9 Binomial and Hypergeometric Probability Distribution

9.1. Introduction

In an experiment of tossing a coin, drawing a card from a pack of playing cards repeatedly, etc., each drawing is called a **trial**. The results of each trial is classified as a success or failure. The probability of success is denoted by p and the probability of failure is denoted by q where, $q=1-p$ or $q+p=1$. Let an experiment be repeated n times, the number of successes obtained in a trial of the experiment is denoted by x and the number of failures by $n-x$.

A trial having two possible outcomes i.e., only success and failure is called **Bernoulli trials.** For each bernoulli trial, the probability of success remains the same and the successive trials are independent.

An experiment in which the outcomes can be classified as success or failure and in which the probability of success remains constant from trial to trial is called **Binomial experiment**. A binomial experiment possesses the following properties:

i) Each trial of the experiment results in an outcome which can be classified into two categories i.e., success and failure.

ii) The probability of success remains constant from one trial of the experiment to the next.

iii) The repeated trials are independent.

iv) The experiment is repeated a fixed number of times.

9.2 Binomial Probability Distribution

Suppose we have n independent trials for each of which the probability of success is p and the probability of failure is q and $q+p=1$. The probability of exactly x success is given by

$$P(X = x) = \binom{n}{x} p^x q^{n-x} \qquad (9.1)$$

where $x = 0, 1, 2, 3, \ldots, n$. The random variable x is called the binomial variable and the distribution of x is called the binomial distribution, the quantities n and p are called the parameters of the binomial distribution.

The binomial probability distribution is generally denoted by $b(x, n, p)$. The probability $\binom{n}{x} q^{n-x} p^x$ are obtained by expanding the binomial expansion $(q + p)^n$

i.e.,

$$(q+p)^n = \binom{n}{0} q^{n-o} p^o + \binom{n}{1} q^{n-1} p^1 + \binom{n}{2} q^{n-2} p^2 + \ldots + \binom{n}{n} q^{n-n} p^n.$$

where $\binom{n}{0}\binom{n}{1}, \binom{n}{2}, \ldots \binom{n}{n}$, are called the binomial co-efficients.

If $p = q = \frac{1}{2}$, the binomial distribution is a symmetrical distribution.

If $p \neq q$, the binomial distribution is a skewed distribution.

If $p > \frac{1}{2}$, the distribution is negatively skewed .

If $p < \frac{1}{2}$, the distribution is positively skewed .

Example 9.1: A fair coin is tossed 4 times. Find the probabilities of obtaining various number of heads.

Solution: $n = 4,\ p = \frac{1}{2},\ q = \frac{1}{2}$

$$\therefore \quad P(X=x) = \binom{n}{x} q^{n-x} p^x,\ x = 0, 1, 2, \ldots, n$$

where x denotes the number of heads.

$$\therefore P(X = 0) = \binom{4}{0} \left(\frac{1}{2}\right)^{4-0} \left(\frac{1}{2}\right)^0 = \frac{1}{16}$$

$$P(X=1) = \binom{4}{1}\left(\frac{1}{2}\right)^{4-1}\left(\frac{1}{2}\right)^{1} = \frac{4}{16}$$

$$P(X=2) = \binom{4}{1}\left(\frac{1}{2}\right)^{4-2}\left(\frac{1}{2}\right)^{2} = \frac{6}{16}$$

$$P(X=3) = \binom{4}{3}\left(\frac{1}{2}\right)^{4-3}\left(\frac{1}{2}\right)^{3} = \frac{4}{16}$$

$$P(X=4) = \binom{4}{4}\left(\frac{1}{2}\right)^{4-4}\left(\frac{1}{2}\right)^{4} = \frac{1}{16}$$

Example 9.2: A fair coin is tossed 5 times. What is the probability of getting

i) Exactly 3 heads ii) at least 3 heads iii) at the most two heads.

Solution: $n = 5,\ p = \frac{1}{2},\ q = \frac{1}{2},$

and $P(X = x) = \binom{n}{x} q^{n-x} p^{x} \quad x = 0, 1, 2, 3, 4, 5$

i) Exactly 3 heads

$$P(X=3) = \binom{5}{3}\left(\frac{1}{2}\right)^{5-3}\left(\frac{1}{2}\right)^{3} = \frac{10}{32}$$

ii) at least 3 heads

$$\begin{aligned} P(X \geq 3) &= P(X=3)+P(X=4)+P(X=5) \\ &= \binom{5}{3}\left(\frac{1}{2}\right)^{5-3}\left(\frac{1}{2}\right)^{3} + \binom{5}{4}\left(\frac{1}{2}\right)^{5-4}\left(\frac{1}{2}\right)^{4} + \binom{5}{5}\left(\frac{1}{2}\right)^{5-5}\left(\frac{1}{2}\right)^{5} \\ &= \frac{10}{32} + \frac{5}{32} + \frac{1}{32} \\ &= \frac{16}{32} = 0.5 \end{aligned}$$

iii) at the most two heads.

$$P(X \le 2) = P(X=2)+P(X=1)+P(X=0)$$

$$= \binom{5}{2}\left(\frac{1}{2}\right)^{5-2}\left(\frac{1}{2}\right)^{2}+\binom{5}{1}\left(\frac{1}{2}\right)^{5-1}\left(\frac{1}{2}\right)^{1}+\binom{5}{0}\left(\frac{1}{2}\right)^{5-0}\left(\frac{1}{2}\right)^{0}$$

$$= \frac{10}{32}+\frac{5}{32}+\frac{1}{32}$$

$$= \frac{16}{32} = 0.5$$

Example 9.3: If you toss a fair die 6 times. What is the probability of getting no even number.

Solution: $n = 6$, $p = \frac{3}{6} = \frac{1}{2}$, $q = \frac{1}{2}$

$$P(X=0) = \binom{6}{0}\left(\frac{1}{2}\right)^{6-0}\left(\frac{1}{2}\right)^{0} = \frac{1}{64}$$

9.2.1 Binomial frequency Distribution

If the binomial probability distribution is multiplied by the number of experiments N then the distribution is called the binomial frequency distribution. Expected frequency of x successes in N experiments of n trials is given by

$$f(X=x)\ \ Np(X=x) = N\binom{n}{x}q^{n-x}\,p^{x},$$

Example 9.4: Out of 800 families with 5 children each. How many would you expect to have

i) 4 boys ii) at least 3 boys iii) at the most one boy.

Solution: $n = 5, N = 800, p = \frac{1}{2}$

$$q = 1 - p = \frac{1}{2}$$

i) With four boys

$$P(X = x) = \binom{n}{x} q^{n-x} p^x$$

$$P(X = 4) = \binom{5}{4}\left(\frac{1}{2}\right)^{5-4}\left(\frac{1}{2}\right)^4$$

$$= \frac{5}{32}$$

Hence expected number of families with 4 boys $= 800 \times \frac{5}{32} = 125$

iii) with at least 3 boys

$$P(X \geq 3) = P(X = 3) + P(X = 4) + P(X = 5)$$

$$= \binom{5}{3}\left(\frac{1}{2}\right)^{5-3}\left(\frac{1}{2}\right)^3 + \binom{5}{4}\left(\frac{1}{2}\right)^{5-4}\left(\frac{1}{2}\right)^4 + \binom{5}{5}\left(\frac{1}{2}\right)^{5-5}\left(\frac{1}{2}\right)^5$$

$$= \frac{10}{32} + \frac{5}{32} + \frac{1}{32}$$

$$= \frac{16}{32} = 0.5$$

Hence expected number of families with at least three boys

$$= 800 \times \frac{16}{32} = 400$$

iii) with at the most one boy.

$$P(X \leq 1) = P(X = 1) + P(X = 0)$$

$$= \binom{5}{1}\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)^{5-1} + \binom{5}{0}\left(\frac{1}{2}\right)^0\left(\frac{1}{2}\right)^{5-0}$$

$$= \frac{5}{32} + \frac{1}{32}$$

$$= \frac{6}{32}$$

Hence expected number of families with at the most one boy will be $800 \times \frac{6}{32} = 150$

Example 9.5: Five dice are tossed 96 times. Find the expected frequencies when throwing of a 4, 5 or 6 is regarded as a success.

Solution: Here,

$$n = 5,\ N = 96,\ p = \frac{3}{6} = \frac{1}{2},\ q = 1 - p = \frac{1}{2}$$

X	$P(x) = \binom{n}{x} q^{n-x} p^x$	Expected Frequencies $NP(x) = f_e$
0	$\binom{5}{0}\left(\frac{1}{2}\right)^{5-0}\left(\frac{1}{2}\right)^0 = \frac{1}{32}$	$96 \times \frac{1}{32} = 3$
1	$\binom{5}{1}\left(\frac{1}{2}\right)^{5-1}\left(\frac{1}{2}\right)^1 = \frac{5}{32}$	$96 \times \frac{5}{32} = 15$
2	$\binom{5}{2}\left(\frac{1}{2}\right)^{5-2}\left(\frac{1}{2}\right)^2 = \frac{10}{32}$	$96 \times \frac{10}{32} = 30$
3	$\binom{5}{3}\left(\frac{1}{2}\right)^{5-3}\left(\frac{1}{2}\right)^3 = \frac{10}{32}$	$96 \times \frac{10}{32} = 30$
4	$\binom{5}{4}\left(\frac{1}{2}\right)^{5-4}\left(\frac{1}{2}\right)^4 = \frac{5}{32}$	$96 \times \frac{5}{32} = 15$
5	$\binom{5}{5}\left(\frac{1}{2}\right)^{5-5}\left(\frac{1}{2}\right)^5 = \frac{1}{32}$	$96 \times \frac{1}{32} = 3$

9.2.2 Mean And Variance Of The Binomial Distribution

We find the mean and variance of the binomial distribution given by $(q + p)^n$. We know that

$$\text{Mean} = E(X) = \sum x P(x) \tag{9.2}$$

$$\text{Variance} = E(X^2) - [E(X)]^2$$

$$= \sum x^2 P(x) - [\sum x P(x)]^2 \tag{9.3}$$

The necessary calculations are shown in the table given below:

Number of Successes (x)	$P(x)=\binom{n}{x} q^{n-x} p^x$	$xP(x)$	$x^2P(x)$
0	$\binom{n}{0} q^{n-1} p^0 = q^n$	0	0
1	$\binom{n}{1} p\, q^{n-1} = npq^{n-1}$	nqp^{n-1}	nqp^{n-1}
2	$\binom{n}{2} q^{n-2} p^2 = \frac{n(n-1)}{2!} q^{n-2} p^2$	$n(n-1)p^2q^{n-2}$	$2n(n-1)p^2q^{n-2}$
3	$\binom{n}{3} p^3 q^{n-3} = \frac{n(n-1)(n-2)}{3!} q^{n-3} p^3$	$\frac{n(n-1)(n-2)}{2!} p^3q^{n-3}$	$\frac{3n(n-1)(n-2)}{2!} p^3q^{n-3}$
⋮	⋮	⋮	⋮
n	$\binom{n}{n} p^n q^{n-n} = p^n$	np^n	$n^2 p^n$

$$E(X) = \sum x\, P(x)$$

$$= npq^{n-1} + n(n-1)p^2q^{n-2} + \frac{n(n-1)(n-2)}{2!} p^3 q^{n-3} + \ldots + np^n$$

$$= np\left[q^{n-1} + (n-1)pq^{n-2} + \frac{(n-1)(n-2)}{2!} p^2 q^{n-3} + \ldots + np^{n-1} \right]$$

$$= np\left[(q+p)^{n-1}\right]$$

$$= np(q+p)^{n-1} \qquad [\because p+q=1]$$

$$\therefore E(X) = np$$

$$E(X^2) = \sum x^2\, P(x)$$

$$= npq^{n-1} + 2n(n-1)p^2q^{n-2} + \frac{3n(n-1)(n-2)}{2!} p^3 q^{n-3} + \ldots + n^2 p^n$$

$$= np\left[q^{n-1} + 2(n-1)pq^{n-2} + \frac{3(n-1)(n-2)}{2!}p^2 q^{n-3} + \ldots. + p^{n-1} \right]$$

$$= np\left[\left\{ q^{n-1} + (n-1)pq^{n-2} + \frac{(n-1)(n-2)}{2!}p^2 q^{n-3} + \ldots. + p^{n-1} \right\} \right.$$

$$\left. + \left\{ (n-1)pq^{n-2} + \frac{2(n-1)(n-2)}{2!}p^2 q^{n-3} + \ldots. + (n-1)p^{n-1} \right\} \right]$$

$$E(X^2) = np\left[(q+p)^{n-1} + (n-1)p\{q^{n-2} + (n-2)pq^{n-3} + \ldots. + p^{n-2}) \right]$$

$$E(X^2) = np\left[(q+p)^{n-1} + (n-1)\ p(q+p)^{n-2} \right]$$

$$= np\left[1 + (n-1)p \right]$$

$$E(X^2) = np + n^2p^2 - np^2$$

$$\text{Variance} = E(x^2) - [E(x)]^2$$

$$= np + n^2p^2 - np^2 - (np)^2$$

$$= np - np^2$$

$$= np(1-p) \quad \text{Variance} = npq \quad (\because q = 1-p)$$

And standard deviation $\sigma = \sqrt{npq}$

Example 9.6: In a binomial distribution $n = 20$ and $P = \frac{3}{5}$. Find the mean, variance and standard deviation of the binomial distribution.

Solution: Here $n = 20$, $p = \frac{3}{5}$, $q = 1 - p = \frac{2}{5}$

we know that

$$\text{Mean} = np = 20\left(\frac{3}{5}\right) = 12$$

$$\sigma^2 = \text{Variance} = npq = 20\left(\frac{3}{5}\right)\left(\frac{2}{5}\right)$$

$$\sigma^2 = 4.8$$

$$\text{or } \sigma = S.D. = \sqrt{npq} = \sqrt{4.8}$$

Example 9.7: In a binomial distribution, mean and standard deviation were found to be 38 and 5.6 respectively find p and n.

Solution:

Mean $= np = 38$ (i)

Standard deviation $= \sqrt{npq} = 5.6$ (ii)

Squaring equation (ii)

$$npq = 31.36 \quad \text{(iii)}$$

Dividing equation iii) by i), we get

$$\frac{npq}{np} = \frac{31.36}{38}$$

$$q = 0.83$$

$$\because p = 1-q$$

$$\therefore P = 1-0.83$$

$$P = 0.17$$

Putting the value of $p = 0.17$ in equation (i)

$$np = 38$$

$$n(0.17) = 38$$

$$n = 224$$

$$n = 224 \text{ and } p = 0.17$$

Example 9.8: Is it possible to have a binomial distribution with mean = 5 and $S.D. = 4$.

Solution:

$$\text{mean} = np = 5 \quad (i)$$

$$S.D. = \sqrt{npq} = 4 \quad (ii)$$

$$\text{Variance} = npq = 16 \quad (iii)$$

$$\frac{npq}{np} = \frac{16}{5}$$

$$q = 3.2$$

Since p or q cannot be greater than one. So it is not possible to have a binomial distribution with mean 5 and with standard deviation 4.

9.3 Hypergeometric Distribution and Hypergeometric Experiment

When the successive trials are without replacement then they are dependent and the probability of success changes from one trial of the experiment to the other. Such an experiment in which a random sample is chosen without replacement from a finite population is said to be hypergeometric experiment.

9.3.1 Properties

A hypergeometric experiment has the following properties:

i) The experiment is repeated a fixed number of times.

ii) The successive trials are dependent.

iii) The probability of success varies from trial to trial (it in not fixed).

iv) The outcome of an experiment can be classified as success or failures.

The random variable X representing the number of successes in a hypergeometric experiment is called a hypergeometric variable and probability distribution of the hypergeometric variable is called hypergeometric distribution.

9.3.2 Hypergeometric Probability Distribution:

Suppose that there are N total number of items out of which k are classified as successes and $(N\text{-}k)$ as failures. n items are to be selected at random without replacement $n \leq N$.

Let X denotes the number of successes and we can obtain exactly "x" successes and $(n\text{-}x)$ failures as follows:

$$P(X = x) = \frac{\binom{k}{x}\binom{N-k}{n-x}}{\binom{N}{n}} \qquad (9.4)$$

where $x = 0, 1, 2, \ldots, n$. The hypergeometric probability distribution has 3 parameters n, k and N.

$$\text{Mean of the hypergeometric distribution} = \frac{nk}{N} \qquad (9.5)$$

$$\text{variance of the hypergeometric distribution} = \frac{nk}{N}\left(1-\frac{k}{N}\right)\left(\frac{N-n}{N-1}\right)$$

Example 9.9: Determine the probability distribution for the number of white beads. Among 5 beads drawn at random from a bowl containing 4 white and 7 black beads. Compute mean, variance and check the results by using the formula.

Solution:

$$N = 11, \quad n = 5$$

Taking white beads as success

$$k = 4$$

$$N-k = 11-4 = 7 \text{ (black beads)}$$

$$x = 0, 1, 2, 3, 4$$

$$P(X = x) = \frac{\binom{k}{x}\binom{N-k}{n-x}}{\binom{N}{n}}$$

$$P(X = 0) = \frac{\binom{4}{0}\binom{7}{5}}{\binom{11}{5}} = \frac{(1)(21)}{462} = \frac{21}{462}$$

$$P(X = 1) = \frac{\binom{4}{1}\binom{7}{4}}{\binom{11}{5}} = \frac{(4)(35)}{462} = \frac{140}{462}$$

$$P(X = 2) = \frac{\binom{4}{2}\binom{7}{3}}{\binom{11}{5}} = \frac{(6)(35)}{462} = \frac{210}{462}$$

$$P(X = 3) = \frac{\binom{4}{3}\binom{7}{2}}{\binom{11}{5}} = \frac{(4)(21)}{462} = \frac{84}{462}$$

$$P(X = 4) = \frac{\binom{4}{4}\binom{7}{1}}{\binom{11}{5}} = \frac{(1)(7)}{462} = \frac{7}{462}$$

Probability distribution is given below:

x	$f(x)$	$x f(x)$	$x^2 f(x)$
0	21/462	0	0
1	140/462	140/462	140/462
2	210/462	420/462	840/462
3	84/462	252/462	756/462
4	7/462	28/462	112/462
Total	462/462=1	840/462	1848/462

$$\text{Mean} = E(X) = \sum x f(x)$$

$$= \frac{840}{462}$$

$$= 1.8182$$

$$\text{Variance} = E(X^2) - [E(X)]^2 = \sum x^2 f(x) - \left[\sum x f(x)\right]^2$$

$$= \frac{1848}{462} - \left(\frac{840}{462}\right)^2$$

$$= 0.6942$$

Checking The Results

$$\text{Mean} = \frac{nk}{N} = \frac{5\times 4}{11} = 1.8182$$

$$\text{Variance} = \left(\frac{nk}{N}\right)\left(\frac{N-k}{N}\right)\left(\frac{N-n}{N-1}\right)$$

$$= (1.8182)\left(\frac{7}{11}\right)\left(\frac{6}{10}\right)$$

$$\text{Variance} = 0.6942$$

Example 9.10 Ten vegetable cans, all of the same size, have lost their labels. It is known that 5 contain tomatoes and 5 contain corns. If 5 are selected at random, what is the probability that all contain tomatoes? What is the probability that 3 or more contain tomatoes?

Solution: $N = 10, \quad n = 5$

considering the tomatoes cans as success.

$$k = 7 \text{ (Tomatoes) and } N - k = 10 - 5 = 5$$

i) All contain tomatoes.

$$x = 5$$

$$n - x = 5 - 5 = 0$$

$$\because P(X = x) = \frac{\binom{k}{x}\binom{N-k}{n-x}}{\binom{N}{n}}$$

$$\therefore P(X = 5) = \frac{\binom{5}{5}\binom{5}{0}}{\binom{10}{5}} = 0.005968$$

ii) 3 or more contain tomatoes

$$x = 3, 4, 5$$

$$n - x = 2, 1, 0$$

$$P(X \geq 3) = P(X = 3) + P(X = 4) + P(X = 5)$$

$$= \frac{\binom{5}{3}\binom{5}{2}}{\binom{10}{5}} + \frac{\binom{5}{4}\binom{5}{1}}{\binom{10}{5}} + \frac{\binom{5}{5}\binom{5}{0}}{\binom{10}{5}}$$

$$= \frac{100}{252} + \frac{25}{252} + \frac{1}{252}$$

$$= \frac{126}{252}$$

$$\because P(X \geq 3) = 0.5$$

Exercise 9

Ans on Page 260

9.1 Define the Binomial Probability Distribution.

9.2 What is a Binomial experiment? Give its properties?

9.3 An event has the probability $p = \frac{3}{8}$. Find the complete binomial distribution for $n = 5$ trials.

9.4 Find the probability i.e., tossing a fair coin four times there will appear

i) 4 heads ii) 1 tail and 3 heads

iii) at least 2 heads iv) at the most 2 heads.

9.5 If 20% of the bolts produced by a machine are defective, determine the probability that out of 4 bolts chosen at random

i) zero ii) 2 bolts are defective.

9.6 Given that the probability of passing an examination is 0.75. What is the probability of

i) passing at least two examinations if you take six ?

ii) failing at least two examinations if you take four ?

9.7 The experience of a house agent indicates that he can provide suitable accommodation for 75% of the clients who come to him. If on a particular occasion 6 clients approach him independently, calculate the probability that

i) less than 4 clients will get satisfactory accommodation.

ii) at least 5 clients will get satisfactory accommodation.

9.8 The incidence of an occupational disease in an industry is such that the workers have 20% chance of suffering from it. What is the probability that out of 6 workmen:

i) not more than 2 will catch the disease ?

ii) 4 or more will catch the disease ?

9.9 If 8 coins are tossed what is the probability that there are

i) Exactly 5 heads ii) 1 to 7 heads.

9.10 If X = binomially distributed with $n=10$, $P=0.4$, then find Mean and variance of $Y=\frac{X-10}{6}$.

9.11 If X is the number of successes with probability of success as $\frac{1}{4}$ in each of 5 independent trials. Find

i) $P(X=0)$ ii) $P(X \le 3)$.

9.12 If 5 true dice are thrown once, determine the probability of getting 0, 1, 2, 3, 4, 5 sixes. Find the mean and variance of the probability distribution so obtained.

9.13 Five dice are tossed 96 times. Find the expected frequencies when throwing of a 4, 5 or 6 is regarded as success.

9.14 Four dice are thrown and the number of sixes in each throw is recorded. This is repeated 180 times. Write down the theoretical frequencies 0,1,2,3 and 4 sixes. Calculate the mean number of sixes in a single throw.

9.15 The mean and variance of the binomial distribution are 6 and 2.4 respectively. Find p and n, the two parameters of the binomial distribution.

9.16 If the probability of a defective bolt is 0.1. Find the mean and standard deviation of the distribution of defective bolts in a total of 500.

9.17 If in a binomial distribution, the mean is 3 and the standard deviation is 1.5. Find its parameters.

9.18. Is it possible to have a binomial distribution with mean = 5 and S.D. = 3.7.

9.19 In binomial distribution with $n=5$, what is the value of other parameters of the binomial. If P(X = 0) = P(X = 1) find mean of the distribution.

9.20 Discuss the statement that in a binomial distribution $\mu = 6$ and $\sigma = 2.5$.

9.21 Find the binomial distribution whose mean is 12 and standard deviation is 3.

9.22 Find the mean and variance of the binomial $(q + p)^3$.

9.23 Find the mean and standard deviation of the Binomial distribution

$$(q + p)^n$$

9.24 Fill in the Blanks.

i) Mean of the binomial distribution is ____________ and its variance is ________________.

ii) Binomial distribution is symmetrical when ________.

iii) Binomial distribution is used when n is __________.

iv) Binomial distribution has _________ parameters.

v) Binomial distribution is positively skewed when _________.

vi) Binomial variable is a ____________ variable which can assume any of the values $X = 0, 1, 2, 3, \ldots, n$.

vii) The shape of binomial distribution depends upon the values of ______.

viii) In a binomial distribution, the experiment consists of a __________ number of _________ trials.

ix) Mean, median and mode for binomial distribution will be equal when ________________.

9.25 Five balls are drawn from a box containing 4 white and 7 black balls. If X denotes the number of black balls drawn, then obtain the probability distribution of X. Find the mean and variance of this distribution and verify the results using the formula.

9.26 Determine the probability distribution for the number of white beads among 5 beads drawn at random from a bowel containing 4 white and 7 black beads use this distribution to compute the mean and variance. And check the results by using the formula.

9.27 A committee of size 3 is selected from 4 men and 2 women. Find the probability distribution by hypergeometric experiment for the number of men on the committee.

9.28 A committee of size 5 is to be selected at random from 3 women and 5 men. Find the expected number of women on the committee.

9.29 Ten vegetable cans, all of the same size have lost their labels. It is known that 5 contain tomatoes and 5 contain corns. If 5 cans selected at random what is the probability that:

i) all contain tomatoes. **(ii)** 3 or more contain tomatoes.

9.30 Write T for true and F for false against each statement.

i) An experiment is called Bernolli experiment if it has two possible outcomes.

ii) The binomial distribution has three parameters *n, p* and *q*.

iii) A paper has 10 multiple-choice questions with 3 alternatives. Answering these questions by guesswork is a binomial experiment.

iv) If $p \neq q$, then the binomial Probability Distribution is skewed.

v) A binomial random variable is a continuous random variable.

vi) The binomial distribution is symmetrical distribution if $p = q = \frac{1}{2}$.

vii) The binomial distribution is negatively skewed distribution if $p > q$.

viii) The trials are independent in the hypergeometric distribution.

ix) Hypergeometric probability distribution has three parameters *n*, *N* and *k*.

x) The Binomial Distribution is used when '*n*' is large.

xi) The variance of the Binomial Distribution is '*npq*'

xii) The binomial random variable cannot assume the negative values

xiii) The binomial distribution is positively skewed distribution if $p < q$

Answers

$0 \leq P(A) \leq 1$

Exercise 1

1.9 i) Population ii) less iii) parameter iv) statistic v) Constant

vi) zero vii) random viii) quantitative Ix) Inferential statistics x) primary data.

1.10 i) T ii) T iii) F iv) F v) F vi) T vii) F viii)F ix) T x) T

Exercise 2

2.8 ii. a) 3.45 – 3.95 class boundaries of 3.7

b) 3.455 – 3.945 class limits of 3.7

2.12 (i)

Class intervals	Frequency	Cumulative frequency
0.1 –0.8	5	5
0.9 –1.6	9	14
1.7 –2.4	15	29
2.5 –3.2	10	39
3.3 – 4.0	6	45
4.1 – 4.8	2	47
4.9 – 5.6	1	48
5.7 – 6.4	2	50

2.17 (i) process (ii) four (iii) lower, first, last

(iv) proportional (v) Cumulative frequency. (vi) Histogram

(vii) Vertical, no (viii) Proportional (ix) Frequency, Polygon

2.18 i) F ii) T iii) F iv) T v) F vi) F vii) T viii) T ix) F x) F

Exercise 3

3.2 Geometric Mean = 47.5675

3.3 Average: 14.2614

3.4 Mean = 1.5, Median = 1, Mode = 1

3.5 Mean = 11.67, Median = 11.5, Mode = 12.8

3.6 (i) Mean = 4.405, (ii) Median = 4.533

3.9 (ii) Mode = 18

3.10 increase is 18.2%

3.11 Mean = 74.7024

3.12 Mean = 21.2, Geometric mean = 20.06

3.13 Mean = 42.12, Harmonic Mean = 35.56

3.14 Harmonic Mean = 50.51

3.15 Harmonic Mean = 37.5

3.16 Arithmetic Mean = 5; Numbers are $b = 2,8$ and $a = 2,8$

3.17 Three numbers are: $X_1 = 3$, $X_2 = 27$, $X_3 = 72$

3.18 (i) Harmonic Mean = 7.66

(ii) (Geometric Mean) increase is 15.98%

3.19 Mean = 11.09, Median = 11.07 Mode = 11.06

3.20 Mean = 0.7317, Median = 0.7318, Mode = 0.7319

$D_6 = 0.73192$ $P_{74} = 0.7319$

3.21 $Q_3 = 31.01$ $D_5 = 23.783$ $P_5 = 6.0125$ Mode = 23.28

3.22 Mode = 0

3.23 Average = 24.003

3.24 Harmonic Mean = 134.607

3.25 Weighted Mean = 72.57

3.26 Weighted Mean = 203.4

3.27 Mean = 18 because $\sum(x-\bar{x}) = 0$ $\therefore$ $\sum(x - 18) = 0$

3.28 1st value is 30, 2nd value is 20, 3rd value is 10

3.29 i) T ii) T iii) F iv) T v) F vi) T vii) F viii) F ix) F

x) T xi) F xii) T xiii) T xiv) F xv) F xvi) T xvii) T

3.30 i) Arithmetic Mean ii) Harmonic Mean

iii) Arithmetic Mean iv) Median

v) Geometric Mean vi) Arithmetic Mean

3.31 (i) Measures of Central Tendency (ii) Extreme (iii) Zero (iv) two equal

(v) G.M (vi) Equal (vii) Mode (viii) Bimodal (ix) Identical (x) Median and Mode.

Exercise 4

4.5 ii. a) Q.D = 5.38 b) Coefficient of SK = 0.05

4.6 M.D = 5.73, Coefficient of M.D = 0.09, Variance = 47.855

Range = 30, Q.D = 5.3, Coefficient of Q.D = 0.08

4.7 Q.D = 33.25, Coefficient of Q.D = 0.302

4.8 S.D = 44.687 Variance = 1996.925 C. V = 9.914%

4.9 Combined SD=2.49

4.10 Combined SD=8.75, Combined mean = 57.06

4.11 $\overline{Y} = 2(\overline{X}) + 5$ and $S_Y = S_X$

4.12 $\overline{Y} = \overline{X} + 10$ $S_Y = S_X$

$\overline{Z} = \frac{110}{100}\overline{X}$ $S_Z = \frac{110}{100} S_X$

4.13 $\overline{X}_A = 44.47$ $C.V_A = 15.38$

$\overline{X}_B = 47.56$ $C.V_B = 13.32$

4.14 Median = 12.7, M.D=2.86

4.15 $M.D_{(Med)} = 21.65$

4.16 S.D = 7.99

4.17 (i) Tube B has greater absolute dispersion

(ii) Tube B has greater relative dispersion

4.19 C.V= 14.64%

4.20 C. V = 44.122%

4.21 $\mu_1 = 0,$ $\mu_2 = 25.7,$ $\mu_3 = 20.66,$ $\mu_4 = 1189.7$

4.22 $\mu'_1 = 0.061,$ $\mu'_2 = 2.64,$ $\mu'_3 = 0.564,$ $\mu'_4 = 28.38$

$\mu_1 = 0,$ $\mu_2 = 2.637,$ $\mu_3 = 0.0811,$ $\mu_4 = 28.301$

4.23 C.V=34.64%, Distribution is Symmetric

4.24 Distribution is platy kurtic as $\beta_2 = 2.158$

4.25 (i) S.K=–1.118, – vely Skewed (ii) – vely Skewed

4.26 $\mu_1=0$, $\mu_2=7.139$, $\mu_3=-5.364$, $\mu_4=125.5705$

Distribution is +vely Skewed

4.27 -vely Skewed

4.28 Coefficient of SK=0.1218

4.29 Coefficient of SK=0.299

4.30 $\overline{X} = 3, C.V = 41\%$

4.31 $\mu_2=11$, $\mu_3=49$, $\mu_4=192$, $\beta_1=1.8$, $\beta_2=1.59$

4.33 LeptoKurtic

4.34 (i) –vely Skewed (ii) –vely Skewed (iii) +vely Skewed

4.35 (i) platy kurtic (ii) $S.D = 3$

4.36 (i) $\mu_4=1875$

4.37 (i) +vely Skewed (ii) –vely Skewed (iii) +vely Skewed

4.38 (i) Symmetric (ii) + vely Skewed (iii) – vely Skewed

(iv) Symmetric (v) Lepto Kurtic

4.39

i)	F	ii)	F	iii)	T	iv)	F	v)	T
vi)	T	vii)	T	viii)	F	ix)	F	x)	F
xi)	T	xii)	F	xiii)	F	xiv)	T	xv)	T

4.40 (i) Called Scatter (ii) Relative (iii) Median

(iv) Change (v) Free measurement (vi) Consistent

(vii) Skewness (viii) Equidistant (ix) Zero

(x) Ratio

Exercise 5

5.7 48.39, 61.29, 67.74, 96.77, 119.35, 122.58, 129.03, 154.84

5.8 (i) 100, 149.37, 155.12, 106.3, 123.23, 172.05, 164.057, 155.91,

186.22, 193.37, 123.23, 127.17, 159.06, 198.82, 255.12

(ii) 63.31, 94.57, 98.20, 67.30, 78.08, 108.92, 104.19, 98.70,

117.9, 122.38, 78.02, 80.51, 100.70, 125.87, 161.52.

5.9 (i) 100, 111.11, 122.22, 133.33, 155.56, 144.44, 166.67, 211.11, 200

(ii) 66.94, 74.38, 81.82, 89.26, 104.14, 96.70, 111.57, 141.33, 133.89

5.10 (i) 100, 125, 150, 175, 200, 250, 225, 250, 275

(ii) 80, 100, 120, 140, 160, 200, 180, 200, 220.

(iii) 40, 50, 60, 70, 80, 100, 90, 100, 110.

5.11 100, 103.85, 100, 88.46, 94.23, 86.54, 80.77, 96.15, 78.84, 80.77

5.12 100, 104, 110, 114, 124, 144, 146, 150, 142, 140.

5.13 100, 100 , 101.07, 104.68, 108.90, 100.58.

5.14 100, 106.96, 103.175, 110.095

5.15 100, 109.0, 125.53, 145.94

5.16 100, 112.66, 132.49, 136.20, 146.74

5.17 (i) 100, 91.59, 82.08, 81.55, 115.57, 113.37

(ii) 100, 89.07, 68.48, 69.17, 114.32, 107.66

5.18 100, 114.3, 119.4, 128.7, 134.9

5.19 (i) 400 (ii) 400

5.20 101.2508

5.21 86.77

5.22 71.83

5.23 162.62

5.25 Laspeyre's=108.78 Paasche's=109.21

Fisher's=108.99

5.26 100, 99.1, 109.97, 104.1, 105.53, 126.66, 128.93, 122.67, 140.2

5.27 73.002

5.28 98.15

5.29 (i) Price-numbers

(ii) Volume index numbers

(iii) Aggregative index numbers

(iv) Forecasting seasonal, cycles

(v) General or special

(vi) Fixed base method, chain base method

(vii) A normal year

(viii) Chaining process, chain base method

(ix) Chain base method

(x) Simple aggregative, simple average of relatives

5.30 i) T ii) T iii) F iv) T v) T vi) T vii) F viii) F ix) F x) F

Exercise 6

6.1 (ii) (a) $A \cap B = \{\ \}$ (b) $\{1,2,3,4,5,7\}$ (c) $\{2,4,6,7,8\}$ (d) $\{2,4\}$

6.2 (i) 5040 (ii) 518918400 (iii) 367567200 (iv) 60480

(v) 3603600 (vi) 120 (vii) 10 (viii) 635013559600 (ix) 330

6.3 (i) {(1,2), (1,3), (4,2), (4,3)} (ii) {(2,1), (2,4), (3,1), (3,4)}

(iii) {(1,4), (1,1), (4,1), (4,4)} (iv) {(2,2), (2,3), (3,2), (3,3)}

6.8 (i) $\frac{1}{2}$ (ii) $\frac{1}{6}$ (iii) $\frac{1}{2}$ **6.9** $\frac{1}{4}$

6.10 $P(\text{sum} < 7) = P(\text{sum} > 7) = \frac{15}{36}$ **6.11** $\frac{1}{22}$ **6.12** $\frac{1}{4}$

6.13 $\frac{4}{33}$

6.14 (i) $\frac{1}{22}$ (ii) $\frac{7}{22}$ (iii) $\frac{9}{44}$

6.15 (i) $\frac{5}{36}$ (ii) $\frac{8}{36}$ (iii) $\frac{9}{18}$

6.16 (i) $\frac{16}{1326}$ (ii) $\frac{650}{1326}$ (iii) $\frac{676}{1326}$

6.17 $\frac{792}{20349}$ **6.18** $\frac{13}{28}$ **6.19** $\frac{5}{8}$ **6.20** $\frac{16}{1326}$

6.21 (i) $\frac{33}{54145}$ (ii) $\frac{33}{54145}$

6.22 (i) $\frac{4}{364}$ (ii) $\frac{100}{364}$ **6.23** (i) $\frac{25}{75}$ (ii) $\frac{50}{75}$

(iii) $\frac{55}{75}$ (iv) $\frac{60}{75}$

6.26 0.92 **6.27** $\frac{2}{3}$ **6.28** $\frac{5}{18}$

6.29 $\frac{7}{429}$ **6.30** (i) 0.0355 (ii) 0.04395

6.31 (i) 0.25 (ii) 0.2083 **6.32** i) $\frac{207}{625}$

6.33 (i) $\frac{6}{15}$ (ii) $\frac{3}{15}$ (iii) $\frac{4}{15}$ (iv) $\frac{13}{15}$ (v) $\frac{2}{15}$ (vi) $\frac{9}{15}$

6.34 $\frac{64}{29100}$ **6.35** (i) $\frac{1}{169}$ (ii) $\frac{1}{221}$ **6.36** (i) $\frac{7}{16}$ (ii) $\frac{55}{784}$

6.38 i) $\frac{9}{14}$ ii) $\frac{2}{7}$ **6.37** i) 0.5 **6.39** 4 to 3

6.40 $\frac{53}{80}$ **6.41** i) $\frac{19}{42}$ ii) $\frac{23}{42}$ **6.42** $\frac{4}{9}$

6.43 0.23 **6.44** $\frac{7}{8}$ **6.45** (i) $\frac{1}{8}$ (ii) $\frac{5}{72}$

(iii) $\frac{5}{36}$ (iv) $\frac{19}{27}$ **6.46** 0.1

6.47 $\frac{1}{12}$ **6.48** $\frac{1}{17}$ **6.49** $\frac{36}{91}:\frac{30}{91}:\frac{25}{91}$

6.50 $\frac{38}{63}$ **6.51** $\frac{16}{36}$ **6.52** $\frac{13}{32}$

6.53 (i) $\frac{11}{25}$ ii) $\frac{60}{100}$ **6.54** i) well defined, distinct

ii) Singelton or unit iii) Power iv) Compound v) Mutually Exclusive

vi) Collectively exhaustive iiv) Exhaustive viii) Equally likely ix) 2^n

x) Permutation, ${}^nP_r$ xi) Combinatoin

6.55 i) T ii) F iii) F iv) F v) F vi) T vii) F viii) T ix) F x) T

Exercise 7

7.1 (iii)a) Discrete random variable

b) Discrete random variable

c) Continuous random variable

d) Continuous random variable

7.2 $m = 100$ $a = 21$ $b = 7$ $x = 10$

The numbers are: 17, 64, 51, 78 45 and 52

7.3 Let the one digit random numbers from table are:

8 3 2 6 9 8 2 8 1 2

so

H T H H T H H H T H

The frequency of 1 head = 7 and frequency of 0 head = 3

7.4 S={TT, HT, TH, HH}

7.6 S= {BB, BM, MB, MM} and

S={0, 1, 2}

7.7 S={TT, HT, TH, HH} and

S={0, 1, 2}

7.8

y	0	1	1	1	2	2	2	3
Outcomes	GGG	BGG	GBG	GGB	BBG	BGB	GBB	BBB

7.9

x	1	2	3	4
$f(x)$	5/70	30/70	30/70	5/70

7.11 (i) Generated, random (ii) Equal (iii) Chance or Stochastic

(iv) Discrete (v) Range, Continuous (vi) 1

(vii) A mathematical (viii) Mathematical expectation

(ix) Converges (x) Discrete, Continuous.

Exercise 8

8.2

x	0	1	2	3
$P(x)$	1/56	15/56	30/56	10/56

8.3

x	0	1	2	3
$P(x)$	1/35	12/35	18/35	4/35

8.4 (ii)

x	0	1	2	3	4
$P(x)$	1/16	4/16	6/16	4/16	1/16

(iii)

x	0	1	2	3	4
$P(x)$	15/210	80/210	90/210	24/210	1/210

8.6 (ii) a) No b) No c) Yes d) No

8.7 a) $C=\frac{1}{15}$ b) $0<C<1$

8.8 $-\frac{1}{3}<Q<\frac{1}{4}$

8.9 i) 7.44 ii. *a*) 0.55, 1.3475 b) 2.1, 5.39

8.10 i) 0.2 ii) 0.6 iii) 0.6 iv) 0.7 v) 0.9

8.11 i) {3, 4, 5, 6, 7, 8, 9} (ii) {1, 2, 3, 4}

8.13 y takes the value between 0 and 30, continuous

8.14 Mean = 2.3 Variance = 2.01

8.15 i. a) $a=1/8$ b) $\frac{5}{16}$

ii. a) $c=1/2$ b) 0.3125 c) 0.5625

iii. a) $k=2$, 13/216

8.17

x	0	1	2	3
$P(x)$	1/8	3/8	3/8	1/8

8.18 (i) Probability Distribution of x

x	0	1	2	3	4	Total
$P(x)$	56/792	280/792	336/792	112/792	8/792	792/792

(ii) 336/792

8.19 ii) $k=b-a$

8.20 ii) Mean=1.94, S.D (x) = 1.43, C.V. = 73.71%, 8.21 Mean= 2.6, Var= 1.34

8.22 i) a) 0.7031 b) 0.543125 c) 0

ii) a) $\frac{1}{4}$ b) $\frac{15}{16}$ c) $\frac{3}{16}$

8.23 i) $a=1/210$ ii) 0.4928 iii) 0.6476 iv) 0.6851

8.24 352.5

8.25 1.875

8.27 ii)

x	0	1	2	3	4	5
$P(x)$	1/252	26/252	126/252	226/252	251/252	252/252

8.28 ii) Mean= 1.875

8.29 A's expectation=Rs.6.0; B's expectation=5.0

8.30 A's expectation=Rs.6.0; B's expectation=4.0

C's expectation=Rs.2.0

8.31

x	0	1	2	3	Sum
$P(x)$	1/56	15/56	30/56	10/56	56/56

Mean = 105/56, Variance=0.5024

8.32 Probability Distribution

x	0	1	2	3	4
$p(x)$	1/256	12/256	54/256	108/256	81/256

Mean=3 Variance=0.75

8.33

x	0	1	2
$P(x)$	0.81	0.18	0.1

Mean = 0.2, Variance = 0.18

8.34 $E(X)$ =8/5 Variance (X)=8.24

$E(5X+10)$=18 Variance ($5X+10$)=206

1:25

8.35 Mean=7 Variance=5.83

8.36 i) $E(X)$=7 ii) $E(X^2)$=590 iii) Variance (X)=541

8.37 $E(X)$=0.75 $E(X^2)$=1.125

8.38 i) T ii) F iii) F iv) F v) F vi) T vii) F viii)T ix) F x) T

Exercise 9

9.3

x	0	1	2	3	4	5
$P(x)$	3125/32768	9375/32768	11250/32768	6750/32768	2025/32768	243/32768

9.4 (i) $\frac{1}{16}$ (ii) $\frac{1}{4}$ (iii) $\frac{11}{16}$ (iv) $\frac{11}{16}$

9.5 (i) 0.4096 (ii) 0.1536 **9.6** (i) 0.995 (ii) 0.2617

9.7 (i) 0.1694 (ii) 0.5340 **9.8** (i) 14080/15625 (ii) 265/15625

9.9 (i) 56/256 ~~(ii)~~ 254/256 **9.10** Mean = −1, Variance = 0.067

9.11 (i) 243/1024 (ii) 1008/1024

9.12

x	0	1	2	3	4	5
$P(x)$	3125/7776	3125/7776	1250/7776	250/7776	25/7776	1/7776

Mean = 0.833 Variance = 0.701

9.13 3, 15, 30, 30, 15, 3

9.14

x	0	1	2	3	4
f	87	69	21	3	0

Mean = 0.67

9.15 $P = 0.6$ $n = 10$ **9.16** Mean = 40 S.D = 6.

9.17 $P = 0.25$ $n = 12$

9.18 No possible, q cannot be greater than 1

9.19 $P = \frac{1}{6}$ Mean = 5/6

9.20 Not possible, q cannot be greater than 1.

9.21 $p = 0.25$ $q = 0.75$ $n = 48$

$P(X = x) = {}^{48}C_x (0.25)^x (0.75)^{48-x}$

9.22 Mean = $3p$ Variance = $3pq$

9.23 Mean = np S.D = $\sqrt{npq}$

9.24 (i) np, npq (ii) $p = q = \frac{1}{2}$ (iii) n is small (iv) Two (v) $p < q$

(vi) Discrete *r.v.* (vii) *S.D.* (viii) fixed, independent (ix) $p = q$

9.25

x	1	2	3	4	5
$P(x)$	1/66	12/66	30/66	20/66	3/66

Mean = 70/66 = $\frac{35}{11}$ Variance = 0.6942

Mean = $n.\frac{k}{N} = \frac{35}{11}$, Variance= $n.\frac{k}{N}(1-\frac{k}{N})\left(\frac{N-n}{N-1}\right) = 0.6942$

9.26

x	0	1	2	3	4
$P(x)$	21/462	140/462	210/462	84/462	7/462

Mean = 1.8181 Variance = 0.6942

Mean = $n\frac{K}{N} = 1.8181$ Variance = 0.6942

9.27

x	1	2	3
$P(x)$	4/20	12/20	4/20

9.28 Expected number of women = 15/8

9.29 (i) 1/252 (ii) 126/252

9.30 i) F ii) F iii) T iv) T v) F vi) T vii) F viii) T ix) T x) T

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