8 Probability Distributions

$0 \le P(A) \le 1$

8.1 Introduction

Whenever we talk about random experiments, there is the need to associate a numerical value with each of their outcomes, in order to study them. As a result, two types of variables arise.

i. Discrete random variable. ii. Continuous random variable.

A discrete random variable almost always arises in connection with counting and a continuous random variable is one whose values are typically obtained by measurements.

In case of a discrete random variable, its probability distribution describes how much of the probability is placed on each of its possible values with the total of all these probabilities equal to 1. The probability distribution of a discrete random variable is usually called its probability mass function.

In case of a continuous random variable, we cannot talk about the probability on a point instead we talk about the probability on any interval of the values the random variable takes, with the total of the probabilities equal to 1. The probability distribution of a continuous random variable is called its probability density function.

The probability distribution of a discrete random variable is usually written with the help of a function, called its formula or it can be described with the help of a two column table (like frequency distribution) where one column gives the values (intervals of values in case of continuous random variables) and the other column gives the probabilities.

8.2 Probability Mass Function

As the value of a discrete random variable is determined by the outcome of a random experiment, one can associate with each possible value of the discrete random variable a probability that a random variable will take on that value. The probability mass function of a discrete random variable Y describes the values of Y and the probability associated with each value of Y. Usually, it is written in a two column table where one column gives the values of the random variable Y and the other column gives the probabilities associated with each value.

Example 8.1: A coin is tossed three times and let the random variable Y denote the number of upturned heads. Find the probability mass function?

Solution: There are 8 possible outcomes. The possible outcomes and the values assigned to them (according to the number of upturned heads) are given in the adjoining table:

Table A

Outcomes	Value of Y
H H T	2
H T H	2
H T T	1
T H H	2
T H T	1
T T H	1
T T T	0
H H H	3

It is clear that Y takes the values 0,1,2,3 because in three tosses of a coin (or three coins are tossed at one times) there are 8 possible columns and their detail is

No head (all tails) (T T T)
One head (H T T, T H T, T T H)
Two heads (H H T, H T H, T H H)
Three heads (H H H)
No head (all tails) can occur only once so, the probability of no head is 1/8 according to the definition of probability.

One head can occur 3 times so, the probability is 3/8. Two heads can occur 3 times so the probability is 3/8. Three heads can occur once so the probability is 1/8.

We can write $P(Y=0)$, $P(Y=1)$, $P(Y=2)$, $P(Y=3)$ read as probability of Y equals to no head, one head, two heads and three heads respectively. So, the probability function is given in the adjoining table:

Y	$P(Y=y)$
0	1/8
1	3/8
2	3/8
3	1/8

8.3 Probability Density Function

The probability density function of a continuous random variable Y is specified by a smooth curve such that the total area under the curve is unity. The probability that Y falls in any particular interval is the area under the curve against the interval.

Consider the weight (in kilograms) of a student in a class of 30 students taking Statistics. The weights measured to the nearest hundred of a kilogram are 60.50, 60.80, 55.40, 53.70, 50.75,...., 45.00 and 49.80. The minimum weight is 45.00 and maximum is 60.80. In this situation the probability histogram approaches a smooth curve. The area under the curve is unity and it cannot go below the horizontal scale. The probability that the weight is between 55 and 57 kg is the area under the curve and above this interval.

Let a and b be two numbers and Y is the random variable. Define the following events:

i) $a < Y < b$ is the event that the value of Y is between a and b.

ii) $Y < a$ is the event that the value of Y is less than a.

iii) $Y > b$ is the event that the value of Y is greater than b.

8.4 Simple Univariate Discrete And Continuous Distributions

The probability distributions of the discrete random variables are represented in a tabular form by the values of the random variable and the corresponding probabilities. For example, when a die is thrown then each upturned face (1,2,3,4,5 and 6) has the same probability of 1/6 of its occurrence. Thus the probability distribution in the tabular form is given in the adjoining table.

Y	$P(y)$
1	1/6
2	1/6
3	1/6
4	1/6
5	1/6
6	1/6

This probability distribution can be expressed in the form of the following formula such that the probabilities $P(Y=y)$ can be expressed by the function $f(y)$

$f(y) = 1/6$ for $y=1,2,3,\ldots,6$

$= 0$ otherwise

This is probability distribution for the number of upturned points when a die is thrown. This is known as discrete uniform distribution. It should be noted that every function defined for the values of a random variable cannot serve as a probability distribution unless it satisfies the condition given under 8.4.1.

The simplest form of the continuous distributions is the continuous uniform distribution prepared by.

$$f(y) = 1/b\,(b-a) \qquad a < y < b \qquad (8.1)$$

Note that y takes the values between a and b.

If $a = 0$ and $b = 1$ then

$f(y) = 1 \qquad 0 < y < 1$

$= 0$ otherwise.

This function is shown in figure 8.1.

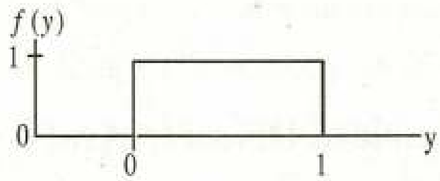


figure 8.1: - Continuous uniform distribution

The area under the probability density function is also 1. So, the width of rectangle is unit interval with height 1. To calculate the probability, area under the curve can be calculated which would be the required probability. Very often these distributions are based on the empirical evidence or prior knowledge.

It should be clear that in case of discrete distributions, the probability of an event is obtained by inserting the value of the random variable in the probability function but in case of continuous distributions, the probability is obtained by calculating the area under the curve and above the interval on which the event is defined.

It is to be noted that P probability of Y greater than P or equal to a and less than or equal to b written as $P(a \leq Y \leq b)$ is not equal to $P(a < Y < b)$ in case of discrete distributions because the probabilities at a and b are not included. In case of continuous distributions these probabilities are equal because area at a point is always zero so the probability at a and at b is always zero i.e.,

zero. So, the probability at a and at b is always zero i.e.,

8.4.1 Properties of Probability Mass Function And Probability Density Function

Probability Mass Function

Let Y be a discrete random variable and $P(y)$ be its probability mass function. The $P(y)$ must satisfy the following two conditions

i) $0 \leq P(y) \leq 1$ for each possible value of Y.

i.e., the probability is a number between 0 and 1.

ii) $\Sigma P(y) = 1$

i.e., the summation over probabilities for all possible values y of the random variable Y should add up to 1.

Example 8.2: A committee of size 3 is to be selected at random from 3 women and 5 men. Obtain a probability distribution for the number of women selected for the committee.

Solution:

No. of Women	No. of Men	Total
3	5	8

Number of women = 3 Number of Men = 5 so total = 8

Number of selected persons =3, So total number of sample points = $\binom{8}{3}$ = 56

Let X be the number of women in the committee, these can be 0,1,2,3 and their respective probabilities are

$$P\text{ (no woman)} = \frac{\binom{3}{0}\binom{5}{3}}{\binom{8}{3}} = \frac{10}{56},\quad P\text{ (one woman)} = \frac{\binom{3}{1}\binom{5}{2}}{\binom{8}{3}} = \frac{30}{56}$$

$$P\text{ (two women)} = \frac{\binom{3}{2}\binom{5}{1}}{\binom{8}{3}} = \frac{15}{56},\quad P\text{ (three women)} = \frac{\binom{3}{3}\binom{5}{0}}{\binom{8}{3}} = \frac{1}{56}$$

The probability distribution is given by:

x	$P(x)$
0	$\frac{10}{56}$
1	$\frac{30}{56}$
2	$\frac{15}{56}$
3	$\frac{1}{56}$

Example 8.3: From an urn containing 4 red and 6 white round marbles. A man draws three marbles at random without replacement. If X is a random variable which denotes the number of red marbles drawn, what is the probability distribution of X?

Solution:

Number of red marbles $= 4$ Number of white marbles $= 6$ total marbles = 10

$$\text{Number of sample points} = \binom{10}{3} = 120$$

If the random variable X denotes the number of red marbles, the possible values of X are 0, 1, 2, 3 with their respective probabilities as:

$$P(X=0)=\frac{\binom{4}{0}\binom{6}{3}}{\binom{10}{3}}=\frac{5}{30},\quad P(X=1)=\frac{\binom{4}{1}\binom{6}{2}}{\binom{10}{3}}=\frac{15}{30}$$

$$P(X=2)=\frac{\binom{4}{2}\binom{6}{1}}{\binom{10}{3}}=\frac{9}{30},\quad P(x=3)=\frac{\binom{4}{3}\binom{6}{0}}{\binom{10}{3}}=\frac{1}{30}$$

The probability distribution of red marbles in a tabular form is:

X	0	1	2	3
$P(x)$	5/30	15/30	9/30	1/30

Example 8.4: Given the discrete probability mass function: -

$$P(x)=\binom{4}{x}\left(\frac{1}{2}\right)^{x}\left(\frac{1}{2}\right)^{4-x}\qquad \text{for } x=0,1,2,3,4$$

Find probability distribution.

Solution:

$$P(x)=\binom{4}{x}\left(\frac{1}{2}\right)^{x}\left(\frac{1}{2}\right)^{4-x}\qquad \text{For } x=0,1,2,3,4$$

$$P(X=0)=\binom{4}{0}\left(\frac{1}{2}\right)^{0}\left(\frac{1}{2}\right)^{4}=\frac{1}{16},\quad P(X=1)=\binom{4}{1}\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)^{3}=\frac{4}{16}$$

$$P(X=2)=\binom{4}{2}\left(\frac{1}{2}\right)^{2}\left(\frac{1}{2}\right)^{2}=\frac{6}{16},\quad P(X=3)=\binom{4}{3}\left(\frac{1}{2}\right)^{3}\left(\frac{1}{2}\right)=\frac{4}{16}$$

$$P(X=4)=\binom{4}{4}\left(\frac{1}{2}\right)^{4}\left(\frac{1}{2}\right)^{0}=\frac{1}{16}$$

The probability distribution of X in tabular form is:

X	0	1	2	3	4
$P(x)$	1/16	4/16	6/16	4/16	1/16

Example 8.5: A random variable X has the following probability distribution:

X	−2	−1	0	1	2	3
$P(x)$	0.1	k	0.2	$2k$	0.3	$3k$

Find:

i) k ii) $P(X < 2)$ iii) $P(X = 2)$ iv) $P(-2<X<2)$ v) $P(X < 1)$.

Solution: Since $\sum_{x=-2}^{3} P(x) = 1$ gives

$0.1 + k + 0.2 + 2k + 0.3 + 3k = 1$ or $0.6 + 6k = 1$ or $6k = 1 - 0.6 = 0.4$

so that $k = \frac{0.4}{6} = \frac{4}{60} = \frac{1}{15}$

$P(x < 2) \quad = P(x = -2) + P(x = -1) + P(x = 0) + P(x = 1)$

$$= 0.1 + k + 0.2 + 2k = 0.3 + 3k = 0.3 + 3\left(\frac{1}{15}\right) = 0.3 + 0.2 = 0.5$$

$P(x \geq 2) \quad = P(x = 2) + P(x = 3) = 0.3 + 3k = 0.3 + 3\left(\frac{1}{15}\right) = 0.3 + 0.2 = 0.5$

$P(-2 < x < 2) = P(x = -1) + P(x = 0) + P(x = 1) = k + 0.2 + 2k = 0.2 + 3k$

$$= 0.2 + 3\left(\frac{1}{15}\right) = 0.2 + 0.2 = 0.4$$

$P(x \leq 1) \quad = P(x = -2) + P(x = -1) + P(x = 0) + P(x = 1)$

$$= 0.1 + k + 0.2 + 2k = 0.3 + 3k = 0.3 + 3\left(\frac{1}{15}\right) = 0.8 + 0.2 = 0.5$$

Example 8.6: Check whether the function given by

$$f(y) = \frac{(y+1)}{14} \qquad \text{for } y = 1, 2, 3, 4$$

$$= 0 \quad \text{otherwise.}$$

is a probability function?

Solution: Here,

$$f(1) = 2/14,\ f(2) = 3/14,\ f(3) = 4/14 \text{ and } f(4) = 5/14$$

Since the values are all non negative and add up to 1 as

2/14+3/14+4/14/+5/14=1

So, both conditions are satisfied, concluding thereby that the function is a probability function.

Probability Density Function:

Let Y be a continuous random variable and $f(y)$ be its probability density function. Then $f(y)$ must satisfy the following conditions.

i) $f(y) \geq 0$ for all y ii) $P(-\infty < Y < \infty) = 1$

It means that the total area under the curve should be 1.

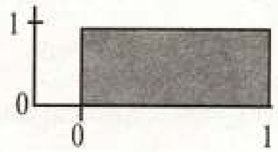
Of course, not every function defined for the values of a random variable can serve as a probability distribution unless it satisfies the above two conditions.

Example 8.7: Verify whether the function

$f(y) = 1, \quad 0 < y < 1$

$= 0$ otherwise

is a density function?



Solution: It is density function of a continuous random variable because y takes all values between 0 and 1. To calculate area we have width of rectangle as 1 and height of rectangle as 1, so

$$\text{Area} = (\text{width})\ (\text{height})$$

$$= (1)\ (1) = 1$$

The function is also positive, thus both conditions are satisfied and we conclude that the function is a density function.

8.4.2 Applications

Once the probability distribution for a random variable has been defined, very often, it becomes easier to calculate the probabilities. In case of discrete random variables, probabilities of the events are obtained by adding the corresponding probabilities but in case of continuous random variables the probability that a random variable falls in a certain interval is computed by calculating the area above that interval.

Example 8.8: The following table gives the probability distribution for the number of courses enrolled during spring semester 1995 by 50 M.Sc. Statistics students.

Y	1	2	3	4	5	6	7
P(*y*)	.02	.03	.16	.40	.25	.16	.05

i) Find the probability that

a) A student enrolled 3 courses?

b) A student enrolled less than 3 courses?

c) A student enrolled atleast 4 courses?

d) A student enrolled at the most 4 courses?

e) A student enrolled between 4 and 6 courses inclusive.

i.e., $P(4 \leq Y \leq 6)$

f) Student enrolled between 4 and 6 courses? (exclusive)

Solution:

a) The probability that a student enrolled three courses is
$P(Y=3) = 0.16$

b) The probability that a student enrolled less than 3 courses means by probability that he enrolled 1 course or 2 courses i.e.,

$P(Y = 1) + P(Y = 2) = 0.02 + 0.03$

$= 0.05$

c) The probability that a student enrolled atleast four courses means the probability of 4 or more courses i.e.,

$P(Y = 4) + P(Y = 5) + P(Y = 6) + P(Y = 7)$

$= 0.40 + 0.25 + 0.16 + 0.05$

$= 0.86$

d) $P(Y \leq 4) = P(Y = 1) + P(Y = 2) + P(Y = 3) + P(Y = 4)$

$=.02+0.3+0.16+.40=0.61$

e) $P(4\leq y\leq 6) = P(Y = 4) + P(Y = 5) + P(Y = 6)$

$= 0.40 + 0.25 + 0.16 = 0.81$

f) Between 4 and 6 there is only one number i.e., 5, so

$P(Y = 5) = 0.25$

Example 8.9: The amount of time (minutes) taken by a doctor to attend a patient is between 5 to 10 minutes. If we assume that the distribution followed is uniform, then calculate Probability that doctor

i) Takes between 6 and 8 minutes

ii) Less than 8 minutes.

Solution: Let Y denote the random variable. The amount of time taken by a doctor to attend a patient.

Here the value of $a = 10$, $b = 5$, so probability distribution is

$f(y) = 1/(b-a) \qquad a< y< b$

$= 1/5 = 0.2 \qquad 5<y<10$

i) The $P(6<y < 8)$ is the area between 6 and 8 and is shown in figure 8.2. Width is 8-6 = 2 and height is 0.2, so

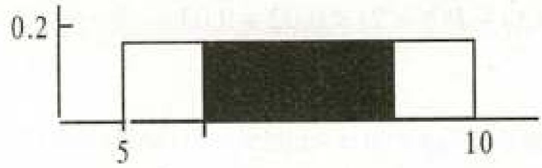


Fig. 8.2 The area between 6 and 8.

$P(6<Y<8)$ = (width of rectangle).(height)

$(2).(0.2) = 0.4$

ii) $P(Y<8) = P(5<Y<8)$

= (width of rectangle). (height)

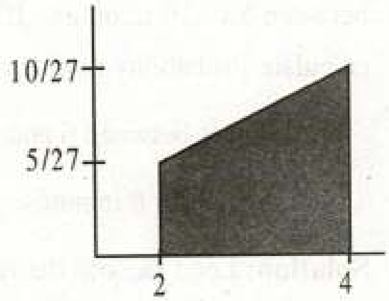
$= (3).(0.2) = 0.6$

Example 8.10: A continuous random variable X that can assume values between 2 and 5 has a density function given by: $f(x) = \frac{2(1+x)}{27}$

Find

i) $P(x<4)$ ii) $P(3 \leq x \leq 4)$.

Solution: We are given that:



i) $P(X<4)$?

$$f(x) = \frac{2(1+x)}{27}, \quad 2 \leq x \leq 5$$

$$f(2) = \frac{2\,(1+2)}{27} = \frac{6}{27}$$

$$f(4) = \frac{2(1+4)}{27} = \frac{10}{27}$$

Base $= 4-2 = 2$

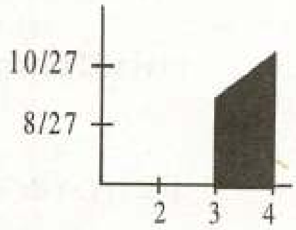
$$P(X<4) = \frac{\text{(Sum of parallel sides)}}{2} \times \text{Base}$$

$$= \frac{f(2) + f(4)}{2} \times \text{Base}$$

$$= \frac{\left(\frac{6}{27} + \frac{10}{27}\right)}{2} \times 2 = \frac{16}{27}$$

(ii) $P(3 \le X \le 4)$?

$$f(3) = \frac{2(1+3)}{27} = \frac{8}{27}$$



$$f(4) = \frac{2(1+4)}{27} = \frac{10}{27}$$

Base = 4 − 3 = 1

$$P(3 \le x \le 4) = \frac{f(3) + f(4)}{2} \times \text{Base}$$

$$= \frac{\left(\frac{8}{27} + \frac{10}{27}\right)}{2} \times 1 = \frac{18}{54} = \frac{1}{3}$$

Example 8.11:

i) A continuous random variable X has a density function $f(x) = 2x$ when $0 \le x \le 1$ and zero otherwise. Find

(a) $P\left(X < \frac{1}{2}\right)$, (ii) $P\left(\frac{1}{4} < X < \frac{1}{2}\right)$.

ii) If $f(x)$ has probability density kx^2, $0 < X < 1$, determine k and find the probability that $\frac{1}{3} < X < \frac{1}{2}$

Solution:

i) $f(x) = 2x,\ 0 \le X \le 1,$

$= 0,$ otherwise.

a) $P\left(X < \frac{1}{2}\right) = \int_0^{1/2} f(x)\,dx = \int_0^{1/2} 2x\,dx = 2\left[\frac{x^2}{2}\right]_0^{1/2} = \left[\left(\frac{1}{2}\right)^2 - 0\right] = \frac{1}{4}$

b) $P\left(\frac{1}{4} < x < \frac{1}{2}\right) = \int_{1/4}^{1/2} f(x)\,dx = \int_{1/4}^{1/2} 2x\,dx = 2\left[\frac{x^2}{2}\right]_{1/4}^{1/2}$

$$= \left[\left(\frac{1}{2}\right)^2 - \left(\frac{1}{4}\right)^2\right] = \left[\frac{1}{4} - \frac{1}{16}\right] = \frac{3}{16}$$

ii) $f(x)$ will be a probability density function, if $\int_{1/4}^{1/2} f(x)\,dx = 1$, i.e.,

$$1 = \int_0^1 f(x)\,dx = \int_0^1 kX^2\,dx = k\left[\frac{x^3}{3}\right]_0^1 = k\left[\frac{1}{3} - 0\right] = \frac{k}{3}$$

So $1 = k/3 \Rightarrow k = 3$

Hence the probability density function $f(x) = 3x^2$, $0 < X < 1$

Now

$$P\left(\frac{1}{3} < x < \frac{1}{2}\right) = \int_{1/3}^{1/2} f(x)\,dx = \int_{1/3}^{1/2} 3x^2\,dx = 3\left[\frac{x^3}{3}\right]_{1/3}^{1/2}$$

$$= \left[\left(\frac{1}{2}\right)^3 - \left(\frac{1}{3}\right)^3\right] = \left[\frac{1}{8} - \frac{1}{27}\right] = \frac{19}{216}$$

Example 8.12: A continuous random variable x has probability density function. $f(x) = cx$ for $0 < X < 2$. Determine

i) c, ii) $P(1 < X < 1.5)$ iii) $P(X < 1.5.)$

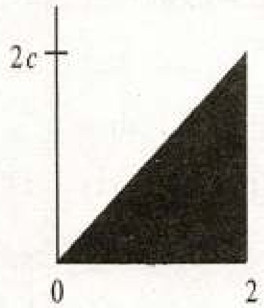
Solution: $f(x) = cx$, $0 < X < 2$

i) We know that Area $= \frac{f(a) + f(b)}{2} \times$ Base $= 1$

$f(0) = 0$, $f(2) = 2c$

Base $= 2 - 0 = 2$

$\frac{\text{base} \times \text{height}}{2}$



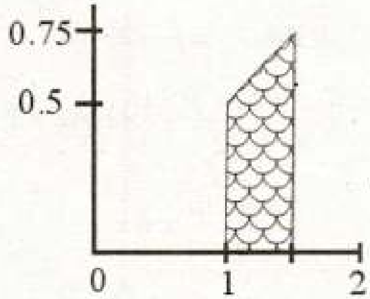
$$\frac{f(0) + f(2)}{2} \times \text{Base} = 1$$

$$\Rightarrow \frac{0 + 2(c)}{2} \times 2 = 1$$

$$\Rightarrow 2c = 1 \quad \text{or} \quad c = \frac{1}{2}$$

$$f(x) = \frac{1}{2}x, \; 0 < X < 2$$

ii) $P(1 < X < 1.5) = ?$



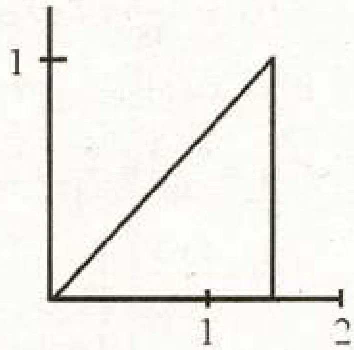
$$f(1) = \frac{1}{2} = 0.5, \;\; f(1.5) = \frac{1.5}{2} = 0.75$$

$$\text{Base} = 1.5 - 1 = 0.5$$

$$P(1 < X < 1.5) = \frac{f(1) + f(1.5)}{2} \times \text{Base}$$

$$= \frac{0.5 + 0.75}{2} \times 0.5 = 0.3125$$

iii) $P(X < 1.5) = ?$



$$f(0) = 0, \;\; f(1.5) = \frac{1.5}{2} = 0.75$$

$$\text{Base} = 1.5 - 0 = 1.5$$

$$P(X < 1.5) = \frac{f(0) + f(1.5)}{2} \times \text{Base}$$

$$= \frac{0 + 0.75}{2} \times 1.5 = 0.5625$$

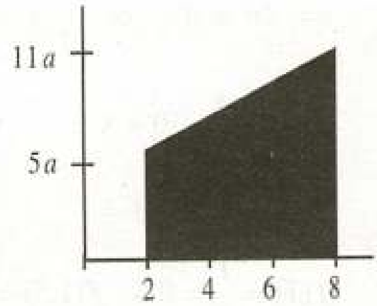
Example 8.13: A continuous random variable X, which can assume values between 2 and 8 inclusive has a density function given by $a\,(x + 3)$ where a is a constant then find:

i) a ii) $P\,(3 < X < 5)$

Solution: $f(x) = a\,(x + 3)$ $2 < X \leq 8$

i) We know that

$$\text{Area} = \frac{f(a) + f(b)}{2} \times \text{Base} = 1$$



$f(a) = f(2) = 5a$

$f(b) = f(8) = 11a$

Base $= 8 - 2 = 6$

$$\frac{f(2) + f(8)}{2} \times \text{Base} = 1$$

$$\frac{5a + 11a}{2} \times 6 = 1$$

or $48\,a = 1$

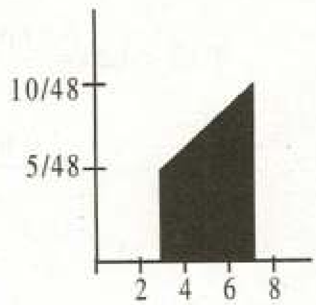
or $a = \dfrac{1}{48}$

$$\therefore f(x) = \frac{x+3}{48},\ 2 < X \leq 8$$

(ii) $P\,(3 < X < 5) = \ ?$

$$f(3) = \frac{3+3}{48} = \frac{6}{48}$$

$$f(5) = \frac{5+3}{48} = \frac{8}{48}$$



$$P(3 < X < 5) = \frac{[f(3) + f(5)]}{2} \times (5 - 3)$$

$$= \frac{\left(\frac{6}{48}+\frac{8}{48}\right)}{2} \times 2 = \frac{14}{48}$$

8.5 Drawing of Probability Mass Function and Probability Density Function

The Probability Function: The probability functions can be presented graphically in the following two ways:

i) Probability Histogram

To draw probability histogram, we take values of the random variable along x–axis and probabilities along y-axis. Adjacent rectangles are drawn against each value such that the height of each rectangle is equal to the probability at that point and the width of each rectangle is one taking 0.5 units to the left of value and 0.5 units to the right of value. Since, the width is one unit so the area of the rectagles is equals to their probabilities. The advantage of drawing probability histogram is that the discrete probability distribution can be approximated by a continuous curve.

Example 8.14: Consider the following probability distribution and draw a probability histogram.

Y	$P(Y = y)$
0	0.08
1	0.26
2	0.34
3	0.23
4	0.09

To construct a probability histogram, the first step is to mark values of the random variable i.e., 0,1,2,3 and 4 on x-axis as mid points. The second step is to draw adjacent rectangles of width 1 on each point going half way to each side from the mid points and with heights such that the heights represent the corresponding probabilities of 0.08, 0.26, 0.34, 0.23 and 0.09 respectively. It is shown in figure 8.3

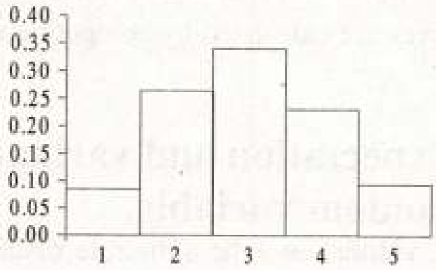


Figure 8.3: Probability Histogram

ii: Bar Chart

A bar chart is drawn with the values of the random variable along x-axis and probabilities along y-axis. The height of each bar equals the probability of the corresponding value.

Data of the above example is taken to explain the procedure. The values of the random variable are 0, 1, 2, 3 and 4. As a first step these are taken along x-axis. The second step is to draw bars against these points with a height equal to the corresponding probability. The probability corresponding to 0 is 0.08 so a bar is drawn on 0 with height 0.08: a bar of height 0.26 is drawn on point 1; a bar of height 0.34 is drawn on point 2; a bar of height 0.23 is drawn on point 3 and a bar of height 0.09 is drawn on point 4. It is shown in figure 8.4

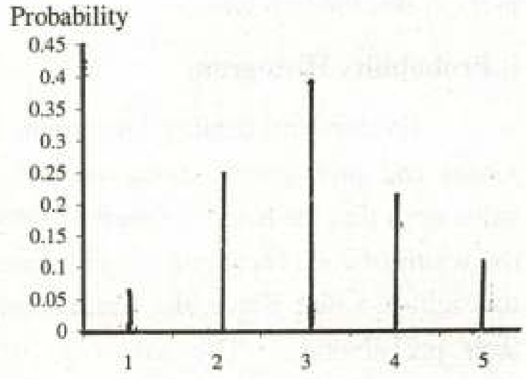


Fig 8.4: Bar Chart

Probability Density Function:

If values of the random variable are very close to each other then the probability histogram of the discrete random variable can be approximated by a smooth curve. So, the probability corresponding to each interval on x-axis will be the area under the curve. This is the situation in case of continuous random variable. So, the probability density functions are presented by smooth curves and the probabilities of the curves are calculated by computing area under the curves corresponding to the events.

8.6 Expectation and variance of the simple discrete random variable.

Expected value: Let Y be a discrete random variable with probability function $P(y)$. The mathematical expectation or expectation of the discrete random variable Y. denoted by $E(Y)$ is defined by:

$$E(Y) = \Sigma\, y\, P(y), \text{ sum is over all values of } Y \qquad (8.2)$$

This expectation of a random variable Y is the mean of its probability distribution i.e., $E(Y)$ is an alternative notation for the population mean μ. Similarly, $E(Y^2) = \Sigma\, y^2 P(y)$, sum over all y. (8.3)

Variance:

The variance of a random variable Y is the variance of the probability distribution and is

$$\sigma^2 = E(Y - \mu)^2 = \Sigma\,(Y - \mu)^2\, P(y) \text{ for all } Y \qquad (8.4)$$

Where σ^2 denotes variance.

The variance of a random variable Y is the variance of its probability distribution. It should be noted that the variance is expected value of $(Y - \mu)^2$.

We know that $E(y) = \Sigma Y\, P(Y) = \mu$ so

$$\begin{aligned}
\sigma^2 = E(Y-\mu)^2 &= \sum (Y-\mu)^2 P(y) \\
&= \Sigma (Y^2 + \mu^2 - 2\mu Y) P(y) \\
&= \Sigma Y^2 P(y) + \mu^2 \Sigma P(y) - 2\mu \Sigma Y P(y) \\
&= \Sigma Y^2 P(y) + \mu^2 - 2\mu^2 \,(\text{as } \Sigma P(y) = 1 \text{ and } \Sigma YP(y) = \mu) \\
&= \Sigma (Y^2) P(y) - \mu^2 \\
&= \Sigma Y^2 P(y) - ((\Sigma YP(y))^2 \\
&= E(Y^2) - [E(Y)]^2
\end{aligned}$$

8.6.1 Properties of Expectation:

i) If c is a constant, then

$$E(c) = c$$

ii) If a and b are constants and Y is a random variable, then

$$E(bY \pm a) = b\,E(Y) + a$$

If $a = -\mu$ and $b = 1$, then $E(Y - \mu) = 0$

iii) If X and Y are two random variables, then the expected value of their sum is the sum of their expected values i.e.,

$$E(X+Y) = E(X) + E(Y)$$

For the difference of two variables, the following result holds, true

$$E(X - Y) = E(X) - E(Y)$$

iv) If X and Y are two independent random variables, then the expected value of their product is the product of their expected values i.e.,

$$E(XY) = E(X)\,E(Y)$$

Example 8.15: The staff of the Department of Mathematics and Statistics at university of agriculture Faisalabad reckon that the number of microcomputers getting out of order in a year is well approximated by the following probability distribution.

No. out of order: y	0	1	2	3	4
Prob. $P(y)$:	0.3	0.3	0.2	0.1	0.1

Find the average and variance of the number of computers getting out of order? Also find the $E(5 - 3Y)$, by using the properties of expectation.

Solution: Average $= E(Y) = \Sigma y P(y)$

Thus $\mu = \Sigma Y_i\, P(y_i) = 1.4$

Variance $= \sum_{i=0}^{4} [Y_i - E(Y_i)]^2\, P(y_i)$

Y_i	$P(y)$	$yP(y)$
0	0.3	0.0
1	0.3	0.3
2	0.2	0.4
3	0.1	0.3
4	0.1	0.4
	Total:	1.4

Calculations for the variance are:

y	$P(y)$	$y-E(y)$	$[y-E(y)]^2$	$[y-E(y)]^2 P(y)$
0	0.3	-1.4	1.96	0.588
1	0.3	-0.4	0.16	0.048
2	0.2	0.6	0.36	0.072
3	0.1	1.6	2.56	0.256
4	0.1	2.6	6.76	0.676
------	------	------	**Total:**	**1.640**

thus

Variance = 1.64

Using property ii) $E(5-3Y) = 5\text{-}3\,E(Y)$ we have, $E(Y) = 1.4$, so

$E(5-3Y) = 5-3\,(1.4)$

$= 5-4.2$

$= 0.8$

Example: 8.16 For the probability distribution of X given below find that;
(i) E (X), (ii) $E(X^2)$

x	**0**	**1**	**2**	**3**
P (x)	$\frac{3}{10}$	$\frac{4}{10}$	$\frac{2}{10}$	$\frac{1}{10}$

Solution:

x	***P*(*x*)**	***xP*(*x*)**	**$x^2P(x)$**
0	3/10	0	0
1	4/10	4/10	4/10
2	2/10	4/10	8/10
3	1/10	3/10	9/10
Total	10/10 = 1	11/10	21/10

$$E(X) = \Sigma x P(x) = \frac{11}{10} = 1.1$$

$$E(X^2) = \Sigma x^2 P(x) = \frac{21}{10} = 2.1$$

Example 8.17: A random variable X has the probability distribution given below

X	0	1	2	3
$P(x)$	$\frac{3}{10}$	$\frac{4}{10}$	$\frac{2}{10}$	$\frac{1}{10}$

Find i) $E(X)$ (ii) $E(3X + 5)$ iii) $E(X^2)$

iv) Show that $E(3X + 5) = 3E(X) + 5$

x	$P(x)$	$xP(x)$	$x^2P(x)$	$3x + 5$	$(3x+5)P(x)$
0	3/10	0	0	5	15/10
1	4/10	4/10	4/10	8	32/10
2	2/10	4/10	8/10	11	22/10
3	1/10	3/10	9/10	14	14/10
Total	10/10 = 1	**11/10**	**21/10**	-	**83/10**

i) $E(X) = \Sigma x P(x) = \frac{11}{10} = 1.1$

ii) $E(3X + 5) = \Sigma (3x + 5) P(x) = \frac{83}{10} = 8.3$

iii) $E(X^2) = \Sigma x^2 P(x) = \frac{21}{10} = 2.1$

iv) $E(3X + 5) = 3E(X) + 5$

$8.3 = 3(1.1) + 5$

$8.3 = 3.3 + 5$

$8.3 = 8.3$

Example 8.18: *A, B* and *C* in the order cut a pack of cards, replacing them after each cut with condition that first who cuts a heart shall win a prize of Rs. 37. Find their respective expectation.

Solution: Let P be the probability of getting heart $= \frac{1}{4}$

And q be the probability of not getting heart $= \frac{3}{4}$

A can cut a heart on 1st, 4th, 7th… drawing with respective probabilities.

$p, q^3 p, q^6 p, \ldots..$

B can cut a spade on 2nd, 5th, 8th, …. drawing with respective probabilities.

$qp, q^4 p, q^7 p, \ldots.$

C can cut a spade on 3rd, 6th, 9th …. drawing with respective probabilities.

$q^2 p, q^5 p, q^8 p \ldots.$

Then the probability that *A* cuts the heart is

$$P(A) = P + q^3 p + q^6 p + \ldots..$$

$$P(A) = \frac{a}{1-r} = \frac{p}{1-q^3} = \frac{\frac{1}{4}}{1-\left(\frac{3}{4}\right)^3} = \frac{16}{37}$$. Expected amount for $A = 37 \times \frac{16}{37} = \text{Rs.}16$

$$P(B) = qp + q^4 p + q^7 p + \ldots.$$

$$P(B) = \frac{a}{1-r} = \frac{qp}{1-q^3} = \frac{\left(\frac{3}{4}\right)\left(\frac{1}{4}\right)}{1-\left(\frac{3}{4}\right)^3} = \frac{12}{37}$$, Expected amount for $B = 37 \times \frac{12}{37} = \text{Rs.}12$

$$P(C) = q^2 p + q^5 p + q^8 p + \ldots.$$

$$P(C) = \frac{a}{1-r} = \frac{q^2 p}{1-q^3} = \frac{\left(\frac{3}{4}\right)^2\left(\frac{1}{4}\right)}{1-\left(\frac{3}{4}\right)^3} = \frac{9}{37}$$. Expected amount for $C = 37 \times \frac{9}{37} = \text{Rs.}9$

Example 8.19: A bag contains 6 Red and 4 white balls. A person draws 2 balls at random without replacement being promised 15 rupees for each red ball and 20 rupees for each white ball he draws. Find his expectation.

Solution: Red balls = 6, White balls = 4, Total = 10, Drawn balls = 2

Rupees for each red ball = 15

Rupees for each white ball = 20

The respective probabilities of the drawing are:

$$P\,(2 \text{ red balls}) = \frac{\binom{6}{2}\binom{4}{0}}{\binom{10}{2}} = \frac{15}{45}$$

$$P\,(\text{one red and one white ball}) = \frac{\binom{6}{1}\binom{4}{1}}{\binom{10}{2}} = \frac{24}{10}$$

$$P\,(2 \text{ white balls}) = \frac{\binom{4}{2}\binom{6}{0}}{\binom{10}{2}} = \frac{6}{45}$$

Hence the required expectations is

$$E\,(X) = 30\left(\frac{15}{45}\right) + 35\left(\frac{24}{45}\right) + 40\left(\frac{6}{45}\right)$$

$$= 10 + 18.67 + 5.33$$

$$= 34$$

Example 8.20: If $f(x) = \dfrac{6 - |7 - x|}{36}$ for $X = 2, 3, 4, 5, \ldots.., 12$ then find the mean and variance of the random variable x.

Solution:

x	$f(x)$	$xf(x)$	$x^2 f(x)$
2	1/36	2/36	4/36
3	2/36	6/36	18/36
4	3/36	12/36	48/36
5	4/36	20/36	100/36
6	5/36	30/36	190/36
7	6/36	42/36	294/36
8	5/36	40/36	320/36
9	4/36	36/36	324/36
10	3/36	30/36	300/35
11	2/36	22/36	242/36
12	1/36	12/36	144/36
Total	**36/36 = 1**	**252/36**	**1974/36**

$$\text{Mean} = E(X) = \Sigma x f(x) = \frac{252}{36} = 7$$

$$\text{Variance}(X) = \Sigma x^2 f(x) - [\Sigma x f(x)]^2$$

$$= \frac{1974}{36} - \left(\frac{252}{36}\right)^2 = 54.83 - 49 = 5.83$$

Example 8.21: Find the missing value such that the given distribution is a probability distribution of X.

X	2	3	4	5	6
$f(x)$	0.01	0.25	0.4	A	0.4

If $Y = 2X - 8$, then show that

i) $E(Y) = 2E(X) - 8$

ii) $\text{Var}(Y) = 4\,\text{Var}(X)$

Solution: We know that sum of probabilities is one

$$\Sigma f(X) = 1$$

$$0.01 + 0.25 + 0.4 + A + 0.04 = 1$$

$$A + 0.7 = 1$$

$$A = 1 - 0.7$$

$$A = 0.3$$

x	$f(x)$	$x f(x)$	$x^2 f(x)$	$y = 2x - 8$	$f(y)$	$y f(y)$	$y^2 f(y)$
2	0.01	0.02	0.04	–4	0.01	0.04	0.16
3	0.25	0.75	2.25	–2	0.25	–0.50	1.00
4	0.40	1.6	6.40	0	0.40	0	0
5	0.30	1.5	7.50	2	0.30	0.60	1.20
6	0.04	0.24	1.44	4	0.04	0.16	0.64
Total	**1.00**	**4.11**	**17.63**	**----**	**1.00**	**0.22**	**3**

$$E(X) = \Sigma x f(x) = 4.11$$

$$E(X^2) = \Sigma x^2 f(x) = 17.63$$

$$\text{Var}(X) = \Sigma x^2 f(x) - [\Sigma x f(x)]^2 = 17.63 - (4.11)^2 = 0.7379$$

$$E(Y) = \Sigma y f(y) = 0.22$$

$$E(Y^2) = \Sigma Y^2 f(y) = 3$$

$$\text{Var}(Y) = \Sigma y^2 f(y) - (\Sigma y f(y))^2 = 3 - (0.22)^2 = 2.9516$$

i) $\therefore$ $E(Y) = 2E(X) - 8 = 2(4.11) - 8 = 8.22 - 8.00$

$0.22 = 0.22$

ii) $\therefore$ $\text{Var}(Y) = 4\,\text{Var}(X) = 4(0.7379)$

$2.9516 = 2.9516$

Example 8.22

i) Given a random variable X with $E(X) = 0.63$ and $\text{Var}(X) = 0.2331$.

Find $E(X^2)$.

ii) Given that $E(X^2) = 400$ and S.D. $(X) = 12$. Find $E(X)$

iii) Given the information that $E(X) = 200$, C.V. $(X) = 7\%$. Find Var (X).

Solution:

a) We have, $\text{Var}(X) = E(X^2) - [E(X)]^2$

$$E(X^2) = \text{Var}(X) + [E(X)]^2 = 0.2331 + (0.63)^2 = 0.63$$

b) We have, $\text{S.D.}(X) = \sqrt{E(X^2) - [E(X)]^2}$

$$12 = \sqrt{400 - [E(X)]^2}$$

Squaring on both sides, we get

$$144 = 400 - [E(X)]^2$$

or $[E(X)]^2 = 400 - 144 = 256$

so that $E(X) = \sqrt{256} = 16$

c) We have, $\text{C.V.}(X) = \dfrac{\text{S.D.}(X)}{E(X)} \times 100$

$$7 = \frac{\text{S.D.}(X)}{200} \times 100 = \frac{\text{S.D}(X)}{2}$$

$$\text{S.D.}(X) = 7\,(2) = 14$$

so that $\text{Var}(X) = (14)^2 = 196$

8.9 Distribution Function

Very often we are interested in calculating the chances that the values of a random variable will remain at or below a certain fixed value. For example, what are chances that a student will get not more than 80% of marks? What are the chances that 5 tosses of coin will produce not more than three heads? In these situations, we are concerned with the probability that a given random variable Y will take on values that are less or equal to some fixed value of Y. Mathematically, it is written as $P(Y \leq y)$. This probability is called distribution function or cumulative distribution of the random variable Y.

The probability $P\ (Y \leq y)$ for the possible values of y is called distribution function (DF) or cumulative distribution function (cdf). It is usually denoted by $F(x)$, so we can write.

$$F(Y) = P\ (Y \leq y) \text{ over possible values of } Y \tag{8.5}$$

In any density function the integral from - ∞ to y is called a distribution or cumulative distribution function is given as:

$$F(Y) = \int_{-\infty}^{Y} f(Y)\, dy$$

The function has the following properties:

i) $F(-\infty) = 0$

ii) $F(+\infty) = 1$

iii) $F(y_1) \leq F(y_2)$ if $y_1 \leq y_2$

(iv) F (y) is continuous atleast on the right of each y.

Example 8.23: A coin is tossed three times. Probability distribution for number of heads (Y) is given below:

Y	$P(y)$
0	1/8
1	3/8
2	3/8
3	1/8

Find distribution function $F(y)$?

Solution: From the definition of $F(y)$, we have

i) $F(Y) = P(Y \leq y)$ and Y takes values 0, 1, 2, 3. No value of Y is less than 0. So, $P(Y < 0) = 0$

ii) for $Y = 0$, $P(Y = 0)$ 1/8, there is no integral value between 0 and 1. So, this probability remains 1/8 till Y approaches 1.

iii) for $Y = 1$, $F(Y \leq 1) = P(Y = 0) + P(Y = 1) = \frac{1}{8} + \frac{3}{8} = \frac{4}{8}$

iv) for $Y = 2$, $F(Y \leq 2) = P(Y = 0) + P(Y = 1) + P(Y = 2) = \frac{1}{8} + \frac{3}{8} + \frac{3}{8} = \frac{7}{8}$

It remains 7/8 until Y reaches $Y = 3$

at $Y = 3$, $F(Y \leq 3) = P(Y = 0) + P(Y = 1) + P(Y=2) + P(Y=3)$

$= 1/8 + 3/8 + 3/8 + 1/8 = 1$

So, this probability is 1 at $Y = 3$

As Y does not take any value beyond 3. So, There is no probability beyond 3. Thus $F(Y \geq 3)$ remains 1.

These results are summarized below and its graph is as in figure 8.5

$F(y) = 0$ for $Y < 0$

$= 1/8$, for $0 \leq Y < 1$

$= 3/8$, for $1 \leq Y < 2$

$= 7/8$, for $2 \leq Y < 3$

$= 1$, for $Y \geq 3$

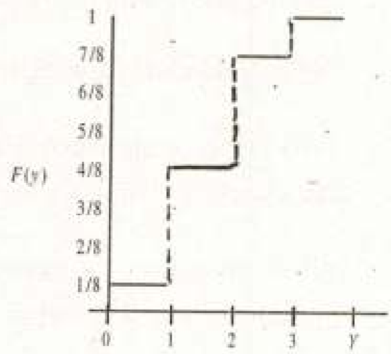


Figure 8.5: Distribution function

Exercise 8

Ans on Page 258

8.1 What are random numbers? How can they be generated? Explain the applications of random numbers.

8.2 Three balls are drawn from a bag containing 5 white and 3 black balls. If X denotes the number of white balls drawn from the bag, then find the probability distribution of X.

8.3 There are seven candidates for three positions of typist. Four of the candidates know Urdu typing while the other three do not know it. If the three candidates are selected at random, find the probability distribution of the number of persons knowing Urdu typing among those selected.

8.4 i) What is meant by probability distribution. Distinguish between discrete and continuous random variables by giving examples.

ii) A coin is tossed 4 times. If X denotes the number of tails, what is the probability distribution of X? Draw a probability histogram.

iii) A bag contains 4 red and 6 black balls. A sample of 4 balls is selected from the bag without replacement. Let X be the number of red balls, find the probability distribution of X.

8.5 i) Define probability function and give an example to explain it.

ii) Two fair dice are thrown and Y denote the product of the two scores. Obtain the probability distribution of Y

8.6 i) What properties a mathematical function should possess to be a probability function and probability density function?

ii) Check whether the following functions satisfy the conditions of a probability function?

a. $f(y) = 1/4$ for $y = 1,2,3,4,5$

b. $f(y) = 1/y$ for $y = 1,2,3,4$

c. $f(y) = y/15$ for $y = 0,1,2,3,4,5$

d. $f(y) = (5 - y^2)/6$ for $y = 0,1,2,3$

8.7 Determine the value of c so that the function can serve as a probability function of a random variable.

a) cy for $Y = 1,2,3,4,5$

b) $(1 - c)\,c^y$ for $Y = 0,1,2\ldots$

8.8 A random variable Y takes values 0, 1, 2, 3 with respective probabilities $1/4\,(1 + 3\theta)$, $1/4\,(1 - \theta)$, $1/4\,(1 + 2\theta)$, $1/4\,(1 - 4\theta)$. For what values of θ is this a valid probability function?

8.9 i) Given the following probability distribution:

x	0	1	2	3	4
$P(x)$	1/126	20/126	60/126	40/126	5/126

Verify that $E\,(2X + 3) = 2E\,(X) + 3$

ii) Let X be a random variable with probability distribution:

x	–1	0	1	2	3
$P(x)$	0.125	0.500	0.200	0.050	0.125

Find

a) $E(X)$ and $\text{Var}(X)$ b) The probability distribution of the random variable $Y = 2X + 1$.

Using the probability distribution of Y, determine $E(Y)$ and $\text{Var}\,(Y)$.

8.10 The following table gives the probability distribution of the random variable Y, the number of courses taught by a teacher during spring semester in the University of Agriculture Faisalabad.

y	1	2	3	4	5
$P(y)$	.1	.2	.3	.3	.1

Find the probability that:

i) Teacher taught 2 courses

ii) Teacher taught less than 4 courses.

iii) Teacher taught between 2 and 5 courses

iv) Teacher taught atleast 3 courses.

v) Teacher taught at the most 4 courses.

8.11 A box contains five slips of paper marked 1,2,3,4 and 5. Two slips are selected without replacement, list the possible values for each of the following random variables:

i) The sum of the two numbers on two slips.

ii) The difference between the first and second number.

8.12 Four randomly selected students from a class are asked their opinion about the teaching system as satisfactory (S) or not satisfactory (NS). Let Y denote the number of students saying satisfactory. Write down the possible outcomes and the possible Y values.

8.13 A point is randomly selected on the surface of a lake that has maximum depth of 30 feet. Let y denote the depth of the lake at the randomly selected point. What are possible values of Y ? Is Y a discrete variable or a continuous variable?

8.14 Calculate mean and variance of the following probability distribution:

y	0	1	2	3	4	5
$P(y)$	0.1	0.2	0.3	0.2	0.1	0.1

8.15 i) A continuous random variable X having values only between 0 and 4 has a density function given by: $f(x) = \frac{1}{2} - ax$, where a is a constant find a) a b) $P(1 < X < 2)$.

ii) A continuous random variable X has probability density function giving $f(x) = cx$ for $0 < X < 2$. Find

(a) c (b) Probability that $1 < X < 1.5$ (c) Probability that $X < 1.5$

iii) If $f(x)$ has probability density kx^2, $0 < X < 1$, determine its kind and find the probability that $\frac{1}{3} < X < \frac{1}{2}$

8.16 What is a random variable? Distinguish between discrete and continuous random variable, giving examples.

8.17 Find the probability distribution of the number of boys in families with three children, assuming equal probabilities for boys and girls.

8.18 From lot containing 12 items, 4 of which are defective, 5 are chosen at random. If X is the number of defective items found in the sample, write down

(i) The probability distribution of X (ii) $P(X \leq 1)$

iii) Verify $\sum_{x=0}^{4}[P(x)] = 1$

8.19 i) Define continuous random variable and its probability distribution.

ii) Find the constant k so that the function $f(x)$ defined as follows may be a density function.

$$f(x) = \frac{1}{k}, \quad a \leq X \leq b$$
$$= 0, \text{ elsewhere}$$

8.20 (i) What do you mean by expected value? What are the properties of expectation?

(ii) Given the following discrete probability distribution:

x	0	1	2	3	4	5
P(x)	6/36	10/36	8/36	6/36	4/36	2/36

Compute its mean, variance, standard deviation and coefficient of variation.

8.21 Let X be a random variable with probability distribution as follows:

x	1	2	3	4	5
$f(x)$	0.125	0.45	0.25	0.05	0.125

Find mean and variance.

8.22 i) A continuous random variable X has a density function

$$f(x) = \frac{x+1}{8} \text{ for } X = 2 \text{ to } X = 4. \text{ Find}$$

a) $P(X < 3.5)$ b) $P(2.4 < X < 3.5)$ c) $P(X = 1.5)$.

(ii) A continuous random variable X has a density function

$f(x) = 2x, \quad 0 \leq x \leq 1$. Find

a) $P(X = \frac{1}{2})$ b) $P(X > \frac{1}{4})$ c) $P(\frac{1}{4} < X < \frac{1}{2})$

8.23 A continuous random variable X which can assume values between $X = 2$ and $X = 8$ inclusive has a density function given by $a\,(x + 30)$, where a is a constant. Find i) a ii) $P(3 < X < 6)$ iii) $P(X \leq 6)$ iv) $P(X \geq 4)$.

8.24 In a summer season, a dealer of desert room coolers can earn Rs. 800 per day if the day is hot and can earn Rs. 200 per day if it is fair and loses Rs. 50 per day if it is cloudy. Find his mathematical expectation if the probability of the day being hot is 0.40 and for being cloudy it is 0.35

8.25 A committee of size 5 is to be selected from 3 female and 5 male members. Find the expected number of female members on the committee.

8.26 What are the properties of probability function and probability density function?

8.27 i) Explain the concept of distribution function.

ii) 10 vegetable cans, all of the same size, have lost their labels. It is known that 5 contain tomatoes and 5 contain corns. If 5 cans are selected at random, then find the probability distribution and the distribution function for the number of tomato cans in the sample.

8.28 i) Define the expected value for a random variable.

ii) The number of automobile accidents in a city are: 1, 2, 3, 4, 4 with corresponding probabilities 1/8, 2/8, 2/8 and 3/8. What is the expected number of daily accidents?

8.29 *A* and *B* throw a die for a prize of Rs. 11. Which is to be won by the player who first throws a 6. If *A* has the firs throw, what are their respective expectations?

8.30 A bag contains 2 white and 2 black balls. Three men *A, B* and *C* draw a ball and don't replace it. The person who draws the white ball first receives Rs. 12. What are their respective expectations?

8.31 Three balls are drawn from a bag containing 5 white and 3 black balls. If X denotes the number of white balls drawn from the bag, then find the probability distribution of X. Also find its mean and variance.

8.32 A coin is biased such that a head is thrice as likely to occur as a tail. Find the probability distribution of heads and also find the mean and variance of the distribution when it is tossed 4 times.

8.33 Approximately 10% of the glass bottles coming from a production line have serious defects. If two bottles are selected at random, find the expected number of bottles that having serious defects.

8.34 A random variable X takes the values –3, –2, 2, 3 and 4 with probabilities $P(X)$ equal to 1/5, 1/10, 1/10, 1/5 and 2/5 respectively. Compute $E(X)$ and show that $E\ (5X + 10) = 5E(X) + 10$. Also compute the variance (X) and variance $(5X + 10)$. Find the ratio of two variances.

8.35 If $f(x) = \dfrac{6-|7-x|}{36}$ for $X = 2,3,4,\ldots, 12$, then find the mean and variance of the random variable X.

8.36 For the following Probability distribution. Find

i) $E(X)$, ii) $E(X^2)$, iii) $E[X - E(X)]^2$.

x	-10	-20	30
$P(X)$	1/5	3/10	1/2

8.37 i) Define expectation of a random variable

ii) The probability distribution of a discrete random variable X is given by

$$f(x)=\binom{3}{x}\left(\frac{1}{4}\right)^x\left(\frac{3}{4}\right)^{3-x},x=0,1,2,3$$

Find $E(X)$ and $E(X^2)$.

8.38 Against each statement, write T for true and F for false statement.

i) A random variable is also named as a chance variable.

ii) The number of accidents occurring on G.T. road during one month is the example of continuous random variable.

iii) The Probability cannot exceed 1.

iv) The range of continuous Random variable is from '0' to 'n'.

v) The Distribution function is an increasing function.

vi) The expectation of a Random variable is also named as the Mean of a Random variable.

vii) The probability function can be negative.

viii) A discrete probability distribution is represented by area graph.

ix) If X and Y are independent random variables then $E(XY)=E(X)\,E(Y)$.

x) If X and Y are independent random variables, then

$$S.D(X-Y)=S.D(X)-S.D(Y).$$