in the figure 16.22(a) below:

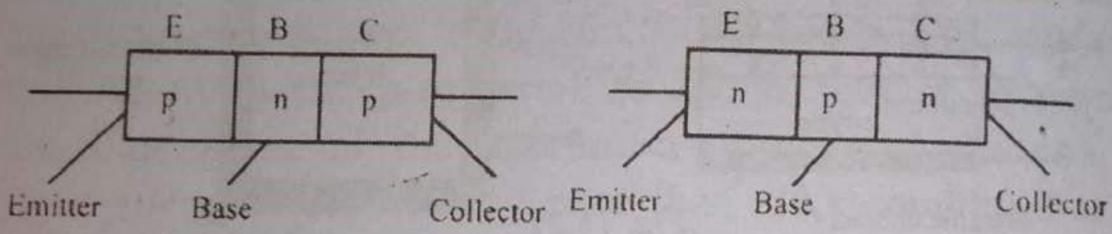


Fig. 16.22 (a)

The central layer between the two is called the base. Among the remaining two, one is called emitter and the other collector, according to the function they perform during operation.These three electrodes are provided with terminals for connection in the circuit. Symbols used for npn and pnp transistors are shown below in Fig. 16.22 (b).

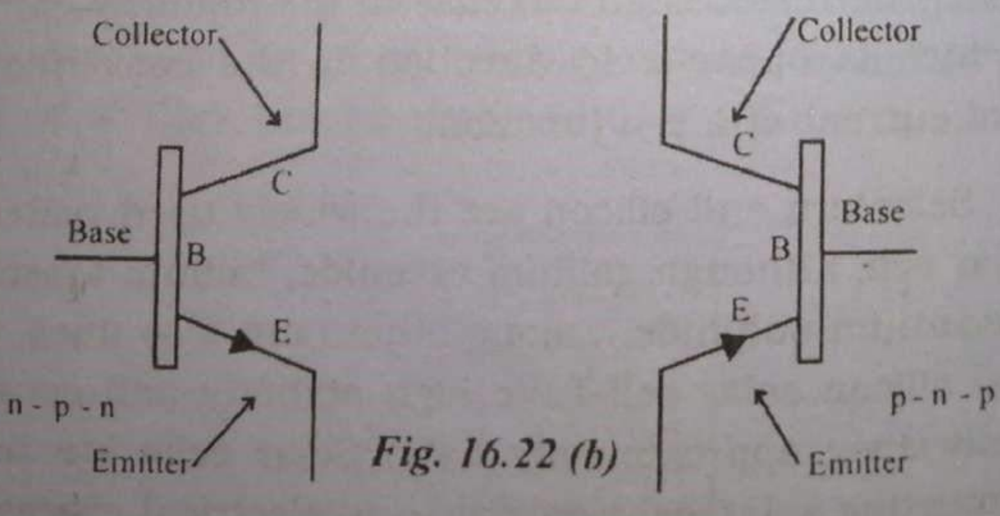


Fig. 16.22 (b)

The arrows of the transistor show the direction of the conventional current from p to n.

16.16 Transistor Operation:

The emitter-base junction of a transistor is forward biased whereas the collector-base junction is reverse biased. If we ignore the presence of emitter-base junction, no current would flow in the collector circuit because of reverse biasing. However, if emitter-base junction is also present, then forward bias on it causes the emitter current to flow. Thus the current in the collector circuit depends upon the emitter current. We shall now discuss the transistor action for npn and pnp transistors.

i) Working of npn transsistor:

Fig.16.23 (a) Shows the npn transistor with forward bias to emitter-base junction and reverse bias to collector-base junction.

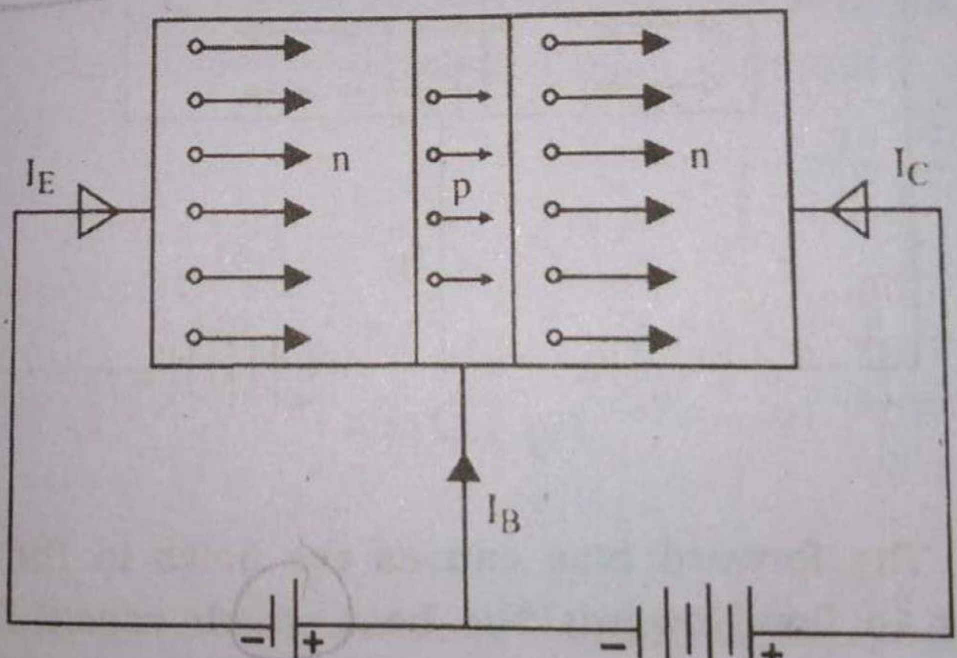


Fig. 16.23 (a)

The forward bias causes the electrons in the n-type emitter to flow towards the base. This constitutes the emitter current I_E. As electrons flow through p-type base, they tend to combine with holes. Since the base is lightly doped and is very thin, therefore, only a few electrons (less than 5%) combine with holes to constitute base current I_B. The remainder (more than 95%) cross over into the collector region to constitute collector current I_C. In this way, almost the entire emitter current flows in the collector circuit. It is clear that

$$I_E = I_B + I_C$$

We define α as the ratio of I_C and I_E i.e.

$$\text{Current gain } \alpha = \frac{I_C}{I_E}$$

ii) **Working of pnp transistor:**

Fig. 16.23 (b) Shows the basic connections of a pnp transistor.

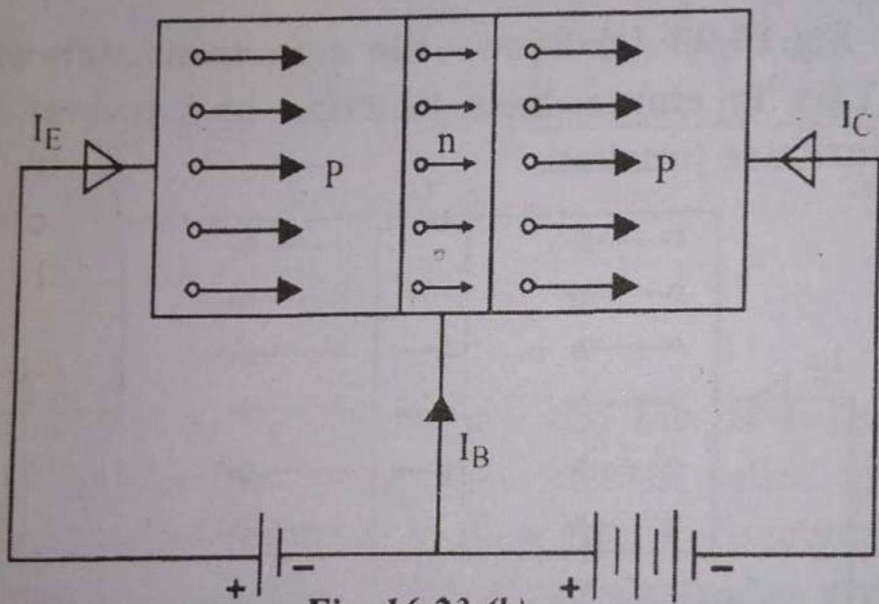
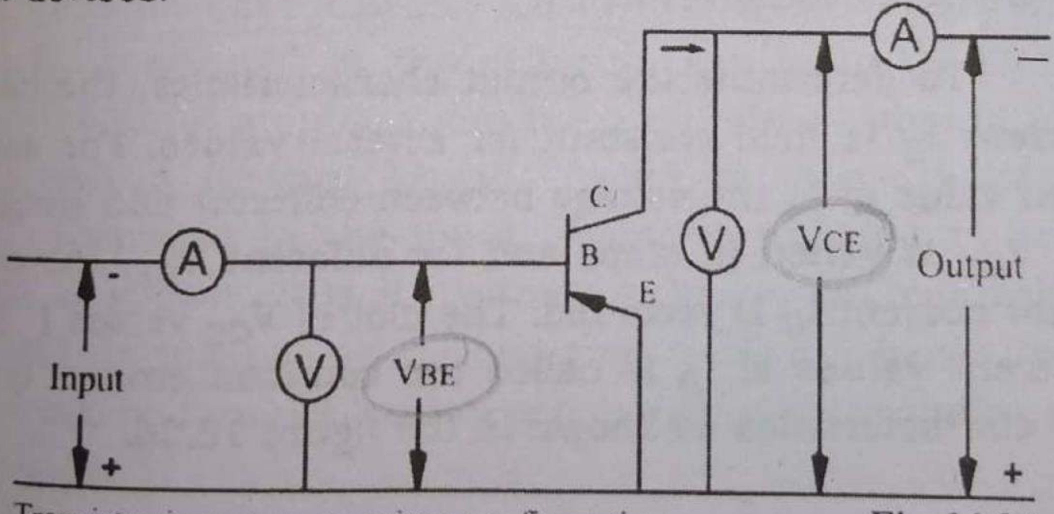


Fig. 16.23 (b)

The forward bias causes the holes in the p-type emitter to flow towards the base which constitutes the emitter current I_E. As these holes cross into the n-type base, they tend to combine with electrons, but since the base is lightly doped and is quite thin, only a few holes combine with electrons, the remaining ones cross into the collector region, to constitute the collector current I_C. In this way almost the entire emitter current flows in the collector circuit. Here the current conduction is by holes, but in the external connecting wires, the current is still by electrons.

16.17 Transistor Characteristics:

When we want to use the transistor as an amplifier, then we apply the input between two terminals of the transistor and obtain the amplified output at the other two terminals.The transistor is basically a three terminal device, so we declare one terminal as being common to both sides.the input and the output. The transistor can be operated in any one of the three useful configurations which is usually named by the common terminal i.e. Common Base (CB), Common Emitter (CE) and Common Collector(CC).**The static characteristics for the transistor contain two sets of curves** - first set, the input characteristics, gives the relationship between input voltage and input current, while the second set is the output characteristics which gives the voltage and current relationship at the output terminal. Let us only consider the common emitter configuration and it's characteristics. This is the configuration most commonly used in devices.



Transistor in common emitter configuration. *Fig:16.24*

The input signal is applied between base and emitter while the output is taken between collector and emitter, thus the emitter is common to both input and output.

(i) **Input Characteristics:**

To draw the input characteristics, the output voltage V_{CE} is held constant and for different values of input voltage V_{BE}, the base current I_B is recorded. The process is repeated for different values of V_{CE}. Now a plot of base emitter voltage V_{BE} versus base current I_B gives the input characteristics for different values of V_{CE} being constant, as shown in Fig. 16.25.

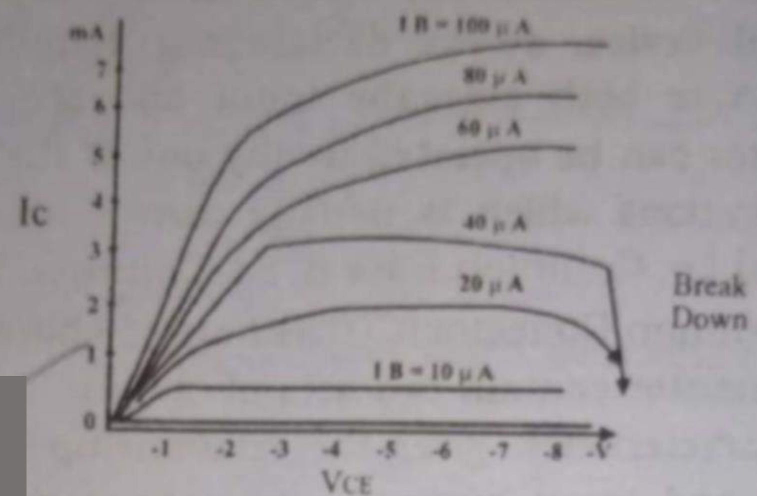


Fig. 16.25

It should be noted that for a given V_{BE}, I_B decreases with the increase in V_{CE}. This is due to increase in junction potential across the collector base junction.

(ii) Output Characteristics:

To determine the output characteristics, the base current I_B is held constant for several values. For each fixed value of I_B the voltage between collector and emitter i.e. V_{CE} is varied in steps and for different V_{CE}, the collector current I_C is recorded. The plot of V_{CE} verses I_c for different values of I_B is called the common emitter output characteristics as shown in the figure 16.26.

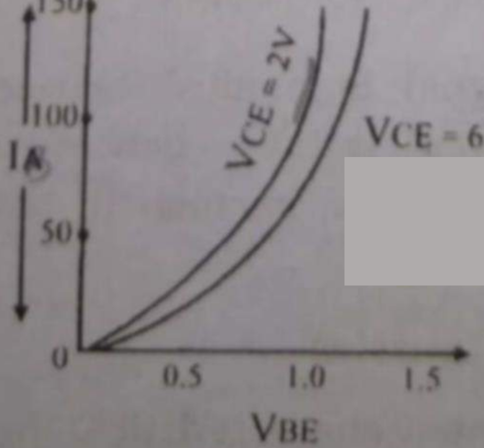


Fig. 16.26

Here the collector base junction is reverse biased and the emitter base junction is forward biased so as we increase the voltage V_{CE}, the majority carriers are emitted from the emitter and also more are collected at the collector. Due to collection of more majority carriers at the collector, the current I_c is increased. It is clear from the figure that the collector current becomes zero when voltage between collector and emitter is zero. Similarly,we can draw the common collector characteristics also.

16.18 Transistor As An Amplifier :

An amplifier is a device that raises the strength of a weak signal.Fig. 16.27 (a). Transistors can be used for the purpose of amplification of weak signals and thus act as an amplifier.

To understand the function of transistor as an amplifier. Fig. 16.27 (b), the weak signal is applied between emitter base junction and output taken across the load R_C connected in the collector circuit. In order to achieve faithful amplification, the input circuit should always remain forward biased. To do so, a d.c voltage V_{EE} is applied in the input circuit in addition to the signal as shown. This d.c voltage is known as bias voltage and its magnitude is such that it always keeps the input circuit forward biased regardless of the polarity of the signal.

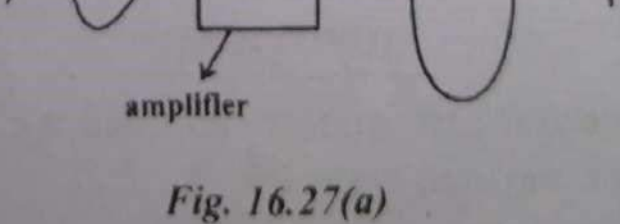


Fig. 16.27(a)

As the input circuit has low resistance, therefore, a small change in signal voltage causes an appreciable change in emitter current.This causes almost the same change in collector current due to transistor action. The collector current flowing through a high load resistance R_C produces a large voltage across it.Thus a weak signal applied in the input circuit appears in the amplified form in the collector circuit. It is in this way that a transistor acts as an amplifier.

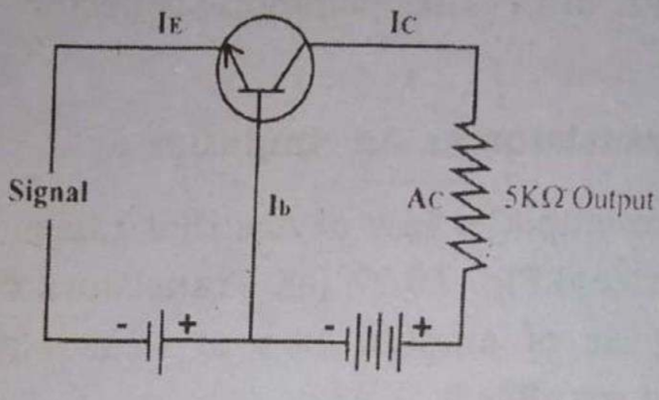


Fig. 16.27(b)

Illustration:

The action of a transistor as an amplifier can be made more illustrative if one considers typical circuit values. Suppose collector load resistance R_C = 5K Ω.Let us further assume that a change of 0.1 V in signal voltage produces a change of 1mA in emitter current. Obviously, the change in collector current would also be approximately 1mA.This collector current flowing through collector load R_C would produce a voltage = 5kΩ x 1mA = 5V. Thus, a change of 0.1V in the signal has caused a change of 5V in the output circuit. In other words, the transistor has been able to raise the voltage level of the signal from 0.1 to 5V i.e. voltage amplification is 50.

QUESTIONS

16.1 Under what circumstances does a charge radiates electromagnetic waves ?

16.2 In an electromagnetic wave, what is the relationship, if any, between the variations in the electric and magnetic fields?

16.3 A radio transmitter has a vertical antenna. Does it matter whether the receiving antenna is vertical or horizontal ?

16.4 Explain, Why are light wave able to travel through a vacuum, whereas sound waves cannot ?

16.5 Explain the condition under which radiation of electromagnetic waves takes place from a certain source ?

16.6 Describe the electromagnetic wave spectrum ?

16.7 What are semiconductors? Describe the structure of germanium crystal ?

16.8 Can a diode be used for amplifying a weak signal?

16.9 What is the difference between n-type and p-type germanium?

16.10 Are radio waves a form of light ?

16.11 Can electromagnet wave be propagated through a piped vacuum?

16.12 What is the difference between amplitude modulation and frequency modulation ?

16.13 Give the energy band description of semiconductors ?

16.14 Discuss the effect of temperature on semiconductors ?

16.15 Give the mechanism of hole current in semiconductors ?

16.16 What is crystallography. Explain the space lattice?

16.17 Explains the lattice unit cell and lattice parameter of a unit cell?

16.18 What is pn - junction? Explain the formation of potential barrier in pn-junction?

16.19 Explain photodiodes and solar cells ?

16.20 Discuss in detail the light emitting diodes ?

16.21 What do you understand by valence band, conduction band and energy gap ?

16.22 What is transistor? Why it is so called.Show diagrammatically the battery connection to a (i) pnp transistor (ii) npn transistor, for it's normal working ?

16.23 Describe the operation of a transistor amplifier?

16.24 In what wave-length range do radar signals lie ?

PROBLEMS

16.1 Light is said to be a transverse wave phenomenon. What is that varies at right angles to the direction in which a light wave travel ?

16.2 A radar sends out 0.05μs pulses of microwaves whose wave length is 2.5 cm. What is the frequency of these microwaves? How many waves does each pulse contain ?

16.3 A nanosecond is 10^{-9}s (a) What is the frequency of electromagnetic wave whose period is 1ns? (b) What is it's wave length? (c) To what class of electromagnetic waves does it belong?

Ans: (10^{9} Hz, 0.3m)

16.4 With a sketch explain the working of (i) Half wave rectifier (ii) Full wave rectifier ?

16.5 Explain the difference between the band structure of a semiconductor and that of a metal. Why does a semiconductor acts as an insulator at 0°K and why does it's conductivity increases with increasing temperature ?

Chapter. 17

ADVENT OF MODERN PHYSICS

The year 1900 not only marked the beginning of this century but also the new era of modern physics. It would be rather difficult to mention all the major developments since 1900, but surely there are quite a few events of pivotal importance which could be narrated vividly. With in a span of few years J.J.Thomson experimentally proved the existence of electron which was taken as the fundamental unit of electricity. Rontgen announced the discovery of X-ray. Henry Bequerel discovered the phenomena of radioactivity.

Meanwhile, Max Planck's put forth his famous hypothesis, that in its interaction with matter, radiant energy, behaved as discrete quanta of energy $E = h\upsilon$ Also, during this period, Albert Einstein gave a reconsideration to the fundamental concepts of classical physics which led to his famous theory of special relativity. This was the turning point which made a line of demarkation between the classical and the modern physics. We will discuss briefly some of the theories and experiments which laid down the foundations of modern physics in this chapter.

17.1 FRAMES OF REFERENCE:

We are familiar that the displacement of a point from some fixed origin is specified by two measurements when the point is located on a particular surface, and by three measurements otherwise. For example, the position of a point on earth's surface is completely specified by its latitude and longitude, these measurements give us the distances from North or South of the equator and East or West of the geographic meridian. The location of an aeroplane or helicopter is completely specified when we state that it is 1000 Km East of Karachi, 300 Km North of that city, and at an elevation of 30 Km above sea level.

The most commonly used set of co-ordinates for the above mentioned purpose is the rectangular cartesian system and is often called as the frame of reference. The reference frame is mathematically expressed interms of a set of three mutually perpendicular lines called axes of the frame of reference as shown in fig.(17.1).

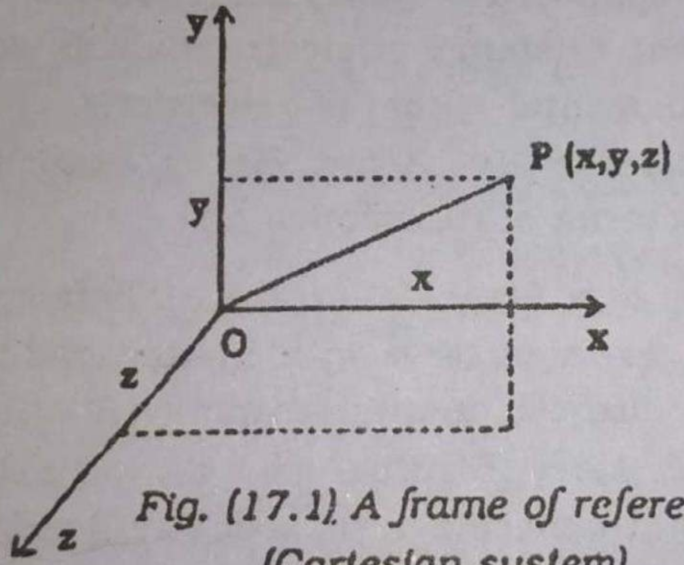


Fig. (17.1) A frame of reference (Cartesian system)

Any point P referred to the frame of reference has its three position coordinates represented by the coordinates (x, y, z).The line joining the origin O to the point P is called the position vector r of the point p with respect to the origin O.

Having seen that displacements needs to be specified relative to a given reference frame we must recognize that velocity must also be specified in the same manner. Hence, it will be worthwhile to know how the same motion will appear to be in two reference frames, one of which is moving relative to the other.

To illustrate the above, let us suppose that a rail car is moving towards East at a speed of 60 Km/h and a train is running from the back to the front of the rail car at a speed of 10Km/h. This means that the rali car covers a distance of 1Km, as measured by the measuring rod or chain laid in the west-east direction along the ground, during every minute, as noted by a clock on the ground along the rail track. On the other hand it means that the man covers a distance of $\frac{1}{6}$ Km, as measured by the measuring rod laid along the floor of the rail car, during every minute, as measured by the clock kept on the rail car. If we make the same assumption made by Galileo and Newton that the measuring rod or chains on the rail car are identical with those on the ground and the two clocks show identical time, then if:

u = Velocity of the rail car relative to the ground.

V' = Velocity of the man relative to the train.

We come across with the situation shown in Fig.(17.2).

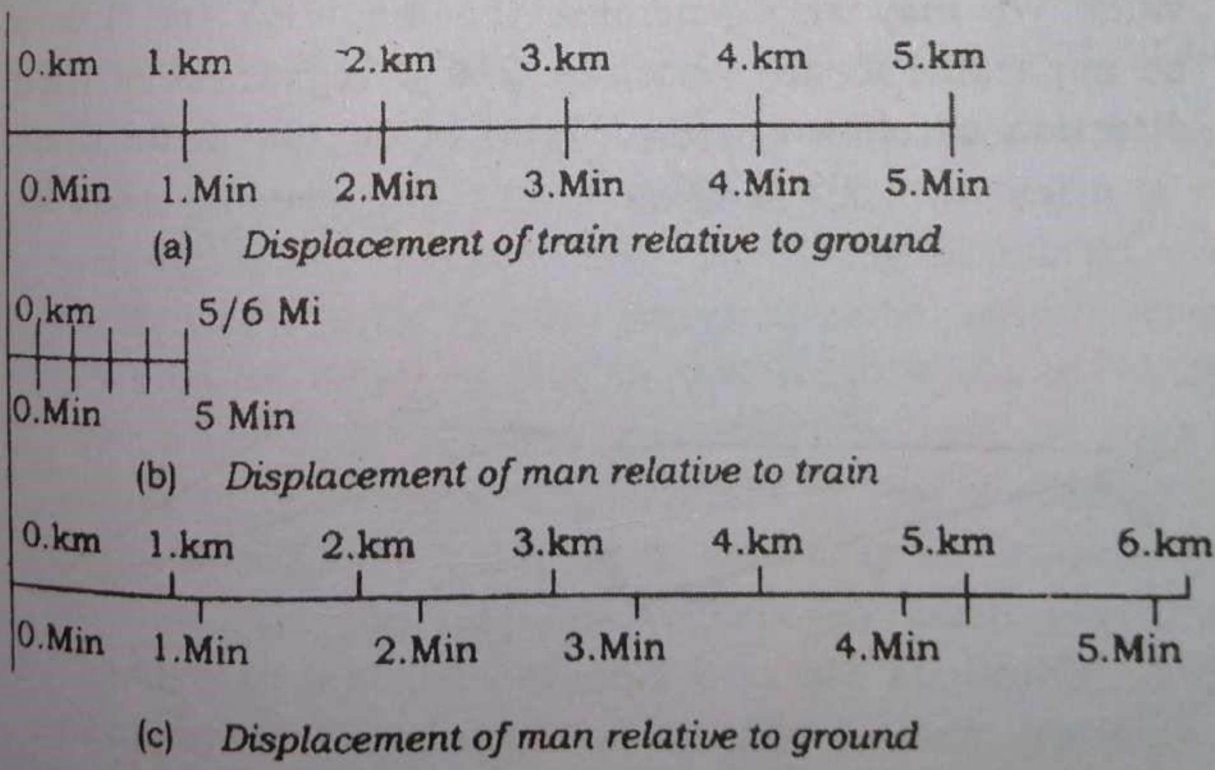


Fig.(17.2)

From the figure of the linear motion as seen in two frames of reference, we see that during an interval of

5 min. the train has covered a distance of 5Km to the East relative to the ground. The man has moved $\frac{5}{6}$ = 0.83 Km East relative to the rail car during the same interval. Hence the displacement of the man relative to the ground during the same interval will be:

$$(5.0 + 0.83)\ \text{Km} = 5.83\ \text{Km towards East.}$$

Similarly, the man's velocity (distance/time) relative to the ground will be

given by:

$$V' = \frac{5.83\ \text{Km}}{5.0\ \text{min}}\ \text{Eastward} = 70\ \text{Km/h Eastward.}$$

We, therefore find that the sum of u and v' is equal to v i.e

$$v = U + v' \qquad \text{------------------} \quad (17.1)$$

Considering any other time interval instead of 5 min, we may establish that the above addition remains valid. We may also generalize that equation (17.1) may be applicable to any velocities $\underline{U}$ & $\underline{v}'$ regardless of their direction as shown in Fig.(17.11) below and write that

$$\underline{v} = \underline{U} + \underline{v}' \qquad \text{-------------------} (17.2)$$

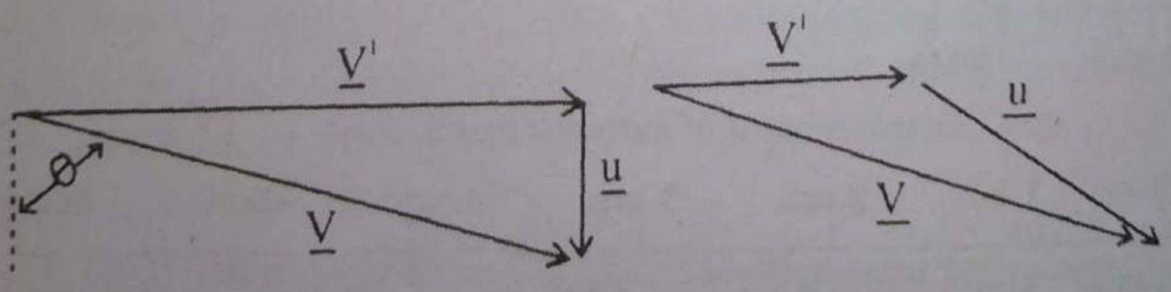


Fig. 17.3 Addition of velocities

Equation 17.2 is known as the Galilean transformation of velocities for two frames one moving relative to the other.

17.2 INERTIAL REFERENCE FRAME:

The word inertial is derived from the Newton's first law of motion which is often called the law of inertia. Inertia is an inherent tendency of inertness possessed by bodies to maintain their state of rest or of uniform motion, unless some external force is applied to change their state of rest or of uniform motion. That is why, we attribute an inertia of either rest or motion to bodies at rest or in motion with a uniform speed in the light of the Newton's first law of motion . In view of this we may define a reference frame moving with constant velocity as an inertial reference frame. With the use of inertial frame we determine whether or not the frame has a non constant velocity i.e whether it is accelerating. Only if the frame is non-accelerating i.e has a constant velocity, will an observer in that frame will experience the validity of the first law: A body at rest will remain at rest in the absence of any unbalanced force acting on it.

For most practical purposes we may consider our earth as an inertial frame. However since the earth is revolving round the sun, and is rotating about its own axis, a coordinate system chosen at earth, require forces to hold an object at rest in this rotating frame. In the strict sense of the word, an object at rest in such a frame of reference will not remain at rest if there is no unbalanced force acting on it. We are however, quite fortunate that the rotational effects on earth are small enough so that the Newton's laws are applicable on the experiments conducted on earth's surface. Thus to a good approximation we may consider that "objects at rest remain at rest" on the surface of the earth even if it is not strictly an inertial frame of reference. Thus, we may also define inertial frame of reference as that in which the Newton's laws are valid.

It can be shown that all inertial frames are equivalent from the point of view of making measurements of physical phenomena. Different observers in different inertial frames may have different values of physical quantities but the basic physical laws.(Relationships between the measured physical quantities) will always remain the same for all observers. For example, let two observers in different inertial frames measure velocity and momentum of two bodies before and after collision. It will be found that they will obtain different values for these quantities for individual bodies. Each observer, will however note that the sum of the velocities and the total momentum is the same before and after the collision in their respective frames. In other wards the law of addition of velocities and the law of conservation will remain the same in both the frames.

17.3. FRAMES OF REFERENCE IN UNIFORM RELATIVE MOTION:

As described in the previous section the Newton's laws of motion remain unchanged in any inertial frame of reference. Hence, if the law of force F= ma is valid in a frame S' at rest at time t = 0, then it will also hold good in another frame S moving at a constant velocity v with respect to the frame S. For simplicity if we assume that the origins of the two frames are coincident at the initial time t =0 the situation will be as shown in Fig 17.4

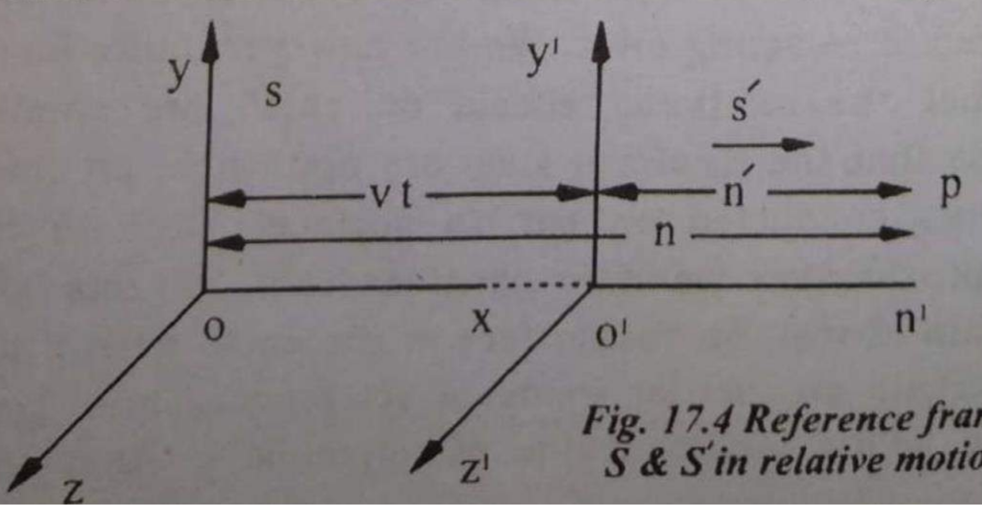


Fig. 17.4 Reference frames S & S' in relative motion

From Fig 17.4 it is seen that the frame S′ moving at a uniform velocity ϑ along the direction of x-axis will be at a distance of ϑt from the origin σ of the frame S at rest. Let us suppose that a measurement is made at the point P designated as (x,y,z) with respect to the frame S, and (x′, y′,z′) from the point of view of the frame S′ to determine the force F = ma on a body of mass 'm' making an acceleration 'a'. From the figure we find that:

$$\left.\begin{aligned} x' &= x - \vartheta t \\ y' &= y \\ z' &= z \end{aligned}\right\} \quad (17.3)$$

and t′= t (i.e.two clocks located in S & S′ are identical in time) from this set of equations, we have transformed the coordinates (x′,y′,z′;t) of the frame S′ in terms of the coordinates (x, y, z, t) of the frame of reference S. This is called the Galilean transformation of space-time coordinates. From this transformation we may infer that:

$$x = x' + \vartheta t \quad \text{---------------(17.4)}$$

Dividing through out by t we get:

$$\frac{x}{t} = \frac{x'}{t} + v$$

or, $$v = v' + v \quad \text{--------------- (17.5)}$$

where, V is the velocity as measured in S and V′ is the velocity from the point of view of S′ .

Equation (17.5) is the same as the Galilean transformation law of velocities obtained in (17.2 of section 17.1)

Let us now consider the measurement of acceleration 'a' of a body in inertial frame S expressed as:

$$a = \frac{V_2 - V_1}{t_2 - t_1} \quad \text{---------------(17.6)}$$

where V_1 and V_2 are the velocities in the frame of reference S during a time interval $(t_2 - t_1)$ for the difference of velocity $(V_2 - V_1)$.

Using equation (17.5) we may write:

$$V_2 = V_2' + v$$

$$V_1 = V_1' + v$$

subtracting the two equations, we get:

$$V_2' - V_1' = V_2 - V_1$$

Dividing both sides by $t' = t$ we have:

$$\frac{V_2' - V_1'}{t'} = \frac{V_2 - V_1}{t}$$

or $$a' = a \qquad \text{------------------ (17.7)}$$

Hence, we have shown that the acceleration a' in the moving frame's is the same as that observed in the stationary frame S.

Since the laws of physics are the same in both the frames of references we assert that:

$$F' = ma'$$

if $$F = ma \qquad \text{------------------(17.8)}$$

provided the mass does not depend on the velocity. But from 17.7 we have $a' = a$, hence from 17.8 we get

$$F' = ma' = ma = F$$

Thus the forces F' and F are equal in the two frames and the Newton's second law remains unaltered under Galilean transformations. Since the first law ($F = 0$, $a = 0$) and the third law involving the forces are contained in the second law of Newton, we therefore conclude that the Newton's laws of motion remain the same under Galilean transformations.

Although the Galilean transformations are correct transformations for accelerated frames. These transformations are also not applicable to electromagnetic phenomenon. Since velocities in different frames in relative motion appears to be different, the speed of light c given by the Maxwell's electromagnetic equation should appear different in different inertial frames. This is in contradiction to the experimental observation of the constancy of speed of light demonstrated by Michelson and Morley (1887). These short comings led Albert Einstein to propose his famous special theory of relativity.

17.4. THE PRINCIPLE OF RELATIVITY:

The choice of the frame of reference to describe the motion of an object is of vital importance in view of the fact that the description of the motion of the object may be different in different frames of reference. It may therefore be quite obvious to expect that a correct description of motion may be obtained in a reference frame which is at rest. But it is almost impossible to determine by means of an experiment performed in a reference frame, whether the frame is at rest or in uniform motion. The only possibility is to detect the motion of a frame relative to another frame. For example, assume an observer sitting in a vehicle moving with a uniform speed. Suppose the observer throw an object vertically upwards. He will observe that the object falls along the same vertical path as shown in Fig (17.5).

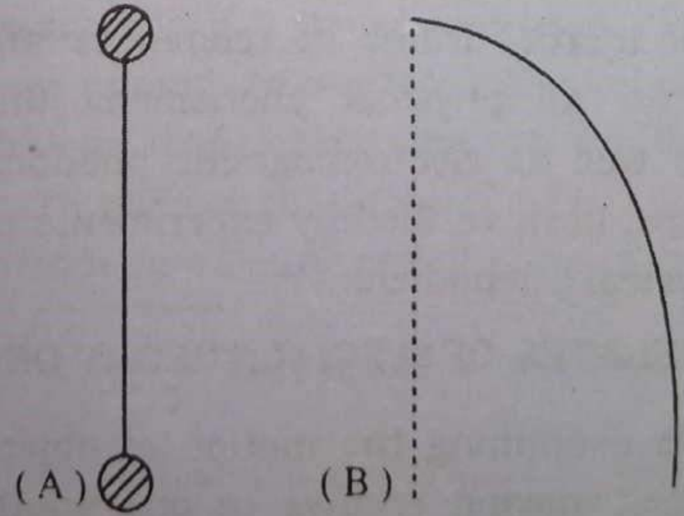


Fig.17.5 Path of an observer as seen in the vehicle (A) and on the ground (B)

For an observer at rest on the ground, the path of the object will appear to be a curve Fig 17.5 (b). The observation will be of opposite nature if the same experiment is conducted on the ground.

As there is no way to find which frame is at rest, there is naturally no means to decide which of the two paths is the correct path. However, we can realize that the motion of the object is much simpler in one frame than that in the other frame of reference. Thus the choice of the frame of reference depends upon the simplicity of the motion of the object. From the above considerations we see that physical phenomena depend, on relative motion. Albert Einstein therefore assumed that all possible reference frames moving at uniform velocity relative one another (i.e inertial frames) are equivalent for the statement and description of physical laws. This simple assumption is called the Principle of relativity.

Based on the constancy of the velocity of light Einstein further generalized this principle to restate it in the form.

All inertial frame of references are completely equivalent for all physical phenomena (including mechanical as well as electromagnetic phenomenon)". This principle have been verified by experiments on a wide variety of physical phenomena.

17.5 POSTULATES OF SPECIAL THEORY OF RELATIVITY;

While examining the motion of objects in frames of references moving relative to one another. Einstein proposed his famous theory of special relativity in the year 1905. The theory is termed as special because it is valid specially for inertial frames and is to be modified into a general theory for accelerated frames of reference. The special theory of relativity is based on two assumptions known as the postulates of special relativity. The two postulates are stated as follows:

(I) There is no preferred or absolute inertial frame of reference i.e all the inertial frames are equivalent for the description of all physical laws.(Newton's laws as well as the Maxwell's Electromagnetic equations).

(II) The speed of light in vacuum is the same for all observers in uniform translational relative motion and is independent of the motion of the observer and the source.

The free space value of the speed of light is a universal constant c which appears explicitly in the Maxwell's electromagnetic equation.

The value of the speed of light is very nearly equal to $3\times 10^{8}\ ms^{-1}$

17.6 CONSEQUENCES OF SPECIAL THEORY OF RELATIVITY

In developing the special theory of relativity we have seen that frames of reference in relative motion with a constant speed V have been used. If the speed V becomes large enough to approach the velocity of light c, then the Galilean transformations are found to be noticeably wrong. To correct the state of affairs it will be necessary to introduce a factor called Lorentz or Relativ istic factor $\sqrt{1-\frac{V^2}{c^2}}$. This factor is infact a measure of departure from Galilean transformation. We see that if $\frac{V}{c}$ where c is the velocity of light is much smaller than as it is in our common situations, then $\frac{V^2}{c^2}$ is so small that the relativistic factor is essentially equal to unity. Under these conditions the classical and the relativity physics predict nearly identical results. However when V approaches c (e.g: $v = \frac{c}{5}$). than the Galilean transformation will be incorrect.

Based on these considerations, if we interpret the results of special theory of relativity we end up in some very interesting consequences. Without going to make actual mathematical calculations, we may summarize the important consequences of the theory of special relativity which are as under:-

1. Mass Variation

According to the special theory of relativity, the mass of an object in a frame of reference at rest is called its rest mass m_0. If this mass is measured by an observer moving with a constant speed v relative to the object, then it will not remain constant if the speed v is comparable to c. The mass m in the moving frame will vary according to the mass variation relation given by:

$$m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}} \qquad \text{-------------------(17.9)}$$

This mass variation formula shows that mass changes with the velocity and is not in general a constant nor the same for all observers but is a quantity that:

(a) depend upon the reference frame from which the body is being observed.

(b) is greater than or equal to the rest mass m_0 when the body is at rest in the frame of reference from which the body is being observed.

Example 17.1

An electron has a rest mass of 9.1×10^{-31} kg when it is at rest relative to an observer. What will be its mass when it is moving at speed one half the speed of light c..

Solution:-

From the mass variation formula we have:

$$m = \frac{m_o}{\sqrt{1 - \frac{v^2}{c^2}}}.$$

Given, $m_o = 9.1 \times 10^{-31}$ kg

$$v = \frac{1}{2} c$$

$$m = \frac{9.1 \times 10^{-31} \text{ kg}}{\sqrt{1-(1/2c)^2/c^2}} = \frac{9.1 \times 10^{-31} \text{ kg}}{\sqrt{1-1/4}} = \frac{9.1 \times 10^{-31}}{1/2\sqrt{3}}$$

or $$m = \frac{18.2 \times 10^{-31} \text{ kg}}{3} = 10.50 \times 10^{-31} \text{ kg}$$

2 LENGTH CONTRACTION

In the theory of special relativity it has been found that the measurement of length of a rod in a stationary reference frame is not the same when the rod is measured by the observer in the moving frame of reference with a velocity relative to the rod, provided the measurement is made along the direction of motion.

Hence, if L_o is the length of the rod in the frame at rest, and L is the length of same rod in the moving frame, them

$$L = L_o \sqrt{1 - \frac{v^2}{c^2}}. \quad \text{.................(17.10)}$$

Since v/c is less than unity, the length L is less than L_o i.e there is a contraction in length along the direction of motion. This is called the Lorentz-Fitzgerald contraction.

Equation: (17.10) tells us that an observer past whom a system is moving with a speed v measures objects in the moving system to be shortened in length along the direction of motion by a factor $1- v^2/c^2$. It is important to note that only the dimension along the line of motion is changed and there is no change in the other two perpendicular direction.

With the development of the special theory of relativity it became apparent that there is no physical contraction of the moving objects. There is, however, an apparent contraction of a body for an observer when there is a relative motion of the object and the observer. In the natural sense the observer in the moving frame can not detect the contraction because in his frame it does not exist; where as in the rest frame, it does exist, but the measuring rod in the moving system has shrunk too further we must note that for moderate velocities ($\frac{v}{c}$ <<1) of the objects the contraction in length is negligible as observed in our every day observation.

Example 17.2

The length of a measuring rod is 1m when it is at rest. What will its length be if it is moving with a velocity one third of the speed of light?

Solution:

We have from the length contraction formula:

$$L = L_o \sqrt{1 - V^2/c^2}$$

Given: $L_o = 1\,m$ and $v = \frac{c}{3}$

$$L = 1 \times \sqrt{1- \frac{1}{9}} = \sqrt{\frac{8}{9}} = \frac{2\sqrt{2}}{3}$$

$= 0.943$ m.

3. TIME DILATION:

Time is regarded as an absolute quantity in classical mechanics whereas in the special theory of relativity it is considered to be a relative entity based on the measurement of time in frame of references in relative motion.

The time interval between two events taking place at the same point in space as timed with a clock at rest with respect to that point is called the proper time interval and is denoted by $\Delta t_o = T_o$. The time measured with a clock in motion with respect to the events is known as the relativistic time it is represented by $\Delta t = T$. Both of the time intervals T_o & T refer to the time elapsed between the same pair of events occurring in the two frames moving with a relative speed v. Then, according to special relativity the two times are related by the formula:

$$T = \frac{T_o}{\sqrt{1 - \frac{v^2}{c^2}}} \qquad \text{----------------(17.11)}$$

Equation:- (17.11) represents, what we call as the time dilation phenomena. According to the time dilation formula we mean that from the point of view of an observer at rest, the time of the observer in motion is dilated i.e. the clocks in moving frame run slowly and the Lorentz factor $\sqrt{1 - \frac{v^2}{c^2}}$ gives us the ratio of the rates of clocks for normal speeds, this factor is so close to unity that we are quite unable to detect time-dilation effect, but for speed comparable to the speed of light c the time dilation effect is quite significant.

We can now conclude that for every observer, his own clocks in his frame of reference run faster than do any other clocks which are moving relative to him. We may also note that every observer may consider himself to be at rest and consider all that moves as moving relative to him. This is actually an outcome of the principle of special relativity stated earlier in section(17.4): Every observer is equivalent to every other observer.

Example17.3

If the average life time of a particle before it decays is 2×10^{-6}s. what will be its average life time if it is moving with respect to an abserver at speed $\frac{c}{2}$?

Solution

From the time dilation formula we have:

$$T = \frac{T_o}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Given: $T_o = 2 \times 10^{-6}s$

$v = c/2$

$$T = \frac{2\times 10^{-6}\ s}{\sqrt{1-1/4}} = \frac{2\times 10^{-6}\ s}{\frac{1}{2}\sqrt{3}} = \frac{4\times 10^{-6}\ s}{\sqrt{3}}$$

i.e $T = 2 - 3 \times 10^{-6}$ s.

4. **MASS ENERGY RELATION:**

In section (17.5) we have stated the postulates of relativity that the speed; of light is a universal constant. We cannot reach speeds greater than the speed of light by the relativistic addition of velocities. The question is. how to reconcile with this result of special relativity with Newton's second law, F = ma? It would be seen that any constant force, no matter how small, applied for a considerably very long time. should continuously accelerate any mass 'm' at a rate $a = -\frac{E}{m}$. until the speed was arbitrarily very large. Einstein, concluded that Energy has inertia i.e the more energy a body possess, the more inertia that body will display. Since, inertia is a property of matter which is associated with mass. Thus from Einstein's argument mass is simply a property attributed to the total energy of the body and only the total energy is required, to know the total mass of the body. thus, in special theory of relativity total energy and mass are related by the famous Einstein's equation.

$$E = mc^2 \qquad \text{------------------}(17.12)$$

from this relation between mass and energy it has been predicted that any process that changed the mass by a detectable amount would involve huge amounts of energy for example, a mass change of 1 gram is equal to an energy change of 9×10^{13} joules. Such energy transfers will be discussed in more detail in Chapter 19, when we will study the atomic nuclear. We must bear in mind that the relation $E = m\ c^2$ is a direct and logical consequence of the mass variation mentioned earlier in section (17.6) under the results of the special theory of relativity.

Example 17.4

Find the mass associated with the energy of a mass of 10 kg moving with a speed of 100 ms^{-1}

Solution:

The kinetic energy of the mass is given by:

$$E = \frac{1}{2}\ m\ v^2$$

or, $$E = \frac{1}{2} \times 10\ kg \times (100)^2\ \frac{m^2}{s^2}$$

or, $$E = 5\ kg \times 10000\ \frac{m^2}{s^2}$$

i.e $$E = 5 \times 10^4 \text{ kg} - \frac{m^2}{s^2}$$

using, now the mass- energy relation:

$$E = m c^2$$

we have:-

$$m = \frac{E}{c^2} = \frac{5 \times 10^4 \text{ kg} - m^2/s^2}{(3 \times 10^8)^2 \text{ kg} - m^2/s^2}$$

or, $$m = \frac{5 \times 10^4}{9 \times 10^{16}} \text{ kg} = 5.5 \times 10^{-13} \text{ kg}$$

17.7. BLACK BODY RADIATIONS AND QUANTUM THEORY

By a black body we mean an object which can absorb all the radiations that falls on it. For all forms of radiations a hollow sphere of metal with a fine hole in it called cavity is approximately a black body as shown in Fig. (17.6).

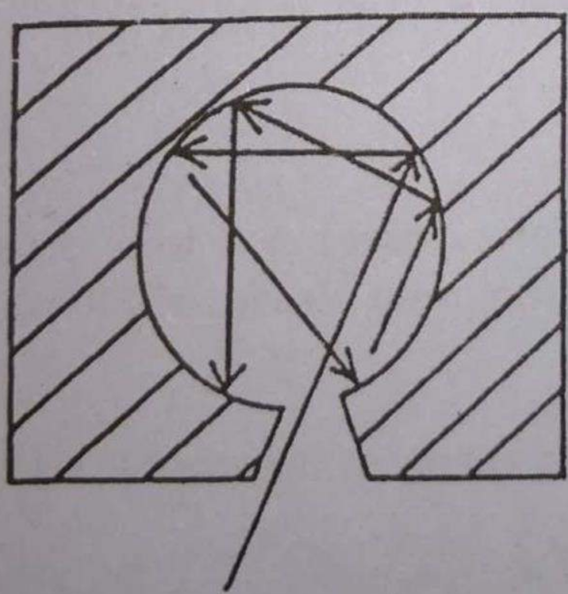


Fig 17.6. A cavity approximately a perfect Black Body

Any radiation entering the hole of the cavity is trapped by multiple reflections inside and very little of it is able to escape. Just as a black body is nearly a perfect absorber, so is the most effective emitter of radiation when heated. Due to this property experiments on black body radiation may be performed by putting such a cavity in a furnace and studying the energy of the emitted radiations from its fine hole as a function of the temperature. The graphs shown in Fig. 17.7.are called Black Body Radiation curves:

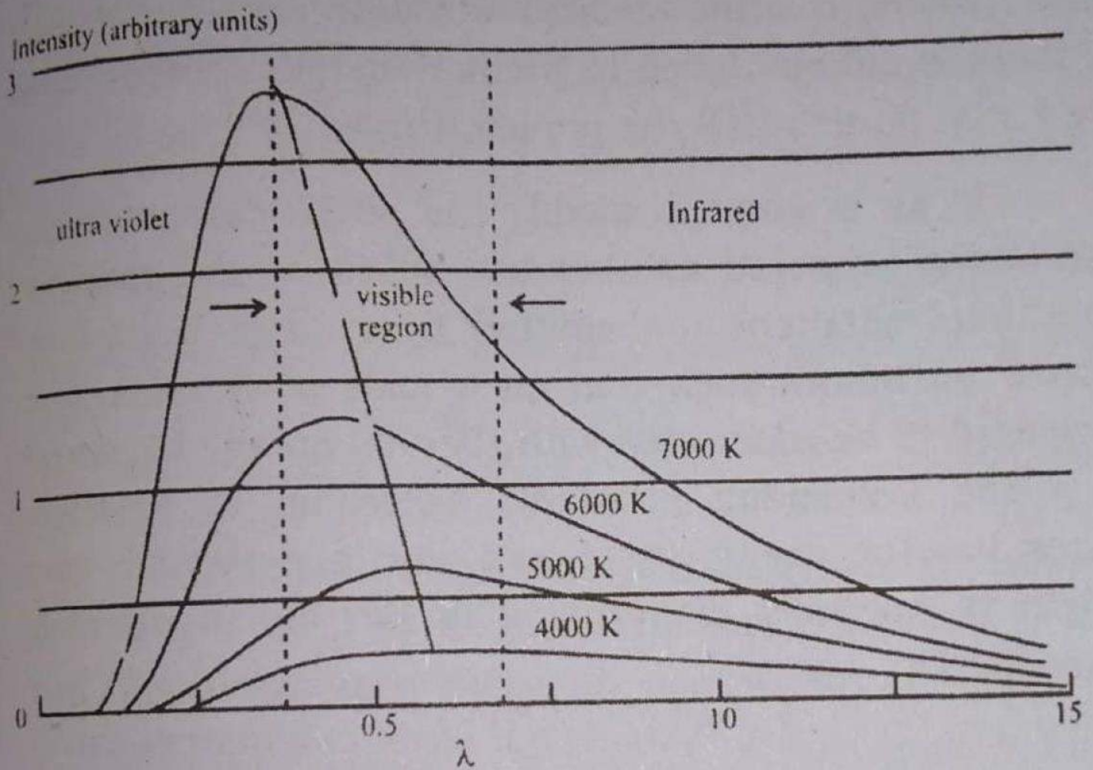


Fig. 17.7

from the blackbody radiation curves we see that the wave length at which maximum energy of radiation occurs is shifted towards the shorter wave length with the rise of temperature. The wave length for maximum radiation λ_{max} is related to the absolute temperature T of the black body by Wien's law given by:

$$\lambda_{max} \times T = \text{constant} \quad \text{----------------} \quad (17.13)$$

The Wien's law is in agreement with the common observation that a white hot furnace is hotter than one which is red hot. This law is valid in regions less than λ_{max} only.

Another relationship known as stefan's law, was also proposed in an attempt to describe the black body radiation curves. This law states that the total energy radiated per second per unit surface area is proportional to the fourth power of absolute temperature i.e

$$E \propto T^4$$

or, $$E = \sigma T^4 \qquad \text{------------------(17.14)}$$

where, σ is the Stefan-Boltzmann constant.However these equations failed to predict energy distribution in the entire curve at all the temperatures.

In an attempt to modify the Wien's law Rayleigh and Jeans proposed another law based on the assumption that radiations are emitted by a large number of atomic oscillators such that each mole of vibration was supposed to be associated with thermal energy kT, where k is the Boltzmann constant. According to Rayleigh-Jeans law the energy associated with a particular wave length is inversely proportional to the fourth power of wave length

i.e. $$E = \frac{\text{(Constant)}}{\lambda^4} \qquad \text{------------------(17.15)}$$

This law has been found to give a good agreement with experimental results at large values of l. For wave lengths near and less than λ_{max} . the Rayleigh-Jean law gave values which were found to be much too large i.e total energy tending to acquire infinite value even at short wave lengths.This is called ultra-violet catastrophe. because it is a serious discrepancy from physical point of view, that the energy can not be infinity. By common experience we know that hot bodies actually emits mostly red light and not ultra violet and x-rays.

17.8. PLANCK'S LAW - QUANTUM THEORY:

To over come the difficulties in providing a successful explanation of the Black body curves Planck (1900) proposed a formula that correctly describe the intensity distribution with respect to wave length of a black body. a perfect absorber or emitter of radiation. Fig.(17.7) the previous section. Planck proposed that radiant energy comes out in discrete amounts or quanta of energy. The energy content of each quantum was directly proportional to the frequency υ

Energy of a quantum = a constant x frequency of quantum

or, $E = h\upsilon$ ------------------(17.16)

Since, $\upsilon = c/\lambda$, where c is the velocity of light.

we have, $E = \dfrac{hc}{\lambda}$ ------------------(17.17)

The constant, h, is known as Planck's constant, has since proved to be a fundamental constant of nature. By matching theory with observations the value of h was determined to be 6.63×10^{-34} J-S.

The success of Planck's Theory is that it avoided the ultra-violet catastrophe by limiting the energy of a hot body to a finite number of sources the high-frequency quanta require more energy; hence, fever of their would be radiated. On the basis of assumption that energy could only be emitted or absorbed by atomic oscillators in discrete quanta, the Pianck's law would be:

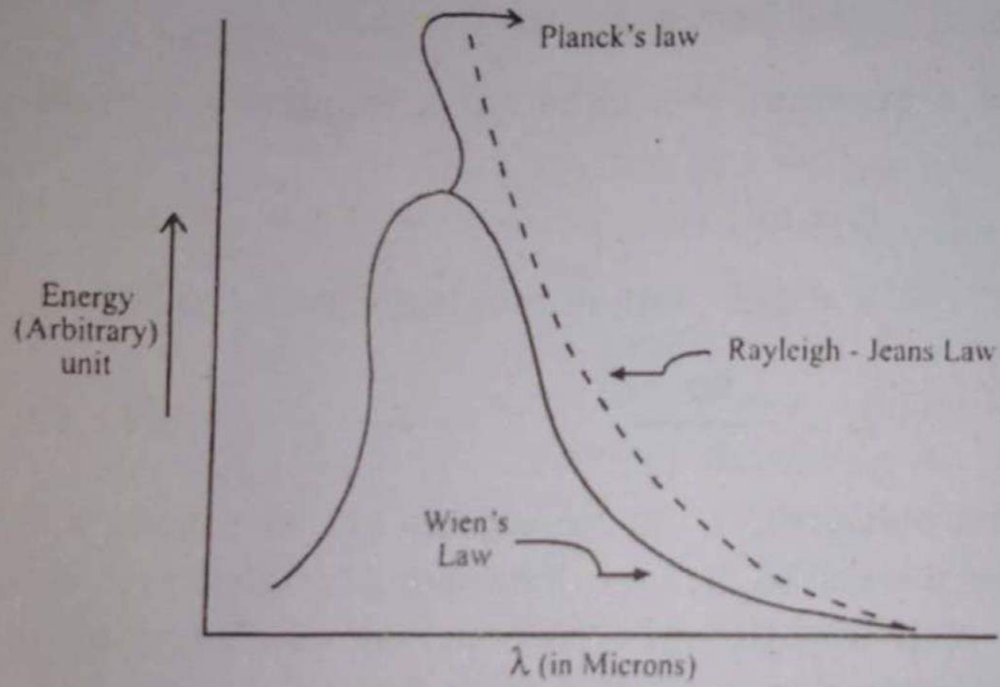
$E = n\,h\upsilon$ ------------------(17.18)

where $E = 0.h\upsilon,\ 2h\upsilon$ ---------- Corresponding to

$n = 0, 1, 2, 3, 4$ - - - - -

Thus we see that the Planck's law had an important virtue; it worked, that is, it agreed very well with experiment i.e theoretical and experimental curves for radiation matched reasonably. The price for this success was a revolution of concept about electromagnetic radiation not as waves, but as discrete quanta of energy.

A comparison of the different radiation laws is given in Fig.17.8 from which it is evident that the Wien's Law and the Rayleigh-Jean's law are just the special cases of the Planck's law



Fog. 17.8 Comparison of Radiation Laws.

To appreciate how great a departure Planck's law is from classical theory, we may recall that classically, the energy of a wave is related to the amplitude. For example, large waves, such as ocean waves have a large energy. That is no need to relate energy with the frequency. We can have waves of low energy and high frequency and vice versa. According to Planck's theory, however, each electromagnetic wave carries with it a minimum energy that is a function of its frequency. This novel idea took about a quarter of century after it's proposal to be fully welcomed in the modern scientific thought. In this way the planck's law created a revolution in modern Physics.

Example: 17.5.

What will be the energy of an x-ray quantum of wave length 1.0×10^{-10}m?

Solution:

Given:

$$h = 6.63 \times 10^{-34} \text{ J-S}$$

$$\lambda = 1.0 \text{ Å} = 10^{-10} \text{ m}$$

$$c = 3 \times 10^{8} \text{ ms}^{-1} \text{ (velocity of light)}$$

To find, E ?

from Planck's law we have:

$$E = h\upsilon$$

But $\upsilon = \frac{c}{\lambda}$ (wave equation)

$$E = \frac{hc}{\lambda} = \frac{6.63 \times 10^{-34} \text{ J-S} \times 3 \times 10^{8} \text{ m/s}}{10^{-10} \text{ m}}$$

Hence, $E = 19.89 \times 10^{-16}\text{J} = 1.99 \times 10^{-15}\text{J}$

17.9 THE PHOTON.

The photon is, a particle that has no charge and no mass. It can interact with all charged particles as well as with some neutral ones it is electromagnetic radiation and the carrier of electromagnetic forces. Every atom is found to emit and absorb photons of the particular energies and frequencies.

The photon is the lightest particle, being a massless particle its energy is:

$$E = mc^2 = p\,c. \qquad \text{------------------(17.19)}$$

where $p = mc$ is the momentum of the photon.

Since $E = h\upsilon$ is the energy of the photon the mass of the photon will be given by

$$mc^2 = h\upsilon$$

or.
$$m = \frac{h\upsilon}{c^2} \quad \text{----------------- (17.20)}$$

from, this it is quite obvious that the mass of photon approaches zero as c^2 is a very large quantity occurring in the denominator of equation (17.20)

The photon is a stable particle and, therefore it does not decay spontaneously into any other particle. Its life time is therefore infinite so long it does not undergo interaction with other particles, and is why photons are supposed to be reaching our earth from the farthest distances of the universe. Thus most of our information regarding the universe is carried by photons.

Example: 17 6

Compare the energy of a x-ray photon of wave length 2.0×10^{-10} m with the energy of 2.0μm infrared photon.

Solution:

We have the energy of the photon of x-ray given by:

$$E_1 = h\upsilon_1$$

But,
$$\upsilon_1 = \frac{c}{\lambda_1}$$

$$E_1 = \frac{hc}{\lambda_1} = \frac{6.63 \times 10^{-34}\ \text{J-S} \times 3 \times 10^8\ \text{m/s}}{2.0 \times 10^{-10}\ \text{m}}$$

i.e.
$$E_1 = 9.94 \times 10^{-16}\text{J}$$

Now, for the infra-red photon we have:

$$E_2 = \frac{hc}{\lambda_2} = \frac{6.63 \times 10^{-34}\ \text{J-S} \times 3 \times 10^8\ \text{m/s.}}{2.0 \times 10^{-10}\ \text{m}}$$

i.e. $E_2 = 9.945 \times 10^{-20} J$

$$\frac{E_1}{E_2} = \frac{9.945 \times 10^{-16} J}{9.945 \times 10^{-20}} = 10^4$$

17.10. THE PHOTO ELECTRIC EFFECT:

Hertz in 1887 discovered that when ultraviolet light falls on certain metals, electrons are emitted. This phenomena in which certain metals emitt electrons when exposed to high frequency light is called photoelectric effect. The experimental arrangement to demonstrate the effect is shown in Fig.(17.9).

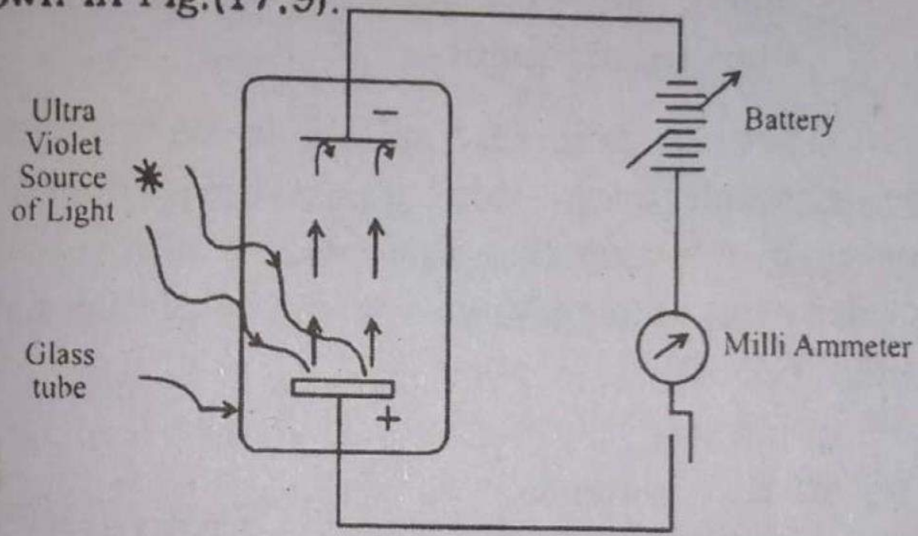


Fig. 17.9 The Photo Electric Effect

The observed photoelectric effect has these features:

(1) Increasing the intensity of the source of light, increases the number of photoelectrons, but not the velocity with which they leave the surface of the metal.

(2) For each substance there is a certain frequency called the threshold frequency below which the effect does not occur.

(3) The higher the frequency of the incident light, the greater the kinetic energy ($\frac{1}{2} mv^2$) of the photoelectrons.

Attempts were made to explain these aspects of the photoelectric effect from the point of view of classical wave theory of light. but no successful explanation was obtained due to the following reasons:

(a) From classical theory. there should be no threshold frequency because at a given time. electrons might absorb enough energy from the incident light to escape from the metal surface at any applied frequency.

(b) The velocity of photoelectrons should depend upon the amplitude of the wave incident on the metal, and therefore upon the intensity rather than the frequency.

Let us first discuss the experimental results of the photoelectric effect. If we draw the photoelectric curves by plotting the photoelectric current I versus the accelerating voltage V we will obtained the curves shown in Fig. (17.10)

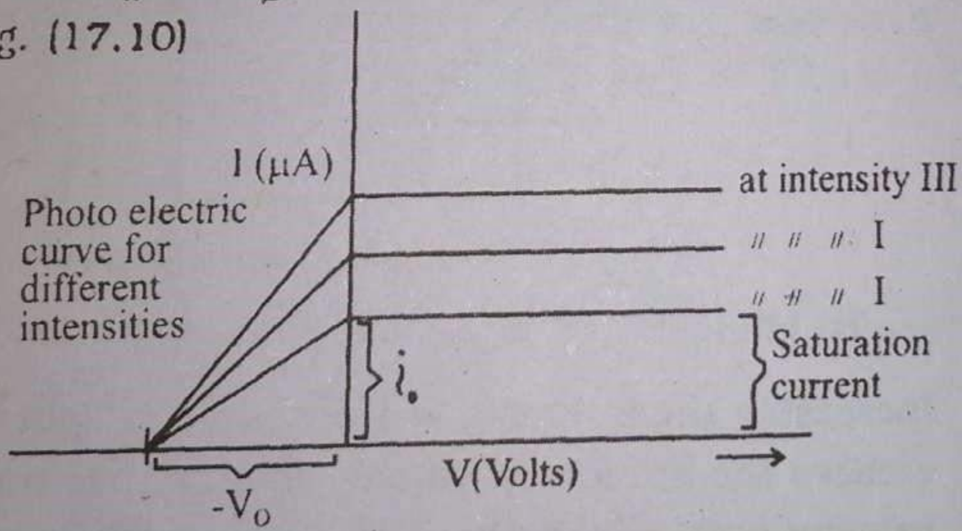


Fig. 17.10

From these curves. It follows that there is a saturation current for different intensities I, II and III etc. and even when the potential V = O. there is some photocurrent I_o and a negative potential $-V_o$ called stopping potential. This behaviour of photoelectric curves indicate that the stopping potential is independent of the intensity of the source and kinetic energy K of the photoelectrons will be maximum for the condition.

$$K_{max} = \frac{1}{2} m\vartheta^2_{max} = eV_o$$

Where, V_{max} = maximum velocity of photoelectrons

e = charge of electron

If photoelectric curves are plotted for different frequencies V_1 and V_2 but the same intensity I of the source, the curves show up the following behaviour Fig. (17.11).

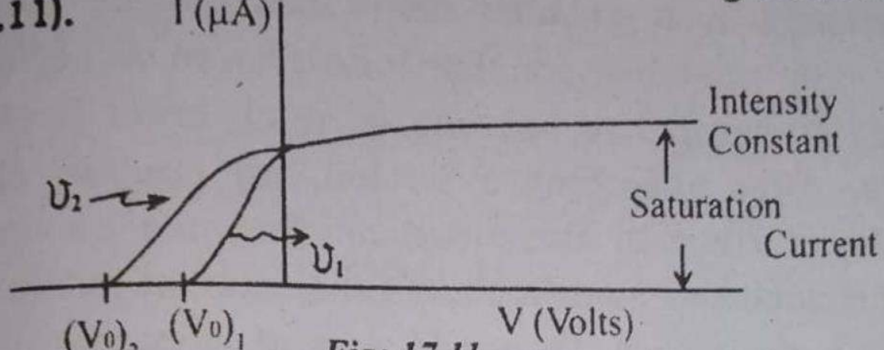


Fig: 17.11

From these curves we infer that the saturation current depends upon intensity and not on the frequency. However, the stopping potential becomes more negative from $(V_o)_1$ to $(V_o)_2$ with the increase in frequency ($V_2 > V_1$). The variation of the maximum energy of photoelectrons K_{max} as a function of frequency υ may be seen from Fig. (17.12) below:

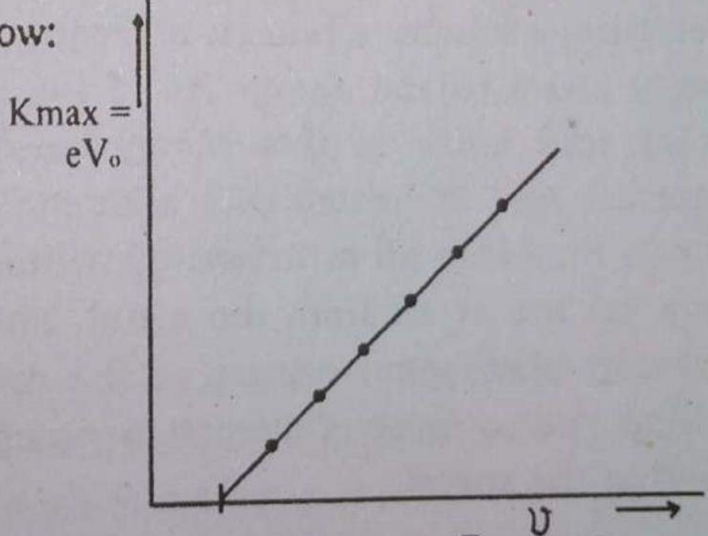


Fig: 17.12

This graph shows a threshold frequency υ_o is a minimum frequency below which no electrons escape the metal surface. The most significant feature of this graph is that the slope of the line gives the ratio h/e. Thus if e is known the measured slope provides a value of Plancks constant h quite independently.

17.11 EINSTEIN'S EXPLANATION OF PHOTOELECTRIC EFFECT ON THE BASIS OF QUANTUM THEORY.

Albert Einstein provided a successful explanation of the photoelectric effect on the basis of quantum theory. He proposed that an electron either absorbs one whole photon or it absorbs none. The chance that an electron may absorb more than one photon is negligible because the number of photons is much lower than the electrons. After absorbing a photon, an electron either leaves the surface of the metal or dissipate its energy within the metal in such a short time interval that it has almost no chance to absorb a second photon. An increase in the intensity of light source simply increases the number of photons and the number of electrons, but the energy for electron remain unchanged. However, the increase of frequency of the light increases the energy of the photons and hence the energy of electrons too.

Thus according to Einstein's quantum theory, an electron absorbs a photon of frequency to acquire an energy equal to the energy $h\upsilon$ of the photon. The electron may lose some of this energy before leaving the metal surface and is ejected with a kinetic energy less than $h\upsilon$, or, it may lose all of its energy within the metal and does not escape at all from the metal. This is true even if the photon absorption occurs at the metal surface because there exist a force of attraction which holds the electrons within the metal.

The energy required to overcome this binding force is called the work function of the particular metal and is denoted by ϕ which is a constant of the metal. Hence the Einstein's equation for photoelectric effect will be written as:

$$K_{max} = \frac{1}{2} m\upsilon^2_{max} = h\upsilon - \phi$$

The graph in Fig. (17.12) shows that the threshold frequency υ_o is a minimum frequency below which no electrons escape out of the metal surface. hence, the condition to find the threshold frequency would be

$$h\upsilon_o = \phi$$

i.e. the Einstein's equation will become:

$$\frac{1}{2} m\upsilon^2_{max} = h\upsilon - h\upsilon_o = h\ (\upsilon - \upsilon_o\ ) \text{ -------}(17.22)$$

The threshold frequency for some metals is shown in table (17.1).

Table 17.1

Photoelectric Threshold Frequency & Work Function of Metals.

Metal	Threshold Freq: υ_o (Hz)	Work Function ϕ in eV
Cesium	4.6×10^{14}	1.9
Beryllium	9.4×10^{14}	3.9
Titanium	9.9×10^{14}	4.1
Mercury	1.09×10^{15}	4.5
Gold	1.16×10^{15}	4.8
Palladium	1.21×10^{15}	5.0

For comparison with these values, it is to be noted that the high frequency end of visible light (violet) has a frequency 8×10^{14} H_z and a photon energy is = 3.3.eV.

Example: 17.7

Sodium light of wave length 5.893×10^{-7} m falls on a photocell. A negative stopping potential of 0.30V is needed to stop the electrons from reaching the collector.

(a) Find the work function of the material of the plate.

(b) **What will be the potential required when light of 4000°A is used?**

Solution:

Data $\lambda = 5.893 \times 10^{-7} m$; $c = 3 \times 10^{8} ms^{-1}$

$V_o = 0.3$ Volt i.e. $= 1.6 \times 10^{-19} C$

$h = 6.63 \times 10^{-34}$ J-S

(a) from the Einstein's Photoelectric Equation, we have

$$V_o e = h\nu - \phi$$

$$\phi = h\nu - V_o e$$

or $$\phi = \frac{hc}{\lambda} - V_o e \text{ - - - - - - - } \therefore c = \upsilon\lambda$$

$$= \frac{6.63 \times 10^{-34} \times 3 \times 10^{8}}{5.893 \times 10^{-7}} - 0.3 \times 1.6 \times 10^{-19}$$

$$= 3.38 \times 10^{-19} - 0.48 \times 10^{-19}$$

$$= 2.90 \times 10^{-19} J$$

$$\phi = 2.90 \times 10^{-19}$$

$$= \frac{2.90 \times 10^{-19}}{1.6 \times 10^{-19}} eV \quad \therefore \phi = 1.8 eV$$

(b) $\lambda = 4000°A = 4000 \times 10^{-10} m = 4 \times 10^{-7} m$:

$\phi = 2.90 \times 10^{-19} J$: $c = 3 \times 10^{8} ms^{-1}$

$e = 1.6 \times 10^{-19} C$; $h = 6.63 \times 10^{-34}$ J-S

$$\therefore V_o e = h\upsilon - \phi$$

or $$V_o e = \frac{hc}{\lambda} - \phi$$

$$\therefore V \times 1.6 \times 10^{-19} = \frac{6.63 \times 10^{-34} \times 3 \times 10^{8}}{4 \times 10^{-7}} - 2.90 \times 10^{-19}$$

$$V \times 1.6 \times 10^{-19} = 4.97 \times 10^{-19} - 2.90 \times 10^{-19}$$

or

$$V \times 1.6 \times 10^{-19} = 2.07 \times 10^{-19}$$

or

$$V = \frac{2.07}{1.6}$$

$$V = 1.29 \text{ Volts}$$

17.12. PHOTO CELL AND THEIR USES.

A simple photocell based on the photo electric effect is shown in Fig. (17.13)

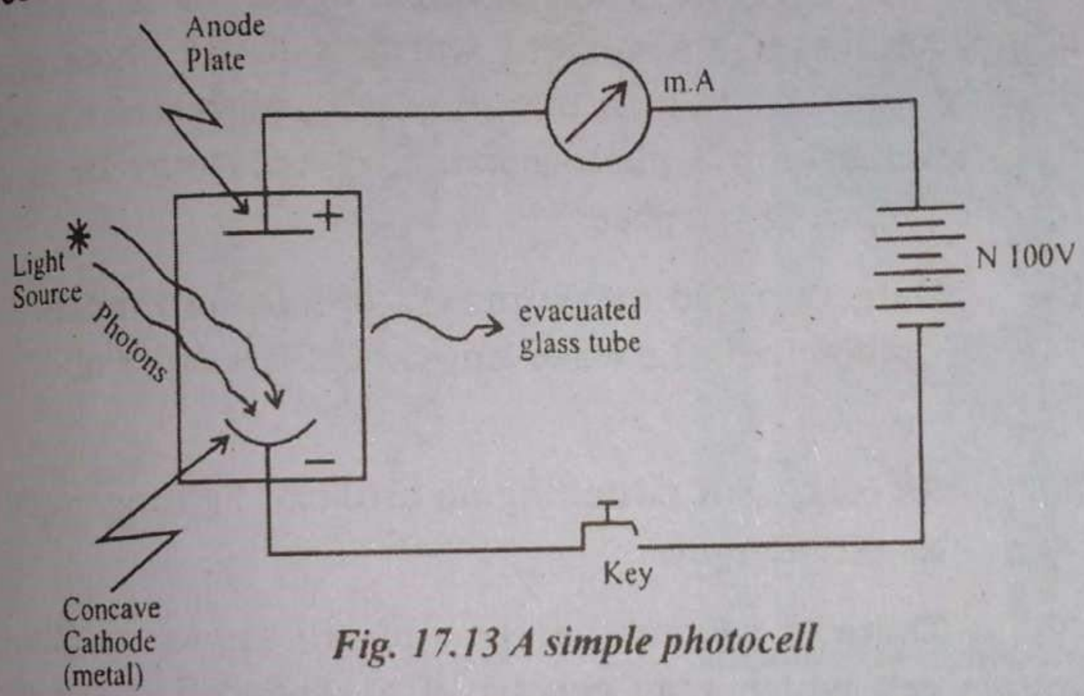


Fig. 17.13 A simple photocell

The photocell or photo tube consist of a evacuated glass tube fitted with an Anode plate and a concave metallic cathode of a appropriate surface. The material of the cathode can be chosen to respond to the frequency range over which the photocell operates. The response can be made proportional to the intensity of the light source. The photocell is connected in a circuit as shown in Fig. (17.13) to operate for a particular use of the cell as a source of photoelectric emission. A photocell can be used in any situation where beam of light falling on a cell is interrupted or broken.

For example:

(a) To count vehicles passing a road or items running on a conveyer belt.

(b) To open doors automatically in a building.

(c) To operate burglar alarms.

Besides Photo-emission cells there are also photo conductive cells in which an internal photoelectric effect may liberate free charge carriers in a material that is otherwise an insulator, and thereby increase its electrical conductivity by as much as 10,000 times when it is illuminated by a light source. Such materials are called photo conductors. A current will flow if the photo conducting material is in a circuit with a source of electromotive force (e.m.f) photo conductive cells may be used for the following purpose:

(i) Detection and measurement of infra-red radiations where the wave length is of the order of 10^{-6} m.

(ii) As relays for switching on artificial lighting, such as street lights.

There is yet another type of cell known as photo voltaic cell which may consist of a sandwich of copper, copper oxide and a thin film (layer) of translucent gold. An e.m.f capable of giving a current of 1mA can be generated when the film is illuminated. Such cells need no source of e.m.f and are frequently used as exposure meters in photography. The exposure meters are required to set the aperture (opening before the camera lens) of the camera compatible to the day light intensity.

Besides the above mentioned uses, the most important use of photocells in of modern everyday life is the production of pictures in television cameras and the sound tracks of motion pictures. In one type of photocell of much usage, the cathode is coated with caesium, a light sensitive material that emit electrons when illuminated. The electrons collect at a second electrode the anode, the current being proportional to the illumination. The sound information is stored on the film in the form of spots of varying widths. When such a film run between the light source the photocell, variations in light intensity reaching the cell cause pulsations in the current that, after being amplified, activates a loudspeaker and reproduces sound. This is diagramatically shown in fig: 17.14

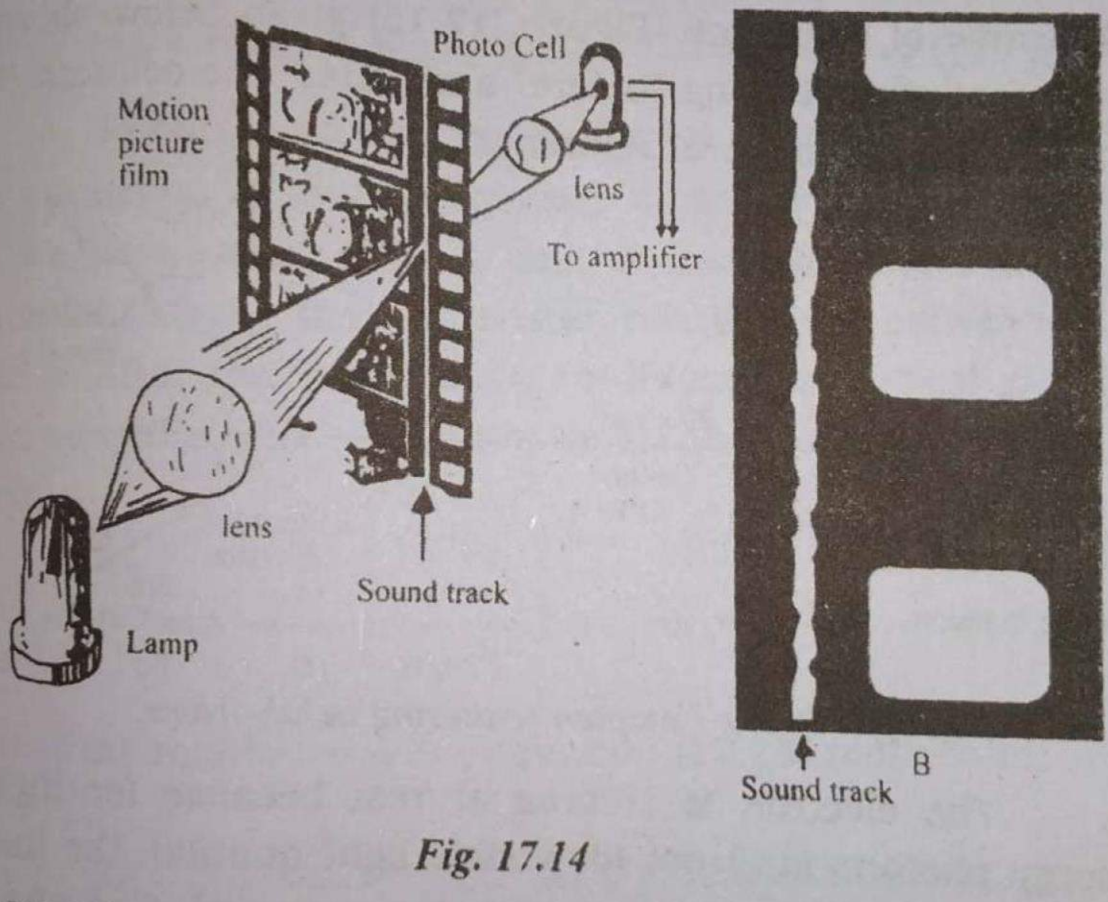


Fig. 17.14

17.13. THE COMPTON EFFECT:

It was reported by many observers that when x-rays are scattered due to interaction with a light body, such as an electron, the scattered rays exhibit lower frequencies (i.e. higher wave lengths) than the incident radiations. Arthur Compton studied this phenomena of change in wavelength in the year 1926. Making accurate measurements, he was the first to propose a theory based on the idea of photon theory of radiation. Since a detailed study of the phenomena was made by Compton, the effect is now called as the compton's effect. Compton effect provided a solid support for photon theory of light since the results obtained were elegantly described by assigning to the photon an energy E =h υ and momentum

$$p = \frac{h\upsilon}{c} \text{ as, } E = h = mc^2 \text{ gives us } \frac{h\upsilon}{c} = mc = p.$$

The scattering is treated to be a two body collision between a photon and an electron.

It is sufficient to analyze the scattering process in a single frame of reference. Figure (17.15) given below shows the event of scattering "before" and "after" the collision in the laboratory frame of reference:

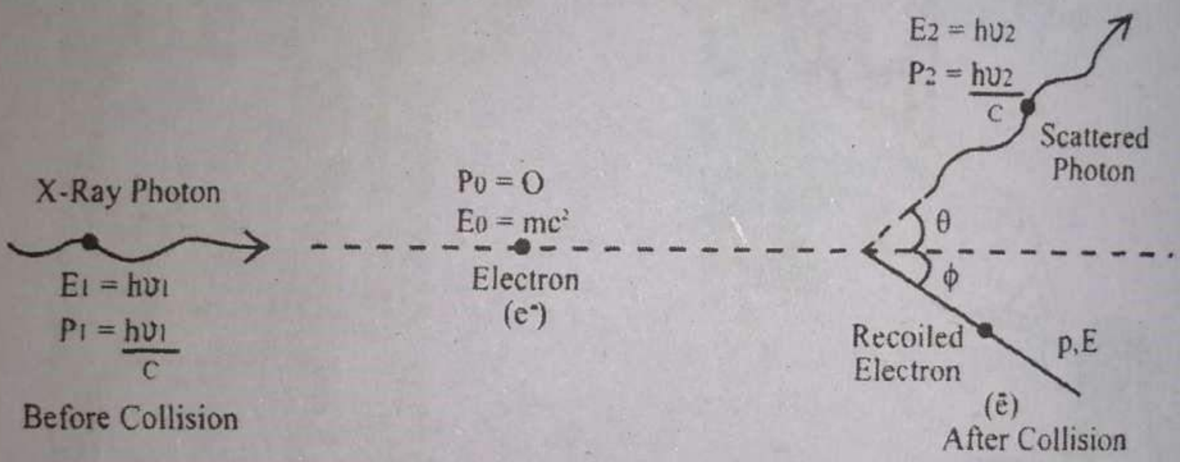


Fig. (17.15) The Compton scattering in lab-frame.

The electron is treated at rest because for high energy photons (and not for visible light quanta), the initial motion of the electrons may be neglected. The photon approaches towards the electron with a frequency υ_1 and is scattered at an angle θ with a lower frequency υ_2. The photon energies before and after collision are $h\upsilon_1$ and $h\upsilon_2$. Whereas the corresponding momentum are $\frac{h\upsilon_1}{c}$ and $\frac{h\upsilon_2}{c}$ The energy and momentum of the recoiled electron are E and P respectively. If the laws of conservation of energy and momentum along and across the direction of approach are applied, we will get the following equations:

Conservation of momntum along the line of impact:

$$\frac{h\upsilon_1}{c} = \frac{h\upsilon_2}{c} \cos\theta + p\cos\phi \quad \text{---------------(17.23)}$$

Conservation of momentum across the line of impact:

$$0 = \frac{h\upsilon_2}{c} \sin\theta - p\sin\phi \quad \text{-----------------(17.24)}$$

Conservation of energy before and after collision

$$h\upsilon_1 + m_0 c^2 = h\upsilon_2 + E \quad \text{--------------- (117.25)}$$

To obtain an expression for the final frequency υ_2 as a function of initial frequency υ_1 and scattering angle θ. We have to eliminate p and E from the above three equation, using the relativistic relationship between E and p. After performing some routine mathematical steps and simplification we will finally get the following expression.

$$\frac{1}{\upsilon_2} - \frac{1}{\upsilon_1} = \frac{h}{m_0 c^2}(1 - \cos\theta) \quad \text{----------(17.26)}$$

using the relation $v = \frac{c}{\lambda}$, equation (17.26) reduces to:

$$\lambda_2 - \lambda_1 = \frac{h}{m_0 c}(1 - \cos\theta) \quad \text{-----------------(17.27)}$$

Equation (17.27) is the famous compton formula for the increase in wave length of the scattered photon.

The quantity $\frac{h}{m_0 c}$ in the compton's equation is called the compton wavelength and is denoted by

$$\lambda_c = \frac{h}{m_0 c} = 2.426 \times 10^{-12} m \quad \text{---------------(17.28)}$$

17.14 Pair Production and Annihilation of Matter

We have already seen that a low energy Photon on striking with a electron loses its entire energy (Photo electric effect) where as a high energy Photon loses a part of its energy (compton effect). A Photon may also lose its energy in another way, that is a Photon in the vicinity of a nucleus may disappear with the Production of an electron-positron pair.

The positron has been identified to be identical with an electron in mass and carries an equal positive charge and is called the anti-particle of the electron. This phenomenon called pair production is shown schematically in Fig. (17.16) below:

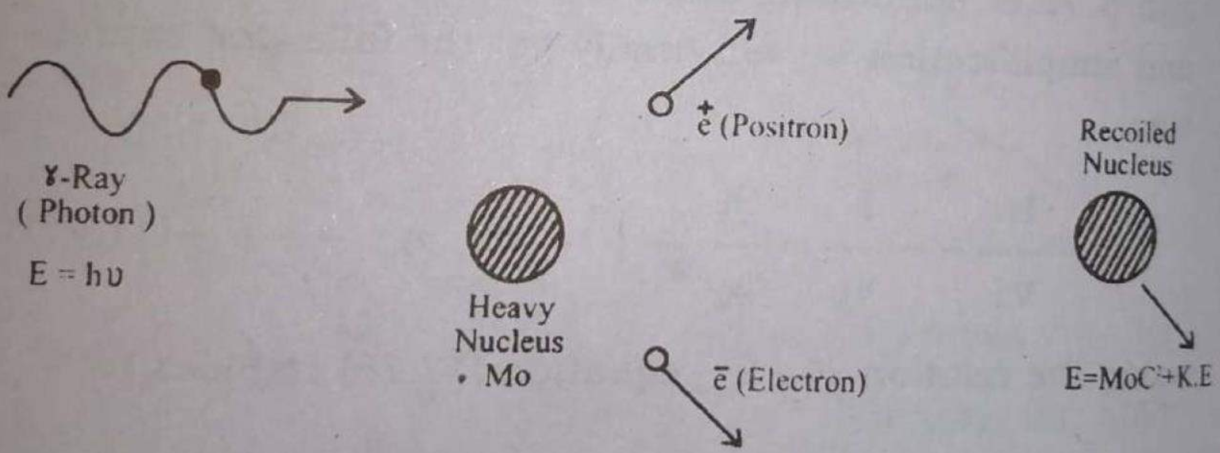


Fig. (17.16) The Pair Production

In order to conserve the charge, the pair production requires that the two particles created by the interaction of photon with matter must have equal and opposite charges and the photon from which the pair was produced must have an energy at least $2m_oc^2$, where m_o is the rest mass of electron and C is the velocity of light. Due to this reason the pair production is not important below 1.02 MeV the energy corresponding to $2m_o$. Note that the role of the heavy nucleus in the vicinity of the incoming photon is just to share some energy and momentum in order to conserve these quantities.

Due to the large mass of the heavy nucleus the recoil kinetic energy of the nucleus $P^2/2m_o$ is negligible as compared with the kinetic energies of the pair. The energy conservation in pair production demands:

$$h\upsilon = 2m_oc^2 + (K.E)\,\bar{e} + (K.E)\,\overset{+}{e} \quad \text{------------------}(17.29)$$

Since the process of pair production involves the creation of a particle & its anti-particle, it is also some times referred to as the materialization of energy in conformity with the mass-energy equivalence to be discussed further in chapter-19.

It has been observed that a process reverse to the pair production may also occur by the destruction or annihilation of the electron-positron pair with the creation of at least two or more photons. The pair annihilation process is schematically shown below in Fig. (17.17).

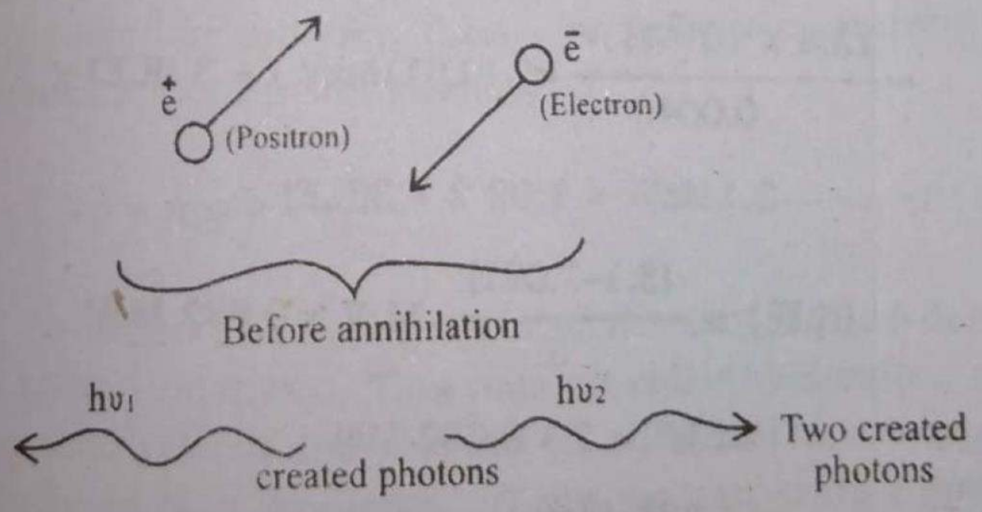


Fig. (17.17) Annihilation of electron positron pair in collision

In pair annihilation a particle and one of its anti-particle come close enough to be converted completely into radiation energy of the two photons moving in opposite direction conserving the total momentum of the creation and annihilation process. Each photon will have an energy equal to rest mass energy m_oc of an electron.

Since the rest mass energy of electron positron pair is 1.02 MeV, each photon created in the annihilation process will have an energy of 0.51 MeV.

The energy conservation equation for the process wil be:

$$(m_o)_{e^+} c^2 + (K.E)_{e^+} + (m_o)_{e^-} c^2 + (K.E)_{e^-} = 2h\upsilon \text{ ------(17.30)}$$

Example: 17.8

A photon of wave length 0.004Å in the vicinity of a heavy nucleus produces an electron positron pair. Find the kinetic energy of each particle if the kinetic energy of positron is twice that of the electron.

Given: $hc = 12.4\times 10^{-3}$ MeV and, $m_o c^2 = 0.511$ MeV

Solution:

From the law of conservation of energy of wave:

Initial Energy = Final Energy

$$= \frac{hc}{\lambda} = 2m_o c^2 + (K.E)_{e^-} + (K.E)_{e^+}$$

or

$$\frac{12.4 \times 10^{-3}\text{ MeV}}{0.0040} = 2\ (0.511\text{MeV}) + 3\ (K.E)\ e^-$$

$$3.1\text{MeV} = 1.02\ 2 + 3(K.E)_{e^-}$$

$$\therefore\ (K.E)_{e^-} = \frac{(3.1-1.022)}{3}\text{ MeV} = 0.692\text{ MeV}$$

Hence

$$(K.E)_{e^+} = 2 \times 0.692\text{ MeV}$$

$$= 1.384\text{ MeV}.$$

Example: 17.9

Pair annihilation occurred due to a head-on collision of an electron and a positron producing 2.5 MeV photon moving in opposite directions. What will be the Kinetic energies of the electron and positron before the collision. Given: $m_o c^2 = 0.511$ MeV.

Solution:

Applying the law of energy conservation we have:

$$2\ m_o c^2 + 2\ K = 2\ (E)$$

When K = K.E of the particle photon before collision.

Thus, we get:

$$2(0.511)\ MeV + 2K = 2\ (2.5MeV)$$

or. $$1.022\ MeV + 2K = 5.0\ MeV.$$

or. $$2K = (5 - 1.022)\ MeV = 3.918\ MeV$$

$$\therefore\ K = \frac{3.918}{2}\ MeV = 1.959\ MeV$$

17.15 THE WAVE NATURE OF PARTICLES AND DE-BROGLIE HYPOTHESIS.

DeBroglie in 1924 put forth a novel idea called the DeBroglie's hypothesis: If light (electromagnetic radiation) can have particle behaviour, then material particles, such as electrons and protons etc. can also behave in a wave like manner. Thus a particle, like electron can possess a momentum given by:

$$p = m\upsilon = \frac{h}{\lambda} \qquad \text{-----------------(17.31)}$$

Where, m is the mass of the particle (as defined in special relativity). This relation called deBroglie's relation has related the electron (a particle), and the wave character of a frequency . Thus we can write down he wave length associated with the particle i.e

$$\lambda = \frac{h}{m\upsilon} = \frac{h}{\lambda} \qquad \text{-----------------(17.32)}$$

Although, the deBroglie's relation was initially developed for the electron, it is valid for all material objects including particles. However, for massive materials the associated wave length is too small to be measured. For example a mass of 20 kg moving with a velocity of 50 m per second will have its wave length:

$$\lambda = \frac{h}{mv} = \frac{6.63 \times 10^{-34}}{20 \times 50} = \frac{6.63 \times 10^{-34}}{1000}$$

$$\text{i.e } \lambda = 6.63 \times 10^{-37}\ \text{m}$$

Hence we see that such a small order of 10^{-37} is not measureable.

On the other hand for light particles like an electron moving with a velocity say $10^7\ \text{ms}^{-1}$ the wave length will be

$$\lambda = \frac{h}{mv} = \frac{6.63 \times 10^{-34}}{9.1 \times 10^{-31} \times 10^7} \equiv 0.72 \times 10^{-10}\ \text{m}$$

this order of magnitude of the wave length falls in the short x-ray wave length and is experimentally measurable.

Example: 17.10

What will be the de Broglie wave length of a mass of 3 kg moving with a velocity of 100 ms^{-1}

Solution:

from deBroglie's relation we have

$$\lambda = \frac{h}{mv} = \frac{6.63 \times 10^{-34}}{3 \times 100} = 2.212 \times 10^{-36}\ \text{m}$$

Example: 17.11

What will be the wavelength of a neutron having an energy equal to 0.06 eV.

Given, rest mass energy of neutron $m_0 c^2 = 940 \times 10^6$ eV.

We know that the kinetic energy is given by:

$$K = \frac{1}{2} m_0 V^2$$

$$\upsilon = \sqrt{\frac{2K}{m_0}}$$, when given K = 0.06 eV

But from, deBroglie's relation we have:

$$\lambda = \frac{h}{m_0 \upsilon} = \frac{h}{m_0 \sqrt{\frac{2K}{m_0}}} = \frac{h}{\sqrt{2m_0K}}$$

$$\lambda = \frac{12.40 \times 10^{-10}}{10.26} = 1.16 \times 10^{-10} \text{ m}$$

17.16 THE DAVISSON AND GERMER EXPERIMENT:

The theoretical prediction of deBroglie's hypothesis $\lambda = \frac{h}{p}$ - was experimentally confirmed by the famous experiment conducted by Davisson and Germer in the year 1927. They were investigating the scattering of an electron beam by the metallic crystal of Nickel. Their experimental set up which was enclosed in a vacuum chamber is schematically shown in Fig. (17.18).

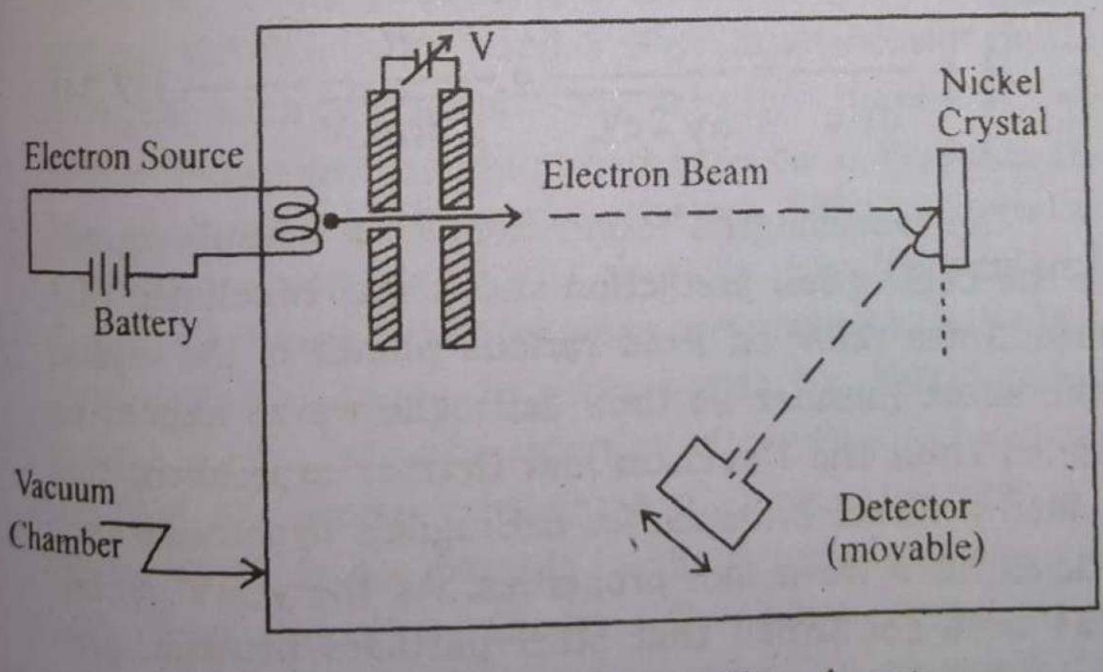


Fig.17.18 Davisson and Germer Experiment

A beam of electron accelerated through the potential V were allowed to strike the nickel crystal. Measurements were made to count the number of electrons scattered by the crystal. Davisson and Germer reported the un-expected result that electrons reflected very strongly at certain angles only and not at other direction. These results remained unexplained for some time until Elasser suggested that perhaps, this was an outcome of deBroglie's relation. Davisson and Germer then further investigated properly oriented crystals to observe if could be possible to interpret that electrons behave as waves of wave length λ as given by the deBroglie's relation (17.32). They calculated the wave length of electron from the known accelerating potential V by applying the relation for kinetic energy of the electron i.e.

$$\frac{1}{2} m_o v^2 = eV$$

so that

$$v = \sqrt{\frac{2eV}{m_o}} \quad \text{----------------- (17.33)}$$

where m_o is the rest mass of electron, e the charge and V its velocity using the deBroglie's relation:

$$\lambda = \frac{h}{m_o v} = \frac{h}{m_o\sqrt{\frac{2eV}{m_o}}} = \frac{h}{\sqrt{2em_oV}} \quad \text{------- (17.34)}$$

The wavelengths found from this formula agreed with the deBroglie's prediction and it was concluded that electrons are reflected from various planes of the crystal in the same manner as their deBroglie waves should be reflected thus the Davisson and Germer experiment has provided a direct evidence for deBroglie's hypothesis that particles have wave like properties. As the years passed it has been confirmed that other particles neutron, protons, atoms and molecules etc, are associated with the same wave effects as that of electrons.

17.17: WAVE PARTICLE DUALITY:

From the ancient times till the time of Newton.

scientists believed that light propagates in the form of corpuscles (particles). Wave theory of light was then established in 1801 when Young's interference experiments was performed. Finally by the year 1900, Planck postulated his law that electromagnetic radiations are emitted in the form of light quanta (photons). It was again demonstrated that particle behaviour had to be assigned to electromagnetic radiations in order to explain the famous photoelectric effect and compton scattering. Finally de-Broglie (1924) proposed his hypothesis in which he suggested that since light much of the character of particles, that particles, like electrons, could exhibit wave like behaviour. hence, it is said that electromagnetic radiation exhibit a wave-particle duality i.e. in certain situations it shows wave like properties while in other circumstances it acts like a particle. Since these are the only two possible modes of propagation of electromagnetic energy. It is essential to have a clear understanding of the distinction between waves and particles. A particle is identified by its position, momentum, mass, energy and charge. On the other hand a wave is associated with attributes such as amplitude, velocity, intensity, wavelength. frequency. energy and momentum. Besides these attributes one of the most distinet difference between wave and a particle is that particles can be localized at certain positions, whereas wave are spread relatively over large region of space. In other words the deBroglie's formula $\lambda = \frac{h}{p}$, may be interpreted physically by proposing that if material particles are allowed to cross a slit whose width is comparable to the wavelength associated with them, then the particles will exhibit diffraction phenomena in exactly the same manner as photons do in the Young's single slit experiment and wave like character predominate. For photons the frequency and wavelength are given by $\upsilon = E/h$ and $\lambda = h/p$. From these expressions it is evident that the left hand sides of these equations involved the wave aspect of photons through υ and λ, whereas the right hand sides display the particle character through E and P. The linking factor between the two aspects is the Planck's constant h, and hence the particle wavelength will be given by

$$\lambda = h/p = \frac{h}{m\upsilon}.$$

There is also another difference between photons and ordinary objects in the manner through which the wave and particle like properties are related. This is because for a photon $\lambda\upsilon = c$, and hence there is only a single rule to obtain both λ and υ from photon's particle like properties of E and p. On the other hand for ordinary objects, separate rules are required to specify its wave length by the relation $\lambda = \frac{h}{p}$. and frequency by $\upsilon = E/h$.

Inspite of the wave-particle duality of radiation and matter (particles) it has not been possible to witness a single phenomena in which radiation or particle exhibit both wave and particle characters simultaneously. All known physical phenomena clearly fall in two Distinct categories. of exhibiting either the particle or wave like behavior of radiation and matter. Thus, a complete description of either radiation or particles requires in either case that both the wave and particle features be considered, but each in its own proper perspective of the particular phenomena exhibited by either of them.

17.18. THE UNCERTAINTY PRINCIPLE:

In classical physics it is generally assumed that position and momentum of a moving object can be simultaneously measured exactly, that is no uncertainties are inolved in its description. For example if we know the initial position and velocity of an object and the net force acting on it, we can apply Newton's Law (Classical mechanics) to predict exactly (with certainty) its final position and velocity. But, can we make similar measurements simultaneously in microscopic world of atoms and subatomic particles?

It is found that however refined we make our instruments there is a fundamental limitation to the accuracy with which the position and velocity of microscopic particle can be known simultaneously. This limitation was first expressed by Heisenberg (1927) and is known as the uncertainty principle. The Heisenberg uncertainty principle states that it is in principle impossible to measure with accuracy both position and momentum of a particle simultaneously. Thus, if we denote Δx and Δp_x the corresponding uncertainties in position; then according to Heisenberg's principle of uncertainty, the product of these uncertainties must always satisfy the inequality:

$$\Delta x \Delta p_x \geq \frac{h}{2\pi} = 1.05 \times 10^{-34} \text{ J S} \quad \text{-----------(17.35)}$$

Similar relations follow for the other two y and z directions.

Although it is difficult to observe this idea in our normal everyday observations but it is necessary and direct consequences of the deBroglie's hypothesis and the wave particle duality.

Let us now examine whether the uncertainty condition is consistent with experiment. Suppose we want to localize a particle in the y direction as shown in Fig. (17.19).

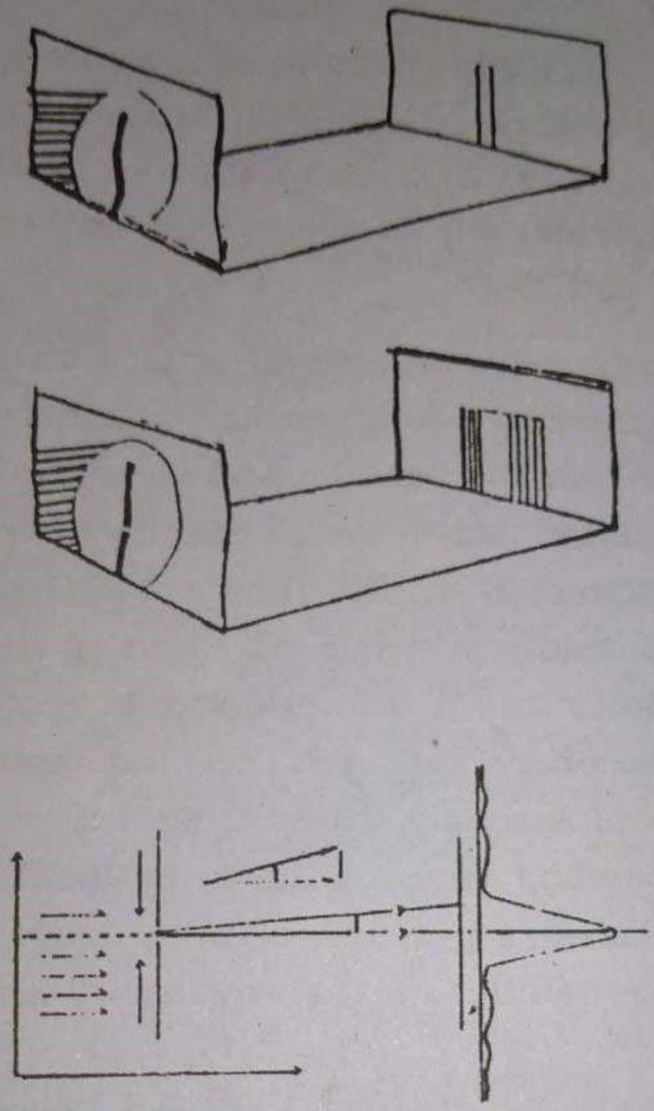


Fig. 17.19

If a particle beam strikes a slit, those passing through the slit must have been localized in a region $\Delta y = d$. where d is the width of the slit. If the particles are ordinary classical particles, they will strike the screen along a thin strip i.e. the projection of the slit on the screen. But if a wave crosses a slit, then there will be a diffraction pattern wider than the width of the slit projected on the screen. The width of the diffraction pattern on the screen will be found to increase as the width of the slit is reduced in size.

A particle arriving at the screen some distance from the centre will have a finite y component of momentum Δp_y along with p_z as shown in figure above.

The relation between the slit width and the distance to 1st minimum i.e. $\theta = \frac{\lambda}{d}$ shows that if we make the slit narrow by reducing d, we increase the diffraction width i.e. increase the uncertainty in p_y hence we may write

$$\theta = \frac{\lambda}{d} = \frac{\lambda}{\Delta y} \qquad \text{------------------- (17.36)}$$

The definition of the angle $\theta << 1$ gives:

$$\theta = \frac{\Delta p_y}{P_x} \qquad \text{------------------- (17.37)}$$

and, by deBroglie's relation we have

$$\lambda = \frac{h}{P_x} \qquad \text{------------------- (17.38)}$$

using the above equations we finally obtain

$$\frac{\Delta p_y}{P_x} = \frac{\lambda}{\Delta y} = \frac{h}{P_x \Delta_y}$$

i.e. $$\Delta p_y \Delta y = h \qquad \text{-------------------(17.39)}$$

Equation (17.39) is therefore consistent with the Heisenberg, uncertainty principle stated in Equation (17.36).

Similar to the uncertainty relation above there is another principle of uncertainty which limits the accuracy in the measurement of time i.e. if ΔE is the energy uncertainty in time Δt then we have an expression similar to (17.39)

i.e $$\Delta E \Delta t > h$$

The reason why the uncertainty principle is of no importance in our everyday life is that Planck's constant h is so small that the uncertainties in position and momentum of even quite light objects are far too small to be experimentally observed for microscopic phenomena such as atomic processes, the displacements and momentum are such that the uncertainty relation is critically applicable.

Example: 17.12

Find the uncertainty in momentum and the kinetic energy of a electron if it is found to exist in a region equal to the diameter of Hydrogen atom.

Solution:

Given: $\hbar = 1.05 \times 10^{-34}$ J-S and $m = 9.1 \times 10^{-31}$ kg.

diameter of hydrogen atom = 10^{-10} m.

i.e. $\quad V = 1.6 \times 10^{-19} J$

using uncertainty relation

$$\Delta p = \frac{\hbar}{\Delta z} = \frac{1.05 \times 10^{-34} \text{ J-S}}{10^{-10} \text{ m}} = 1.05 \times 10^{-24} \text{ kg ms}^{-1}$$

Now, kinetic energy of electron

$$= \frac{(\Delta P)^2}{2m} = \frac{(1.05 \times 10^{-24} \text{ kg ms}^{-1})^2}{2 \times (9.1 \times 10^{-31} \text{ kg})}$$

$$= 6.1 \times 10^{-19} \text{ J} = 3.8 \text{ eV}.$$

Example: 17.13

Determine the minimum uncertainty in the position of a particle of mass 5×10^{-3} kg moving with a speed of $2 ms^{-1}$ The momentum can be determined to a accuracy of one part in a thousand.

Solution:

$(\hbar = 1.05 \times 10^{-34} \text{ J-S})$

Given: $$\frac{\Delta p}{P} = \frac{1}{1000} = 10^{-3}$$

$$\therefore \Delta p = p = 10^{3} p = 10^{-3} mv \quad (\because p = mv)$$

Now using the uncertainty relation

$$\Delta x \, \Delta p \approx \hbar$$

we get. $$\Delta x \approx \frac{\hbar}{\Delta p} = \frac{1.05 \times 10^{-34}\ J.S}{10^{-3} \times 5 \times 10^{-3}\ kg \times 2ms^{-1}}$$

i.e. $$\Delta x = \frac{1.05}{10} = 10^{-28}\ m = 1.05 \times 10^{-29}\ m$$

Example: 17.14

What will be the uncertainty in energy of an electron thrown to a higher state in an atom and falling back to the original state in about 10^{-8} s? Given $\hbar = 1.05 \times 10^{-34}$ J.S.

Solution:

Using the uncertainty relation:

$$\Delta E\ \Delta t \geq \hbar$$

we have, $$\Delta E \approx \frac{\hbar}{\Delta t} = \frac{1.05 \times 10^{-34}\ J.S}{10^{-8}}$$

$$\therefore\ \Delta E \approx 1.05 \times 10^{-26}\ J$$

$$\approx 0.65 \times 10^{-7}\ eV$$

QUESTIONS

17.1 What do you understand by a frame of reference, and what are inertial frames?

17.2 Give the principle of relativity and explain the postulates of special theory of relativity.

17.3 Discuss the black body radiation and the associated difficulties in explaining the radiation curve.

17.4 Give the essential features of the photoelectric effect. How can these features be explained theoretically.

17.5 Discuss briefly the wave particle duality and the principle of uncertainty.

17.6 How can you demonstrate experimentally that particles have wave-like particles?

17.7 What is a photo-cell? mention some of its uses in the modern life.

17.8 Explain, why is compton effect not observable with visible light?

17.9 What phenomena require a wave description of light? What phenomena require a particle picture of light? How are the two aspects related quantitatively?

17.10 In what way do the particles of light (Photons) differ from particles of matter such as electrons and protons.

17.11 In the photoelectric effect the energy of a photoelectron is less than that of incident photon. Explain.

17.12 How did deBroglie's hypothesis help to explain the stability of the atom?

17.13 With the help of Uncertainty principle show that electrons can not exist inside the nucleus of an atom.

PROBLEMS

17.1 In the inertial frame of a pendulum the time period is measured to be 3 s. What will be the period of the pendulum for an observer moving at a speed of 0.95 c with respect to the pendulum?

Ans: (9.6 s)

17.2 What will be the length of a bar in the stationary frame if its length along the x'-direction is 1m and the motion is with a velocity 0.75c with respect to the observer at rest.

Ans: (0.66m)

17.3 Given $m_o c^2 = 0.511$MeV. Find the total energy E and the kinetic energy K of an electron moving with a speed $v = 0.85c$.

Ans: (0.970MeV: 0.459Mev)

17.4 The total energy of a proton of mass 1.67×10^{-27}kg is three times its rest energy.

Find (a) Protons rest energy

(b) Speed of the proton

(c) Kinetic energy k of proton in eV

Given $c = 3 \times 10^8$ ms^{-1} and 1eV $= 1.60 \times 10^{-19}$ J

Ans (939 MeV: 2.82×10^8 ms^{-1} and 1878 MeV)

17.5 A particle of rest mass m_o has a speed $\vartheta = 0.8C$. Find its relativistic momentum, its kinetic energy and total energy ?

Ans: $\left[\frac{4}{3} m_o c; \frac{2}{3} m_o c^2; \frac{5}{3} m_o c^2\right]$

17.6 What will be the velocity and momentum of a particle whose rest mass is m_o and whose kinetic energy is equal to its rest mass energy

Ans : $\frac{\sqrt{3}}{2}$ C, $\sqrt{3}$ $m_o c$

17.7 The sun radiates energy at a rate 3.8×10^{26} w. At what rate the mass of sun diminishes?

Ans: [1.32×10^{17} kg per yr]

(Given $c = 3 \times 10^8$ ms^{-1} 1 yr $= 3.15 \times 10^7$S)

17.8 What will be the work function of a substance for a threshold frequency of 43.9×10^{13} Hz?

Ans: (1.82 eV)

17.9 What will b the value of $\lambda_{min} = \frac{hc}{eV_o}$. if $h = 6.63 \times 10^{-34}$ J.S, $c = 3 \times 10^8$ ms^{-1}.

$e = 1.6 \times 10^{-19}$ C and $V_o = 10^4$ V

Ans: (1.24×10^{-10} m)

17.10 In a compton scattering process, the fractional change in work length of x-ray photons is 1% at an angle $\theta = 120°$, find the wavelength of x-ray used in the experiment.

Ans: (3.63×10^{-10} m)

17.11 Find the wave length of a 2.0g light ball moving with a velocity:

(a) 1.0 mm per century (b) 1.0 ms^{-1}

(Given: $h = 6.63 \times 10^{-34}$ J.S and 1yr = 3.15×10^7 S)

Ans: (1.05×10^{-18} m: 3.3×10^{-31} m)

17.12 An electron exist within a region of 10^{-10} m. Find its momentum, uncertainty and the approximate kinetic energy.

Given $\hbar = 1.05 \times 10^{-34}$ J.S and $m = 9.1 \times 10^{-31}$ kg

Ans: (1.05×10^{-24} kg ms^{-1} · 6.01×10^{-19} J)

17.13 Sodium surface is shined with light of wave length 3×10^{-7} m. If the work function of Na = 2.46eV, find the K.E of the photoelectrons and also the cut off wave length $\lambda c = \frac{hc}{\phi}$

(Given, 1eV = 1.6×10^{-19} J)

Ans: (1.68eV, 5.05×10^{-7}m)

17.14 X-rays of wave length λ_o are scattered from a carbon block at an angle of 45° with respect to the incident beam. Find the shift in wave length.

(Given: $h = 6.63 \times 10^{-34}$ J-S, $m = 9.11 \times 10^{-31}$ kg and $c = 3 \times 10^8$ ms^{-1}.)

Ans: (7.11×10^{-13} m)

17.15 If the electron beam in a T.V picture tube is accelerated by 10,000V what will be the deBroglie's wave length?

Ans: [1.28×10^{-11} m]

17.16 What minimum energy photon can be used to observe an object of size 2.5×10^{-10}m.

Ans: (4.96×10^{3} eV)

17.17 What will be the deBroglie's wave length associated with a mass of 0.01 kg moving with a velocity 10 ms^{-1}?

Ans: (6.63×10^{-33}m)

17.18 Certain excited state of hydrogen atom have a life time 2.5×10^{-19} s. What will be the minimum uncertainty in energy?

Ans: (2.65×10^{-15} J)

17.19 X-rays are scattered from a target material. The scattered radiation is viewed at an angle of 90° with respect to the incident beam. Find the compton shift in wave length.

Ans: (2.42×10^{-12}m)

17.20 Find the frequency of a photon when an electron of 20 KeV is brought to rest in a collision with a heavy nucleus.

Ans: (4.84×10^{18} Hz).

Chapter-18.

THE ATOMIC SPECTRA

Introduction

The subject of atomic spectra deals with the measurement of the wavelengths and intensities of the electromagnetic radiations emitted or absorbed by atoms. A typical arrangement for observing emission in atomic spectra is shown in Fig.18.1(a). The source S may be an electric arc or spark or an electric discharge passing through a monatomic gas or a heated salt of the material. The light emitted by the source after it passes through a system of lenses and collimating slits, falls on a prism (or diffraction grating). The prism disperses the radiation and different wavelengths are recorded at different points on the photographic plate. The impression on the photographic plate appears as lines (because of the rectangular slit in front of the source) corresponding to different wave lengths. It is because of this appearance that the spectrum obtained is called a line spectrum. Line spectra are typical of atoms and each kind of atom has its own characteristic spectrum. The line spectrum contains a series of lines in the visible region of the spectrum as shown in the fig. 18.1 (a).

We mentioned about the emission spectra of atoms. One may observe absorption spectra of atoms by using the arrangement shown in Fig.18.1(b). In this case, light containing all wavelengths is made to pass through a tube containing the gaseous state of the element under investigation. The light coming out is analysed as before. The spectrum is now consisted of dark lines against a bright back ground.

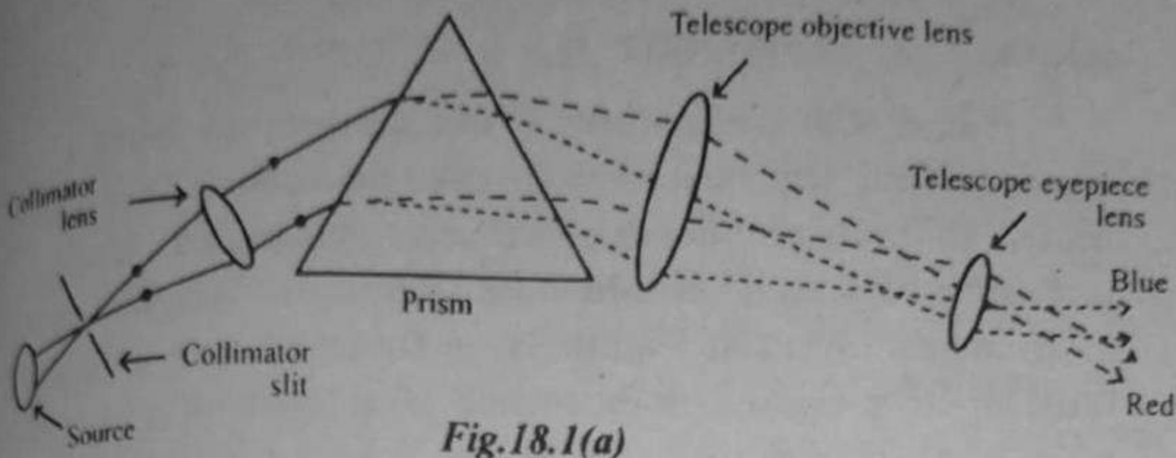


Fig.18.1(a)

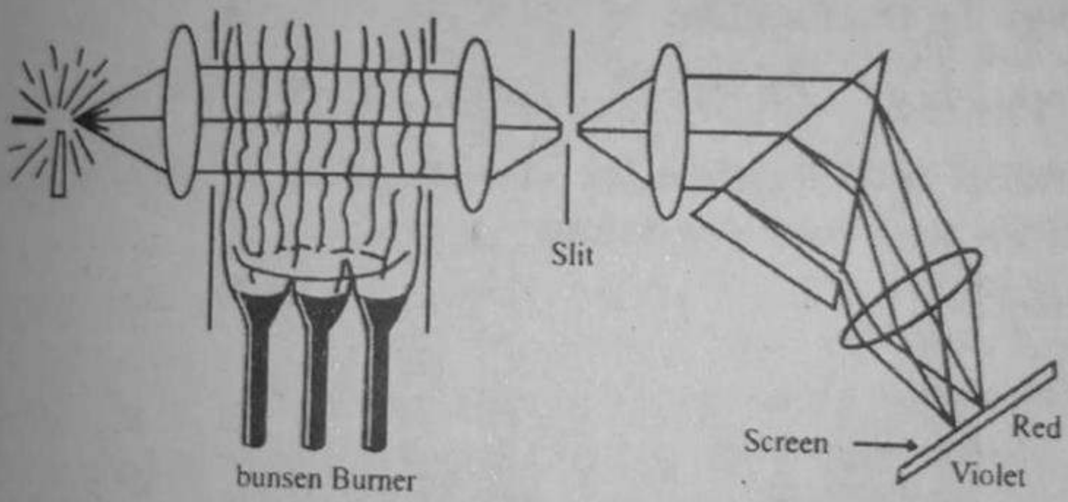


Fig.18.1(b)

18.1. The Spectrum of Hydrogen Atom.

Experimental spectroscopy has been studied since the middle of the nineteenth century, and it was known that atomic spectra of gases consisted of sharp lines of defining frequency. Some atoms had very complicated spectra, though that of hydrogen was simple. The visible spectrum of hydrogen was found to consist of a series of lines shown in Fig. 18.2. Spectroscopists Balmer and Rydberg had been able to fit the frequencies of hydrogen.

Balmer's formula for hydrogen is

Frequency $\quad 1 = CR\ (1/p^2 - 1/n^2)$ .... (18.1)

where $\quad p = 2$ and $n = p+1,\ p+2,\ p+3$ .....

and R is a constant. From the relation eq:18.1, it appears that the observed frequencies could be written as the differences between two term values, i.e. R/p^2 is one term and R/n^2 would be other term. The first term in Balmer's formula, Eq.(18.1), is that with $p = 2$. It was natural to suspect the existence of a term for $p = 1$. In 1906 such a term was in fact found by Lyman. He found a series of lines in the hydrogen spectrum in the far ultraviolet, known as the Lyman series, with frequencies given by the formula:

Frequency $\quad CR\ (1/1^2 - 1/n^2)$ .... $n = 2, 3, 4,$------ (18.2).

And in 1908 Paschen found a series in the infrared given by the formula by taking $\quad p = 3$.

Frequency $\quad 1 = CR\ (1/3^2 - 1/n^2)$. $n = 4, 5, 6.$ ......----- (18.3)

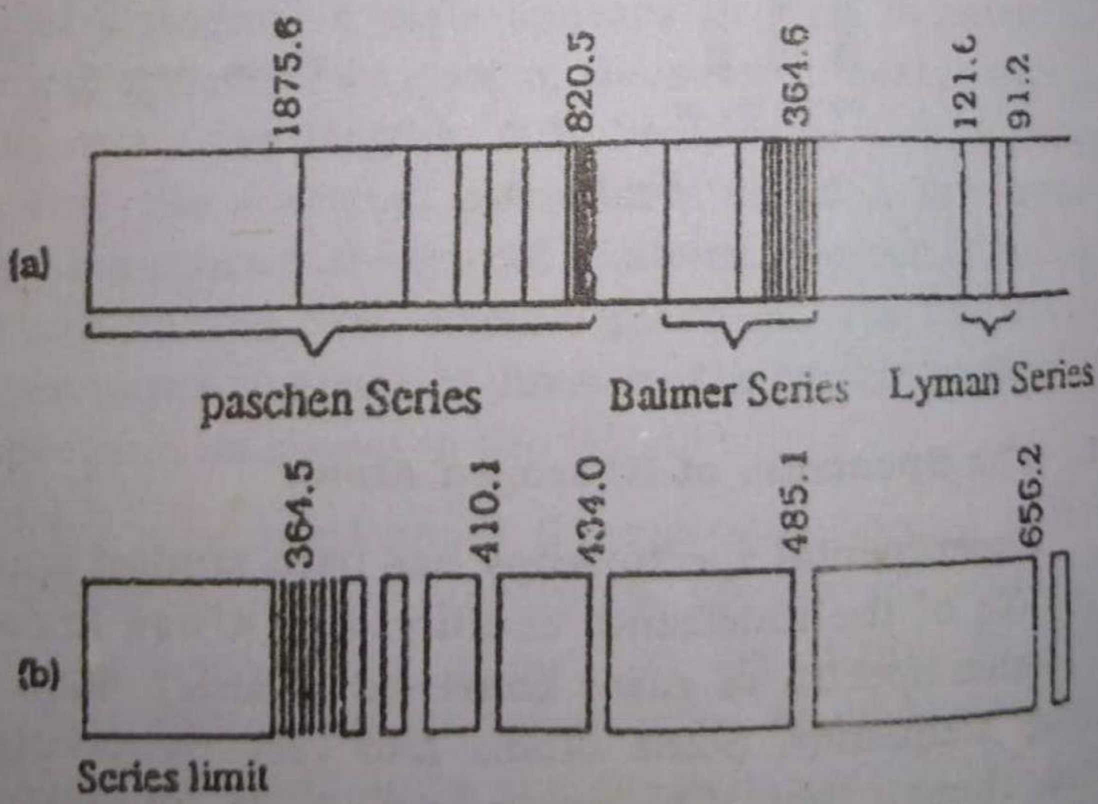


Fig. 18.2

Fig. (18.2) shows these series.

At the end of the first decade of the twentieth century, the significant information at the disposal of Bohr was that the atom radiates fixed frequencies in the form of sharp spectral lines, and have a permanent existence. But this information seemed to be contradicting the picture of the atom proposed by Rutherford (1911) to explain experiments on the scattering of α - particles. According to this model, the atom was assumed to consist of a massive positively charged nucleus surrounded by electrons in orbits of radius of the order of 10^{-10} m. According to classical mechanics, however, an electron moving in the coulomb field of the nucleus would radiate electromagnetic energy at the frequency of its orbital motion. Since no energy is provided to the atom from the outside source so we expect the electrons to lose energy and slowly spiral into the nucleus, emitting radiation of continuously increasing frequency. But real atoms are observed to be stable in their normal state and radiate only certain discrete frequencies when excited. Bohr in 1913 undertook to solve this paradox.

18.2. Bohr's Model for the Hydrogen Atom

In order to develop a quantitative theory for the spectrum of the hydrogen atom, Bohr put forward the following postulates.

i) An electron moves only in those circular orbits for which its orbital angular momentum L is an integral multiple of $h/2\pi$.

ii) The total energy of the electron remains constant as long as it remains in the same orbit.

iii) If the electron jumps from an initial orbit of energy E_i to final orbit of energy E_f ($E_i > E_f$), a photon of frequency is emitted.

$$\nu = \frac{E_i - E_f}{h} \quad \text{----------------- (18.4.)}$$

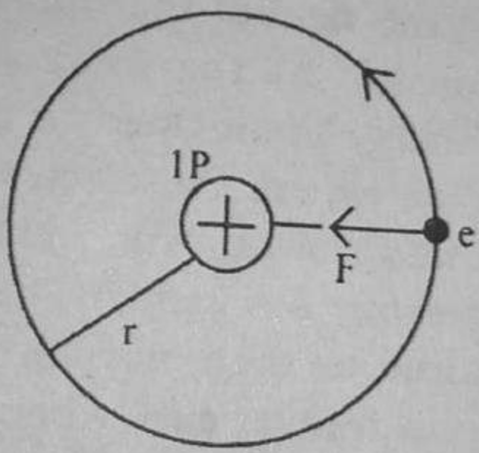


Fig.18.3.

According to Bohr's postulates the hydrogen atom consists of nucleus containing a proton and the electron revolving around the nucleus in definite circular orbits (Fig.18.3).

Since the proton is considered to be stationary being massive and the electron is attracted to it with a force given by Coulomb's law, the attractive force has a magnitude:

$$F = \frac{1}{4\pi\epsilon_o} \frac{e^2}{r^2}$$

where each of the two particles has a charge e and ϵ_o is the permittivity constant and $1/4\pi\epsilon_o$ is given to be $9.0 \times 10^{+9}$ N m^2 C^{-2}. The electron is revolving in a circular orbit of radius r with the velocity v, the Coulomb attractive force is balanced by the centripetal force $\frac{mv^2}{r}$

Therefore

$$\frac{1}{4\pi\epsilon_o} \frac{e^2}{r^2} = mv^2/r \quad \text{------------------} \ (18.5)$$

The electron revolving around the nucleus does not radiate energy as given by classical electromagnetic theory but instead it is assumed that the total energy of the atom remains constant.

The total energy of the atom is the sum of the kinetic energy and the potential energy of the electron. The kinetic energy T of the electron from Eq.(18.5.) is given

$$T = 1/2\ mv^2$$

$$= \frac{e^2}{(4\pi\epsilon_o)2r} \quad \text{(from 18.5)} \quad \text{------------------} \ (18.6)$$

The potential energy V of the proton-electron system also depends on r. We take the potential energy to be zero when the electron is infinitely distant from the nucleus. Thus, from electrostatic theory

$$V = -\frac{e^2}{4\pi\epsilon_o r} \quad \text{------------------} \ (18.7)$$

The potential energy is negative as the Coulomb force is attractive. The total energy of the system is, therefore.

$$E = T + V$$

$$E = -\frac{e^2}{8\pi\epsilon_o r} \quad \text{------------------} \ (18.8)$$

$$= -T$$

In order to decide what particular values of radii of circular orbits are permitted, it was assumed that the angular momentum of the electron must be an integral multiple of $h/2\pi$, where h is the same Planck's constant which relates the energy and frequency of a photon. If L is the orbital angular momentum, then according to this assumption,

$$L = n\ h/2\pi \qquad \text{------------(18.9)}$$

where n has values 1,2,3, ... ∝. Since the electron of mass m is moving in a circular orbit of radius r with velocity v, then

$$L = m\,v\,r$$

and from Eq.(18.9) $L = n\hbar$ where $\hbar = h/2\pi$

$$\therefore\ mvr = n\hbar, \quad n = 1,2,3,......$$

solving for v and putting in Eq. (18.5), we get

$$r = 4\pi\epsilon_o \frac{n^2\hbar^2}{me^2} \text{ where } n = 1,2,3,\text{----------(18.10)}$$

which gives the radii of the "non radiating" orbits. For the ground state n=1 and

$$r_1 = \frac{4\pi\epsilon_o \hbar^2}{me^2}$$

Substituting the values, we get

$$r_1 = 0.53 \times 10^{-10}\ m = 0.053\ nm$$

which is called the Bohr radius. From Eq. (18.10)

It is seen that the radii are proportional to the square of the integer number n, called the principal quantum number. Now if r in Eq. (18.8) is replaced by Eq. (18.10) we get,

$$E = E_n = -\frac{m e^4}{32\pi^2 \epsilon_0^2 \hbar^2}(1/n^2) \quad \text{--------(18 .11)}$$

The only allowed values of the energy are those given by Eq. (18.11) when n takes the values 1,2,3,...... This shows that the energy is quantized.

Substituting the values of various constants in Eq. (18.11), we get.

$$E_n = -\frac{13.6}{n^2}\ \text{eV where } n = 1,2,3 \text{----------- (18.12)}$$

The state of the lowest energy or ground state corresponds to n=1, and its energy is–13.6 eV. The energy of the electron corresponding to n = 2,3,4,...... is given by

E_2 = -13.6/4 = -3.40 eV

E_3 = -13.6/9 = -1.51 eV

E_4 = -13.6/16 = -0.85 eV

. = -----------------

. = -----------------

. = -----------------

These energy levels can now be represented graphically in Fig. (18.4). The quantum numbers are shown at the left and the corresponding energies of hydrogen in electron volts are given at the right. Note that all the states from n = 1 to n = ∞ are bound states, since they have negative energies. The separation of the levels decreases towards the top of the diagram and converges to zero for ∞. Above the line given by n = ∞, the energy states have positive energy, E > 0. The system is then unbound, meaning that the electron is free.

Now according to Bohr's postulate 3, if one electron jumps from an initial state n_i (energy E_i) to another state of lower energy n_f (energy E_f), the frequency of the emitted photon is from Bohr's formula:

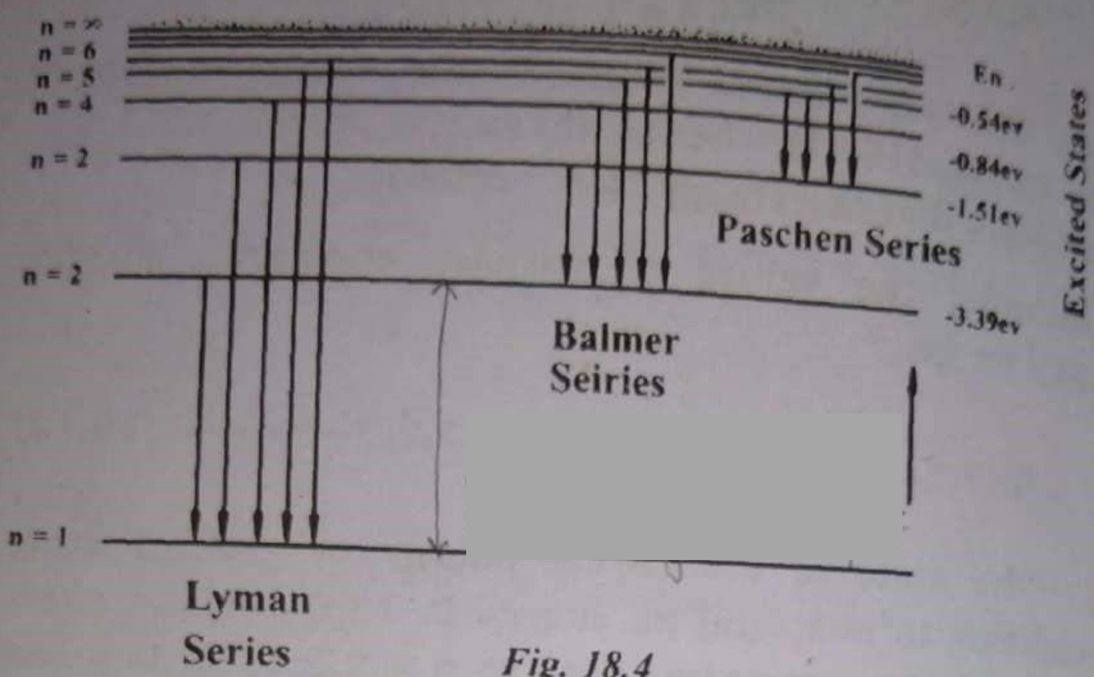


Fig. 18.4

$$\upsilon = \frac{E_i - E_f}{h}$$

$$\frac{E_i - E_f}{2\pi h} \quad (\text{since } \hbar = \frac{h}{2\pi})$$

Substituting for the energies from Eq:(18.8),

we have

$$\upsilon = c/\lambda = \frac{me^4}{64\pi^3 h^3 \epsilon_o^2} \left(\frac{1}{n_f^2} - \frac{1}{n_i^2} \right)$$

In terms of wave number $\bar{\upsilon}$ or the wavelength λ of the emitted photon is

$$\bar{\upsilon} = \frac{1}{\lambda} = \frac{me^4}{64\pi^3 h^3 \epsilon_o^2 c} \left(\frac{1}{n_f^2} - \frac{1}{n_i^2} \right) \quad \text{----(18.13)}$$

Comparing Eq. (18.13) with Eq. (18.1), both have the same form. Therefore from this comparison we find that

$$R_\infty = \frac{me^4}{64\pi^3 h^3 \epsilon_o^2 c}$$

is the theoretical value of the Rydberg constant.

Eq. (18.13) can now be written as

$$\bar{\upsilon} = 1/\lambda = R\infty\ (1/n^2_f - 1/n^2_i) \text{-----------------} (18.14)$$

Hydrogen Series

If we put $n_f = 2$ in Eq: (18.14), we get

$$\bar{\upsilon} = R_\infty\ (1/2^2 - 1/n'^2)$$

where $n' = 3; 4,5,$- - - - - - - - - -

This relation is identical with the series formula for the Balmer series of the hydrogen atom if $R_\infty = R_H$. The value of $R\infty$ is calculated to be $1.0970 \times 10^7 m^{-1}$ which is in quite good agreement with the experimental value of $R_H = 1.0968 \times 10^7 m^{-1}$. According to Bohr's model, the known series of the hydrogen spectrum arise from transitions in which the electron goes to a certain final quantum state n′. The relations for the different series are:

Lyman series $\bar{\upsilon} = R\infty\ (1/1^2 - 1/n'^2)$ where $n' = 2,3,4...$

Balmer series $\bar{\upsilon} = R\infty\ (1/2^2 - 1/n'^2)$ where $n' = 3,4,5...$

Paschen series $\bar{\upsilon} = R\infty\ (1/3^2 - 1/n'^2)$ where $n' = 4,5,6..$

Brackett series $\bar{\upsilon} = R\infty\ (1/4^2 - 1/n'^2)$ where $n' = 5,6,7,8.$

P'fund series $\bar{\upsilon} = R\infty\ (1/5^2 - 1/n'^2)$ where $n' = 6,7,8,...$

As shown in the energy level diagram (Fig.18.4). the region extends beyond $n = \infty$ which corresponds to a state in which the electron is completely removed from the atom and in such a case $E = 0$ and $n = \infty$.

Example 18.1.

An electron in the hydrogen atom makes a transition from the n = 2 energy state to the ground state (corresponding to n = 1). Find the wavelength and frequency of the emitted photon.

Solution:

We can make use of the equation:

$$1/\lambda = R_\infty (1/n_1^2 - 1/n_2^2)$$

$$1/\lambda = R_\infty (1/1^2 - 1/2^2) = 3/4\ R$$

$$\lambda = (4/3)\ 1/R_\infty = \frac{4}{3(\ 1.097 \times 10^7 m^{-1}\ )}$$

$$= \quad 1.215 \times 10^{-7} m$$

The wavelength lies in the ultraviolet region.

Since $c = \upsilon\lambda$ so the frequency of the photon is

$$\upsilon = c/\lambda = \frac{3 \times 10^8 ms^{-1}}{1.215 \times 10^{-7} m} = \ 2.47 \times \ 10^{15} H_z$$

Example 18.2.

Calculate the binding energy of the hydrogen atom (the energy binding the electron to the nucleus).

Solution:

The binding energy is numerically equal to the energy of the lowest state. The largest negative value of E in equation.

$$E = -\frac{me^4}{8\epsilon_o^2 h^2}\ 1/n^2\ (n = 1,2,3........)$$

is found for n=1. This gives

$$E = -\frac{(9.11 \times 10^{-31} kg)\ \ (1.60 \times 10^{-19} C)^4}{8(8.5 \times 10^{-12} C^2 - N.m^2)^2\ (6.63 \times 10^{-34} J.S)^2}$$

$= - 2.17 \times 10^{18}$ J

$= - 13.6$ eV. (Since 1 e V = 1.6×10^{-19} J)

Example 18.3.

Find the shortest wavelength photon emitted in the Balmer series and determine its energy.

Solution:

The shortest wavelength photon in the Balmer series is emitted when the electron makes a transition from $n = \infty$ to n = 2. Therefore

$$1/\lambda_{min} = R\,(1/2^2 - \infty^2) = R/4$$

$$\lambda_{min}\ 4/R = \frac{4}{1.097 \times 10^{-7} m^{-1}} = 364.6 nm.$$

This wavelength is in the ultraviolet region and corresponds to the series limit. The energy of a photon with this wavelength is:

$$E_{photon} = hc/\lambda_{min} = 3.41 eV$$

This is the maximum-energy of photon in this series, since it involves the largest energy change.

18.3. Excitation and Ionization Potential.

The Bohr model predicts that the total energy of an atomic electron is quantized. For example Eq: (18.11) gives the allowed energy values for the electron in a one electron atom. State for n = 1 is said to be in the ground state whereas states with n = 2,3,4,.... are called the excited states. If an atoms is supplied energy so that it reaches one of its allowed values, then the atom is said to have gone to one of its excited states. The excitation potential is defined to be the accelerating potential which moves the electron of the atom from the ground state to the higher state Since the atom has discrete states so definite amounts of energy are required to take the electron to the various excited states of the atom.

There are many ways by which the electron of an atom can be excited to different states. If the atomic gas is heated the electron may be excited by thermal motion collision. If the gas is in an electric discharge, a free electron which has been accelerated by the electric field may hit the atom of the gas and excite it to a higher state. The atoms of the gas when illuminated may absorb energy from a photon and are excited.

If an electron lying in the ground state of the atom is given sufficient energy so that it is raised to the orbit for which $n = \infty$, it will disengage itself from the atom.The atom will then become positively charged. It is then said to be ionized. The ionization potential is therefore, defined to be the accelerating potential which removes an electron completely from an atom.

For hydrogen atom the energy needed to ionize it is 13.6 electron volts and the corresponding ionization potential is 13.6 volts.

Example: 18.4.

(a) What is the longest wavelength of light capable of ionizing a hydrogen atom ?

(b) What energy is needed to ionize a hydrogen atom?

Solution:

(a) The wavelength of light capable of ionizing the hydrogen atom will make the electron to raise it from $n = 1$ to $n = \infty$ So using the relation

$$1/\lambda = R\,(1/n^2_1 - 1/n^2_2)$$

we have

$$1/\lambda = 1.097 \times 10^7\,(1/1 - 1/\infty)\ m^{-1}$$

$$\lambda = 9.12 \times 10^{-8}\ m$$

(b) The energy required is the energy of the photon in (a). It is

$$Energy = h\upsilon = hc/\lambda = 6.63 \times 10^{-34} Js \left(\frac{3 \times 10^{8} ms^{-1}}{9.12 \times 10^{-8}\ m}\right)$$

$$= 2.18 \times 10^{-18}\ J = 13.6\ eV.\ (Since\ 1\ eV = 1.6 \times 10^{-19} J)$$

18.4 X-Ray spectra

By X-rays, we usually mean electromagnetic radiation (light) which has a wavelength shorter than that of ultraviolet light though, there is no sharp boundary. The range is usually considered to be 0.1 to 1 nm, which corresponds to quantum energies of 1——— 100 k eV

X-rays are produced if heavier atoms are bombarded by electrons which have been accelerated through thousands of volts. They were first observed by Wilhelm K.Roentgen in 1895, (and are also called Roentgen rays) using an apparatus similar in principle to that shown in Fig.(18.5.). Electrons released from the heated cathode by thermionic emission are accelerated towards the anode by a large potential difference. It is found that at sufficiently high potentials (several thousand volts) a very penetrating radiation is emitted from the surface of the anode. These rays are of the same nature as light or any other electromagnetic wave. The X-rays are detected by photographic plates, film, counting tubes, or more recently, by semiconductor detectors. In 1913, W.H.Bragg discovered the phenomenon of X-ray diffraction by crystals. This technique permitted the precise measurement of the wavelength of X-rays and thus became the basis for a study of X-rays spectrum.

Spectral analysis of x-rays shows that:

i) there is always a continuous. the X-ray bremsstrahlung

ii) and under certain conditions. there is in addition a line spectrum. the characteristic spectrum.

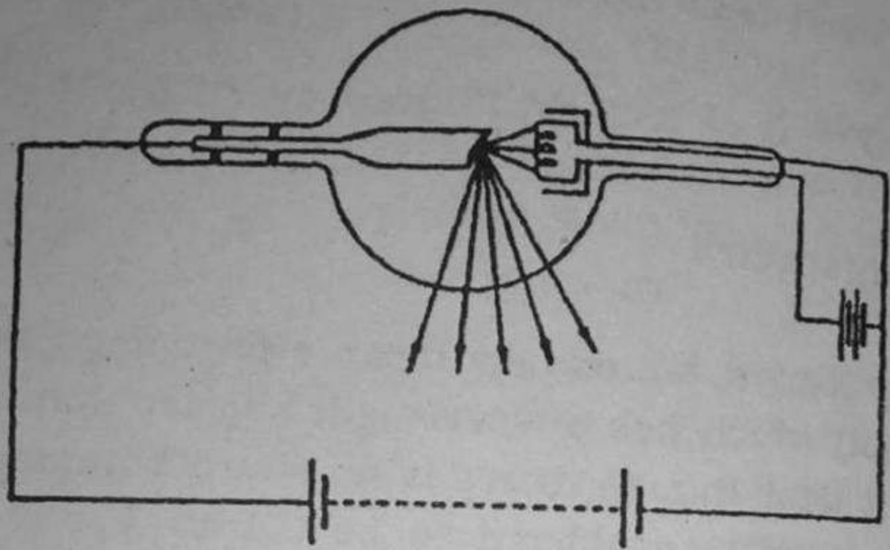


Fig.18.5.

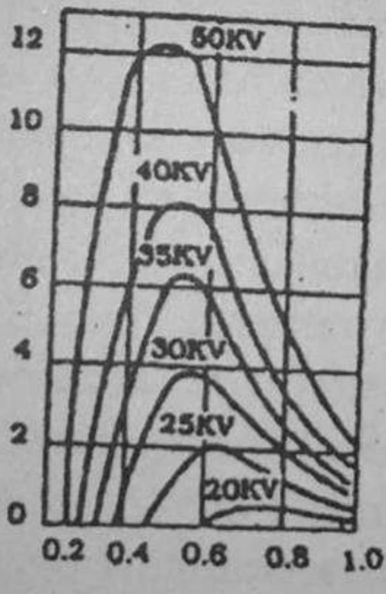


Fig.18.6.

18.5. X-ray Continuous Spectra or X-ray Bremsstrahlung

Ordinarily, a continuous spectrum of frequencies of X-rays is emitted, but the maximum frequency or minimum wavelength was observed to be always directly proportional to the accelerating voltage between the electrodes in the manner indicated in Fig. (18.6.).

Furthermore, this maximum frequency was found to be very nearly independent of the material of which the electrodes were made. These observations can be understood on the basis of quantum hypothesis. The bremsstrahlung spectrum is a result of the fact that when electrons pass close to the atomic nuclei, they are deflected and slowed down (Fig.18.7).

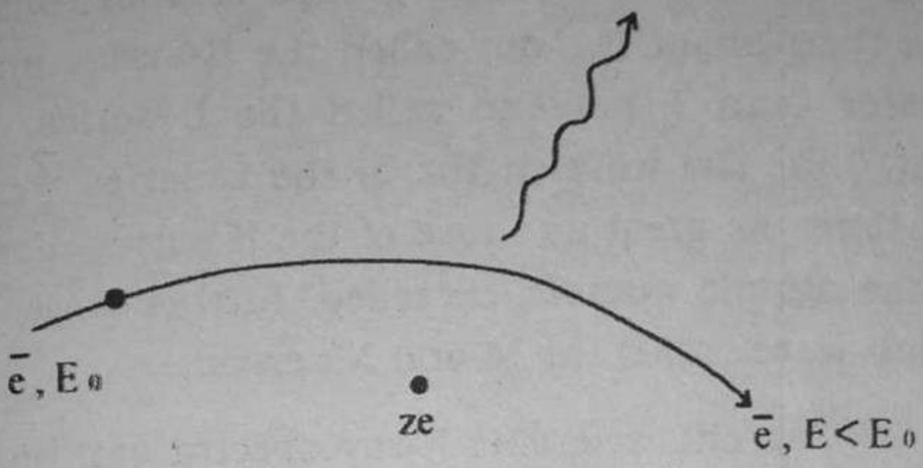


Fig.18.7

A positive or negative accelerated charge will according to classical electrodynamics, emit electromagnetic radiation.This is continuous x-rays bremsstrahlung. In terms of quantum theory, this can be understood as follows: for each braking incident, a quantum of light $h\upsilon = E_0 - E$ is emitted. However, since the beginning and end states are not quantized-the electrons are free, not quantized, The process is represented as

$$\text{Atom} + \bar{e}\,(\text{fast}) \rightarrow \text{Atom} + \bar{e}\,(\text{slow}) + h\upsilon$$

Characteristic X-Ray Spectra:

We have mentioned that the spectrum of X-rays consists of a continuous spectrum upon which is superposed a line spectrum (Fig.18.8). The distribution of energy in the continuous spectrum depends only upon the potential difference across the X-ray tube while the line spectrum is the characteristic of the target. These are called characteristic spectra and were investigated by Moseley in 1913 by making each element in turn the target in an X-ray tube. Thirty nine elements extending from aluminium to gold were examined in this way. The X-rays were analyzed with a Bragg crystal spectrometer. Most elements showed two groups of lines, one generally less than about 0.1 nm called the K-series and another greater than 1 nm and called the L series, similar to Fig.(18.8). The wavelengths of the L series were roughly ten times as great as those of the K series. For elements whose atomic number exceeded, furthçr series appeared which were called the M and N series.

The characteristic X-ray spectra can be explained from the principle of inner shell transitions. The electrons of an atom are ordered according to their arrangement in shells about the nucleus. Each shell has a certain maximum number of electrons. There is a rule that the number of electrons in the nth orbit is equal to $2n^2$. According to this rule, there cannot be more than 2,8,18,32,..... electrons in the orbits or shells for n=1,2,3,4,..... These orbits or shells are called K,L,M,N,.... for n =1,2,3,4,.....respectively.

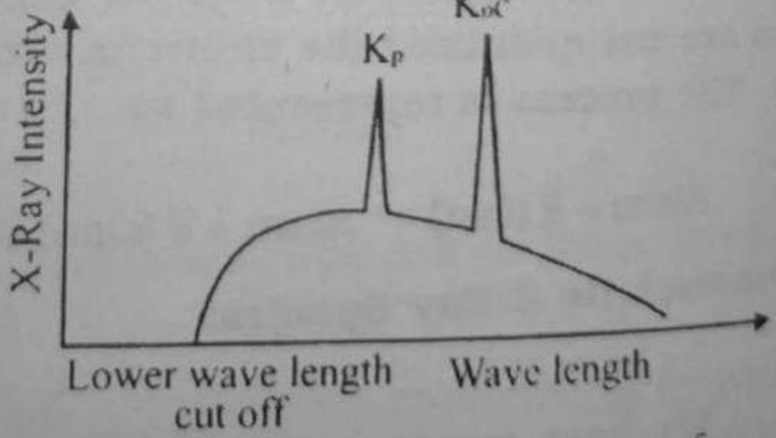


Fig.18.8

N	1	2	3	4	5
Spectroscopy Notation	K	L	M	N	O

When highly energetic incident electrons knock an electron from the K shell, there occurs a vacancy in that shell. This vacant space in the K shell is filled by an electron from the L shell giving up energy in the form of an X-ray. This radiation, which is characteristic of the target material, is labeled the K_α line. The electron in the M shell that fills a vacancy in the K shell gives up energy as an X-ray called the $K\beta$ line. These transitions from the shells L,M,N and so on to the K shell give rise to a series of lines $K\alpha$, $K\beta$, $K\gamma$, and so on called the K series. When incident electrons dislodge electrons from the L shell, these are filled by electrons from the remaining M,N,O, shells etc. These transitions give rise the L series, the first line of which is L_α The nomenclature of these transitions is illustrated in Fig.18.9.and 18.10. Upon closer observation each of the characteristic X-ray line is found to be composed of a number of closely spaced lines called the X-rays fine structure, the explanation of which is not taken up here, since this topic is beyond the level of this book.

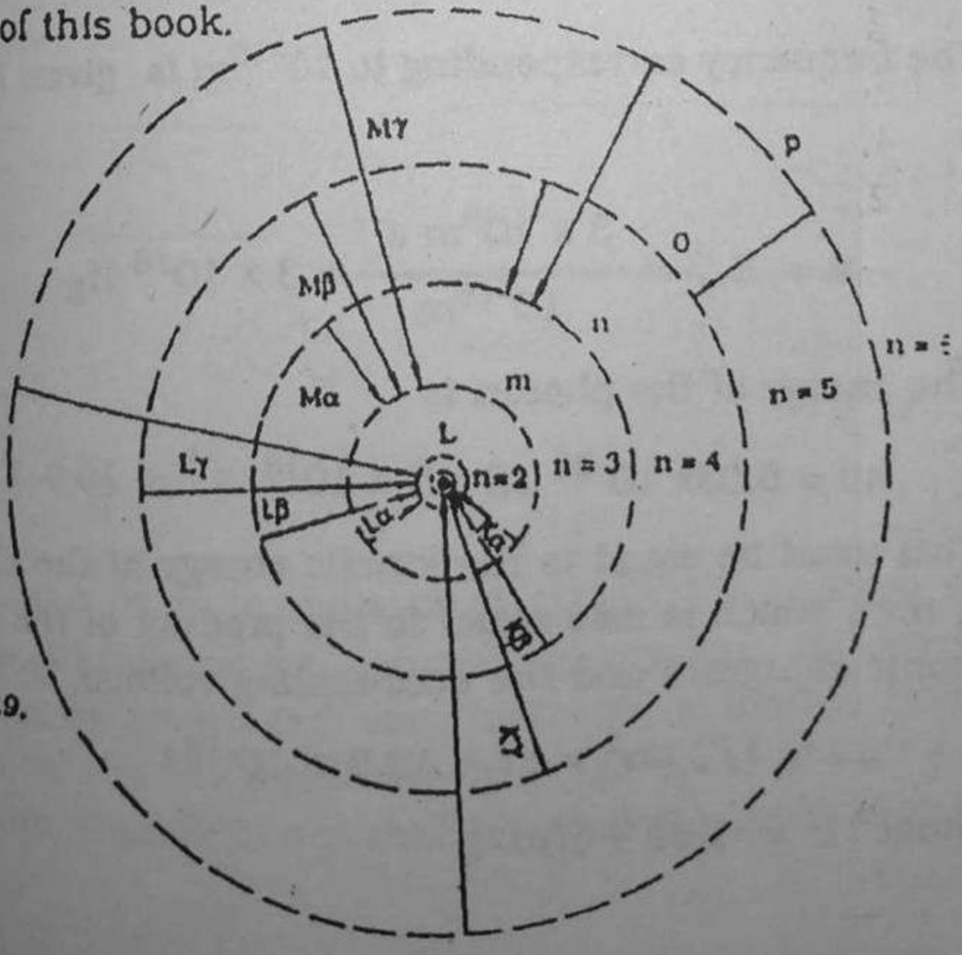


Fig.18.9.

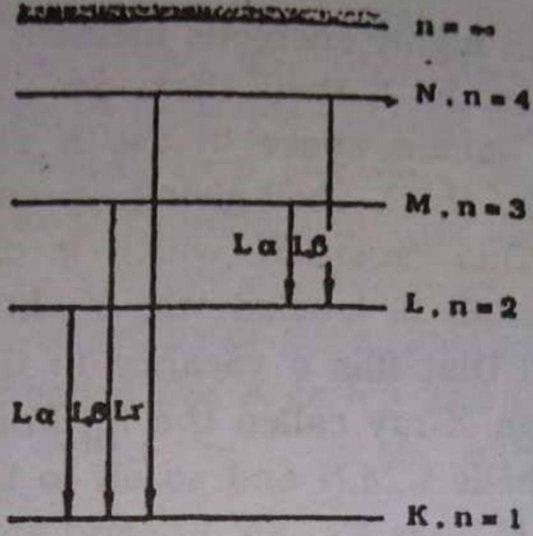


Fig.18.10.

Example: 18.5

Calculate the potential difference through which an electron must be accelerated in order that the short wave limit of the continuous X-ray - spectrum shall be exactly 1×10^{-10}m.

Solution:

The frequency corresponding to 10^{-10}m is given by

$$\upsilon = E/\lambda = \frac{3 \times 10^{8} \text{m s}^{-1}}{10^{-10}\text{m}} = 3 \times 10^{18} \text{H}_z$$

The energy of the photon is

$$h\upsilon = 6.63 \times 10^{-34} \text{ JS} \times 3 \times 10^{18} \text{ s}^{-1} = 19.9 \times 10^{-16} \text{J}$$

This must be equal to the kinetic energy of the electron $\frac{1}{2}mv^2$, which is also equal to the product of the electronic charges e and the accelerating voltage, V:

$$\text{i.e. } 1/2\, mv^2 = eV = 19.9 \times 10^{-16}\text{J}$$

Since $e = 1.60 \times 10^{-19}$C

so $V = 19.90\times 10^{-16} J / 1.60\times 10^{-19} C = 12.400$ V.

18.6. Introduction to Lasers

Lasers are one of the most important discoveries made in the second half of the twentieth century. The laser is a device for producing very intense, highly directional, coherent and monochromatic light beams. The name laser stands for Light Amplification by Stimulated, Emission of Radiation. Different types of lasers: solid state, gas, semiconductor, liquid-are used for producing light at frequencies from the far infrared to the ultraviolet regions. In order to understand the basic principle of lasers, we must first discuss the three radiation processes.

Suppose that a beam of photons of energy $\upsilon h = E_2 - E_1$ is incident on a sample in which atoms are in the ground state. If the photons interact with an atom in the ground state, the atom absorbs the photon and reaches the excited state E_2 (Fig.18.11) as the atom was induced

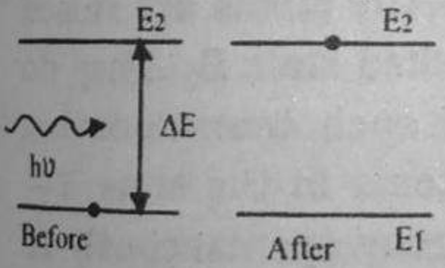


Fig. 18.11

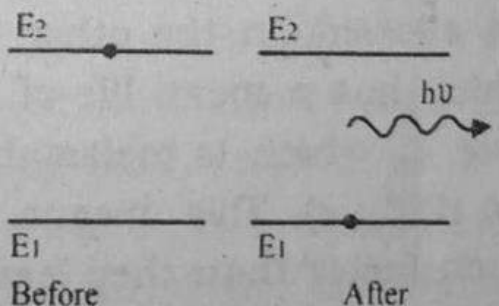


Fig. 18.12

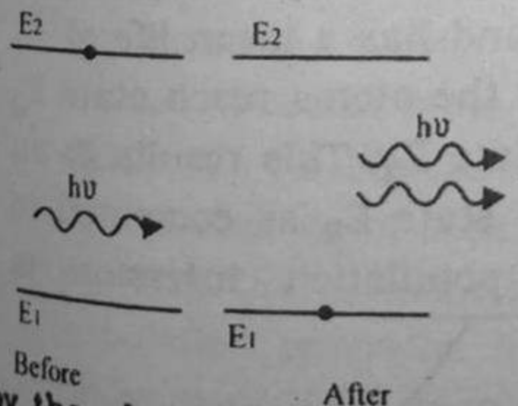


Fig. 18.13

by the photon to go to the excited state. This process is known as stimulated or induced absorption. An atom can remain in an excited state only for a limited time called life time of the state-usually of the order of 10^{-8} S. There exists, therefore, a probability that the atom in the excited state E_2 will fall back to the lower state E_1. Since this transition is spontaneous (not of external origin), then the radiation emitted is called spontaneous emission (Fig.18.12). The emitted radiation is of random character and is incoherent. The third process is called stimulated emission. In this process, an atom in an excited state E_2 (Fig.18.13) absorbs a photon of energy $h\upsilon = E_2 - E_1$. This incident photon will increase the probability that the atom will go back to its ground state and thereby emits a second photon of energy $h\upsilon$. The two identical photons emitted in this process-the incident photon and the emitted photon are shown in the Fig. The emitted photon will be exactly in phase with the incident photon.

18.7. The Laser Principle

Principle of a laser is explained by considering that atoms of a material have a number of energy levels and atleast one of which is metastable, the state having much longer life time than 10^{-8} s. We consider a three level system as shown in Fig.18.14.The atoms are raised from the ground state E_1 to the excited state E_3.They do not fall back to state E_1 because such transitions are not allowed.On the other hand, atoms in the state E_3 (which has a mean life of 10^{-8}s) decay spontaneously to state E_2 which is metastable (and has a mean life of 3×10^{-3} s). This means that the atoms reach state E_2 much faster than they leave state E_2. This results in an increased number of atoms in state E_2 as compared to the number in E_1. Thus population inversion is

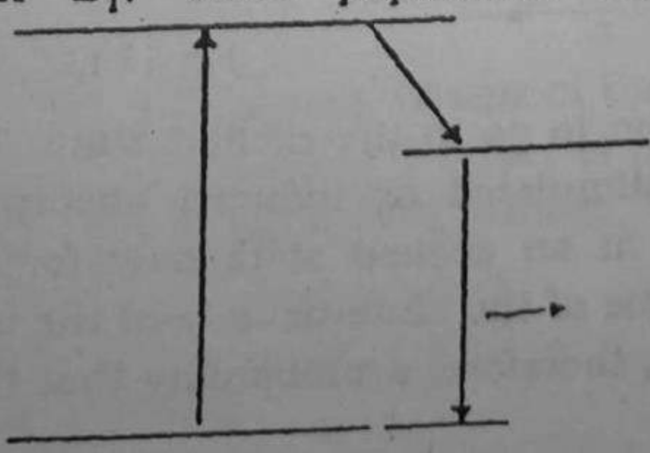


Fig.18.14

achieved. After population inversion is obtained, the state E_2 is exposed to beam of photons of energy $h\upsilon = E_2 - E_1$, which causes induced emission. In order to sustain this process, some method is employed to maintain the population inversion in the states. This is achieved by confining the emitted radiations in an assembly. The ends of which are fitted with mirrors. One end mirror is totally reflecting while the other is made partially reflecting for the laser beam to be taken out.

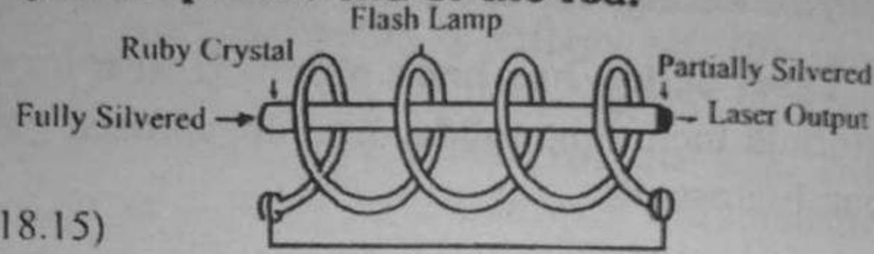
Laser action has been obtained in a large variety of materials including solids, liquids, ionized gases, dyes, semiconductors etc.

We give a brief account of a typical laser called ruby laser.

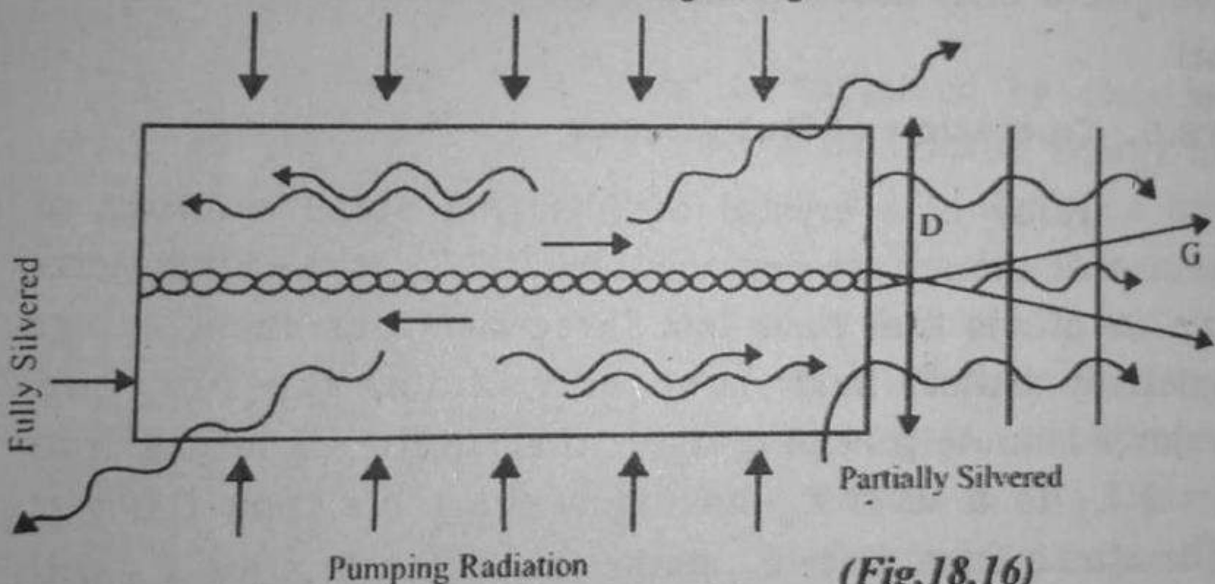
18.8. Operation of Ruby Laser

Ruby is a crystal of Al_2O_3, a small number of whose Al atoms are replaced by Cr^{+++} ions. Such ions are Cr atoms that have lost three electrons each. A high intensity helical flash lamp surrounding the ruby provides adequate pumping light to raise the Cr atoms from level E_1 to a level E_3 having a short life time (10^{-8} s). The atoms from state E_3 make transition to state E_2 with spontaneous emission making the number of atoms larger than those in state E_3, since E_2 is a metastable state which has its life time of the order 10^{-3}. In this process the number n_1 of atoms from state E_1 are going faster to E_3 than the number n_2 of atoms leaving the state E_2. Population inversion has been created. A few Cr atoms make transitions spontaneously from level E_2 to level E_1 and these emitted photons of $\lambda = 694.3$ nm stimulate further transitions. Stimulated emission will dominate stimulated absorption (because $n_2 > n_1$) and we obtain an intense coherent monochromatic red beam of light.

In practice, the ruby laser is a cylindrical rod with parallel, optically flat reflecting ends, one of which is only partly reflecting as shown in Figure.18.15'. These emitted photons which travel exactly in the direction of the axis are reflected several times, and they are capable of estimulating emission repeatedly. Those photons not in the direction of axis leave through the sides. Thus the number of photons is built up rapidly and leave through the partially transparent end of the rod.



(Fig.18.15)



(Fig.18.16)

18.9. Applications of Laser

There are a number of applications of Laser technology. Some applications are as follows:-

1. Three dimensional images of objects obtained by using lasers in a process called Holography.
2. As surgical tool for "welding" detached retina.
3. To perform precision surveying and length measurements.
4. As a potential energy source for inducing nuclear fusion reactions.

5. For telephone communications along optical fibers.

6. For precision cutting of metals and other materials. These and other applications are possible only because of the unique characteristics of Laser light.

QUESTIONS.

18.1. The Bohr theory of hydrogen atom is based upon several assumptions. Do any of these assumptions contradict classical physics?

18.2. Why does the hydrogen gas produced in the Laboratory not glow and emit radiations?

18.3. Why are the energy levels of the hydrogen atom less than zero.

18.4. If hydrogen gas is bombarded by electrons of energy 13.6 eV would you expect to observe all the lines of hydrogen spectrum.

18.5. Hydrogen gas at room temperature absorbs light of wavelengths equal to the lines in the Lyman Series but not those of the Balmer Series. Explain.

18.6. How X-rays are different from the visible radiations.

18.7. What property of X-rays makes them so useful in seeing otherwise invisible internal structures?

18.8. Explain the difference between laser light and light from an incandescent bulb.

18.9. Name some applications of Lasers.

PROBLEMS

18.1. Calculate the following (a) the orbit radius (b) the angular momentum (c) the linear momentum (d) the kinetic energy (e) the potential energy (f) the total energy for the Bohrs hydrogen atom in ground state.

Ans.[(a) 5.3×10^{-11}m (b) 1.1×10^{-34}Js
(c) 2.1×10^{-24}kgm/s (d) 13.6eV (e) −27.2eV
(f) 13.58 eV]

18.2. What is the wavelength of the radiation that is emitted when a hydrogen atom undergoes a transition from the state n=3 to n=1

Ans.(103 nm.)

18.3. Light of wavelength 486.3 nm is emitted by a hydrogen atom in Balmer series. what transitions of the hydrogen atom is responsible for this radiation.

Ans.(n = 4.)

18.4. In the hydrogen atoms an electron experiences a transition from a state whose binding energy is 0.54 eV to another state whose excitation energy is 10.2 eV (a) What are the quantum numbers for these states? (b) Compute the wavelength of the emitted photon. (c) To what series does this line belong.

Ans (a) 5,2 (b) 434 nm (c) Balmer Series.

18.5. Photon of 12.1 eV absorbed by a hydrogen atom, originally in the ground state, raises the atom to an excited state. What is the quantum number of this state:

Ans: (3.)

18.6. (a) Find the wavelength of the first three lines of the Lyman series of hydrogen.

Ans: [121.5 nm, 102.5 nm, 97.2nm]

18.7 In an experiment, the excitation potentials of hydrogen are found at 10.21 V and 12.10 V. three different spectral lines are emitted. Find their wavelengths.

Ans: [102.55 nm, 121.54 nm, 655.92 nm.]

18.8. What minimum energy is needed in an X-ray tube in order to produce X-rays with a wavelength of 0.1×10^{-10} m.

Ans: [1.99×10^{-16} J]

18.9. A certain atom emits spectrum lines at 300, 400 and 1200 nm. Assuming that three energy levels are involved in the corresponding transitions, calculate the quantum of energy emitted at each wave length:

Ans: [4.14 eV, 3.1eV, 1.03 eV]

18.10. Calculate the energy of a photon whose frequency is

(a) (i) 4×10^{14} Hz (ii) 20 GHz

(iii) 30 MHz. Express your answer in eV.

Ans: [(i) 1.65 eV (ii) 8.28×10^{-5} eV]

(iii) 1.24×10^{-7} eV.]

(b) Describe the corresponding wavelengths for the photons described in (a)

Ans:[(i) 750 nm, (ii) 0.015m (iii) 10m.]

Chapter -19

THE ATOMIC NUCLEUS

19.1. Introduction

The beginning of twentieth century not only witnessed the successful development of atomic hypothesis,the old quantum theory of radiation by planck, and Einsteins special theory of relativity;these years also marked the beginning of nuclear physics as a new field of scientific activity.

In 1896 Henri Bacqueral first observed the phenomena of radioactivity. A year after J.J Thomson discovered the electron. Later on Thomson demonstrated that cathode rays were constituted of electrons.In this way the notion of the atom as the ultimate indivisible unit of matter had to be discarded, and it became evident that atoms included electrons among their structure, and these electrons could be liberated by electromagnetic excitation as in a gas discharge, or spontaneously as in radio activity.

In 1902 Rutherford and Soddy, showed that in radioactive decay an atom transform itself into a different chemical element. This discovery led to the development of models for nuclear structure,and after a period of three decades James Chadwick made another breakthrough by discovering the neutron which laid the foundation of a nuclear model,namely a very small (of the order of 10^{-14}m),roughly spherical in shape,and highly dense object,comprising of protons and neutrons.

19.2. Nuclear Structure

If the nucleus has any sort of structure,then we must argue what are its constituents? To answer this question, let us make a study of the known chemical elements in the order of their atomic numbers. The hydrogen atom which is the lightest atom has one electron and its atomic number Z=1, helium has two electrons and its Z=2, silver has forty seven electrons and its Z=47,and uranium has ninety two electrons and its atomic number Z=92 etc. The increase of atomic mass with atomic number suggests that all atoms are simply combinations of hydrogen. Thus the helium atom with Z=2 should have 2 protons in its nucleus, silver atom should have 47 protons,etc in order to make the atom neutral in charge.However, atomic masses are not found to increase in steps of the mass of hydrogen atom. Helium atom for example,has a mass four times that of hydrogen, lithium has a mass of about seven times etc. The proton is the nucleus of hydrogen atom. Its mass is 1836 times of the mass of the electron, and it carries a positive charge $e=1.60 \times 10^{-19}$C.

One possible argument to overcome. the above discrepancy in the behavior of atomic masses is that there may be enough protons in the nucleus to account for its atomic mass and several electrons may also be present in the nucleus inorder to neutralize the positive charge of the protons that are in excess of the required number. Thus the helium nucleus whose mass is four times that of proton, through its charge is only +2e may be considered to have four protons and two electrons. This explanation gets support from the observed phenomena of beta activity in which nuclei spontaneously emit electrons and whose occurrence can not be easily explained if the electrons are not assumed to be present in the nuclei.

However there are many arguments against this concept of the existence of electrons in the nuclei. One of the objection against the presence of electron in nucleus is that, inorder to confine an electron inside a nucleus, the electron must have an energy of about 10^3 Mev, whereas the observed energies of the electrons in beta activity is of the order of 2 to 3 Mev only. As such the presence of electron inside the nucleus is ruled out.

The problem was resolved by the Chadwicks discovery of neutron. The neutron has a mass slightly greater than that of the proton and it carries no charge. The term nucleon is used as a generic name for either proton or neutron. The masses of the proton and neutron are determined to be:

$$m_p = 1.6726 \times 10^{-27} \text{ kg}$$

$$m_n = 1.6750 \times 10^{-27} \text{ kg} \quad \text{------------(19.1)}$$

The atomic number Z of an atom is the number of atomic electrons in the neutral atom; it is also the number of protons in the nucleous. The atmic mass number A equals the number of nucleons in the nucleous often called as the nucleons number. Since Z is the atomic number (or proton number), the neutron number N is therefore defined as N = A - Z.

A nucleous is completely specified by any two of those three numbers. To identify a nucleous the conventional symbol is $_ZX^A$ where X is the chemical abbreviation for the particular element.

19.3. Isotopes

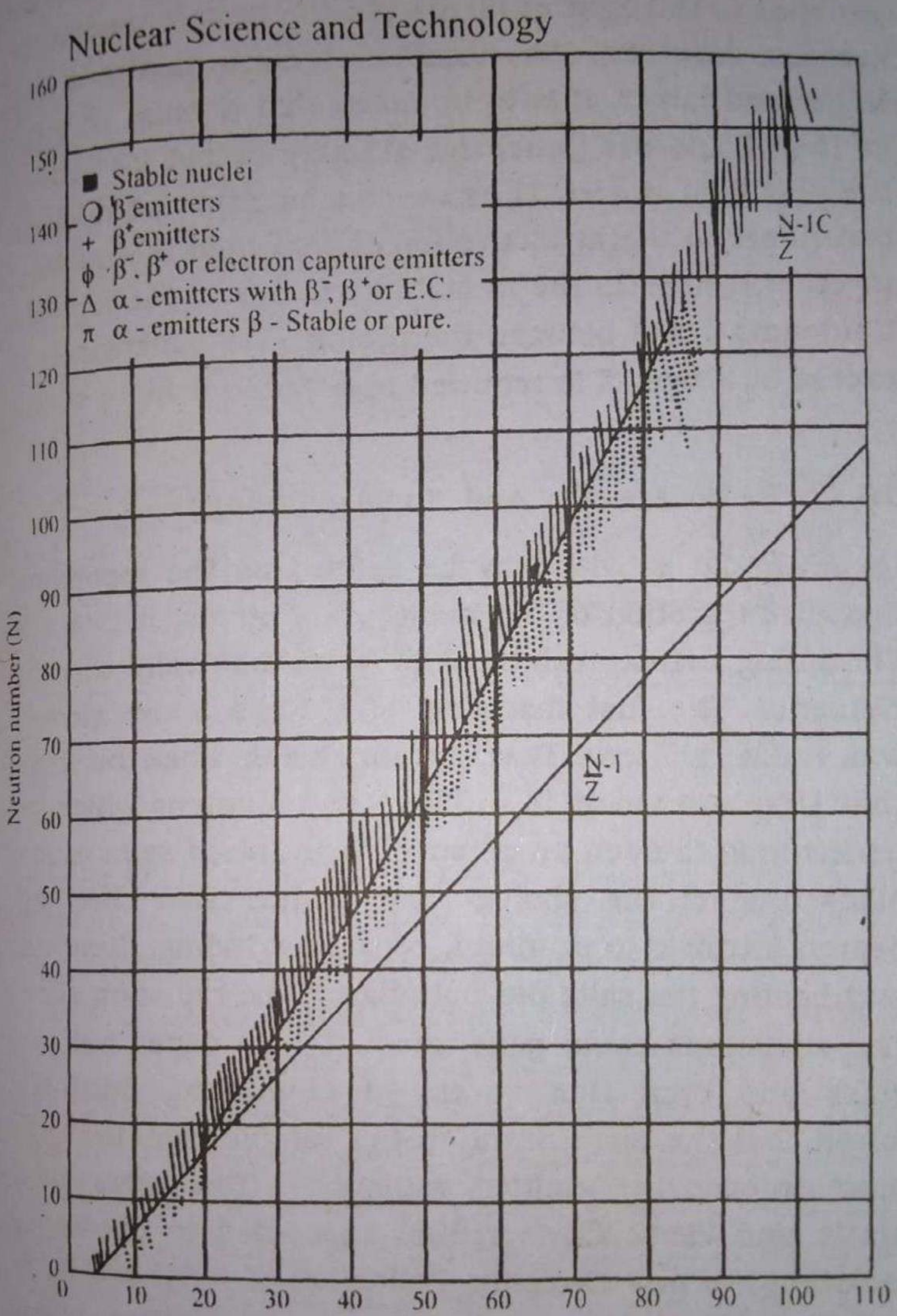
The atomic or proton number Z determines the chemical properties of the element. Many chemical elements have nuclei with more than one value of the mass number A. For example, we have hydrogen, deuterium and tritium for Z = 1, where :

A = 1, Z = 1 and N = 0 is Hydrogen

A = 2, Z = 1 and N = 1 is Deuterium

A = 3, Z = 1 and N = 2 is tritium ------------(19.2)

All such nuclei having same Z but different values of N are called ISOTOPES. Deuterium is sometimes expressed by the symbol $_{1}^{2}D$ or $_{1}D^{2}$.



Out of about 1400 nuclei which are known to exist, only about one-fifth are found to be stable. Let us now, analyze them on a chart according to their number of protons and neutrons. As shown in the figure (19.1), if a nucleous has an equal number of protons and neutrons, its position on the graph will be on the 45° line. This is approximately the situation for the commonest isotopes of the light elements. As the atomic number increases, however, the number of extra neutrons gets larger and larger. It is to be noted that N must be greater than Z inorder to achieve stability except for some of the very light nuclei. Thus we conclude that all nucleous contribute to the attractive forces that hold the nucleous together, where as the in stability is due to the repulsive Coulombs forces between the protons. As Z increases, an excess of N over Z is required in order to achieve stability.

19.4. Radio Activity and Nuclear Changes

Radio activity may be defined as the sopontaneous disintegration of the nucleous of atoms. It is a self-disrupting activity exhibited by some naturally occuring elements. The first discovery of a radio active element was made by Henri Bacqueral in 1896, when he found that Uranium atoms (Z = 92) emits radiations which are penetrating to such an extent that uranium salts causes blackening of the photographic plates. This effect appeared intrinsic to uranium, because grinding, dissolving and heating the salts did not change the radiation effect. The radiations could penetrate not only paper but also glass and even thin sheets of aluminum. Bacqueral found that the more uranium the sample contained, the more intense the emitted radiations. Two years later, Marie and Pierre Curie (1898) succeeded in chemically isolating two new elements, Pollonium (Z = 84) and Radium (Z = 88) which were found to be radioactive. It has been demonstrated that all the elements with Z greater then 83 are radioactive. Rutherford and his co-workers were able to prove with the help of experiments that the radiations emitted by radioactive substances are of three different types.

This can be demonstrated with the help of a simple experiment illustrated in Fig. 19.2 - (a) & (b).

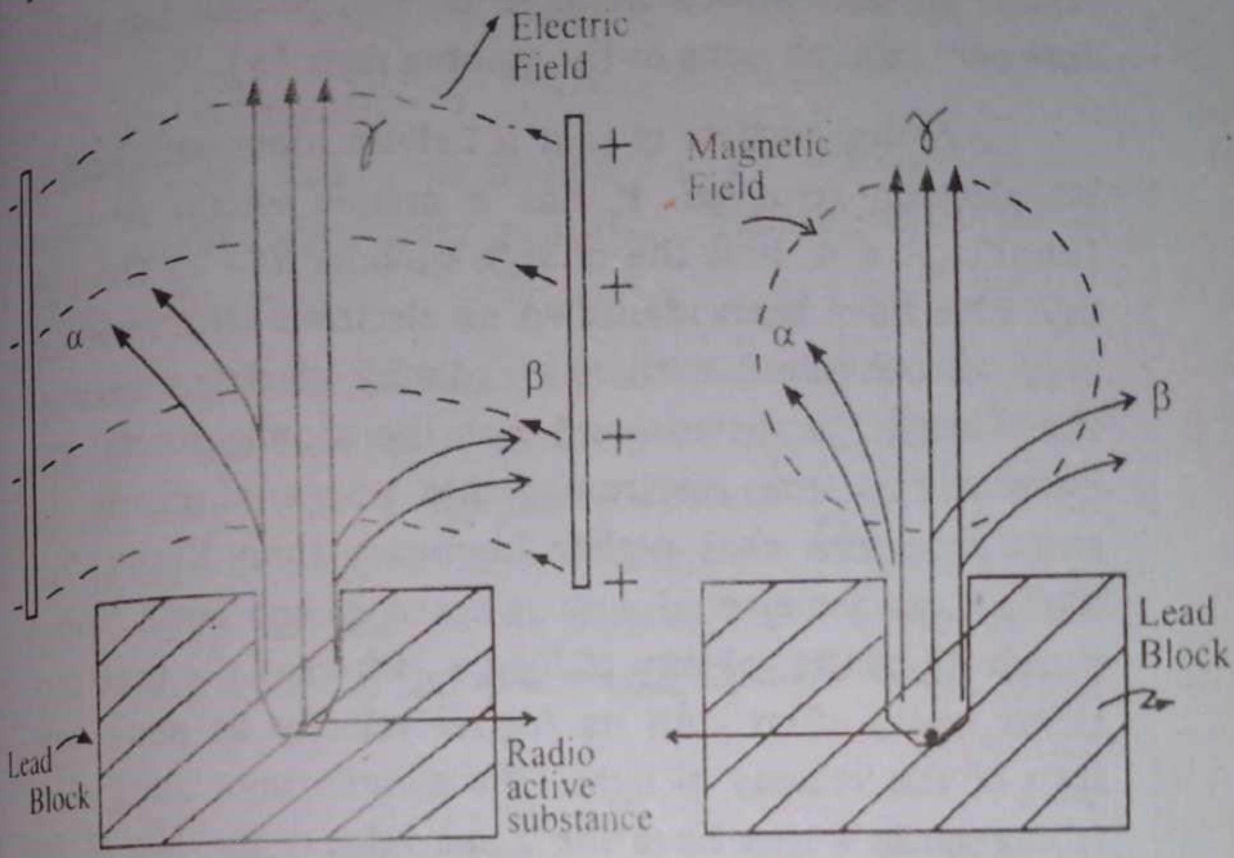


Fig.19.2 (a) Rutherford Experiment *Fig.19.2 (b)*

A small amount of Radio active substance is placed at the bottom of a Cavity drilled in a block of Lead. When the narrow beam of radio active rays is allowed to pass through the space between two charged plates as shown in fig.19.2 (a) the path of some of the rays bends towards the positive plate and some rays sends towards the negative plate while others go undeflected by the influence of electric field between the plates. Similar effect is observed in the presence of a transverse magnetic field as shown in fig.19.2 (b). The rays bending towards the negative plate indicate that they consist of positively charged particles, while those bending towards the positive plate indicate negatively charged particles.

Further experiments confirmed that the positively charged particles are nuclei of Helium atoms called Alpha particles (α), while the negatively charged particles are electrons, and are called Beta particles (β). The rays which go undeflected indicate no charge and are therefore energetic photons or the gamma rays (γ).

Alpha particle is just a helium atom with both of its electron removed. It has a atomic weight or mass number $A = 4$, and the charge number $Z = 2$. The beta particles have been identified as electrons with more energy as compared with the ordinary electrons because their origin is nucleus and not the atomic orbits. The gamma rays are electromagnetic waves of nearly the same or some what higher frequency than X-rays. The alpha rays are ejected with a speed of one tenth to one hundreth of the velocity of light c. whereas the beta particles travel often with as higher velocity as about one fifth of the velocity of light. The gamma rays being electromagnetic waves have the same velocity as that of the velocity of light.

19.5. The Disintegration Of Radioactive Elements

A nuclear species corresponding to given values of A & Z is called a nuclide and is denoted by $_ZX^A$, where X is the chemical symbol for the particular element, e.g carbon is denoted by $_6C^{12}$ etc.

The nuclear volume is directly proportional to A, which leads to the important result that the density of nuclear matter is essentially constant for all nuclides. The main cause of radio activity is the instability of nuclides of heavy elements. The unstable nuclides in nature decay by the phenomena of radio activity taking place by the process of emission of α or β particles, which may be accompanied by γ-rays. As the mass number of α-particle is $A = 4$ and the charge number $Z = 2$, the decay product after alpha particle emission will have as mass number (A–4) and charge number (Z–2) i.e a parent nucleus $_{Z}X^{A}$ will decay into $_{Z-2}X^{A-4}$ due to α - emission viz:

$$_{Z}X^{A} \text{ ---------- } _{Z-2}X^{A-4} + {_2}He^4 \text{ (α-particle) ------(19.3)}$$

(Parent Nucleus) (Daughter Nucleus)

In this process the daughter-nucleus may also remain unstable and under go further disintegration till it attains stability.

The process of beta particle emission involves no effect on mass number A, but it does change the charge number Z by +1 or -1 depending upon whether the emitted particle is negative beta particle β^- (electron) or a positive beta particle β^+ (positron). Thus beta activity may lead to either of the following disintegrations:-

$$_{Z}X^{A} \text{ ——— } _{Z-1}X^{A} + \beta^{+}$$

or, $$_{Z}X^{A} \text{ ——— } _{Z+1}X^{A} + \beta^{-} \text{ ----------------(19.4)}$$

Most frequently the α or β emission leaves the daughter nuclide in an excited state, and that one or more γ-rays are emitted as it goes back to the ground or normal state (un excited state). Since the gamma rays are massless photons, γ-emission will cause no change on either A or Z. These decay schemes are shown below fig.19.3-(a) and (b) by the respective energy level diagrams indicating decay directly to the ground state or to an excited state of the daughter nucleus.

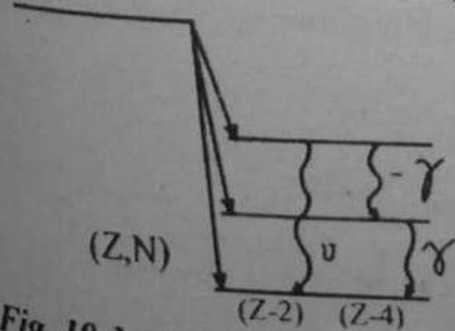


Fig. 19.3 (a) An Alpha-emitter

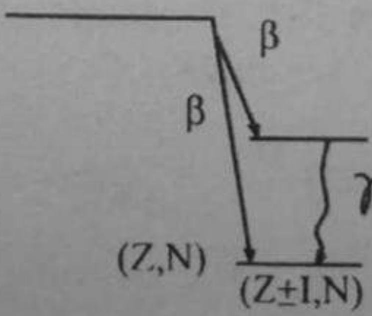


Fig. 19.3 (b)-A Beta emitter.

Example 19.1.

A nucleus consists of 11 protons and 12 neutrons. What is the conventional symbol of this nucleus?

Solution

Given that, there are 11 protons: hence the atomic number Z is 11. From the periodic table the chemical element of atomic number Z = 11 is Sodium, whose chemical symbol is Na. The mass number of the nucleus is therefore: A = N + Z = 12 + 11 = 23 Hence, the symbol of the given nucleus will be $_{11}Na^{23}$

Example 19.2.

What element will be formed due to the emission of β- particle from the nuclide $_{82}Pb^{214}$

Solution

In β- emission the charge number Z is enhanced by 1 where as the mass number A remains unaltered. Hence we have:

$$_{82}Pb^{214} \xrightarrow{\bar{\beta}} {}_{82+1}X^{214} \text{ or } {}_{83}X^{214} \quad \text{---------(19.5)}$$

But, from the periodic table of elements the chemical element with Z = 83 and A = 214 is bismuth whose symbol is Bi. Hence, the element formed in the given decay will be $_{83}Bi^{214}$.

Example 19.3.

Find the element formed due to the disintegration of $_{88}Ra^{226}$ by the emission of an alpha particle.

Solution

Since the α - emission involves a change in Z by 2 & A by 4. We have

$$_{88}Ra^{226} \xrightarrow{\alpha} {}_{88-2}X^{226-4} \text{ or } {}_{86}X^{222} \quad \text{--------(19.6)}$$

But, from the periodic table the element with Z =

86 and A = 222 is Radon with symbol R_n. Hence the element formed will be ${}_{86}R_n{}^{222}$.

19.6. The Law Of Radioactive Decay

The rate of decay in a radio active process is experimentally found to be directly proportional to the number of parent nuclides present in the unstable nuclides of a given species, then the number of disintegration ΔN occuring in a time interval Δt will be given by :

$$\Delta N \propto N_o$$

$$\propto \Delta t$$

or,

$$\Delta N = -\lambda N_o \Delta t \quad \text{-----------------(19.7)}$$

where, λ is the constant called the disintegration or the decay constant. The negative sign in Equation (19.7) is introduced to indicate the decrease in the number N with time.

From equation (19.7) it is seen that if λ is large, more nuclei will be decaying in the same time interval i.e the element decays rapidly. On the other hand if λ is small, the element will decay slowly. The decay constant is a characteristic of the substance that decays and is absolutely independent of all external conditions such as temperature, pressure etc. from equation (19.7) we may also write:

$$\frac{\Delta N}{\Delta t} = -\lambda N_o \quad \text{-------------(19.8)}$$

The number of disintegrations per second is called the activity A and is taken as a positive number. Hence, from (19.8) the activity at any time t may be expressed as :

$$A = \lambda N_o \quad \text{-------------(19.9)}$$

The ratio N/N_o is defined as the relative activity, where N_o is the number of the parent nuclei at the initial or starting time t = 0'. Thus, if we plot the relative activity versus time we get the following trend of the decay curve fig. (19.4).

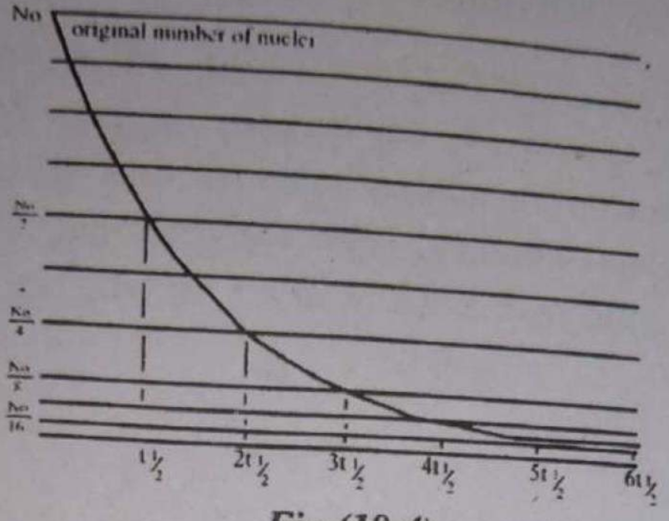


Fig.(19.4)

The above curve shows that the radio active decay process is exponential in nature i.e the number of radio active atoms decreases rapidly in the beginning and then the decay slows down as the time passes. From this nature of decay we may infer that the activity may be mathematically expressed in the following form:-

$$\frac{N}{N_o} = e^{-\lambda t} \qquad \text{------------------}(19.10)$$

where, e is the Natural Logarithm base. Thus we have the following famous form of the exponential law of radio active disintegrations:-

$$N = N_o e^{-\lambda t} \qquad \text{------------------}(19.11)$$

19.7. The Half Period Or The Half Life Of The Radio Active Nuclide

Equation (19.11) established in article 19.6 above indicates that an infinite time is required for the radio activity to disappear completely, since all the substances are the same in this regard, a more qualitative and useful term Half Life is often used to distinguish one radio active substance from the other. Half Life time or simply the Half Life denoted by $T_{1/2}$ is defined as the time required for the radio active element to decay to one half of its initial number N_0. Thus, to determine the Half Life, we can solve equation (19.11) to get

$$T_{1/2} = \frac{0.693}{\lambda} \quad \text{------} \quad (19.12)$$

Equation (19.12) shows that the half life of a radio active element is inversely proportional to the decay constant λ, which can be used to distinguish one substance from the other eg: Half life of Radium is 1590 years while for Radon it is only 3.825 days. This mean that Radium is long lived where as Radon is short lived.

19.8. Nuclear Changes And The Conservation Laws

All types of radio active decays obey some simple and basic rules which are based on conservation laws. The common conservation laws are as follows:-

(1) Conservation of Nucleon Number A.

(2) Conservation of Charge Number Z.

(3) Conservation of Energy.

(4) Conservation of Linear Momentum.

(5) Conservation of Angular momentum. (including spin)

The first rule is a sort of generalization of a variety of observations, whereas by all the other conservation laws we simply mean that the total charge, total energy, and total momenta of the system remains the same before and after the concerned change or transformation.

We may illustrate these rules by considering the example of the disintegration of $_{90}Th^{227}$ into $_{88}Ra^{223}$ and a alpha particle i.e

$$_{90}Th^{227} \longrightarrow {}_{88}Ra^{223} + {}_2He^4 \ (\alpha\text{-particle}) \quad \text{---} (19.13)$$

Using the mass of $_{90}Th^{227}$ = 227.027 U, the mass of $_{88}Ra^{223}$ = 223.018 U and mass of $_{2}He^{4}$ = 4.002 U, where atomic mass unit (U) is defined as one twelfth of the mass of an atom of $_{6}C^{12}$.

The kinetic energy of the α - particle have been found to be 6.04 MeV We may now examine how these facts agree with the rules.

1. By adding the number of nucleons on both sides of equation (19.13) we may verify that no nucleus have been increased or decreased, i.e

$$A = 227 = 223 + 4.$$

2. The total charge number is Z = 90 before and after the decay i.e

$$Z = 90 = 88 + 2.$$

3. The difference in mass before and after the decay is :

$$227.027 \text{ U} - 223.018 \text{ U} - 4.002 \text{ U} = 0.007 \text{ U}$$

This mass difference corresponds to a difference of energy given by :

$$(0.007 \text{ U})\,(931 \text{ MeV/U}) = 6.517 \text{ MeV}$$

This energy of 6.517 MeV should show itself as the Kinetic Energy of the decay product. It has been observed that α - particle come out with an energy of 6.04 MeV. This experimental value of the energy of α - particle is this in close agreement to our calculated energy 6.517 MeV with a small difference of 0.11 MeV. This apparent difference is taken care of in the fourth and fifth rules of the conservation of momenta. In our present example this energy of 0.11 MeV is nothing but the Kinetic energy of recoil of the daughter nucleus Radon.

All the radioactive decays or nuclear reactions involves the release of some energy which is commonly denoted by the symbol Q, and in the general terminology of Nuclear physics it is refered to as the Q-value of the reaction. If Q is positive, energy is released and the reaction is exothermic; whereas for negative Q-value, energy is required to be supplied in order to let the reaction go and the reaction is said to be endothermic.

Example 19.4.

Find the Q-value of the reaction, when ${}_{94}Pu^{239}$ makes a alpha decay.

Solution

The reaction in the above decay may be written as:

$$ {}_{94}Pu^{239} \rightarrow {}_{2}He^{4} + {}_{92}U^{235} + Q $$

The isotopic masses in the above reaction are :

$$ {}_{92}U^{235} = 235.0439 \text{ u} $$

$$ {}_{2}He^{4} = 4.0026 \text{ u} $$

i.e sum of the masses = 239.0465 u

Now, mass of ${}_{94}Pu^{239}$ is given by :

$$ {}_{94}Pu^{239} = 239.0522 \text{ u} $$

$\therefore$ Change in mass (Δm) = 239.0522 - 239.0465

= 0.0057 u

Hence, the Q - value will be :

$$ Q = (\Delta m)c^2 = (0.0057)\ 931\ \text{MeV}/\mu $$

$$ = 5.7 \times 10^{-3} \times 931 $$

$$ = 5.306\ \text{MeV}. $$

19.9. Mass Energy Relation and The Mass Defect:

We have learnt in mechanics that the Kinetic energy associated with a body of mass m moving with velocity V is given by $\frac{1}{2}mv^2$ This expression is only an approximation suitable for moderate range of velocities.

However for atoms and subatomic particles which can be accelerated to velocities approaching Velocity of light c, this approximation breaks down, but Einstein showed that

$$m = \frac{m_o}{\sqrt{1 - \frac{v^2}{c^2}}} \quad \text{-------------------(19.14)}$$

In view of the mass variation formula (19.8) Einstein proposed that the expansion for the Kinetic energy (K.E) of an object moving at high velocities ought to be of the form :

$$K.E = (m - m_o)c^2 = (\Delta m)c^2 \quad \text{-------------(19.15)}$$

In other words, the change in mass of the object (Δm), when multiplied by the square of the speed of light, gives the energy of the object. Einstein showed that equation (19..15) is applicable to any form of energy, and this relation predicts that energy has mass.

This suggests that mass and energy are interconvertable and the relation provides us an equivalence of mass and energy.

Example 19.5.

The rate of radiation of energy from the same is 3.80×10^{26} W. Determine the change in mass of the sun and calculate the rate at which the mass of the sun diminishes.

Solution

From the mass energy relation we have,

$E = (\Delta m) c^2$.

where Δm is the change in mass.

Hence, we have :

$$\Delta m = \frac{E}{c^2} = \frac{3.80 \times 10^{26} J}{(3 \times 10^8 m/s)^2}$$

$$\Delta m = \frac{3.80 \times 10^{26} J}{9 \times 10^{16} m^2/s^2} = 4.20 \times 10^9 kgs^{-1}$$

or

Now, one year consists of $365 \times 60 \times 60 \times 24$ s

i.e 3.16×10^7 s/ year

∴ The rate of loss of Sun's mass will be given by:

$(\Delta m)\ (3.16 \times 10^7\ s/yr) = 1.32 \times 10^{17}\ kg/yr.$

19.10. Mass Defect and Binding Energy

According to the accepted model of the nucleus, it consist of protons and neutrons. But the electrostatic repulsive force between two protons with in the nucleus $(\sim 10^{-15}$ m) is so strong that nucleus should be blown apart. But we observe that nucleus of elements with atomic number as high as 82 are stable. The gravitational force of attraction is far too weak to hold the nucleus together. There ought to exist a force strong enough to over power the repulsive electrostatic force. This is called strong nuclear force. Experiments using protons and neutrons as bombarding particles have shown that the strong nuclear force is independent of the charge. This nuclear force is stronger than the electric force, but is effective only at extremely short range.

Another interesting feature emerges, when we measure the nuclear masses and compare them with the masses of the constituent nucleus in free states. The mass of the nucleus is always less that the mass of the constituent nucleons. This difference in mass, Δm is known as the MASS DEFECT. As shown earlier, Δm is equivalent to an energy $(\Delta m)\ c^2$. This difference in energy between the stable nucleus and the free constituents nucleon, is called the BINDING ENERGY of the nucleus.

We can illustrate the case by the following example. For deuteron $_1H^2$ which consist of one proton and one neutron we have :

Mass of proton $_1H^1 = 1.6724 \times 10^{-27}$ kg

Mass of neutron $_0n^1 = 1.6748 \times 10^{-27}$ kg

i.e the sum of the proton and neutron mass will be 3.3472×10^{-27} kg.

But, mass of deuteron $_1H^2 = 3.343 \times 10^{-27}$ kg.

∴ The mass defect Δm will be given by:

$$\Delta m = (3.3472 - 3.3431) \times 10^{-27}\ \text{kg} = 0.0041 \times 10^{-27}\ \text{kg}$$

The mass defect must appear as the binding energy can be calculated :

$$E = (\Delta m)\ c^2 = (0.0041 \times 10^{-27}\ \text{kg})\ (3 \times 10^8\ \text{m/s})^2$$

or, $E = 3.69 \times 10^{-13}$ J

Since $1\ \text{eV} = 1.6 \times 10^{-19}$ J

The binding energy of deuteron will be:

$$(B.E)_{Deuteron} = \frac{3.69 \times 10^{-13}\ \text{J}}{1.60 \times 10^{-19}\ \text{J/eV}} = 2.3 \times 10^{-6}\ \text{eV}$$

$$= 2.3\ \text{MeV}$$

Another useful quantity for a nucleus is the binding energy per nucleon or the PACKING FRACTION. The value of packing fraction fordeuteron is 1.1MeV, a large value consistent with the observation that the combination of two protons and two neutrons is very stable. Packing fraction for various nuclei has been plotted against graph - Fig. (19.5):

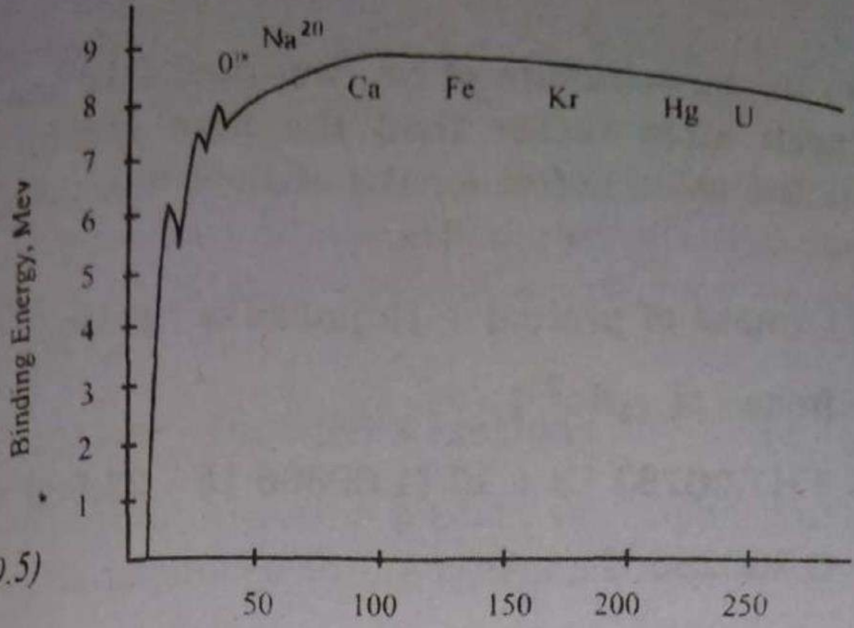


Fig.(19.5)

It can be noted from the graph that the binding energy per nucleon increases rapidly at first and then slowly decreases as the number of nucleous increases beyond about 60. Apparently the nuclei with largest binding energy per nucleon and hence most tightly bound are those in the middle region of the periodic table of elements - iron, cobalt and nickel. These nuclei have considerably less mass than the sun of the masses of their constituent nucleons. It would take a large amount of energy to pull the nucleous apart. For these elements the amount of binding energy per nucleon is over 8 MeV. The graph can also be interpreted as a rough graph of nuclear stability.

Example 19.6.

Sodium ($_{11}Na^{23}$) has a atomic mass of 22.989 u. Find the total binding energy of the sodium nucleus and estimate the binding energy per nucleon.

Solution

The atomic number of sodium is $Z = 11$

The mass number of sodium is $A = 23$

$\therefore$ The neutron number $N = A - Z$

$= 23 - 11 = 12.$

Because the atomic mass includes the mass of 11 electrons in the structure of Na, we must take the mass of hydrogen atom rather than the bare proton mass. Therefore the mass defect for the sodium nucleus will be given by:

$$11 \text{ (mass of proton)} + 12 \text{ (mass of neutron)}$$

$$- (\text{mass of } {}_{11}Na^{23})$$

$$= 11(1.00782\ U) + 12\ (1.00866\ U) - 22.989\ U$$

$$= 0.200285\ U$$

$$\therefore B.E = (0.200285\ U)\ (931.5\ MeV/U) = 186.6 MeV$$

$$\text{and B.E per Nucleon} = \frac{B.E}{A} = \frac{186.6\ MeV}{23} = 8.11 MeV$$

19.11. Nuclear Reactions:

Nuclear reactions in which alpha and beta particles are emitted by unstable nuclei were introduced earlier. These nuclear reactions are however spontaneous and uncontrollable. But they provided the first rich source of nuclear particles, which may be used to bombard other nuclei and initiate new nuclear reactions. One of the first nuclear reaction (artificially induced) was observed by Rutherford in 1919. He used alpha particles (obtained from radioactive nuclei) to bombard Nitrogen nuclei to produce an Oxygen isotope and energetic protons. This reaction is as follows:

$${}_7N^{14} + {}_2He^4 \text{ (α-particle)} \longrightarrow {}_8O^{17} + {}_1H^1 \text{ (proton)}$$

Oxygen Isotope

In nuclear physics this reaction is commonly abbreviated in the form ${}_7N^{14}(\alpha,p)\ {}_8O^{17}$. Just like the balancing of a chemical reaction equation, here the nuclear reaction equation is also balanced. The charge number Z on both the sides of the equation is the same i.e $Z = 7+2 = 9$ on the left hand side and $8 + 1 = 9$ on the right hand side. Similarly, the nucleon number A is the same on both sides of the equation i.e $A = 14+4 = 18$ and $17+1 = 18$.

The discovery of this nuclear reaction was a land mark in physics and opened a new era for producing other types of nuclear reaction which may be summarized as below:

1. **Protons - Induced Reactions**

If lithium absorb a proton, two alpha particles are found to be produced in the reaction.

$$_3Li^7 + {}_1H^1 \text{ (proton)} \longrightarrow {}_2He^4 + {}_2He^4$$

Two alpha particles.

This reaction is of great historical importance because it provided the earliest experimental verification of the Einstein's mass - energy relationship.

2. **Deutron - Induced Reactions**

High energy deuterons may be absorbed by $_3L_1^6$ to produce two alpha particles i.e

$$_3Li^6 + {}_1H^2 \text{ (deuteron)} \longrightarrow {}_2He^4 + {}_2He^4$$

3. **Gamma - Induced Reactions**

High energy gamma rays also have been found to induce nuclear reactions by a process which is usually known as photo disintegration. Examples of such reactions are :

$$_{13}AL^{27} + {}_1H^2 + \gamma \text{ (50 Mev)} \longrightarrow {}_{11}Na^{25} + 2\,{}_1H^1$$

Sodium isotope   Two protons

19.12. Nuclear Fission

A remarkable nuclear reaction was discovered in 1934 when Fermi, Segre, and Co-workers bombarded Uranium by neutrons and noticed the production of several beta activities with different half lives. Four years later in the year 1938 two German scientists Hahn and Strassman showed that one of the radioactive elements produced when uranium is bombarded by neutrons is the Barium isotope $_{56}Ba^{141}$. Soon after in the year 1939 Frisch and Meitner suggested that in the above neutron bombardment experiment the uranium nucleus undergoes a nuclear fission process producing two fragments $_{56}Ba^{141}$ and $_{36}Kr^{92}$ of roughly equal size. Such a process of splitting of a heavy nucleus into smaller fragments is called Nuclear Fission. Each fission process also involves the production of some smaller particles also in addition to the bigger fragments. Typical nuclear fission reactions are the following:

$$_0n^1 + _{92}U^{235} \rightarrow (_{92}U^{236})^* \rightarrow _{56}Ba^{144} + _{36}Kr^{89} + 3_0n^1 + Q$$

Excited Compound nucleus. Barium. Krypton. Neutrons. Energy

where, Q is the energy released in the reaction. Thus the general scheme of the nuclear fission reaction is of the following form :

$$_0n^1 + _{92}U^{235} \rightarrow (_{92}U^{236})^* \rightarrow X + Y + \text{Neutrons} + \text{Energy } (\gamma\text{-Ray})$$

where, $(_{92}U^{236})^*$ is the excited nucleus after the capture of neutron. X and Y are the fission fragments. The fission process may be schematically represented as shown in Fig - (19.6) below:

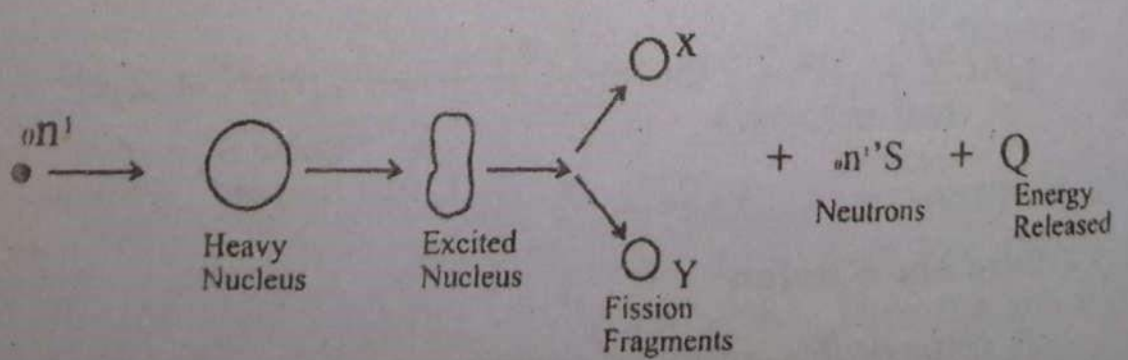


Fig.(19.6)

The most important aspect in the above reaction is the liberation of atleast one or more energetic neutrons. Which may further induce fission in the additional heavy nuclei in the form of a chain reaction resulting in the sudden release of a huge amount of energy (a nuclear bomb). The energy release in the fission process is due to the conversion of mass defect between the mass of the heavy nucleus and the resulting fragments, into energy.

The nuclear chain reaction may be schematically represented as shown in Fig.(19.7) below :

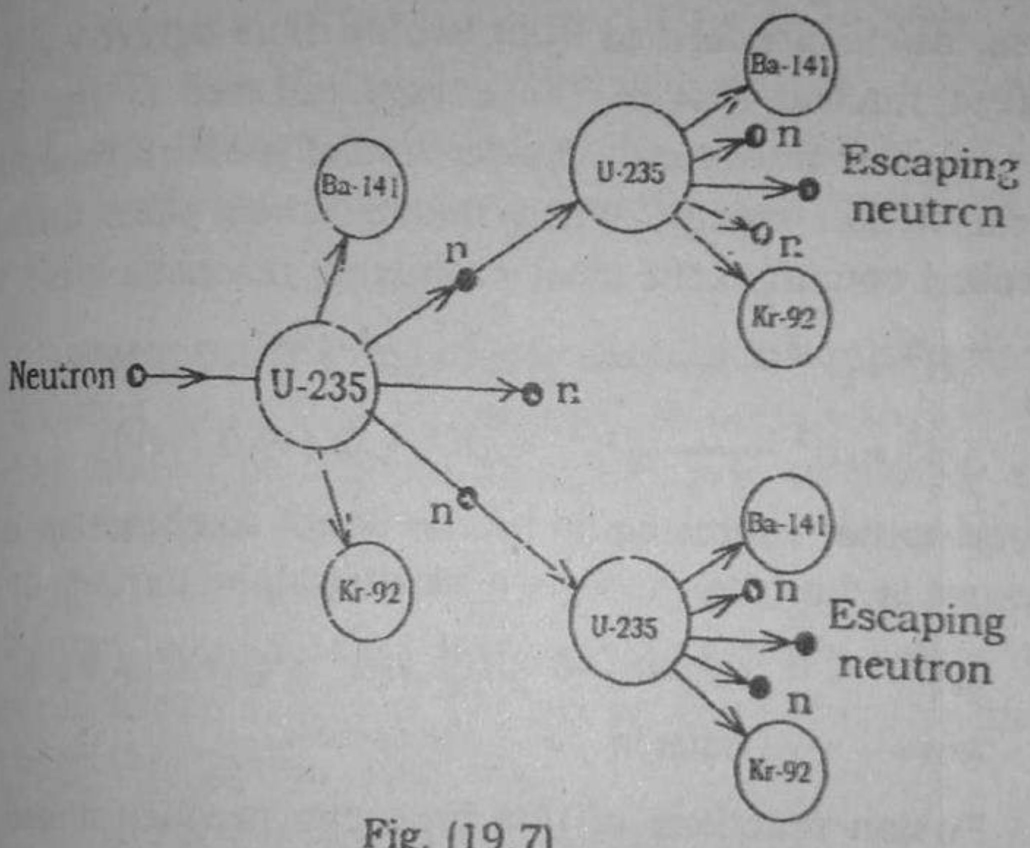


Fig. (19.7)

The chain reaction described above is a self-sustaining reaction. If such a reaction is allowed to proceed rapidly it would lead to a huge explosion because of the unchecked release of energy. The uncontrolled chain reaction may be checked if some device is used to absorb some of the neutrons produced in fissions. A suitable device is the use of some material as a moderator to slow down the reaction. Commonly used materials as good moderation are graphite, and cadmium. Which may keep the chain reaction well below the self-sustaining level. Moderators are used in the form, numerous rods inserted into the reaction chamber.

19.13. Nuclear Fusion

From the binding energy curve Fig.(19.5) of Section 19.10 we may note that nuclei with mass number A less then 30 or 40 have smaller binding energies per nucleon than havier nuclei. For example, the binding energy per nucleon of Hydrogen is 1.12 MeV, whereas it has a value of 7.07 MeV for Helium. This suggests that, in principle, a process inverse to fission is energetically possible for lighter nuclei. Hence, the process in which heavier nuclei are formed from two or more lighter nuclei is called nuclear fusion. The energy released in the fusion of lighter nuclei into heavier nuclei is called thermonuclear fusion energy. When fusion taken place under controlled condition the most promising reactions are :

$$_1H^2 + _1H^2 \longrightarrow _2He^3 + _0n^1 + Q \ (=3.3 \text{ MeV})$$

and, $$_1H^2 + _1H^2 \longrightarrow _1H^3 + _1H^1 + Q \ (=4.0 \text{ MeV})$$

Another source could be the direct combination of a deutron and a triton to form a heavier alpha particle i:e

$$\underset{\text{Triton}}{_1H^3} + \underset{\text{Deuteron}}{_1H^2} \longrightarrow \underset{\text{Alpha particle}}{_2He^4} + _0n^1 + Q(=17.6 \text{ Mev})$$

Fusion reactions of this type can produce abundant energy. The raw material for the reaction is deuteron which is found in abundance in world oceans as heavy water. Fusion reactions are also, the basic source of energy in stars including the sun. One such chain reaction of fusion process is as follows:

$$_1H^1 + _1H^1 \longrightarrow _1H^2 + \beta^+ + \upsilon \text{ (neutrino)}$$

$$_1H^1 + _1H^2 \longrightarrow _2He^3 + \gamma \text{ (gamma ray photon)}$$

$$_1H^1 + _2He^3 \longrightarrow _2He^4 + \beta^+ + \upsilon$$

In the first and the third reaction above, neutrino (υ) is a electrically neutral particle of very small mass (very small as compared with that of electron or may be even zero) that carries away the missing energy in the concerned nuclear reaction, and fulfills the requirements of the conservation laws of momentum and energy. This means that in a processes involving beta decay the disintegration energy is shared between the beta particle, the neutrino, and the recoil nucleus with the division of energy among three particles.

The above fusion reactions means that the net effect of the proton-proton cycle of three fusion reactions is the conversion of four protons ($_1H^1$) into an alpha particle ($_2He^4$), two positrons (β^+), two neutrinos (υ) and a gamma ray photon. In this fussion process the amount of energy released have been calculated to be of the order of 25 MeV.

Fusion of Hydrogen into Helium can also proceed in another way. This is assumed to occurs in the Sun and is known as carbon -Nitrogen cycle or simply Carbon Cycle. This cycle of fusion reactions was proposed by Bethe in the year 1938. In this cycle four protons are converted into an alpha particle with Carbon acting as a catalyst in the reaction. The sequence of reaction taking place in the Carbon cycle are :

$$_6C^{12} + {_1H^1} \rightarrow {_7N^{13}} + \gamma \text{ (gamma Ray)}$$

$$_7N^{13} \rightarrow {_6C^{13}} + \beta^+ + \upsilon \text{ (neutrino)}$$

$$_6C^{13} + {_1H^1} \rightarrow {_7N^{14}} + \gamma \text{ (gamma Ray)}$$

$$_7N^{14} + {_1H^1} \rightarrow {_8O^{15}} + \gamma \text{ (gamma Ray)}$$

$$_8O^{15} \rightarrow {_7N^{15}} + \beta^+ + \upsilon \text{ (neutrino)}$$

$$_7N^{15} + {_1H^1} \rightarrow {_6C^{12}} + {_2He^4} \text{ (alpha particle)}$$

Carbon Reappears

(I.e it acts as a catalyst)

The energy released after a complet...

which four protons ($_1H^1$) combine to form a nucleus of Helium (an alpha particle, $_2He^4$) and two positrons (β^+) have found to be more than 26.7 MeV. There has been a concentrated effort to produce controlled thermonuclear fusion in the laboratory on earth and some progress toward it has been made since 1948. There seems to be a promise of Virtually unlimited useful source of energy through the development of controlled fusion reactors in future.

19.14. Nuclear Reactors

A seemingly in exhaustible source of energy is locked up inside the atomic nucleus. A nuclear fission reaction produces on the average 10^9 times more energy compared to the energy released when a carbon atom in the conventional coal furnace combine with oxygen in the air to form carbon dioxide.

Thus, a great energy resource may be opened up by the possibility of developing easier means to extract the hidden energy from the nucleus. Reactors are the possible devices for achieving this objective. Different kinds of nuclear reactors have been developed in our present day world. Inspite of a number of possible variation in the design and components of nuclear reactors there are quite a few general features which are to almost all types of reactors. These features may be summarized as under:

1. **Nuclear Fuel**

A material consisting of the fissionable (or fissile) isotope is called the reactor fuel. The fuels that may be used in a reactor are Uranium U^{235} in its natural abundance of 0.715% or in an enriched proportion i.e. U^{235} and U^{239} etc.

2. **Moderators**

In the nuclear fission process at least one or more energetic neutrons are produced per fission. To reduce the energy of neutrons some suitable materials are required which are known as moderators. The good moderating materials possess usually low mass number and large slowing down power. The ordinary water (light water) is an attractive moderator material because of its easy supply at low cost. Heavy water is supposed to be the best suited material for neutron moderation inspite of its greater cost as compared with light water. Other moderators which may be used in a reactor are graphite. Beryllium and its oxide and certain organic compounds.

3. Coolants

A huge amount of heat is generated in the reactor core as a result of fission taking place in the nuclear fuel. To remove this large quantity of heat materials are required which are called coolants. These materials are usually circulated through the core in order to absorb heat and transfer it to the outside core. The properties of a good coolant are :

(a) It should have as little effect on neutrons as possible i.e. it should not absorb nor moderate the neutrons.

(b) It should not induce any chemical effect with other materials in contact with system.

(c) The coolant should not breakup under the effect of radiations.

(d) The coolant material should be capable of acquiring long lived radio activity during its circulation through the reactor.

(e) It should have low vapour pressure at the operating temperature of the reactor.

(f) The material should be able to remove large quantities of heat for a small input of pumping power.

In fact, no single coolant possess all the above properties simultaneously, and the choice of a coolant depends upon the type of reactor. The materials commonly used as coolants are light water, heavy water, liquid metals such as sodium or sodium potassium alloy or mercury. Certain organic liquids and gases are also used as coolants.

4. Control Materials

In order to control the nuclear fission in a reactor suitable neutron absorbing materials are required to be placed in the core region. The control material should be such that it does not become radioactive by the neutron capture. Cadmium has been found to be a good control material. Because of its low melting point cadmium can be used as a control material at low temperature.For higher temperature an alloy of silver with 15% indium and 5% cadmium is more suitable because of its higher melting point. Boron is also a very good control material due to its very high melting point and large neutron absorbing capability. Some times Boron is mixed with stainless steel, aluminium or carbon is successfully used as a control material for some reactors.

5. Shielding

With the exception of reactors operating at very low powers all reactors are sources of intense neutron and gamma ray radiations. These radiations are hazardous in the vicinity of the reactor. Hence proper shielding material is always required to protect the persons working in the reactor area. A shielding material used for such a protection is called the biological shielding because its purpose is to protect health. Generally a layer of concrete, about six to eight feet thickness has been found to absorb both neutrons and gamma rays and acts as a good biological shield. For reactors operating at high powers another type of protection known as thermal shield is required to eliminate excessive heat produced in the immediate neighbourhood or contact of the core of the reactor. For the purpose of thermal shielding of this type special cooling arrangements are required to prevent the shield in contact with the core from cracking or suffering from any other heat damage. A few centimeters of iron or steel very close to the core of the reactor provides the required thermal shield.

19.15 Breeder Reactor

In the natural occurrence of uranium, the isotope $_{92}U^{238}$ makes up only 0.72% whereas the rest 99.28% is the other isotope $_{92}U^{238}$. The least abundant isotope $_{92}U^{235}$ is used as the fuel for conventional reactors. If some how we could make use of $_{92}U^{238}$ as a fuel as a nuclear reactor, we could be better off in increasing the life span of the existing uranium fuel supplies of the world deposits of uranium ore. Due to this reason our interest in breeder type of reactors has increased significantly. The far more available isotope, $_{92}U^{238}$ has been found more likely to undergo the following reaction:

$$_{92}U^{238} + {}_0n^1 \rightarrow {}_{93}Np^{239} + \beta^- + \upsilon^- \text{ (anti neutrino)}$$

$$\longrightarrow {}_{94}Pu^{239} + \beta^- + \upsilon^-$$

In this nuclear reaction $_{94}Pu^{239}$ is an alpha emitter with a half life of 2.4×10^4 yr. It is responsive to undergo fission by slow neutrons. In a breeder reactor some of the neutrons from the fission of $_{92}U^{235}$ are used to transmute $_{92}U^{238}$ into $_{94}Pu^{239}$ as indicated in the above reaction. We see that the breeder reactor does not create material capable of undergoing fission, but it simply converts the unuseable most abundant isotope $_{92}U^{238}$ into fissionable isotope $_{94}Pu^{239}$ with a very long half life of 2.44×10^4 year. The isotope $_{94}Pu^{239}$ is also radio active and can decay into $_{92}U^{235}$ with the emission of alpha particle, but due to its long half life large quantities of $_{94}Pu^{239}$ can be collected and used for power reactors where it fissions under neutron bombardment with the release of huge amounts of energy through the following nuclear reaction:

$$_0n^1 + {}_{94}Pu^{239} \longrightarrow 2\,{}_0n^1 + X + Y + 200 \text{ MeV}$$

Where, X and Y are fission products consisting of a variety of isotope near the middle of periodic table. The term breeder is used to signify that, starting with a nonfissile abundant isotope $_{92}U^{238}$, we are able to breed a fissile nucleus $_{94}Pu^{239}$, which can be used in a reactor to produce almost the same amount of energy as is available from the fission reaction of $_{92}U^{235}$

There are however, still some problems of technical nature with operating a breeder reactor, which may be resolved in due course of time. Another serious problem is that $_{94}Pu^{239}$ is highly toxic and is also a potent material for producing fission bombs. This is of course, a matter of great anxiety and concern for many nations of the world.

In a Fast Breeder Reaction (FBR) more fissionable material is produced than consumed by the capture of fast neutrons from fertile material. In such reactors the energy of the neutrons should not be lowered to decrease otherwise the neutrons will be absorbed as slow neutrons in the structured materials. Thus, use of lighter elements should be avoided in these reactors eg. water used as coolant in thermal reactors is not a suitable coolant in FBR, because of its high slowing power. In fast breeder reactors sodium is widely used as coolant and reactors so designed are called Liquid Metal Fast Breeder Reactors (LMFBR). A schematic diagram of a LMFBR is shown in Fig. (19.8)

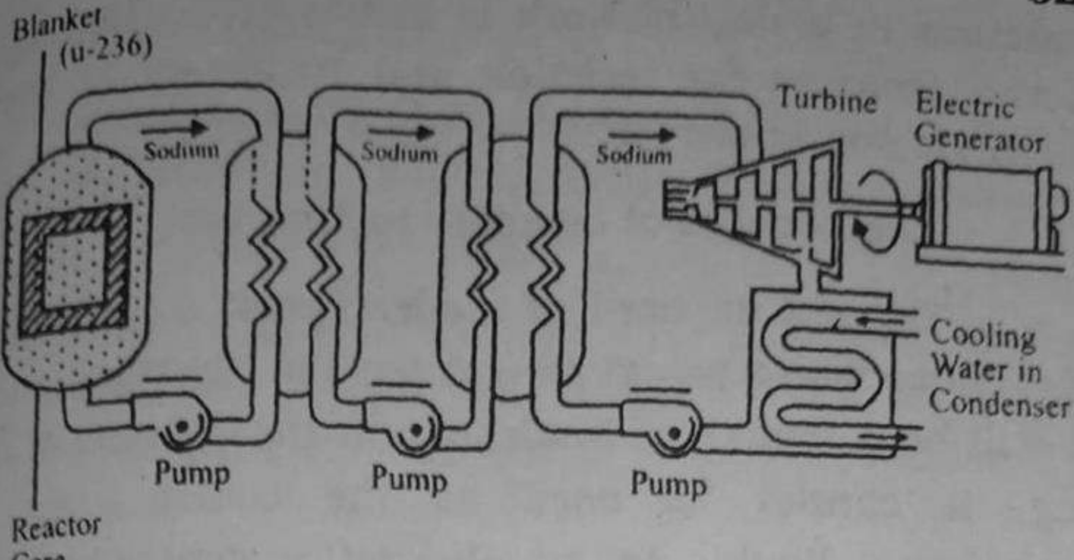


Fig. (19.8)

From the above diagram it may be seen that this type of reactor is just like a conventional reactor except that the reactor core is made up of 15 to 30% $_{92}U^{235}$ surrounded by a blanket of $_{92}U^{238}$. Since the fast neutrons are more efficient in converting $_{92}U^{238}$ to $_{94}Pu^{239}$ there is no need to use a moderator in this reactor to slow down the liberated neutrons. Reactions in a fast breeder reactor are shown in Fig. (19.9).

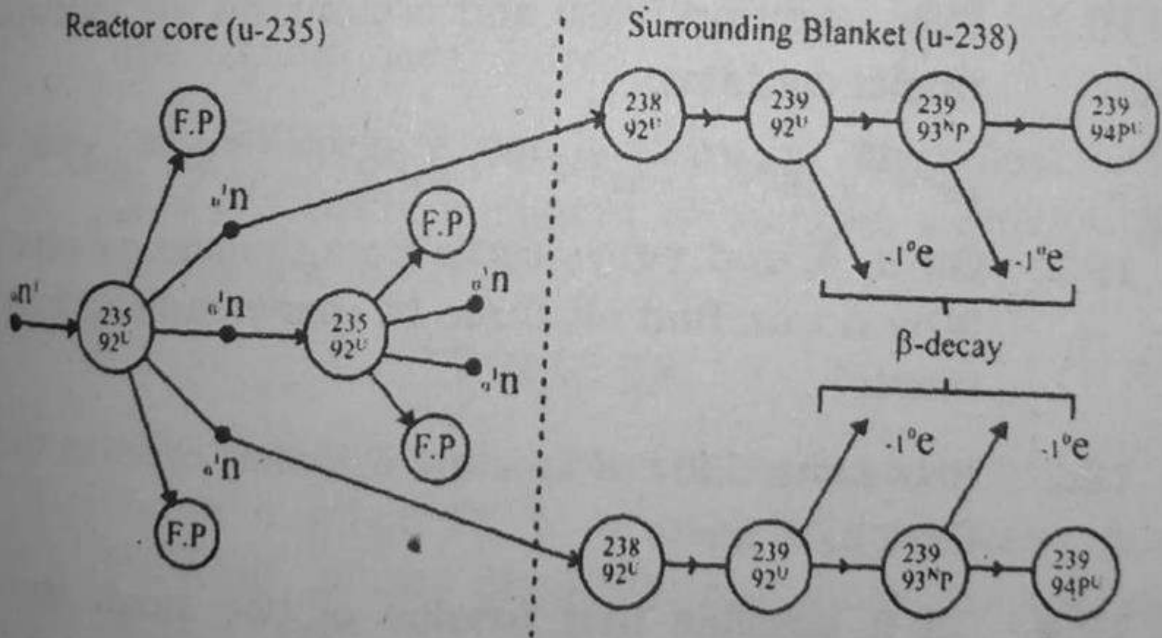


Fig. (19.9)

Nuclear reactors are not only used as useful sources of power generation, but also they are sources of useful neutrons which acts as research tools in physics, biochemistry, biology, medicine and many other related decciplines.A very useful and important utilization of Nuclear topes (Ans: 52.2% and 49.8%).

19.4 The half life of $_{104}Po^{210}$ is 140 days. By what per-cent does its activity will decrease per week? (Ans: 3.46%).

19.5. If a neutron would be entirely converted into energy, how much energy would be produced? Express your answer in Joules as well as electron volts. (Ans: 1.50×10^{-10} J: 942.0 Mev).

19.6. Find the binding energy of $_{52}Te^{126}$. Given:

$$m_p = 1.0078\,u,\ m_n = 1.0086\,u,$$

$$m_{Te} = 125.9033\ u \text{ and } 14 = 931.5\ MeV$$

(Ans: 1.066×10^3 MeV

19.7. If the number of atoms per gramme of $_{88}Ra^{226}$ is 2.666×10^{21} and it decays with a half life of 1622 years. Find the decay constant and the activity of the sample.

(Ans: 1.355×10^{-11} S^{-1}: 3.612×10^{10} disintegrations/S).

19.8 What will be the maximum energy of the electron in the beta decay of $_1H^3$ through the reaction.

$$_1H^3 \text{-------} {}_2He^3 + \beta^- + \gamma$$

(Ans: - 0.0186 Mev)

19.9 Find the Q-value for the nuclear reaction.

$$_{20}Ca^{42}\ (P.d)\ _{20}Ca^{41}$$

(Ans:- 9.25 MeV)

19.10. Find the energy released when two deuterium ($_1H^2$) nuclei fuse together to form an alpha particle ($_2He^4$).

(Ans:- 23.80 MeV

Chapter. 20

NUCLEAR RADIATIONS

20.1. Interaction of Nuclear Radiations with matter.

In the previous chapter we have discussed natural radioactivity in which α and β particles and γ-rays are emitted from the disintegrating nucleus of an atom. In nuclear fission and fusion reactions neutrons and other particles together with certain radiations are emitted from the nucleus. Moreover, interaction of high energy particles (natural or artificial) with matter produce certain nuclear or atomic reactions with the emission of particles like protons, deuterons, neutrons and ionising radiations like γ-rays from the nucleus and x-rays and ultraviolet rays from the atom. These particles and radiations have been studied carefully for their properties and effects which are as follows:

(i) Alpha Particle

It shoots out from the nucleus with a high velocity (0.1×10^8 m/s). Thus it possesses very high energy (7.7 MeV for the most energetic from R_{ac} (i.e: Bismuth-214). Due to its large size, more charge and high energy it can make very large number of collisions with the atoms and ionise them as it passes through them, before it stops. Head-on collisions are rare, however, if an α-particle passes close to an atom, the strong electrostatic attraction between it and an electron tears the electron off from the atom and ionises it. An α-particle loses about 35 eV energy in each collision. Thus a 7.7 MeV α-particle from RaC (Bi 214) produces about 0.2×10^6 ions before it stops. The range of α-particle is small (about 7×10^{-2}m in air and only 4×10^{-5}m in aluminium for the 7.7 MeV α-particle). Thus metal sheets form good shields for α-particle: The number of ions produced by an α-particle or its range in air is a measure of its energy. Alpha particles produce fluorescence on striking certain substances such as zinc sulphide and bariumplatinocynide.

(ii) Protons

A proton is also a positively charged particle with properties similar to the α-particle. Its mass is one-fourth and charge is one half of that of an α-particle. It is smaller in size and carries less energy at the same velocity. Obviously, it suffers fewer collisions with the atoms of the medium as compared with the α-particle and penetrates the medium to a greater distance (about 5 to 10 times) before stopping. Its ionising power is also much less, about one-fifth that of the α-particle. The mechanism of ionisation is however identical.

(iii) Beta Particles.

A β-particle also ionises the atoms of the medium along its path but this ionisation is much less than that produced by an α-particle or a proton. The reason is that due to ts very small size the collisions are fewer and farther apart. Even in a single collisions most of its energy is lost. Head-on collisions being rare, it can ionise an atom by strong electrostatic repulsion when it passes close to its electron. The range of β-particle in a medium is very large, nearly 100 times that of an α-particle of the same energy. The ionisation produced by it is less than one-hundredth of that by the α-particle. Alpha particles are stopped by an ordinary paper, but the β-particles may pass through a thick book. However, a small thickness of a heavy metal rich in electrons is enough to stop the β-particles e.g. 5×10^{-3} m of aluminium. Fluorescence is also produced when β-particles strike calciumtungstate and barium platinocynide.

(iv) Gamma Rays

Gamma rays are very high energy electromagnetic radiations of extremely short wavelength emitted from the nuclei of radioactive atoms originating from the high energy transitions of the nucleons in the nuclei. They are accompanied with the emission of α- or β-particles. They carry no charge and have no rest mass but possess very high energy of the order of several MeV. They penetrate far greater distance in material media as compared to α-or β-particles. Very energetic γ-rays are capable of penetrating several centimeters of concrete.

Like ultraviolet rays and x-rays,γ-rays are also capable of ionising even far more strongly the atoms of the medium they pass through. Being a photon, a γ-ray can produce ionisation in three ways:

(i) It may lose all its energy in a single encounter with the electron of an atom (Photoelectric effect).

(ii) It may lose only a part of its energy in an encounter (Compton effect).

(iii) Very few of very high energy, γ-ray photons may impinge directly on heavy nuclei, be stopped and annihilated giving rise to electron-positron pairs (the materialization of energy).

Through a gas many of its photons may pass several meters without any encounter. A good many, however, do have encounters with the electrons of the atoms which are knocked off with the production of ions. This ionisation is much less strong than that produced by α- or β-particles. Since most of the photons are absorbed by electrons and substance rich in electrons, e.g. lead will stop most of the γ-ray photons and serve as a good shield against γ-rays.

(v) Neutrons

A neutron is essentially emitted from the nucleus of an atom. It is so called because it is electrically neutral and carries no charge. Its mass is very nearly equal to that of a proton. Consequently, unlike charged particles it can neither experience or exert any electrostatic force of attraction or repulsion. Therefore, it can interact with an electron or the nucleus of an atom only by direct impact. When it hits an electron, it knocks it out from the atom (ionisation) with practically no change in its own energy or direction of motion. However, when it hits a nucleus, appreciable changes in its energy and direction of motion are likely. Nevertheless, such direct collisions are very rare. Hence a neutron is a highly penetrating but very slightly ionising particle.

It is evident from the above discussion that nearly all the particles and radiations ionise the atoms in their path. This effect, therefore, is used as the basis for most of the detection devices, a few of which will now be discussed here.

20.2 Wilson Cloud Chamber

Wilson cloud chamber is a device for making visible the paths of ionising particles. It helps to examine the mechanism of ionisation of various ionising radiations and the products of their interaction with the material inside the chamber. This device was devised in 1895 by the British physicist, C.T.R. Wilson.

It consists of a closed cylindrical chamber with transparent glass top,T, and a movable piston,P, at the bottom (Fig.20.1). On the sides near the top, the cylinder is provided with a glass window, L, for admitting illuminating light and an inlet, I, for the ionising particles or

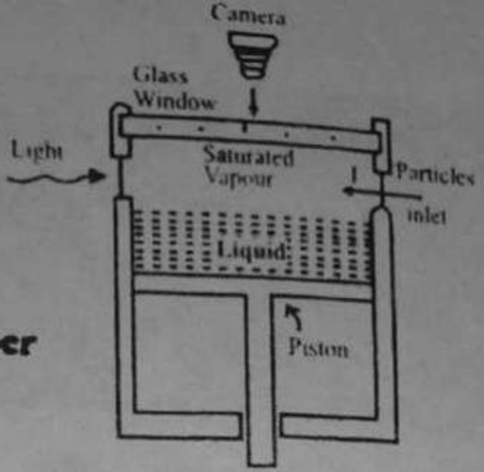


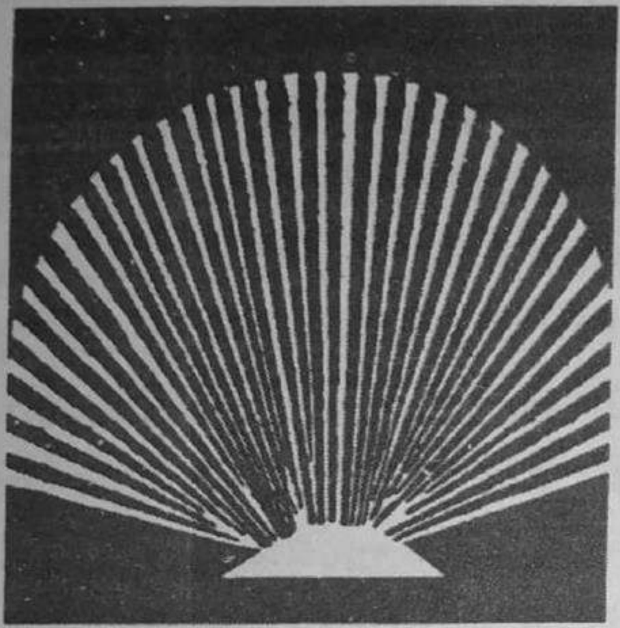
Fig.20.1. Wilson Cloud Chamber

radiations. The piston can be moved up or down by a lever attached to it (not shown in the figure).

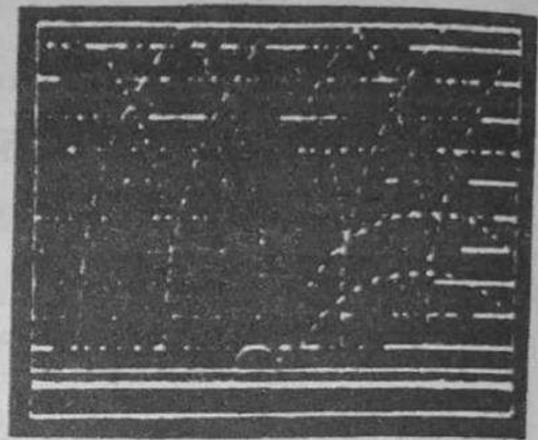
Before making the enclosed space above the piston airtight, enough quantity of a low boiling point liquid such as water or alcohol is introduced in the space to produce its saturated vapours. A small quantity of the liquid stay on the piston. The vapour of a liquid usually condense at its dew point but the condensation never takes place in the absence of some particles— dust particles or ions— which are essential to form the nuclei (centers) of condensation. In particle-free space the saturated vapour may cool much below the dew point. Then they are called super saturated vapour. Under this condition if some ions are incidentally produced amidst these vapours,condensation immediately takes place a round them forming tiny droplets of fog which shows itself when illuminated. This explains the underlying principle of the cloud chamber.

To investigate any ionising particle or radiation, the particle source is mounted in the chamber at the inlet, or the radiation may be admitted through the inlet window. An intense beam of light is projected into the chamber through the window,L, to illuminate the fog track, and a photographic camera is mounted above the glass top of the chamber.

(a) (b) (c)



(d)



(e)

With the above pre-setting of the apparatus, the piston is pulled down suddenly with the help of the lever. The saturated vapours cool down below the dew point into supersaturated vapours. If an ionising particle or radiation passes into the chamber at the same time, the gas molecules all along its path ionise into a trail behind it. The super saturated vapours immediately condense round these ions forming tiny droplets of fog which becomes visible by the reflection of light from them. The track of the particle shows as a bright line which can be photographed at the proper instant.

An α-particle is highly ionising. The ions produced are so numerous that its track is a thick and continuous line, Fig. 20.2 (a). The β-particle is much less ionising. Its track is, therefore, a thin and broken line. Fig. 20.2(b). Gamma rays are photons emitted in a widening cone of some angle. They produce ionisation by photoelectric effect distributed over a wide space. Some of the high energy photoelectrons ejected by them give tiny line-tracks in random directions like the β-particles. The overall effect of γ-rays (as also of x-rays and ultraviolet rays) is that the whole region exposed to radiations shows scattered dots and small lines rather like a fog and no well-defined line track, Fig.20.2(c)

Often a magnetic field is applied vertically across the cloud chamber to cause the particles to deflect. From the deflection, its direction and magnitude, and the length and curvature of the paths, additional information about the charged and uncharged nature, the magnitude of the charge, the charge to mass ratio (e/m), etc of the incident particle or the particle produced by their interaction with the atoms, can be obtained By this very method a number of particles have been discovered.

20.3 Geiger Counter

Geiger counter is a portable device which is widely used for the detection of ionising particles or radiations.Fig.20.3 shows its basic construction. It consists of a hollow metal cylinder one end of which is closed by an insulating cap. At the centre of the cap is fixed a stiff straight wire along the axis of the cylinder. A thin mica or glass disc closes the other end which also serves as the entrance window for the ionising particles or radiations. The sealed tube usually contains a special mixture (air, argon, alcohol, etc.) at a low pressure of 50 to 100 millimetres of mercury. A potential difference of the order of one thousand volts is applied between the metal cylinder and the axial wire through a suitable series resistor, R (about 10^9 – ohms). The potential difference is only slightly less than that necessary to start a discharge between the wire and the cylinder.

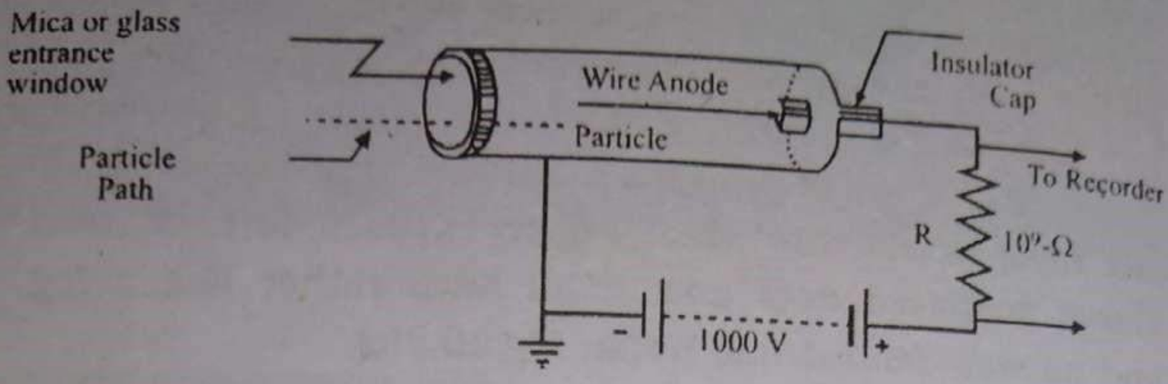


Fig.20.3 Geiger Counter

When an ionising particle enters the tube through the window, it ionises some gas molecules in it. These ions are accelerated by the strong radial electric field producing more ions by collision with the atoms and causing the ionisation current to build up rapidly. So a momentary surging current flows between the wire and the cylinder and through the resistor,R, producing a momentary potential difference across R. The ends of R are connected to a loudspeaker or an electronic counter. Thus each time a particle enters the counter an ionisation current pulse occurs which gives a click in the loudspeaker or a count in the electronic counter. The ionisation current, however, decays rapidly in a small fraction of a second since the circuit has a small time constant and the counter is ready to register another particle almost immediately.

In the case of ionising radiations, the number of counts registered by the counter measures the intensity or ionising power of the incident radiation.

20.4. Solid-State Detectors.

These devices basically make use of the solid-state semiconductor diodes i.e. the p-n junction. It may be recalled that no current passes through a semiconductor diode when it is reverse biased. However, if an energetic ionising particle (or radiation) passes through the p-n junction region, a reverse current pulse passes through the diode due to the ionisation of the atoms of the region.

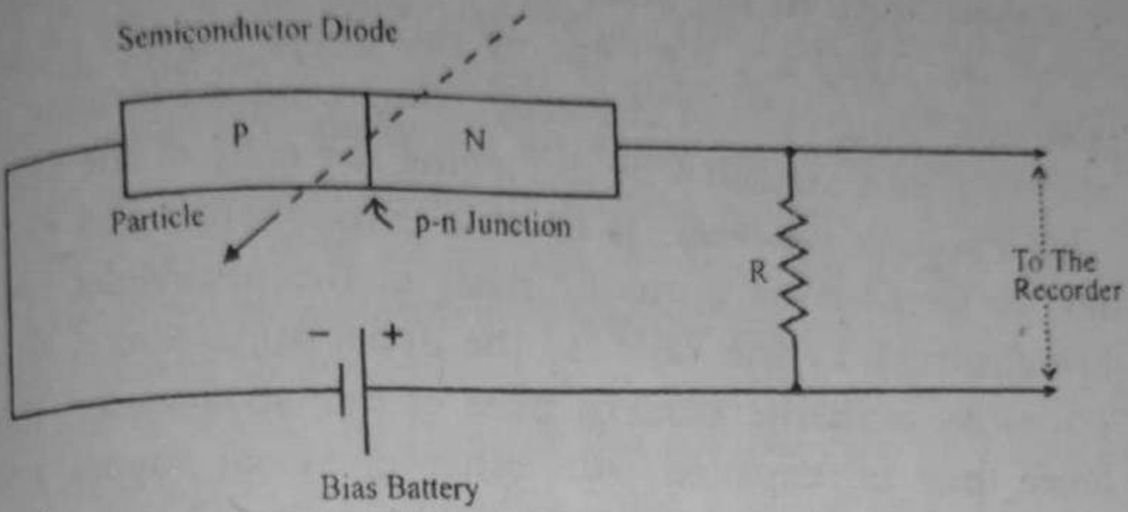


Fig.20.4 Solid State Detector

This can be made to produce a small potential difference across a suitable resistor,R, connected in series with the diode and the battery. The current pulse so produced is then fed into an amplifier and the amplified pulse is applied to a loudspeaker or an electronic counter which can register the number of clicks or counts respectively as the Geiger counter. Fig.20.4 shows the schematic arrangement of the device.

The advantage of this device over the Geiger counter is that it eliminates the use of Geiger tube, the special gaseous mixture in it and the low pressure that it demands. Moreover, the dangerously high potential difference (~ 1000 volts) between the axial wire and the metal cylinder and the precaution of earthing the cylinder for safety are done away with, since this device works at low potential differences up to 9 volts. The size and cost of the apparatus also are very much reduced. It can detect particles having energy only a few electron volts.

20.5 Radiation Exposure

The most common type of damage is due to the ultraviolet rays in sunlight. Ultraviolet rays are electromagnetic radiations of frequencies higher than that of the violet rays in the solar spectrum but lower than that of x-rays. They are high energy radiations capable of destroying living cells of the body. They produce tanning of the skin and sunburn by damaging the cells of the skin. This damage, however, is of little consequence and need not be avoided as normally most of the ultraviolet rays are absorbed by the layer in the upper atmosphere. Nevertheless, with the modern pace of industrialization this layer may be depleted with other chemical vapors and gases resulting in the increased intensity of ultraviolet rays reaching us. There might, then, be an increased danger of the incidence of skin cancer in the human beings and animals which the rays can cause.

In addition to the ultraviolet rays in the sunlight we are constantly exposed to other ionising radiations. All the substances in the earth contain atoms of the radioactive species. Consequently, our bodies are all the time exposed to a low level of background radiations. The other sources of such radiations are the cosmic ray showers from space and the x-ray exposures for diagnostic purposes. Moreover, our food and drinking water also contain some radioactive atoms from which we receive these radiations. Lastly, the nuclear centres and power stations, and the nuclear explosions may create a lot of these radiations. These are all unavoidable. However, normally the overall intensity of the background radiation is within a safe limit, not causing us very serious harm. Besides affecting the living organisms, constant exposure to these radiation also likely to affect the non-living materials. It is found that materials used in construction and manufacture suffer loss in strength by constant exposure to intense radiations.

20.6. Radiation Damage

As explained in the preceding discussion, ionising radiations are dangerously harmful, though within a certain intensity limit their effects may be repairable. The radiations and their different sources, both natural and artificial, are in wide use today. The interactions of radiation with living tissue are highly complex. It is known that excessive exposure to radiations like sunlight, x-rays and other nuclear radiations, causes destruction of tissues . In mild cases the destruction shows as the burn like the sunburn. A greater exposure can cause very severe illness or death in a variety of ways one of which is the destruction of the components of the bone marrow which produces the blood corpuscles in the body. The apparently safe limit of exposure has been established under a subject called 'dosimeter'. When exposure to radiation is unavoidable, it must never be allowed to exceed the safe limit. The safe level of radiation exposure is also open to question. Available evidence shows that exposure to the extent of 10 to 100 times that from natural sources is rarely harmful.

20.7. Effect of Radiation Damage

A general reference has already been made to the fact that considerable damage can be caused by an ionising radiation when it passes through matter. The damage produced in the biological organisms is of great importance and concern for us. It is mainly due to the ionisation produced in the living cells. Several processes may occur. Highly reactive ions and radicals may be produced in the organism and may take part in chemical reactions that may interfere with the normal functioning of the cells. For example, free radicals H^+ and OH^- may be produced from the water present in the cells which may react and break the chemical bonds and disrupt the vital molecules such as proteins.

All radiations ionise atoms and molecules by knocking out electrons. The molecule may split up or its structure may alter such that it either fails to perform its normal function or starts performing a harmful function. If too many molecules of the cell are damaged it may die altogether. The damage to the vital DNA molecules may be of very serious consequence. The disruption of the cells of an organism may continue producing more defective cells to the detriment of the whole organism. Thus, radiation can cause cancer, (the rapid production of defective cell) even at low levels or doses. Another effect of radiation damage is the 'radiation sickness' characterised by nausea, fatigue, loss of body hair and so on. Radiation can also destroy the components of the bone-marrow that produce red blood cells, thus causing leukemia, the so called 'blood cancer'. Radiation damage from large doses may also bring quick death.

Even more disastrous is the effect of the radiation damages the cells of the reproductive organ's of either sex. Damage to the genes results in mutations which are very harmful. This damage can be transmitted to generation after generation in the form of birth defects and abnormalities. The effect of radiation including that of the x-ray exposure, is a cause of great concern. For this reason no one of the child-bearing age should be exposed to unnecessary radiation treatment of the reproductive organs. Children who are growing rapidly are more vulnerable to radiation. For this reason most good doctors are reluctant to prescribe x-ray exposure to children unless absolutely necessary.

20.8. Biological and Medical Uses of Radiation.

The application of radioactivity and radiation to human beings and other biological organisms is a vastly growing field.

These fall into two categories:

(i) Their use as tracers.

(ii) Their use as therapeutic agents in medicine and as sterilizing agents

(i) Radioactive Tracers.

In biological and medical research radioactive isotopes of different elements are widely used as tracers. First a given compound is artificially prepared using a radioactive isotope e.g $_6C^{14}$ or $_1H^3$. Such 'tagged' molecules containing the radioisotope are then administered to an organism in small doses and as they move or undergo a chemical reaction they are traced by a radiation detector which detects them by the radiation they emit.

In this way the details can be traced of how food molecules are digested; to what parts of the body of an organism they are diverted and how certain essential compounds are synthesized by the organism. Isotopes which are γ-rays emitters are the best for this application.

Tracer studies reveal that if a small quantity of radioactive iodine, $_{53}I^{131}$, is taken in the food, most of it deposits in the thyroid glands of man and animals. If radioactive calcium, $_{20}Ca^{45}$, is taken by man or animals orally or by injection, nearly 90% of it deposits in the bones of the young ones while only 40% in the old individuals. Sodium being an important constituent of the body fluids, the rate of flow of the blood, etc. in the body can be traced by giving radioisotope of sodium, $_{11}Na^{24}$.

In plants also the distribution of different minerals taken through the roots can be traced by using their suitable radioisotopes. The absorption of carbon dioxide, the seat of photosynthesis and the distribution of plant food prepared by photosynthesis can be traced by placing the plant in the atmosphere of carbon dioxide prepared from radioisotope of carbon $_6C^{14}$. By a technique called auto-radiography the position of the radioisotope is detected on the photographic film. In this technique the leaf is firmly placed on a photographic plate or film.

The film is darkened most strongly by the emitted radiations where the isotope is concentrated most densely.

Autoradiograph of a leaf of squash plant exposed to Co_2 containing radio active $_6C^{14}$ atoms, showing abundance of $_6C^{14}$ atoms in the blackened Photosynthetic green tissue region and their absence in the non-blackened region of the Veins

Fig.20.5.

The trace shows the distribution of carbohydrates produced in the leaf from the absorbed carbon dioxide (Fig.20.5)

(ii) Radiation Therapy and Diagnostics

The usefulness of x-rays in medical diagnosis is very well known. The rays can easily pass through less dense tissues of the flesh and the cracks or fractures of bones. However, they are stopped by denser parts like bones and to a less degree by tumors, etc. They can, therefore, give a shadow graph of any internal part of the body on a photographic film from which diagnosis of a crack or fracture in a bone or a tumor in the fleshy part can be made. The diagnostic usefulness of x-rays outweighs the small radiation hazard involved with the exposure. However, unnecessary frequent exposures must always be avoided.

(iii) Treatment of Cancer.

No doubt that radiation can cause cancer, it can also be used to treat it as it can destroy rapidly growing cancer cells. Some of the normal cells surrounding a cancerous tumor are inevitably killed causing side effects characteristic of radiation sickness.

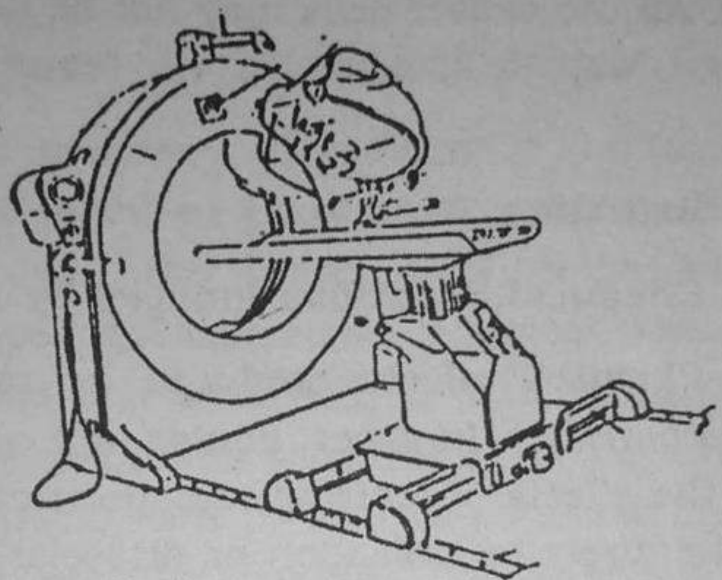


Fig.20.6 Cobalt Therapy Machine.

In treating a localized cancerous tumor a narrow beam of γ-rays from cobalt-60 or energetic x-rays is often used. The beam is directed at the tumor and the source is continuously rotated such that while the beam impinges on the tumor all the time, its surrounding region receives as low a dose as possible. The cancerous cells on the tumor can thus be destroyed to the relief of the patient without causing appreciable damage to the surrounding tissues. This is the so-called external therapy.

The other mode of treatment is called internal therapy. In some cases of internal therapy a tiny γ-ray source of very short half life may be inserted directly inside a tumor by surgical operation. It will kill majority of the cancer cells. This technique is sometimes used to treat cancer or of the thyroid glands with radioisotope of iodine, ${}_{53}I^{131}$. Alternatively the radioisotope ${}_{53}I^{131}$, is injected into the blood. As iodine has special affinity to the thyroid glands, it concentrates there, particularly in the cancerous region. The intense radiation from the radioisotope kills most of the defective cells. Similarly, cancerous affections in other parts of the organism may be treated by using the radioisotope of an element having special affinity to them.

The radiation treatment may increase the life-span of a cancer patient but it may not be completely effective. All the cancer cells may not be destroyed. Hence, there is always a likelihood of the recurrence of the disease.

20.9. Radiation Techniques in Other Fields

(I) Chemical Reactions induced by Radiations

Chemical effects produced by radiations are of great importance. In gases, liquids and covalently bonded solids the effects of ionising radiations can be attributed almost entirely to ionisation or dissociation of the molecules. Consequently a great variety of products may be obtained from organic compounds by radiolysis i.e. decomposition or dissociation by exposure to radiations.

Gases like hydrogen, carbon monoxide and carbon dioxide may be obtained by decomposition using radiation. Carbon dioxide and water molecules may synthesize in presence of chlorophyll into carbohydrates as in photo synthesis.By induced chain reactions many molecules of the same type may link together to give bigger molecules or polymers e.g. styrene and ethylene may polymerize into polystyrene and polyethylene respectively. The exchange of identical (or non-identical) radicals or groups may occur between two types of molecules mixed together. This can be tested by introducing radioactive ${}_6C^{14}$ atoms in one type, mixing the two types together and then separating them after sometime. If both types show radioactivity, exchange of radicals is confirmed.

Radiation-induced change of practical importance can be produced in the mechanical or physical properties of some polymers such as polyethylene which are due to cross linkages between the polymer chains. Irradiation of solids with fast neutrons or ions may alter many physical properties such as thermal and electrical conductivity, hardness and other mechanical properties.

(II) Radio Processes in Space, the Cosmic Rays.

After the discovery of radioactivity, a source of ionising radiations, it was found that radiation-detection instruments showed the presence of radiations even when not exposed to radioactive sources. This background effect was first attributed to natural radioactive substance in the earth. The shielding of the detection device oven with thick walls of lead could never eliminate the effect. Moreover, by removing the detection device several thousand metres from the ground, the effect increased manifold instead of decreasing. The conclusion was that a radiation of very high penetrating power is falling on the earth from outer space. This radiation was, therefore, named 'cosmic rays.

Early investigations were confined to the earth's surface or to low altitudes within earth's atmosphere where the radiations observed are, in fact, not the primary particles but the secondary radiations produced by the interaction of the primary particles with the atoms or molecules of the gases at the top of earth's atmosphere. These secondary radiations were found to consist of very high energy radiations comprising nearly all known elementary particles and the γ-rays.They are capable of penetrating lead more than a metre thick and are observed up to appreciable depths below water and ground.

mostly of protons, having very high energies extending up to 10^{18} e.V A small percentage of heavier nuclei is also present such as those of He (15%) and C, N, O (less than1%).

Most of the primary cosmic rays are believed to originate from the galaxies, shooting out constantly as showers in all directions and filling the space. They are accelerated to high energies by interstellar magnetic fields. During the solar flares the sun contributes significantly to the low energy (mostly< 1 GeV) cosmic rays arriving the earth. During intense sun-spot activity the galactic cosmic-ray flux reaching the earth decreases a bit, which probably, is a magnetic effect.

(iii) Uses of radiation

(i) Polymerisation

Many plastics, synthetic rubbers, synthetic textile fibres,etc. much as polyethylene, polystyrene, polyester, so commonly used today, are composed of the polymers of certain simpler molecules. The important characteristic of a polymer is its long chain of carbon atoms to which various chemical groups are attached. As hinted to earlier, radiations can split long chains of molecules or establish cross-linkages between smaller chains. Breaking of chains shortens the average molecular length whereas cross-linkage increases it and enhances the physical properties like hardness, mechanical strength, temperature resistancy, surface texture and finish, transparency, etc. By proper treatment the radiation polymers with improved characteristics can be synthesized from the parent molecules or their polymers of lower order. Polymerisation by radiation has thus become an important process in industry for the manufacture of many useful materials.

(ii) Sterilization and Food Preservation

Another useful application of radiation is for the sterilization of food and medical supplies. Since radiation kills or deactivates bacteria, medical supplies such as bandages, syringes, needles, surgeon's gloves, catheters, surgical instruments and other hospital equipments can be irradiated with a strong dose to sterilize them, instead of using old-fashioned high temperature treatment or antiseptic medicines. For the preservation of foods, fresh or seasoned, they can be irradiated with appropriate high dose before or after vacuum packing to kill the bacteria responsible for spoilage. Similarly, milk can be pasteurised and drinking water of town supplies freed from harmful germs by irradiation with suitable doses of x-rays or γ-rays.

(iii) Gauging and Control

In industry when a material such as paper, cloth or metal is manufactured in sheet form by a continuous process its thickness can be measure and controlled by a beta-ray thickness gauge without interrupting the production. A beta-ray source is fixed under the moving sheet and a detection device above it which is connected to an amplifying recording device. The reading of the detection device is related to the thickness of the sheet, which may be read all the time on a calibrated dial and hence it can be measured and controlled during the entire production. Alternatively, an automatic system can be installed to maintain the constancy of any desired thickness of the sheet.

(iv) Radiography

For the detection of porosity, cavity or other infirmities in a metal casting and cracks or imperfections of welded joints in engineering and ship-building industries, radiographic examination can be made. X-rays are widely used for this purpose. This is simply an extension of the use of x-rays for diagnostic purposes of the human body discussed in the preceding section.

(v) Radiation Methods in Archaeology

Small amounts of the radioisotope of carbon, $_6C^{14}$, are produced in the carbon dioxide molecules in the upper atmosphere by cosmic-ray neutrons. The half-life of C^{14} is about 5730 years. Assuming that the cosmic ray intensity has not changed during the past 20,000 to 50,000 years, the rate of formation an decay of C^{14} now must nearly be equal and the abundance of C^{14} and C^{12} atoms should be constant throughout the atmosphere. Carbon dioxide circulating through the atmosphere is absorbed by plants. From the plant kingdom it goes to the bodies of men and animals as food. Hence all living things constantly absorb some radioactive C^{14} and are slightly radioactive, the abundance of C^{14} in their bodies being the same as in the atmosphere as long as they are alive. The ratio of C^{14} to C^{12} atoms in plants was found to be 1.5×10^{-12} before the use of nuclear weapons. When plants and animals die the absorption of C^{14} stops while the disintegration of C^{14} in them continuous and their activity goes on decreasing. If the activity of C^{14} in a dead body, such as wood, bone or a fossil, can be measured it can provide a clue to the age of the specimen i.e. the time elapsed after its death. This method of finding the age of a specimen by the C^{14} method is called 'radiocarbon dating which has now become the most powerful tool for the archaeologists and geologists. However, the nuclear explosions and accidents have definitely disturbed the constant activity of C^{14} in the atmosphere and the living things and may do so in future too, rendering this method of dating rather unreliable. Many other systems of dating also exist which make use of other radioactive isotopes of elements viz. U^{238}, Cl^{36}, etc.

(vi) Activation Analysis

The detection and estimation of an element in a mixture is sometimes nearly impossible if it is present in very minute traces or if its chemical properties are very similar to those of the other elements in the mixture. A technique developed in recent years, called activation analysis' is found to be very effective for this purpose.

It is seen that if a mixture containing different elements is subjected to the thermal neutrons inside a nuclear reactor for appropriately chosen lengths of time, some atoms of each element present in the mixture become radioactive beta-emitters. The β-emission from them is usually followed by γ-radiation. The activated atoms of different elements emit γ-rays of different energies from which they can be identified even in concentrations as low as 1 part in 10^5. The energies of the emitted γ-rays can be measured by a γ-ray spectrometer. This method is called neutron-activation analysis'. It has proved to be of great use in the analysis of archaeological and geological objects. Occasionally, charged particles such as protons, and deuterons are also used for activation and investigation carried out by the emitted β-rays with a Geiger Counter.

QUESTIONS

20.1 Describe as many points of difference between α-, β-, and γ- rays as possible.

20.2 Explain how you would test whether the radiations from a radicactive source α-, β-or γ- radiations with the simplest equipment.

20.3 It said that an α - or β- particle carries an atom without colliding with its electrons. How can each do so ? Explain.

20.4 In how many ways can γ-rays produce ionisation of the atoms? Explain.

20.5 In what way does a neutron produces ionisation of an atom?

20.6 Name the different electromagnetic radiations which are capable of producing ionisation of atoms. By what process do they usually ionise?

20.7 Why is lead a better shield against α-, β- and γ-radiations than an equal thickness of water column?

20.8. Lead is heavier and denser than water, yet what is a more effective shield against neutrons? Explian why?

20.9. In an x-ray photograph bones show up very clearly while the fleshly part shows very faintly. Why?

20.10. In a cloud chamber photograph the path of an α-praticles is a thick and continuous line whereas that of a β-particle is a thin and broken line. Explain why?

20.11. Why do γ-rays not give a line-track in the cloud chamber photograph?

20.12. A neutron can produce little ionisation. Is there any sure chance of getting a cloud-chamber track for it or a count in the Geiger-counter?

20.13. A cloud chamber photograph of an α-particle is usually straight. Sometime as abrupt bend accompanied by a small branched track, the so-called forked track is obtained near the end. What could possibly be the cause of this forked track?

20.14. Why is the recommended maximum dose for radiation a bit higher for women beyond the child bearing age that for younger women?

20.15. it is possible for a man to burn his hand with x-or γ-rays so seriously that he must have it amputated and yet may suffer no other consequence. However, a whole-body x- or γ-ray overexposure so slight as to cause no detectable damage might cause birth deformity in one of his subsequent children. Explain why?

20.16. Which of the rays ----- α,β,or γ would you advise for the treatment of:

(i) Skin cancer?

(ii) The cancer flesh just under the skin?

(iii) A cancerous tumor deep inside the body ? Give reasons.

20.17 Two radioisotopes of an element are available one of long half-life and the other of short half-life. Which isotope is advisable for the treatment of patients and why?

20.18 Why are many artificially prepared radioisotopes of elements rare in nature.

20.19. Can radiocarbon dating be used to measure the age of stone-walls of the ancient civilizations?

Some Common Conversions

Length

1 m = 3.281 ft = 39.37 in; 1 cm = 0.3937 in

1 km = 1000 m = 0.6214 mi

1 ft = 30.48cm; 1 in = 2.540 cm

1 yd = 0.9144m

1 mi = 5280 ft = 1.609 km

Area

$1 cm^2 = 0.155 m^2$

$1 m^2 = 10^1 cm^2 = 10.76 ft^2$

$1 in^2 = 6.452 cm^2$

$1 ft^2 = 929.0 cm^2 = 0.09290 m^2$

Volume

$1 m^3$ = 1000 liters = $10^8 cm^3 = 1.308 yd^3 = 35.31 ft^3$

1 liters = $1000 cm^3 = 0.001 m^3 = 61.03 1n^3 = 0.0353 ft^3$

$1 ft^3 = 0.02832 m^3$ = 7.481 gallons = 28.32 liters

Time

1 day = 86,400 s; 1yr = 3.156×10^2s

Velocity

1 m/s = 3.281 ft/s = 3.6 km/h

1 km/h = 0.2778 m/s = 0.6214 mi/h = 0.9113 ft/s

1 mi/h = 1.609 km/h = 0.447 m/s = 1.467 ft/s

Acceleration

$1 m/s^2 = 100 cm/s^2 = 3.281 ft/s^2$

$1 ft/s^2 = 30.48 cm/s^2 = 0.3048 m/s^2$

Mass

1 kg = 1000 g = 0.0685 slug

1 slug = 14.59 kg = 32.17 lb mass

1 metric ton = 1000 kg

Density

$1 g/cm^3 = 1{,}000 kg/m^3 = 1.940 slug/ft^3$
$= 62.43$ lb-mass/ft^3

1 lb-mass/ft^3 = 0.0311 slug/ft^3 = 16.02 kg/m^2
$= 0.01602 g/cm^3$

Force

1 N = 10^5 dyn = 0.2248 lb

1 lb = 4.45 N = 4.45×10^6 dys

1 ton = 2000 lb

Pressure

1 N /m^2 = 1.451×10^{-4} 1h/in^2 = 0.200 1b/ft^2

1 lb/in^2 = 6.89×10^3 N/m^2 = 6.89×10^4 dyn/cm^2

1 atm = 76 cm Hg = 760 torr = 14.70lb/in^2
= 1.013×10^5 N/m^2
= 1.013 bar = 1.013×10^5 dyn/cm^2

Work and Energy

1 J = 10^7 ergs = 0.239 cal = 0.7376 ft-lb

1 ft-lb = 1.356 J

1 cal = 4.184 J = 3.068 ft-lb

1 Btu = 252 cal = 778 ft-lb = 1054 J

1 kilowatt-h [kWh] = 3.60×10^5 J

1 eV = 1.60×10^{-19} J

Power

1 W = 1 J/s = 0.738 ft-lb/s

1 hp = 0.746 kW = 550 ft-lb/s

1 Btu/h = 0.293 W

Specific Heat and Latent Heat

1 cal/g-C^0 = 4.184 J/g = 4184 J/kg-C^0

1 cal/g = 4.184 J/g = 4184 J/kg = 1.80 Btu/kg-C^0

R = 8.314 J/mol-K = 1.99 cal/mol-K
= 0.0621 (atm-l/mol-K)

Multiples, Submultiples, and Prefixes

(Applicable to All SI Units)

Multiples and Submultiples	Prefixes	Symbols
1 000 000 000 000 $= 10^{12}$	tera	T
1 000 000 000 $= 10^{9}$	giga	G
1 000 000 $= 10^{6}$	mega	M
1 000 $= 10^{3}$	kilo	k
100 $= 10^{2}$	hecto	h
10 $= 10^{1}$	deka	da
base unit: 1 $= 10^{0}$		
0.1 $= 10^{-1}$	deci	d
0.01 $= 10^{-2}$	centi	c
0.001 $= 10^{-3}$	milli	m
0.000 001 $= 10^{-6}$	micro	μ
0.000 000 001 $= 10^{-9}$	nano	n
0.000 000 000 001 $= 10^{-12}$	pico	p